

From McGee's puzzle to the Lottery Paradox

by Lina Maria Lissia (University of Turin)

1. *The election scenario*

In a well-known article, McGee (1985, p. 462) has proposed the following scenario:

Opinion polls taken just before the 1980 election showed the Republican Ronald Reagan decisively ahead of the Democrat Jimmy Carter, with the other Republican in the race, John Anderson, a distant third. Those apprised of the poll results believed, with good reason:

[1] If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

[2] A Republican will win the election.

Yet they did not have reason to believe

[3] If it's not Reagan who wins, it will be Anderson.

McGee (1985) speaks of a “counterexample to modus ponens”. In fact, the question whether, and in which sense, (1)-(3) deserves such a label, remains, as of today, highly controversial. Still, there is at least one claim on which students of McGee's example seem to agree, i.e. the claim that the puzzle is dissolved if we assume a material interpretation of the natural language conditional “if ... then ...”. Indeed, if we assume the material conditional, we should interpret (3) as the disjunction “either Reagan wins or Anderson wins”, which is very plausible, for the simple reason that Reagan is hot favourite. That is, as McGee himself specifies, if we interpret (1)-(3) according to the material conditional, we believe both the premises and the conclusion (McGee 1985).

Interestingly, starting from McGee's scenario it is also possible to generate what looks like a counterexample to modus tollens (see Gauker 1994, but also Kolodny and MacFarlane 2010):

(1) If a Republican wins the election, then if it's not Reagan who wins it will be

Anderson.

(4) If it's not Reagan who wins, it's not the case that Anderson will win.

(5) The winner won't be a Republican.

(5) does not seem to follow from (1) and (4). Indeed, "a Republican will win" is very plausible, as long as the winning Republican is Reagan.

In this case as well, it seems that if we assume the material conditional the puzzle disappears. Indeed, we can only apply modus tollens to (1) and (4) if (4) and the nested consequent of (1) contradict each other. However, if we assume the material conditional (4) and the nested consequent of (1) cannot be seen as contradictory.

In this paper I will challenge the idea that modus ponens does not fail in McGee's scenario if we assume the material conditional. Indeed, in what follows I show that even if we give a material interpretation of the conditionals in (1)-(3) and (1)-(5) McGee's puzzle is not dissolved. Before that, however, I will have to provide some clarifications about the nature of McGee's example.

2. McGee on his "counterexample"

Modus ponens for the indicative conditional is the principle according to which if P is true and $P \rightarrow Q$ is also true, then Q is true as well. (In what follows, each time I use the generic label "indicative conditional" I will assume that the latter is not given a material interpretation. Moreover, $P \rightarrow Q$ will denote the indicative conditional, and $P \supset Q$ will denote the material conditional.) For an argument to be a counterexample to this principle, it must be the case that P and $P \rightarrow Q$ are true and Q is false. The title of McGee's paper ("A counterexample to modus ponens") may at first suggest that McGee regards (1)-(3) as a counterexample in this sense. However, in the body of the article, McGee describes (1)-(3) (and the other, structurally similar, examples he provides) as cases in which "one has good grounds for believing the premises of an application of modus ponens but yet one is not justified in accepting the conclusion" (p. 462). Moreover, McGee's example revolves around what "those apprised of the poll results" *had reason to believe*, and not about truth.

A later paper by the author contains some important elucidations. In McGee 1989, he explicitly admits that his examples concern the preservation of acceptability, versus truth preservation: “Such examples show that *modus ponens* fails in English [...] More precisely, the examples show that *modus ponens* does not preserve warranted acceptability. As I [McGee] pointed out (1985, p. 463) and as Sinnott-Armstrong, Moor, and Fogelin (1986) have emphasized, the examples have no direct bearing on the question whether *modus ponens* is truth-preserving” (McGee 1989, p. 512 and fn. 20).

That is, McGee seems to target a principle that may be formulated along these lines:

Epistemic modus ponens. If $Bel(P \rightarrow Q)$, and $Bel(P)$, then $Bel(Q)$, where Bel is a rational belief operator.

If we now turn to (1)-(5), it seems that, by McGee’s own criteria, we should regard it as a failure of the following schema:

Epistemic modus tollens. If $Bel(P \rightarrow Q)$, and $Bel(not\ Q)$, then $Bel(not\ P)$.

As for rational belief (or acceptability) itself¹, McGee does not provide many details about the way it should be defined in order for his examples to go through. However, in McGee 1985, he mentions high probability as a reason for believing the premises of his examples. Although he does not endorse it explicitly, he seems to adopt a principle called “Lockean Thesis”:

Lockean Thesis. If and only if P has high probability, then one should believe P . (Or equivalently: if and only if P has high probability, then it is rational to believe P).

In what follows I argue that McGee’s scenario gives us reasons to believe that, under mild assumptions, epistemic modus ponens and modus tollens fail, *even if natural language conditionals are given a material interpretation* (i.e. even if $P \rightarrow Q$ and $P \supset Q$ are taken to be equivalent). More precisely, under the assumption that the Lockean

¹Even though there may be subtle differences between the concept of (rational) acceptance and that of (rational) belief, these are not relevant for my purposes; as a result, in this paper, I will take the two terms to be synonyms.

Thesis holds, McGee can be taken to show that the two following principles of the logic of belief are falsified (where \supset is, as we already know, the material conditional and \sim is the negation symbol):

*Epistemic modus ponens**. If $Bel(P \supset Q)$, and $Bel(P)$, then $Bel(Q)$.

*Epistemic modus tollens**. If $Bel(P \supset Q)$, and $Bel(\sim Q)$, then $Bel(\sim P)$.

Of course, the failure of epistemic modus ponens* (or modus tollens*) entails the failure of a more general principle, often called “Belief Closure”:

Belief Closure. Rational belief is closed under classical logic.

In the literature on rational belief and rational degrees of belief it is commonly held that the Lockean Thesis and Belief Closure cannot be jointly satisfied. Indeed, joint acceptance of Belief Closure and the Lockean Thesis gives rise to the so-called Lottery Paradox (Kyburg 1961). In this paper, I will show that the latter is intimately linked to McGee’s election scenario.

One more preliminary remark is in order. From now on, when I say “modus ponens” or “modus tollens” I will be talking about the epistemic version of modus ponens (modus tollens) itself; in each different context I will specify whether I am referring to epistemic modus ponens or to epistemic modus ponens*. The same holds for the other logical principles I mention in this paper: for each argument schema I talk about, I am in fact referring to its epistemic version.

3. *The Argument Schema*

In this paper I will make the reasonable assumption that if we are justified in believing (2) above it is because of its high probability (for a similar assumption, see Neth (2019), as well as McGee himself (1985; 1989); also see Stern and Hartmann 2018). Essentially, I will assume that McGee endorses the principle I called “Lockean Thesis”. As a result, if (1)-(3) is to be taken as a potential counterexample to modus ponens, the reason why we should believe (1) must be the same (that is, its high probability); and the reason

why we should not believe (3) must be that its probability is not high enough.

One popular way of interpreting the conditionals in McGee's example is compatible with the author assuming the Lockean Thesis. According to this interpretation, (1) has a probability of 1 because, supposing that a Republican wins, the conditional probability that Anderson will win given that Reagan doesn't win is 1. In this view, (2) is also likely, because the unconditional probability that a Republican will win is high. However, the conditional probability that Anderson wins, given that Reagan doesn't win, is low, that is, (3) is unlikely.

This interpretation of the premises can be made more precise by adopting what is often called, in the literature on conditionals, "Adams' Thesis"; that is, by assuming that the acceptability of an indicative conditional is equal to the probability of its consequent given its antecedent. In the literature on conditionals, many versions of the Thesis can be found, involving subtle differences; however, the one below should be enough for my purposes. Note that it only holds for simple conditionals, $P \rightarrow Q$, such that $P(P) \neq 0$:

Adams' Thesis. The acceptability of $P \rightarrow Q$ is equal to the probability of Q given P (i.e. of Q conditional on P)².

Stern and Hartmann (2018) also adopt an account of the conditionals in (1)-(3) based on Adams' Thesis. However, as they observe, the latter does not provide us with an analysis of (1), as Adams' Thesis only applies to simple conditionals and (1) is an embedded conditional. Indeed, consider an indicative conditional of the form $P \rightarrow (Q \rightarrow R)$: "If we were to apply [Adams' Thesis] to this conditional, it would seem that $Acc(P \rightarrow (Q \rightarrow R)) = P((R|Q)|P)$, but there is no such probability expression as $P((R|Q)|P)$ " (Stern and Hartmann 2018, p. 608; here and below, I modified the authors' notation to make it coherent with mine). However, there is such a probability expression as $P(R|P \wedge Q)$. Here, I will follow Stern and Hartmann (2018) in assuming that choosing to analyse $Acc(P \rightarrow (Q \rightarrow R))$ as $P(R|P \wedge Q)$ is safe. This step can be motivated by a plausible principle of conditional logic, i.e. import-export, according to

²This approach eludes the so-called triviality results, for we are only interested here in the acceptability (or believability) conditions for indicative conditionals, and not in the question whether indicative conditionals are propositions (see Stern and Hartmann 2018).

which $P \rightarrow (Q \rightarrow R)$ is equivalent to $(P \wedge Q) \rightarrow R$ ³. An acceptability version of the principle can be formulated as below (see, again, Stern and Hartmann 2018).

Acceptability Import-Export. $Acc(P \rightarrow (Q \rightarrow R)) = Acc((P \wedge Q) \rightarrow R)$

By Acceptability Import-Export and Adams' Thesis, we obtain that $Acc(P \rightarrow (Q \rightarrow R)) = P(R|P \wedge Q)$ ⁴. That is, our attitudes towards (1), (2), (3) are represented as indicated in (1'), (2'), and (3') respectively:

(1') $P(R|P \wedge Q)$

(2') $P(P)$

(3') $P(R|Q)$

I will call this way of representing our attitudes towards McGee's argument's premises and conclusion "the Argument Schema". According to it, both (1) and (2) have a high degree of acceptability (as (1') and (2') are both high), whereas (3) is only acceptable to a low degree (because (3') is low). The Argument Schema is clearly compatible with McGee assuming the Lockean Thesis: by the latter, we should (fully) accept both (1) and (2), while we should (fully) reject (3).

Note that I do not mean to claim that (1')-(3') is the only representation of our attitudes towards (1)-(3) compatible with McGee assuming the Lockean Thesis. However, it certainly is a natural way of spelling out such attitudes. As a result, even though the point I will make is more general (as it concerns the category of indicative conditionals in general, independently of whether we accept Adams' Thesis), I will use the Argument Schema as the main reference for my discussion.

³Import-export is usually regarded as a very natural principle. For example, (1) clearly seems equivalent to "If a Republican wins the election and it's not Reagan who wins, then it will be Anderson". However, import-export also has its critics; for instance, it is invalid in Stalnaker's theory of conditionals (see Stalnaker 1968). Recently, Mandelkern (forthcoming) has argued in favour of restricting the validity of import-export, based on a surprising result concerning the relation between import-export and classical conjunction. A survey of the putative counterexamples to import-export for indicative conditionals can be found in Khoo and Mandelkern 2019.

⁴As the authors specify, "this follows only when [Acceptability Import-Export] is restricted to settings where $P(P \wedge Q) > 0$ (since [Adams' Thesis] applies only in these settings)" (Stern and Hartmann 2018, fn. 15).

4. A puzzling conclusion (?)

I mean to challenge the idea that modus ponens* does not fail in McGee's scenario after all. My key point here will be that even if we give a material interpretation of the conditionals in (1)-(3) and (1)-(5) McGee's puzzle is not dissolved.

Let us postulate a material interpretation of the conditionals in McGee's scenario: McGee's story allows us to reason from the following three premises, which are the material conditional versions of (1), (2) and (4). (Here again, \supset stands for the material conditional, \sim for the negation and \wedge for the conjunction. Moreover, X is "Carter loses the election", Y is "Reagan loses" and Z is "Anderson loses".)

(a) $X \supset (Y \supset \sim Z)$

(b) X

(c) $Y \supset Z$

Of course, there is a clear difference between premise (a) and premises (b) and (c): (a) has a probability of 1 whereas (b) and (c), in spite of being highly probable, are not certain. (The reason why (a) has a probability of 1 is, of course, that it is equivalent to "either Carter will win or Reagan will win or Anderson will win", i.e. to the assumption that someone will win the election, which does seem to have a probability of 1 for all relevant purposes.) However, by the Lockean Thesis, we should also believe both (b) and (c).

Yet a puzzling conclusion follows from (a), (b) and (c).

(1) $X \supset (Y \supset \sim Z)$ [(a)]

(2) X [(b)]

(3) $Y \supset Z$ [(c)]

(4) $Y \supset \sim Z$ [modus ponens* 1,2]

(5) $Y \supset (Z \wedge \sim Z)$ [conjunction of the consequents*⁵ 3,4]

⁵The inference rule that lets us infer $P \rightarrow (Q \wedge R)$ from $P \rightarrow Q$ and $P \rightarrow R$ is often called "CC" in the literature on conditional reasoning. However, as "CC" sometimes stands for a different principle (i.e. it is

(6) $\sim Y$ [modus tollens* 5]

What the derivation above shows is that from (a), (b) and (c) we should infer that Reagan will certainly win (i.e. that $\sim Y$ is the case)⁶.

However, the conclusion according to which Reagan will certainly win is false: though high, the probability of $\sim Y$ is not 1. Far from dissolving the riddle, the assumption that the conditionals in McGee's scenario are material conditionals leads to a surprising conclusion.

Perhaps you disagree. You may protest that assuming (a), (b) and (c) does not, in fact, lead to anything problematic: "Reagan will win" is a perfectly plausible conclusion. Or at least, you may argue, it is a perfectly plausible conclusion given that I am assuming the Lockean Thesis. Indeed, "Reagan will win" has high probability, i.e. by the Lockean Thesis, we should believe it. After all, the propositional language I have relied on in the derivation above does not include operators such as "probably" or "certainly". That is, the difference between "Reagan will very probably win" and "Reagan will certainly win" cannot be expressed in the language I have used, so my (putative) puzzle about McGee's scenario cannot be expressed either.

Such concerns are legitimate but ultimately inconclusive, or so I submit. To point this out, I will provide an argument that has a structure similar to McGee's, but where it is obviously the case that we should not believe $\sim Y$. Indeed, one can provide at least one argument where, unlike what happens in McGee's original scenario, the probability of $\sim Y$ is less than t , where t is the threshold assumed for rational belief. In the next section, I propose an example that satisfies this requirement.

Yet, you may say, even assuming that one can come up with such an example, we need not regard modus ponens* as responsible for the puzzling conclusion, and modus tollens* is not necessarily guilty either. Actually, there seem to be three possible culprits, which correspond to the three inference rules used in the derivation: modus ponens*, modus tollens* and conjunction of the consequents*. The next section will reveal the identity (identities) of the culprit(s).

used as an abbreviation for "cautious cut"), I decided to choose a more explicit label. (See Douven (2016) and Unterhuber (2013) for more about standard labels for rules of conditional logic.)

⁶A related point is made in Lutskanov 2018 (p. 230).

5. *The restaurant scenario*

I am sitting in a restaurant with my Italian friend Pasquale. I know that Pasquale always orders one of the day's specials. Today's specials are pizza, pasta and roast beef. I know that Pasquale loves both pizza and pasta, and that he does not like roast beef very much. I estimate that there is a 0.4 probability that Pasquale will have pizza, a 0.4 probability that he will have pasta and a 0.2 probability that he will have roast beef.

Assume Adams' Thesis and set $t = 0.6$. In this context, I should believe both (6) "If Pasquale doesn't have pizza, then he will have pasta" and (7) "Pasquale won't have pizza". Indeed, they both have a probability of at least 0.6. Now, from (6) and (7), using modus ponens, I should infer (8) "Pasquale will have pasta". But (8) only has a probability of 0.4; so I should not believe (8), that is, modus ponens fails.

Let us now turn to modus tollens. By the Lockean Thesis, I should believe (9) "Pasquale won't have pasta", which has a probability of 0.6. Now, from (9) and (6) I should draw, by modus tollens, the conclusion that (10) "Pasquale will have pizza". But (10) only has a probability of 0.4; therefore, I should not believe (10), i.e. modus tollens fails.

So we have both a failure of modus ponens and a failure of modus tollens. Or, more precisely, given $t = 0.6$ and Adams' Thesis, in the above examples rational belief is not closed under modus ponens and modus tollens, respectively.

Note that if we assume the material conditional (i.e. if we regard (6) as equivalent to "either Pasquale will have pizza or he will have pasta") both principles still fail; i.e. in the restaurant scenario modus ponens* and modus tollens* also fail. Indeed, given $t = 0.6$, we should believe "either Pasquale will have pizza or he will have pasta", which has a probability of 0.8.

It turns out that the arguments (6)-(8) and (6)-(10) above allow us to identify the culprit(s) among the inference rules used in the derivation I proposed in section 4. Indeed, let us assume that X is "Pasquale doesn't have pizza", Y is "Pasquale doesn't have pasta" and Z is "Pasquale doesn't have roast beef". As in McGee's scenario, in the restaurant scenario we should also believe the relevant instances of (a), (b) and (c).

According to the derivation in section 4, however, from (a), (b) and (c) we should draw the conclusion that $\sim Y$ is the case, i.e. that Pasquale will have pasta. But this is absurd, as “Pasquale will have pasta” only has a probability of 0.4.

In McGee’s example $\sim Y$ was not certain but had, at least, high probability. In the restaurant example, instead, the probability of $\sim Y$ is not even high: it is clear that we should not believe the conclusion.

In section 4, I observed that the derived puzzling conclusion may depend on three different logical principles: modus ponens*, modus tollens* and conjunction of the consequents*. Thanks to the restaurant variant of McGee’s scenario, we now know who the culprit is (or rather, who the culprits are). Indeed, in (6)-(8) and (6)-(10) conjunction of the consequents* does not play any role: (6)-(8) only involves modus ponens*, and (6)-(10) only involves modus tollens*. This shows that conjunction of the consequents* is not needed in order to derive the conclusion; instead, using modus ponens* or modus tollens* is enough to generate it.

6. From McGee’s puzzle to the Lottery Paradox

It can be noted that modus ponens and modus tollens (together with modus ponens* and modus tollens*) are not the only logical principles that fail in the restaurant scenario. Indeed, conjunction introduction does not hold either: given $t = 0.6$, we should believe “Pasquale won’t have pizza”, “Pasquale won’t have pasta” and “Pasquale won’t have roast beef”, but we should not believe the conjunction of these three propositions (actually, we should believe its negation). That is, failure of modus ponens and modus tollens (as well as modus ponens* and modus tollens*) is not the only relevant feature of the restaurant scenario. Indeed, it can be shown that the restaurant example has the same structure as the Lottery Paradox (Kyburg 1961).

Famously, Kyburg’s puzzle goes as follows: suppose that I participate in a fair 1000-ticket lottery with exactly one winner. In this context, I have very good reasons to believe that my ticket will lose. Indeed, the probability that it will win is 0.001. I believe the same about the ticket of the person next to me, and about all the other tickets. Nonetheless, if I apply this reasoning to every ticket from $n^{\circ}1$ to $n^{\circ}1000$ I reach the

conclusion that all tickets will lose, which is false.

In both Kyburg's scenario and mine, a disjunction must be satisfied: in the lottery scenario, one ticket must win; in the restaurant example, it is assumed that Pasquale will pick one of the day's specials. However, at the same time, in both scenarios we should not believe any of the disjuncts: in the lottery scenario each of the 1000 tickets is unlikely to win; in the restaurant scenario, none of the day's specials is likely to be Pasquale's choice. That is, the restaurant scenario is a lottery scenario, at least if we adopt the standard definition of a lottery scenario as a scenario where, given t higher than 0.5, the Lockean Thesis and conjunction introduction come into conflict, i.e. one should end up believing a contradiction. This definition clearly applies to the restaurant scenario, as in it a probability of 0.6 is assumed as a threshold for rational belief.

So if we assume the above definition of a lottery scenario, then a slight modification of McGee's original example leads to a version of the Lottery Paradox, namely one with three tickets and a probability threshold for rational belief of 0.6. All one has to do in order to obtain such a scenario is to decrease the probability of "Reagan will win"; more specifically, one has to assign to "Reagan will win" a probability lower than the threshold: this is enough to generate a probability distribution where the probability of each of the three disjuncts is below t . That is, this is enough to generate a lottery scenario (provided, of course, that some specific proportions are respected between the probabilities of the propositions; the restaurant scenario exemplifies such proportions).

I would like to stress that such a modification (i.e. the one that takes us from McGee's original scenario to the restaurant puzzle) is an innocent one: the fact that in the original scenario "Reagan will win" is very likely can be regarded as a contingent feature of the scenario itself. That is, transforming McGee's original example into the restaurant example by no means betrays the original scenario.

Clearly enough, in the restaurant scenario the relevant properties of McGee's scenario are preserved: this happens because even if in the restaurant example the probability of $\sim Y$ ("Pasquale will have pasta", corresponding to "Reagan will win" in the election scenario) is low, the probability of X ("Pasquale won't have pizza"/"A Republican will win") is high. That is, even if in the election scenario the probability of "Reagan will win" were to be lower than it is, McGee should still have to regard (1)-(3) and (1)-(5) as failures of modus ponens and modus tollens respectively. Removing the contingent fact

that one of the disjuncts in the original scenario has a probability higher than the threshold simply allows us to gain further insight into the puzzle.

Interestingly, this undermines those accounts of McGee's puzzle according to which (1)-(3) is not a modus ponens argument, but rather contains a fallacy of equivocation of certain kinds. Paoli (2005), for instance, argues that (1)-(3) is not an instance of modus ponens because "A Republican wins" should be given a different interpretation in (1) and (2). An essential role in distinguishing the two interpretations is played by the fact that, according to the author, in (2) "A Republican will win" simply stands for "Reagan will win". Fulda (2010) also claims that (1)-(3) is not a modus ponens, but rather an enthymeme, in which "Reagan will win" is the suppressed premise. We now see that both attempts to dismiss McGee's argument are misguided, as they focus on a very contingent feature of the argument: the fact that in McGee's original scenario one of the disjuncts ("Reagan will win") seems rationally acceptable to begin with.

7. The need for a unified solution

My main conclusion will be that it is impossible to solve McGee's puzzle without thereby solving the Lottery Paradox, and the other way around. (A corollary of this conclusion is that if one believes that there is nothing puzzling about McGee's argument, one should also think that the Lottery Paradox is not a paradox after all, and vice versa.)

Let me address one potential objection to this conclusion. The objection is based on a difference between the restaurant example and McGee's original example, i.e. on the fact that in the restaurant scenario both "kinds" of modus ponens (modus ponens and modus ponens*) can only fail if t is relatively low, namely, if it is equal to 0.6. It goes as follows: what makes (1)-(3) interesting is that each of its two premises seems to have a very high probability (higher than 0.6), and still modus ponens seems to fail, although not for the material conditional, but rather for the indicative conditional (i.e. modus ponens seems to fail, unlike modus ponens*). Assume that the premises and conclusion in (1)-(3) are best analysed according to the Argument Schema: if (1) has a probability of 1, (3) can be assigned a low probability even though (2) is assigned a probability higher than 0.6 (provided that such a probability is not 1; for the very minor

probabilistic constraints that (1') and (2') impose on (3') see Stern and Hartmann 2018, fn. 18). That is, the failure of modus ponens does not depend on (2) being assigned a probability of at most 0.6.

In other terms, the complaint is this: if we assume that a probability of 0.6 is not sufficient for rational belief (whereas a greater probability does suffice) the connection between McGee's puzzle and the Lottery Paradox disappears: we would then still be allowed to regard McGee's original puzzle as a genuine puzzle as far as *indicative* conditionals are concerned, but there would be no puzzle about the restaurant scenario.

There is, I think, a clear response to these remarks. Quite simply, it is very rarely the case that defenders of the Lockean Thesis commit to a specific value for t . Actually, there seem to be only very few authors who have a strong preference for a specific threshold⁷.

This notwithstanding, let us go on and assume that the objection can be thoroughly articulated, so that in the restaurant scenario modus ponens and modus ponens* do not really fail, because a higher probability is needed for rational belief. After all, it does not seem unreasonable to think that a probability of 0.6 is not (at least not always) sufficient for rational belief. This is not enough, however, to show that McGee's puzzle and the Lottery Paradox are not to be solved in the same way. Indeed, the only way to escape the conclusion that McGee's puzzle and the Lottery Paradox should get the same solution is by arguing that we should not *explain* in the same way the failure of Belief Closure in the restaurant scenario (which is just, as we have seen, a 3-ticket lottery scenario where $t = 0.6$) and in the original, 1000-ticket version of the same scenario. That is, someone who would like to argue that the specific thresholds assumed in McGee's original puzzle and in the restaurant puzzle are relevant in order to solve the puzzles themselves, should also argue that the failure of Belief Closure in a 3-ticket version of the lottery scenario, and the same failure in the original 1000-ticket version are completely different phenomena, to be explained independently of each other. At this stage of our discussion, I find it fair to say that such an approach to the issue is not very promising, and in fact apparently *ad hoc*.

⁷One of them is Achinstein (2001), who claims that a probability greater than 0.5 is both necessary and sufficient for rational belief. Recently, Shear and Fitelson (2019) have argued that the inverse of the golden ratio ($\phi^{-1} \approx 0.618$) should be regarded as a non-arbitrary bound on the belief threshold.

I conclude that the sensible ways to deal with McGee's scenario are the same as the sensible ways to deal with the lottery scenario. In both cases, we seem to have two main options: giving up either the Lockean Thesis or Belief Closure.

Note that, if we decided to reject Belief Closure, we would be forced to deny at least three principles: modus ponens*, modus tollens* and conjunction introduction. Indeed, I showed that in the restaurant scenario the three of them fail. In fact, this also holds for Kyburg's scenario: even though the Lottery Paradox is generally presented as involving conjunction introduction, we can generate lottery-like paradoxes by using other principles (see Douven 2016). It is instructive to see briefly how.

Let us consider Kyburg's original scenario. In it, we should believe "Ticket n°1 wins \vee ticket n°2 wins... \vee ticket n°1000 wins" (where \vee is the disjunction symbol), which is equivalent to "(Ticket n°1 loses \supset ticket n°2 loses... \supset ticket n°999 loses) \supset ticket n°1000 wins". We should also believe, about each of the tickets between n°1 and n°999, that it will lose. However, we should not believe that ticket n°1000 will win (in fact, we should believe that it will lose). That is, modus ponens* fails.

In this scenario, modus tollens* does not hold either. Indeed, we should accept (Ticket n°1 loses) \supset ticket n°2 loses... \supset ticket n°999 loses \supset ticket n°1000 wins.
Ticket n°2 loses.

...

Ticket n°999 loses.

Ticket n°1000 loses.

Nevertheless, we should reject

Ticket n°1 will win

and accept its negation. That is, in Kyburg's original scenario, exactly as in the restaurant scenario, there are at least three ways to generate an unacceptable conclusion: using modus ponens*, modus tollens*, or (as in the original version of Kyburg's puzzle) conjunction introduction.

8. Back to McGee's original argument

What I just said concerns the way we should deal with the restaurant scenario and the lottery scenario in general. But what can we say concerning specifically the original version of McGee's argument?

In fact, the dilemma raised by the Lottery Paradox applies in a straightforward manner to (1)-(3). That is, if we reject either the Lockean Thesis or Belief Closure McGee's original argument is blocked.

Suppose that we deny the Lockean Thesis: we should then reject (1) and (2), which would block the derivation of (3). If, instead, we denied Belief Closure (i.e. as we have seen, at least modus ponens*, modus tollens* and conjunction introduction), this would also solve the puzzle. The reason is the following: suppose that we reject modus ponens*; it seems that, *a fortiori*, we should reject modus ponens. This is because it is natural to regard the indicative conditional as stronger than the material conditional; i.e. it is generally assumed that if we should believe an indicative conditional, we should also believe the corresponding material conditional. One main argument to this conclusion goes as follows: suppose that we rationally believe the negation of $P \supset Q$, i.e. $P \wedge \sim Q$; it seems natural to infer that we rationally believe the negation of the corresponding indicative conditional. This reasonable assumption entails that if modus ponens* (modus tollens*) turned out to fail, modus ponens (modus tollens) would also fail. That is, if we rejected modus ponens*, McGee's original puzzle would also be solved, as (1)-(3) would not be an instance of a valid logical schema anymore, whether we assume the material conditional or a stronger conditional.

Of course, the same holds for (1)-(5): suppose that we reject modus tollens*: (1)-(5) would not instantiate a valid principle anymore, whether, again, we assume the material conditional or a stronger conditional.

So I showed that the two puzzles (McGee's and the Lottery) have the same structure; i.e. that a slight modification of McGee's election scenario is a lottery scenario. This entails that the two scenarios put us before the same dilemma: should we deny the Lockean Thesis or Belief Closure? I then noted that, no matter which of these two principles we choose to deny, McGee's original argument is blocked. In other terms,

exactly as the Lottery Paradox, McGee's 1985 paper can be taken to show that under the reasonable (and popular) assumption that the Lockean Thesis holds, Belief Closure fails. A straightforward consequence of this conclusion is that it undermines any account of McGee's puzzle that does not involve either giving up the Lockean Thesis or Belief Closure. Remarkably, this includes the vast majority of the existing accounts of McGee's problem⁸.

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⁸Among others, this includes the accounts of the puzzle by Appiah (1987), Bennett (2003), Cantwell (2008), Fulda (2010), Gillies (2004), Katz (1999), Kolodny and MacFarlane (2010), Lowe (1987), Moss (2015), Neth (2019), Paoli (2005), Piller (1996), Schulz (2018), Stern and Hartmann (2018), and Stojnić (2017).

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