Against Belief Closure

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Abstract. I argue that we should solve the Lottery Paradox by denying that rational belief is closed under classical logic. To reach this conclusion, I build on my previous result that (a slight variant of) McGee's election scenario is a lottery scenario (see Lissia 2019). Indeed, this result implies that the sensible ways to deal with McGee's scenario are the same as the sensible ways to deal with the lottery scenario: we should either reject the Lockean Thesis or Belief Closure. After recalling my argument to this conclusion, I demonstrate that a McGee-like example (which is just, in fact, Carroll's barbershop paradox) can be provided in which the Lockean Thesis plays no role: this proves that denying Belief Closure is the right way to deal with both McGee's scenario and the Lottery Paradox. A straightforward consequence of my approach is that Carroll's puzzle is solved, too.

1. Outline of the paper

Students of rational belief and rational degrees of belief generally agree that the two following principles are incompatible:

Belief Closure. Rational belief is closed under classical logic.

Lockean Thesis. If and only if, given one's evidence, P is very probable (where "very probable" means "equal to or higher than a specified threshold value t"), then one should believe P. [1] (Or

[1]In what follows I will apply the Lockean Thesis to (among others) indicative conditionals. That is, I will make use of the principle that if and only if an indicative conditional $P \rightarrow Q$ is very likely, then one should believe, or accept, $P \rightarrow Q$. Now, as a consequence of Lewis' famous triviality results (Lewis 1976), a number of philosophers have argued that indicative conditionals are not propositions. So I avoid committing myself to the controversial claim that indicative conditionals have propositional content. In other words, when I will talk about the probability of $P \rightarrow Q$ I will not be talking about the probability of $P \rightarrow Q$ *being true*. Following Adams (1975), I will take such a probability to be the probability of Q conditional on P. That is, in what follows, the Lockean Thesis (when applied to indicative conditionals) will read: if and only if the probability of Q conditional on P is high, then one should believe, or accept, $P \rightarrow Q$ (provided that $P \rightarrow Q$ is a simple conditional and that $p(P) \neq 0$, see section 2 below). If you think that taking indicative conditionals to be non-propositional is not enough to avoid Lewisian triviality, note that I can propose a stronger line of defence. Indeed, to the best of my knowledge, all Lewis-style triviality results assume a classical account of credence and credal update (namely, a standard Bayesian setting). This means that we can elude triviality by renouncing (or suitably modifying) such a standard framework. For instance, Lassiter (2020) has shown how to escape triviality by replacing the classical bivalent semantics for probability with a three-valued semantics. Goldstein and Santorio (2020) and Santorio (2020) also demonstrate that we can escape triviality by rejecting classical Bayesianism (more specifically, Goldstein and Santorio (2020)

equivalently: if and only if, given one's evidence, P is very probable (where "very probable" means "equal to or higher than a specified threshold value t"), then it is rational to believe P.)[2]

The well-known Lottery Paradox (Kyburg 1961) is an illustration of the incompatibility between these two principles. One way of presenting the puzzle is the following. Consider a fair 1000-ticket lottery with exactly one winner. There is a very low probability, for each ticket, that it will win, namely a probability of 0.001. Consequently, if $t = 0.999$, then, by the Lockean Thesis, it is rational to believe, of each ticket, that it will lose. By multiple applications of Belief Closure, it is also rational to believe the conjunction "ticket n°1 will lose ∧ ticket n°2 will lose . . . ∧ ticket n°1000 will lose" (where "∧" is the conjunction symbol). By applying the Lockean Thesis again, this time in the right-to-left direction, this entails that the probability of "ticket n°1 will lose ∧ ticket n°2 will lose . . . ∧ ticket n°1000 will lose" is greater than or equal to t, i.e., 0.999. However, given that the lottery is fair and has exactly one winner, "ticket n°1 will lose ∧ ticket n°2 will lose . . . ∧ ticket n°1000 will lose" has a probability of 0. So we have reached an absurd conclusion, as the probability of one and the same proposition cannot be greater than or equal to 0.999 and equal to 0 at the same time. We have ended up with a contradiction, so we conclude that Belief Closure and the Lockean Thesis are incompatible.

In the present paper I propose a solution to the Lottery Paradox. The solution I put forth consists in rejecting Belief Closure. Indeed, as we will see, a convincing argument to this conclusion can be provided. My proposal builds on my previous result that (a slight variant of) McGee's election scenario is a lottery scenario (see Lissia 2019). This result implies that the

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argue that we should reject Bayesian conditionalization). Note that I am not claiming that we should endorse either of these proposals (Lassiter's or Goldstein and Santorio's); in fact, discussing the possible ways out of triviality, as well as their consequences, would deserve a separate paper. What I want to highlight here is simply that abandoning classical Bayesianism could offer some interesting escapes from triviality, which may be especially worth exploring if you dislike the non-propositional view, or are unconvinced that it suffices to avoid triviality. (For more on this point, see fn. 3 below.)

[2] In spite of the subtle differences that may exist between the notion of (rational) acceptance and that of (rational) belief, I will use these two terms interchangeably; indeed, such differences are of no relevance for this paper's purposes. As it is clear from my definition, "it is rational to believe P" and "one should believe P" will be used interchangeably too. Although aware of the discussions surrounding the relation between rational belief and epistemic obligations, I will not deal with them here, as the niceties involved in this discussion are, once again, of no relevance for my purposes.

sensible ways to deal with McGee's scenario are the same as the sensible ways to deal with the lottery scenario: we should either reject the Lockean Thesis or Belief Closure. I show, then, that a McGee-like argument (which is just, in fact, Carroll's 1894 barbershop paradox) can be provided in which the Lockean Thesis plays no role: this proves that denying Belief Closure is the right way to handle both McGee's scenario and the Lottery Paradox. A corollary of this conclusion is that Carroll's puzzle (1894) is also solved: besides McGee's problem and the Lottery Paradox, my proposal accounts for Carroll's barbershop paradox, which turns out to be a simple variant of McGee's argument.

However, I will only attend to these issues (i.e., I will only propose my solutions to these three puzzles) starting from section 5. Sections from 2 to 4 included do not actually introduce new material: they just summarize some results obtained in Lissia 2019, which I need in order to introduce my new conclusions. More precisely, this paper will be structured as follows. In section 2 I recall McGee's scenario and propose my interpretation of it. In section 3 I present the "restaurant scenario". The importance of this scenario lies in that, first, it preserves the relevant features of McGee's election story, and that, second, as I will show, it is just a lottery scenario. Section 4 clarifies the consequences the discovery of this variant of McGee's scenario has on the way we should handle McGee's original argument. The new results, namely my solutions to the three puzzles (the Lottery Paradox, McGee's problem, and Carroll's barbershop paradox), which constitute this paper's original contribution, are described starting from section 5.

2. *McGee's argument*

Famously, McGee (1985, p. 462) has proposed the following scenario:

Opinion polls taken just before the 1980 election showed the Republican Ronald Reagan decisively ahead of the Democrat Jimmy Carter, with the other Republican in the race, John Anderson, a distant third. Those apprised of the poll results believed, with good reason:

[1] If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.
[2] A Republican will win the election.
Yet they did not have reason to believe
[3] If it's not Reagan who wins, it will be Anderson.

In Lissia 2019, I have challenged a standard claim concerning McGee's scenario, i.e., the claim that modus ponens does not fail in it if we assume the material conditional. It is not difficult to see why this claim is widely held: if we assume the material conditional, (3) is equivalent to the disjunction "either Reagan wins or Anderson wins", which is very plausible, for the simple reason that Reagan is "decisively ahead" of his competitors. Now, it can be shown that giving a material interpretation of the natural language (indicative) conditional "if…then…" does not block McGee's puzzle (see Lissia 2019). More precisely, I have shown that, under the supposition that the Lockean Thesis holds, McGee can be regarded as demonstrating that the following principles of the logic of belief are falsified (where "$\supset$" is the material conditional, "~" is the negation symbol, and *Bel* is a rational belief operator). (As in Lissia 2019, I use * to identify those principles in which the material conditional is involved versus those in which the conditional is a non-material indicative, which I denote as $P \rightarrow Q$.)

*Epistemic modus ponens**. If $Bel(P \supset Q)$, and $Bel(P)$, then $Bel(Q)$.

*Epistemic modus tollens**. If $Bel(P \supset Q)$, and $Bel({\sim}Q)$, then $Bel({\sim}P)$.

In other words, in a way similar to the Lottery Paradox, McGee's (1985) article can be taken to show that Belief Closure and the Lockean Thesis are incompatible.

Note that the condition for such a conclusion to follow (i.e., the conclusion that McGee's paper shows that Belief Closure and the Lockean Thesis cannot be jointly satisfied) is that some specific assumptions concerning McGee's scenario are granted. The first (call it "A1") is that the principle applied in (1)-(3) is what I call, in Lissia 2019, "epistemic modus ponens" (with no star):

Epistemic modus ponens. If $Bel(P \rightarrow Q)$, and $Bel(P)$, then $Bel(Q)$.

This principle can be contrasted with truth-preserving modus ponens, in which the notion of truth is involved, instead of that of rational belief:

Truth-preserving modus ponens. If P is true and $P \rightarrow Q$ is true, then Q is true.

A1 just says that McGee targets the former principle.

The second assumption (which I will call "A2") is that McGee endorses the Lockean Thesis.

Both A1 and A2 are well supported by McGee's remarks (see McGee 1985 and 1989), and have been recently endorsed by students of McGee's problem (for a recent example, see Stern and Hartmann 2018). Actually, McGee explicitly acknowledges that epistemic modus ponens is used in (1)-(3). Indeed, referring to (1)-(3) and to the two other (structurally similar) examples he provides in his 1985 paper, he says: "Such examples show that *modus ponens* fails in English […] More precisely, the examples show that *modus ponens* does not preserve warranted acceptability. As I [McGee] pointed out (1985, p. 463) and as Sinnot-Armstrong, Moor, and Fogelin (1986) have emphasized, the examples have no direct bearing on the question whether *modus ponens* is truth-preserving" (McGee 1989, p. 512 and fn. 20). Let me also stress that in presenting (1)-(3) McGee specifies that "those apprised of the poll results" *believed* with good reason (1) and (2), whereas they had no reason to *believe* (3) (McGee 1985, p. 462). Another explicit reference to the fact that epistemic modus ponens, or something along its lines, is applied in (1)-(3) is contained in the author's remark that on some occasions "one has good grounds for believing the premises of an application of modus ponens but yet one is not justified in accepting the conclusion" (McGee 1985, p. 462).

Concerning A2, it seems reasonable to argue that the reason why we should believe (2) is its high probability (besides Stern and Hartmann 2018, Neth 2019, among recent papers, subscribes to this claim). This seems confirmed by the fact that when presenting his counterexamples, McGee speaks of the reasons to believe their premises (or to disbelieve their conclusions) in terms of likelihood ("[i]t is more likely that […]"; "[…] it is virtually certain that […]"; "[…] it is entirely certain that […]"; McGee 1985, p. 463).

However, in spite of my hypothesis that A2 holds being plausible, it is in fact not necessary for my purposes to rely on the claim that McGee indeed assumed the Lockean Thesis, i.e., that in the version of the puzzle McGee had in mind the Lockean Thesis is involved. What only needs to be the case for this article's purposes is that there is a reasonable interpretation of McGee's puzzle in which both epistemic modus ponens and the Lockean Thesis are assumed. Now, as I have just shown, this clearly seems to be the case.

Let us go on. If we take high probability to be what justifies belief in (2), then if (1)-(3) is to be regarded as a potential failure of modus ponens, it must be the case that the reason why we should believe (1) also is its high probability, and that the reason why we should not believe (3) is that its probability is not high enough (see Lissia 2019).

Interestingly, there is one very popular way of interpreting indicative conditionals which is compatible with McGee accepting the Lockean Thesis. This way of interpreting indicative conditionals (or rather, their acceptability) is mostly known as "Adams' Thesis" (see Adams 1975):

Adams' Thesis. The acceptability of $P \to Q$ is equal to the probability of Q given P (i.e., of Q conditional on P), provided that $P \to Q$ is a simple conditional and that $p(P) \neq 0$.

If we also assume import-export, i.e., a principle usually regarded as highly plausible for the logic of conditionals (or, more precisely, if we assume the counterpart principle of acceptability, Acceptability Import-Export: $Acc(P \to (Q \to R)) = Acc((P \wedge Q) \to R)$[3]), we obtain that our attitudes towards (1)-(3) are represented as follows:

(a) $p(R|P \wedge Q)$
(b) $p(P)$
(c) $p(R|Q)$

The reason why our attitude towards (1) is the one indicated in (a) is that, by Acceptability Import-Export and Adams' Thesis, we obtain that $Acc(P \to (Q \to R)) = p(R|P \wedge Q)$. (More precisely, as Stern and Hartmann (2018, fn. 15) specify, "this follows only when [Acceptability Import-Export] is restricted to settings where $p(P \wedge Q) > 0$ (since [Adams' Thesis] applies only in these settings)". Here and below, I modified the authors' notation to make it coherent with mine.)

Note that the above does not rule out that there may be other ways of representing our attitudes towards (1)-(3) compatible with the Lockean Thesis. However, (a)-(c) certainly is one very natural way of representing them, which I will thus take as the main reference here.

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[3]As I note in Lissia 2019 and remind in fn. 1 above, the approach I develop here does not incur the so-called triviality results, for in the present context we are only interested in the acceptability conditions for indicative conditionals, and not in the question whether indicative conditionals are propositions. However, as also made clear in fn. 1, the reasons why triviality results do not affect my argument are actually deeper: triviality results crucially assume a standard Bayesian setting. Indeed, as established in Goldstein and Santorio 2020, Lassiter 2020 and Santorio 2020, these results can be eluded precisely by modifying such a framework (see, again, fn. 1 above). A thorough discussion of the different ways out of triviality falls outside the boundaries of this paper, so the only point I want to make here is that merely assuming Adams' Thesis and Acceptability Import-Export cannot lead to triviality: further assumptions, among which classical Bayesianism, are needed.

In Lissia 2019 I have shown that, if A1 and A2 are granted, a slightly modified version of McGee's election scenario can be provided, in which (i) both epistemic modus ponens* and epistemic modus tollens* fail, and (ii) the relevant features of the scenario are preserved. Here we will have to consider once again that example, which I dub "the restaurant scenario".

3. From McGee's puzzle to the Lottery Paradox

I am sitting in a restaurant with my Italian friend Pasquale. I know that Pasquale always orders one of the day's specials. Today's specials are pizza, pasta and roast beef. I know that Pasquale likes both pizza and pasta very much, and that he does not especially enjoy roast beef. I estimate that there is a 0.4 probability that Pasquale will have pizza, a 0.4 probability that he will have pasta and a 0.2 probability that he will have roast beef.

Set t = 0.6 and assume the material conditional. In this context, it is rational for me to believe both (4) and (5):

(4) If Pasquale doesn't have pizza, then he will have pasta.
(5) Pasquale won't have pizza.

Indeed, they both have a probability of at least 0.6. Now, from (4) and (5), by epistemic modus ponens*, I should draw the following conclusion:

(6) Pasquale will have pasta. (!)

But (6) only has a probability of 0.4; so it is not rational for me to believe (6), that is, epistemic modus ponens* fails.

Turning now to epistemic modus tollens*: by the Lockean Thesis, I should believe (7), which has a probability of 0.6:

(7) Pasquale won't have pasta.

Now, from (7) and (4), by epistemic modus tollens*, I should infer (8):

(8) Pasquale will have pizza. (!)

But (8) only has a probability of 0.4; therefore, it is not rational for me to believe (8), i.e., epistemic modus tollens* fails.

So epistemic modus ponens* and modus tollens* fail in the restaurant scenario. Or, more exactly, assuming t = 0.6, in the above arguments rational belief is not closed under modus ponens* and modus tollens* respectively. But this is not all; a principle I will label "epistemic conjunction introduction" fails too:

Epistemic conjunction introduction. If $Bel(P)$, and $Bel(Q)$, then $Bel(P \wedge Q)$.

Clearly, given t = 0.6, I should believe:

(5) Pasquale won't have pizza.
(7) Pasquale won't have pasta.
(9) Pasquale won't have roast beef.

However, I should not believe the conjunction of these three sentences.

Here we come to a crucial point: assume the standard definition of a lottery scenario as a scenario where, given t higher than 0.5 (and lower than 1), the Lockean Thesis and epistemic conjunction introduction are incompatible (because we end up with a contradiction; see my presentation of the Lottery Paradox in section 1 above). By this definition, the restaurant scenario is just a version of the lottery scenario Kyburg uses in his Lottery Paradox, albeit one with only three tickets and a probability threshold for rational belief of 0.6. In other words, insofar as we adopt the above definition of what a lottery scenario is, the restaurant scenario is a lottery scenario.

So, under the assumption that the Lockean Thesis holds, epistemic modus ponens* and epistemic modus tollens* both fail in the restaurant example, i.e., the condition (i) above is satisfied. Now, it is also possible to show that (ii) holds with regard to the restaurant scenario, i.e., that in it the relevant traits of the election example are preserved. Assume that X = "Carter loses the election" (i.e., "a Republican wins"), Y = "Reagan loses", and Z = "Anderson loses". Given these interpretations of X, Y and Z, (1)-(3) has the following form:

$X \rightarrow (Y \rightarrow {\sim}Z)$

X

$\therefore Y \rightarrow {\sim}Z$

Consider now the restaurant scenario. Let X be “Pasquale doesn’t have pizza”, Y be “Pasquale doesn’t have pasta” and Z be “Pasquale doesn’t have roast beef”; clearly, McGee would still have to regard this instance of the above argument form as a failure of modus ponens:

(1’) If Pasquale doesn’t have pizza, then if he doesn’t have pasta, he will have roast beef.
(5) Pasquale won’t have pizza.
(3’) If Pasquale doesn’t have pasta, he will have roast beef. (!)

(As we did for McGee’s original argument, we can also take (a)-(c) (see section 2 above) to represent our attitudes towards (1’)-(3’): we obtain that the probability that Pasquale will have roast beef, given that he does not have pizza or pasta (i.e., p($R|P \wedge Q$)) is 1, the probability of “Pasquale will not have pizza” (i.e., p(P)) is 0.6, while the probability that Pasquale will have roast beef given that he does not have pasta (i.e., p($R|Q$)) is low (much lower than 0.6). So assuming a 0.6 threshold, we should believe the argument’s premises, and should reject its conclusion.)

The very important point here is that, unlike “Reagan wins” in the election scenario, in the restaurant scenario “Pasquale has pasta” has a low probability (i.e., unlike what happens in the original scenario, in the restaurant scenario ${\sim}Y$ is unlikely). This means that even if in the election scenario the probability of “Reagan will win” were lower than it is, McGee should still view (1)-(3) as a failure of modus ponens. In other words, for modus ponens to fail in McGee’s sense it is not necessary that one of the disjuncts in the probability distribution has a probability higher than t: even if in the restaurant example the probability of ${\sim}Y$ (“Pasquale has pasta”, corresponding to “Reagan wins” in the election scenario) is low, the probability of X (“Pasquale doesn’t have pizza”/“A Republican wins”) is still high. This proves that it is a merely contingent fact that one of the disjuncts in the original scenario has a probability higher than the threshold. Removing this contingent element allows us to gain a deeper understanding of the structure underlying McGee’s scenario.

So both (i) and (ii) hold with respect to the restaurant scenario. Now, as we have seen, the restaurant scenario is just a lottery scenario. My conclusion is that we should expect a unified solution to both McGee's puzzle and the Lottery Paradox; i.e., that the sensible ways to handle McGee's scenario are the same as the sensible ways to handle the lottery scenario.

This conclusion is unscathed, I submit, by a potential objection also addressed in Lissia 2019. According to this objection, the restaurant scenario does not really preserve all the relevant features of McGee's original story, because in the former epistemic modus ponens and epistemic modus ponens* only fail if we set t no higher than 0.6. Now, the election scenario seems different: in it, a threshold higher than 0.6 seems to be compatible with the failure of epistemic modus ponens (versus epistemic modus ponens*). In other words, here is the complaint: if we assume that a probability of 0.6 is not sufficient for rational belief (whereas a greater probability does suffice) it is no longer clear whether epistemic modus ponens* would still fail. And if this principle were unscathed for t higher than 0.6, then we should still regard McGee's original puzzle as a genuine puzzle as far as *indicative* conditionals are concerned, but there would be no puzzle about the restaurant scenario.

A first response to this objection is simply that advocates of the Lockean Thesis are usually not committed to a specific value for t.[4] Even so, in Lissia 2019, I go on and concede that a threshold of 0.6 may not suffice (at least not always) for rational belief (after all, a defender of a contextualist version of the Lockean Thesis may argue that in the restaurant context there are specific reasons to reject a 0.6 threshold). However, actually, both epistemic modus ponens and modus ponens* do fail for t greater than 0.6.

Concerning epistemic modus ponens, an argument by Stern and Hartmann (2018, pp. 609 and 610) provides a formal ground for our intuition that in McGee's original scenario a threshold greater than 0.6 seems to be compatible with the failure of epistemic modus ponens. Indeed, Stern and Hartmann (2018) have proved we can always find $\mathrm{p}(R|P \wedge Q)$ and $\mathrm{p}(P)$ such that they are both high, while at the same time $\mathrm{p}(R|Q)$ is low. Consider, again, (a)-(c):

(a) $\mathrm{p}(R|P \wedge Q)$
(b) $\mathrm{p}(P)$
(c) $\mathrm{p}(R|Q)$

[4]Achinstein (2001), who argues that a probability greater than 0.5 is both necessary and sufficient for rational belief, is one notable exception. Also note that Shear and Fitelson (2019) have claimed that a value they dub "the golden threshold", i.e., a value corresponding to the inverse of the golden ratio ($\phi^{-1} \approx 0.618$), should be regarded as a non-arbitrary Lockean threshold.

Importantly, Stern and Hartmann (2018) provide the following expansion of (c):

$$p(R|Q) = p(R|P \wedge Q)\, p(P|Q) + p(R|{\sim}P \wedge Q)\, p({\sim}P|Q).$$

Though the first probability in the expansion corresponds to (a), none of the other probabilities appear in the premises. This means that you can coherently assign (c) a probability as low as 0 (or as high as 1) even when you regard (a) and (b) as highly acceptable. For example, if you assign .99 to (a) and .99 to (b), you can coherently judge (c) to be utterly unacceptable when your estimate for $p(R|{\sim}P \wedge Q)$ is low. (Stern and Hartmann 2018, p. 610).

The only exceptions are cases in which $p(R|Q) = p(R|P \wedge Q)$, as obviously, if $p(R|Q) = p(R|P \wedge Q)$, we cannot have that both (a) and (b) are high and (c) is low.[5]

Let us now turn to epistemic modus ponens*. As is well known, failure of Belief Closure in cases in which the Lockean Thesis is assumed is not limited to cases in which $t = 0.6$. As long as a suitable number of tickets is also chosen, failure of Belief Closure can always be observed, no matter the specific threshold t (the only proviso is that t must be strictly between 0.5 and 1). Kyburg's original scenario is one such case: in it t is greater than 0.6, but Belief Closure still fails. As a result, no matter which version of the Lottery Paradox we are considering (with 3 tickets, 1000 tickets, or with a still different number of tickets), accusing the specific threshold value assumed (0.6, 0.999, etc.) of being responsible for the Paradox itself seems hopeless. As I will clarify below, this point extends to epistemic modus ponens* specifically.

Note that I am not denying that an account of both the restaurant scenario and Kyburg's scenario could be given in contextualist terms. What I only want to stress here is that the same kind of account should be provided for both scenarios, which entails nothing concerning the specific account that we should provide. A clarification, though: an account involving a context-dependent version of the Lockean Thesis would entail a *modification* of the Lockean Thesis, whose original form would be rejected.[6] The most famous treatment of this kind is found in

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[5]Actually, in specifying the very minor probabilistic constraints that (a) and (b) impose on (c) the authors focus on the case in which $p(P) = 1$ (Stern and Hartmann 2018, fn. 18), but actually $p(P) = 1$ is only a special case of that in which $p(R|Q) = p(R|P \wedge Q)$.

[6]It is possible to distinguish a strong, context-independent version of the Lockean Thesis from a weaker, context-dependent version (see Genin 2019, p. 478). The version of the Lockean Thesis I am assuming in this article (see section 1 above) closely matches what Genin (2019, p. 478) calls "The Strong Lockean Thesis", which may be

Leitgeb (2014; 2015; 2017), according to which we should supplement the Lockean Thesis with the condition that the probability of P should remain higher than 0.5 when the agent gains evidence which is consistent with P. So endorsing Leitgeb's proposal would still come down to adopting one of the two "classical" options regarding the Lottery Paradox: denying the Lockean Thesis or denying Belief Closure (more on these two options below).

I conclude that McGee's original scenario and Kyburg's scenario should be dealt with in the same way. More specifically, it looks like we have two main options: rejecting the Lockean Thesis or rejecting Belief Closure. (Note that, conforming to what is standard in the literature, and as I have just suggested regarding Leitgeb's account, I take rejections of the Lockean Thesis to include modifications of the latter; on this point, also see section 6 below.) It can be observed that denying Belief Closure would in fact entail denying at least three principles: epistemic modus ponens*, epistemic modus tollens*, and epistemic conjunction introduction. Indeed, as we have seen, in the restaurant scenario the three of them fail. And this is also the case in Kyburg's scenario: even though the original version of the Lottery Paradox involves (epistemic) conjunction introduction, we can generate lottery-like paradoxes by using other principles, notably (epistemic) modus ponens* and modus tollens* (see, for instance, Douven 2016).[7]

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defined as follows. (In the definitions below I follow Genin (2019) in taking the domain of the quantifier to be the set of all belief states a *particular* agent may find herself in, or as the set of all belief states whatsoever.)

Strong Lockean Thesis. There is a threshold $0.5 < t < 1$ such that all rational belief states satisfy $Bel(P)$ if and only if $p(P) \geq t$.

We may contrast the Strong Lockean Thesis with the Weak Lockean Thesis:

Weak Lockean Thesis. For every rational belief state, there is a threshold $0.5 < t < 1$ such that $Bel(P)$ if and only if $p(P) \geq t$.

As Genin (2019, p. 478) observes, the Strong Lockean Thesis is generally taken to be the standard definition of the Lockean Thesis. It is also the one that allows us to generate the Lottery Paradox. By contrast, Leitgeb (2014; 2015; 2017), to take a famous example, endorses a precise specification of the Weak Lockean Thesis, which does not give rise to the Lottery Paradox (the details of his proposal see Leitgeb 2014, 2015 and 2017).

[7]It may be helpful to see how these lottery-like paradoxes are constructed. In the original scenario, it is rational to believe "Ticket n°1 wins ∨ ticket n°2 wins… ∨ ticket n°1000 wins", which is equivalent to "(Ticket n°1 loses ⊃ ticket n°2 loses… ⊃ ticket n°999 loses) ⊃ ticket n°1000 wins". It is also rational to believe, about each ticket

4. Back to McGee's argument

So far, my proposal concerns the way we should handle McGee's and Kyburg's scenarios *in general*; however, I now would like to dwell on the consequences of what I have shown concerning (1)-(3) specifically. Of course, such consequences are not especially relevant if our only aim is to solve the Lottery Paradox. Still, this question is of the utmost importance if, as I plan to do here, we also wish to provide an account of McGee's original argument. Now, what I have shown so far straightforwardly applies to (1)-(3). No matter whether we decide to drop the Lockean Thesis or Belief Closure, in both cases the derivation of (3) from (1) and (2) is blocked. Suppose that we drop the Lockean Thesis: we are no longer compelled to believe (1) and (2). (This clearly follows from the fact that the Lockean Thesis is an implicit assumption in McGee's puzzle, or at least in the version of McGee's puzzle I am considering here (see my interpretation of (1)-(3) in section 2 above); as a result, if we abandon the Lockean Thesis, the puzzle disappears.) Suppose, instead, that we give up Belief Closure, i.e., as we have seen, at least epistemic modus ponens*, epistemic modus tollens* and epistemic conjunction introduction: this also blocks (1)-(3). The reason is that if epistemic modus ponens* turns out to fail, epistemic modus ponens also fails. Or at least, this is the case under the reasonable (and popular) assumption that $Bel(P \rightarrow Q)$ entails $Bel(P \supset Q)$. Here is a convincing argument to this conclusion I also provide in Lissia 2019: suppose that we rationally believe $P \wedge {\sim}Q$ (i.e., the negation of $P \supset Q$); this seems sufficient for rationally believing the negation of $P \rightarrow Q$.

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between n°1 and n°999, that it loses. However, it is *not* rational to believe that ticket n°1000 wins (in fact, it is rational to believe that it loses). That is, epistemic modus ponens* fails.

In this scenario, epistemic modus tollens* fails too. Indeed, it is rational to accept

(Ticket n°1 loses) ⊃ ticket n°2 loses… ⊃ ticket n°999 loses ⊃ ticket n°1000 wins.

Ticket n°2 loses.

…

Ticket n°999 loses.

Ticket n°1000 loses.

Nevertheless, it is rational to reject

Ticket n°1 wins

and accept its negation.

So I showed that (a slight variant of) McGee's scenario is just a lottery scenario. This implies that the only sensible ways to deal with McGee's scenario are either by denying Belief Closure or by denying the Lockean Thesis. As we have seen, giving up either Belief Closure or the Lockean Thesis also blocks (1)-(3). A consequence of this result is that it undermines any account of McGee's puzzle that does not involve dropping either Belief Closure or the Lockean Thesis. Indeed, as I have argued above, there is a reasonable and easily accessible interpretation of (1)-(3) which assumes both epistemic modus ponens and the Lockean Thesis. As a result, even though other interpretations of (1)-(3) are perhaps possible, any student of McGee's problem should deal at least with this one. Now, from my interpretation of (1)-(3), it follows, as we have seen, that (a slight variant of) McGee's scenario is actually a lottery scenario, so that we should to deal with it either by dropping Belief Closure or by dropping the Lockean Thesis. This leads us to the conclusion that any author tackling McGee's puzzle should renounce either the Lockean Thesis or Belief Closure. What is interesting and important here is that the great majority of the existing discussions of McGee's puzzle do not address the rejection of either principle.[8]

Now of course the next question is: which is the "culprit" between Belief Closure and the Lockean Thesis? Answering this question would solve both the Lottery Paradox and McGee's puzzle. Now suppose that an argument can be provided which has the same structure as McGee's, but in which the Lockean Thesis plays no role: clearly, this would show that denying Belief Closure is the right way to react to both McGee's scenario and the Lottery Paradox. In the next section, I will argue that an argument proposed by Carroll in 1894 is such an argument.

[8]I am thinking, *inter alia*, of the accounts proposed by Appiah (1987), Lowe (1987), Piller (1996), Katz (1999), Bennett (2003), Gillies (2004), Paoli (2005), Cantwell (2008), Fulda (2010), Kolodny and MacFarlane (2010), Moss (2015), Stojnić (2017), Schulz (2018), Stern and Hartmann (2018), Neth (2019), Edgington (2020), and Williamson (2020).

5. The barbershop and the election

In 1894, Lewis Carroll proposed his famous “barbershop paradox” (Carroll 1894). The scenario can be summarized as follows: Carr, Allen and Brown are three barbers who never leave their shop at the same time, as one of them has to be there in order to keep the shop open. Moreover, due to the consequences of an illness, Allen never goes out without Brown. On the one hand, we should believe (10):

(10) If Carr is out, then if Allen is out, Brown is in.

Nevertheless, we should also believe (11):

(11) If Allen is out, then Brown is out.

(11) and the nested consequent of (10) seem to contradict each other and therefore seem to imply, by (epistemic) modus tollens, that we should believe (12) Carr is in. However, it looks like we should not accept this conclusion, for it is perfectly possible that Carr is out, provided that Allen is in. That is, intuitively, epistemic modus tollens (i.e., if $Bel(P \rightarrow Q)$, and $Bel({\sim}Q)$, then $Bel({\sim}P)$) fails[9] (recall that “→” is the non-material, indicative conditional).

Assuming that (11) and the nested consequent of (10) are contradictory requires a principle mainly known as “conditional non-contradiction”: $P \rightarrow {\sim}Q$ if and only if ${\sim}(P \rightarrow Q)$. Luckily, this assumption seems unproblematic: as far as (non-material) indicative conditionals with a possible antecedent are concerned, conditional non-contradiction is one of the most natural and solid principles of conditional logic; in fact, it has been claimed that it is “almost indisputably true” that it holds (Bennett 2003, p. 84).

In what follows, I mean to show that the culprit behind McGee’s puzzle is epistemic modus ponens, and not the Lockean Thesis. To do this, I will have to demonstrate that our intuition

[9] Actually, Carroll formulates his puzzle in terms of truth, not in terms of rational belief. Indeed, he specifies that the *truth* of (12) does not seem follow from the *truth* of (10) and (11) (Carroll 1894). My rendering of his puzzle, however, is in terms of rational belief. The reason is that this makes the analogy with McGee’s scenario emerge more clearly. Note that introducing this variant of the puzzle is perfectly legitimate, as in Carroll’s scenario it clearly seems that we should believe both (10) and (11), and that we should not believe (12).

that epistemic modus ponens fails in (1)-(3) does not depend on the assumption that the Lockean Thesis holds. One way of reaching this conclusion is by showing that there is at least one argument such that (i) it has the same structure as (1)-(3) and that (ii) even if we assume a different norm of belief (different from the Lockean Thesis) we still have the intuition that epistemic modus ponens fails in that argument. This is exactly what I plan to do here. First, I will demonstrate that starting from either of the two scenarios (McGee's or Carroll's) we can generate both what looks like a failure of epistemic modus ponens and what looks like a failure of epistemic modus tollens. Such putative failures clearly have the same structure in the two scenarios. I will then show that in the arguments generated starting from Carroll's scenario the Lockean Thesis does not play any role. This conclusion is a straightforward consequence of the fact that Carroll's story does not involve probabilities.

Consider, first of all, the logical structure of Carroll's argument. Let X, Y and Z be "Carr is out", "Allen is out" and "Brown is out" respectively. (10)-(12) has the following form:

$X \rightarrow (Y \rightarrow {\sim}Z)$
$Y \rightarrow Z$
$\therefore {\sim}X$

Now, it can be noted that starting from McGee's scenario we can also generate what looks like a counterexample to epistemic modus tollens:[10] one just has to replace "Carr is out" with "Carter loses the election" (i.e., "a Republican wins"), "Allen is out" with "Reagan loses" and "Brown is out" with "Anderson loses". What we obtain is an argument having exactly the form as (10)-(12):

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.
(13) If it's not Reagan who wins, it's not the case that Anderson will win.
(14) The winner won't be a Republican. (!)

It seems rational to believe both (1) and (13); however, we should not believe (14). Indeed, "a Republican will win" is highly plausible, as long as the winning Republican is Reagan.

[10]For a discussion of the modus tollens version of McGee's argument see Gauker 1994, but also Kolodny and MacFarlane 2010.

There is an important difference, though, between (1)-(14) and Carroll's argument. This difference has to do, as announced, with the role played by the Lockean Thesis in McGee's scenario: in (1)-(14), it seems that the reason why we should believe (13) is that it is very likely. Or at least, this is the case if we stick to the assumption I have made throughout this paper that McGee endorses the Lockean Thesis, or, more exactly, that there is a reasonable interpretation of his puzzle in which the Lockean Thesis is assumed. Now, given this interpretation, if we are to regard (1)-(14) as a potential failure of epistemic modus tollens the reason why we should believe (1) must be the same (i.e., that it is very likely), while the reason why we should not believe (14) must be that it is not likely enough.[11]

The point I will make below is that Carroll's story, by contrast, does not involve probabilities, which entails, of course, that the Lockean Thesis plays no role in Carroll's argument, where by "plays no role" I mean: it is not essential to our intuition that epistemic modus tollens fails in the argument. Notably, I will show that we can perfectly assume another norm of belief (other than the Lockean Thesis) and still have the intuition that epistemic modus tollens fails in (10)-(12). The truth norm of belief will serve as a key illustration.

The Lockean Thesis certainly is a very popular norm of belief. However, it does have some (also very popular) competitors: one of them is the so-called "truth norm". According to one possible definition of this norm, we should believe *P* if and only if *P* is true. The opposition between the truth norm and the Lockean Thesis is the contemporary heir to a very classical debate in epistemology, whose origins may be traced back to the opposition between William James' alethic approach to belief (James 1896) and Clifford's evidential conception of it (Clifford 1877). Several different versions of the truth norm have been proposed over the years. Advocates of some kind of truth norm include Wedgwood (2002; 2009), Boghossian (2003), Shah (2003), Gibbard (2005), O'Hagan (2005), Shah and Velleman (2005), Engel (2007),

[11]By proceeding in a similar way as for McGee's original argument, our attitudes towards (1)-(14) may be represented as follows:

(a) $p(R|P \wedge Q)$

(d) $p(\sim R|Q)$

(e) $p(\sim P)$

Clearly, by the Lockean Thesis, we should believe both (a) and (d), because they are both very likely; however, we should not believe (e), as its probability is low.

Whiting (2010), Littlejohn (2012; 2014), McHugh (2012; 2014), Raleigh (2013), Turp (2013), and Greenberg (2020). I will not dwell here on these different proposals, as the subtleties they involve are not relevant in the present context. Rather, the definition I have given above will be enough for my purposes.

Now suppose that we accept the truth norm. As applied to indicative conditionals, this norm of course implies that indicative conditionals have truth conditions. Both claims are controversial (both the claim that the truth norm is the right norm of belief and that indicative conditionals have truth conditions). I am not saying that we should endorse these claims though, just that we can assume them for the sake of the argument. Now, it can be noted that were we to accept the truth norm, we would still have the intuition that in (10)-(12) epistemic modus tollens fails. Indeed (as Carroll himself points out, see fn. 9 above), (10) and (11) both seem true; (12), however, does not seem true.[12]

It is also the case that, starting from Carroll's scenario, we can generate a modus ponens version of (10)-(12):

(10) If Carr is out, then if Allen is out, Brown is in.
(15) Carr is out.
(16) If Allen is out, Brown is in. (!)

As in (1)-(3), we regard the premises as rationally acceptable but we reject the conclusion. The only (relevant) difference between (1)-(3) and (10)-(16) concerns, again, the role of the Lockean Thesis. As noted in section 2, it seems that in McGee's story what grounds our acceptance of "A Republican will win" (i.e., of (2)) is its high probability. As a result, if we are to look at (1)-(3) as a potential failure of (epistemic) modus ponens, the reason why we should accept (1) and should reject (3) respectively must be that (1) is likely enough, while (3) is not (keep in mind, once again, my interpretation of (1)-(3) in section 2 above). In his scenario, Carroll explicitly assumes, instead, that (10) and (15) are both true (Carroll 1894, pp. 436 and 437). That is, were we to assume the truth norm, we would still have the intuition that (10)-(16)

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[12]Let us briefly consider, in passing, another well-known norm of belief (actually one which is becoming increasingly popular among epistemologists), i.e., the so-called "knowledge norm". According to one possible definition of it, one should believe *P* if and only if one knows *P*. It is noteworthy that were we to adopt this norm, my point would still hold: it clearly seems that we know both (10) and (11); however, we do not know (12).

is a counterexample to epistemic modus ponens.

Again, let me stress that my claim is not that in Carroll's scenario truth-preserving modus ponens and modus tollens fail.[13] My only claim here is that were we to accept the truth norm of belief we would still have the intuition that (10)-(16) and (10)-(12) are failures of epistemic modus ponens (If $Bel(P \rightarrow Q)$, and $Bel(P)$, then $Bel(Q)$) and epistemic modus tollens (if $Bel(P \rightarrow Q)$, and $Bel({\sim}Q)$, then $Bel({\sim}P)$) respectively[14] (again, recall that "→" is the non-material, indicative conditional.)

In more general terms, it can be observed that McGee's and Carroll's scenarios have the same structure or, more precisely, that McGee's scenario is just a "weaker" version of Carroll's. Why "weaker"? Consider Carroll's story and replace, as above, "Carr is out" with "Carter loses the election" (i.e., "a Republican wins"), "Allen is out" with "Reagan loses", and "Brown is out" with "Anderson loses". If probabilities are "injected" "in the right way" (i.e., if the right proportions are respected in the probability distribution) what we obtain is McGee's election story. It is in this sense that we may regard McGee's scenario as "weaker" than Carroll's: at least if we accept the construal of the election scenario I have assumed throughout this article, McGee's scenario involves a probabilistic component which is absent in Carroll's, meaning that it can only generate "counterexamples" in which the Lockean Thesis is involved. This is not the case for the barbershop scenario: (10)-(12) and (10)-(16) are stronger because they do not involve the Lockean Thesis (they only involve epistemic modus tollens and modus ponens respectively).

6. *Rejecting Belief Closure*

The above provides the basis for a historical point. Much before McGee, Carroll had already presented a "counterexample to modus ponens" (and modus tollens) very similar to McGee's, and actually even stronger than McGee's. Indeed, as we have just seen, in Carroll's scenario the Lockean Thesis does not play any role, i.e., cannot be regarded as the "culprit" for our

[13] I have already defined truth-preserving modus ponens (see section 2). Unsurprisingly, I take truth-preserving modus tollens to be the principle according to which if $P \rightarrow Q$ is true and ${\sim}Q$ is true, then ${\sim}P$ is true.

[14] Interestingly, here too, my point would also hold if we were to accept the knowledge norm, instead of the truth norm. Imagine that you just saw Carr sitting at the bar: intuitively, you would then know both (10) and (15); however, you would not know (16).

intuition that epistemic modus ponens and modus tollens fail in it.

Let us take stock. I conclude that Belief Closure should be blamed for the Lottery Paradox. Indeed, I have shown that (a slight variant of) McGee's scenario is just a lottery scenario. This implies that the only sensible ways to deal with McGee's scenario are either by rejecting Belief Closure or by rejecting the Lockean Thesis. The fact that an argument structurally identical to (1)-(3) can be provided in which the Lockean Thesis plays no role can be regarded as proof of the fact that denying Belief Closure is the right response to both McGee's scenario and the Lottery Paradox.

Lottery Paradox scholars are usually regarded as belonging to two categories: those who reject (or at least propose to modify) the Lockean Thesis[15] and those who reject Belief Closure. Both categories have a long history: the former, which is the largest one, includes authors like Lehrer (1975; 1990), Kaplan (1981a; 1981b; 1996), Stalnaker (1984), Pollock (1995), Ryan (1996), Evnine (1999), Nelkin (2000), Adler (2002), Douven (2002), Smith (2010; 2016), Lin and Kelly (2012a; 2012b), and Kelp (2017). Among the Belief Closure deniers are, instead, Klein (1985), Foley (1992), Hawthorne and Bovens (1999), Kyburg and Teng (2001), Christensen (2004), Hawthorne and Makinson (2007), Kolodny (2007), Easwaran and Fitelson (2015). Contextualist accounts seem to form a category of its own: according to contextualists, one or more terms featuring in the formulation of the Paradox are ambiguous (or, alternatively, the truth conditions for attributions of rational belief are context-dependent). Typically, these authors seem to focus on the Lockean Thesis (or on some related principle, depending on the specific version of the Paradox they discuss, which may differ in some respects from the one I am considering here). Notably, they focus on the ambiguities (putatively) contained in the definition of the Lockean Thesis/the related principle, or on its (alleged) context-dependence (see Lewis 1996, Cohen 1998, Leitgeb 2014, 2015, 2017, and Logins 2020; on Leitgeb's approach also see section 3 above). However, note that in fact these (purported) solutions to the Lottery Paradox are not really an independent category (see, again, Logins 2020). Indeed, they reduce to the claim that the Lockean Thesis/the related principle does not hold unrestrictedly, but only on some of its readings, or in certain contexts, which entails that their advocates should be viewed as Lockean Thesis deniers, or modifiers (first category above).

[15] I take this category to include those authors who restrict the Lockean Thesis to the effect that we should set $t = 1$. Levi (1980), Gärdenfors (1986), Van Fraassen (1995), Arló-Costa (2001), and Arló-Costa and Parikh (2005) are among the proponents of such a restriction.

Anyway, I have shown that the Lockean Thesis does not play any role in the Lottery Paradox: this clearly undermines, among others, a contextualist approach according to which we should restrict the truth of the Thesis to some of its readings, or to certain contexts.

Note that a straightforward account of McGee's puzzle results from what I have shown in this paper. Indeed, as we have seen, it follows from the failure of epistemic modus ponens* that epistemic modus ponens fails too. This accounts for (1)-(3), i.e., McGee's original puzzle is solved. (Of course, the modus tollens version of McGee's puzzle is also solved, for from the failure of epistemic modus tollens* it follows that epistemic modus tollens fails too.)

As a final remark, let me stress that if the assumptions I made concerning McGee's argument (viz., A1 and A2 above) did not hold this would not undermine my account of the Lottery Paradox. Of course, it would then be true that I did not solve McGee's original puzzle, but only a version of McGee's puzzle with respect to which A1 and A2 hold. However, this would not be a problem for my account of the Lottery Paradox, as in order to derive the conclusion that Belief Closure is responsible for the latter it is enough to assume a version of McGee's puzzle with respect to which A1 and A2 do hold, and which does not necessarily coincide with the one McGee had in mind. However, as already specified, mine would still be a very reasonable and easily accessible interpretation of McGee's problem, thus one that students of the latter should account for.

A proviso, though: it suffices that A1 holds for McGee's original puzzle to be solved. The reason is the one above: under the supposition that $Bel(P \rightarrow Q)$ entails $Bel(P \supset Q)$, if epistemic modus ponens* fails, then epistemic modus ponens fails. The same applies, of course, to Carroll's arguments, or at least to the epistemic versions of them I consider in this article: on the assumption that the principles involved in (10)-(12) and (10)-(16) are epistemic modus tollens and modus ponens respectively, if epistemic modus tollens* and modus ponens* fail, both arguments are blocked.

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