The Epistemic and the Deontic Preface Paradox

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Abstract

This paper generalizes the preface paradox beyond the conjunctive aggregation of beliefs and constructs an analogous paradox for deontic reasoning. The analysis of the deontic case suggests a systematic restriction of intuitive rules for reasoning with obligations. This proposal can be transferred to the epistemic case: it avoids the preface and the lottery paradox and saves one of the two directions of the Lockean Thesis (i.e., high credence is sufficient, but not necessary for rational belief). The resulting account compares favorably to competing proposals; in particular, we can formulate the rules of correct doxastic reasoning without reference to probabilistic features of the involved propositions.

Keywords: preface paradox, belief aggregation, Lockean Thesis, doxastic logic, deontic logic, rational akrasia

1 Introduction

The *preface paradox*, first articulated by Makinson (1965), presents a challenge to the combination of two ideas: (i) an agent may have sufficient evidence for believing propositions that stand in logical tension to each other; and (ii) we can validly infer to the conjunction of any two propositions we believe.

In this paper, we develop a novel take on that paradox, based on the analysis of an analogous deontic scenario. Incompatible obligations have

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been studied thoroughly in deontic logic, and we show that this literature suggests promising strategies for dealing with incompatible beliefs, too. Specifically, *restricting* rather than dismissing rules such as closure of rational belief under conjunction and Modus Ponens yields an attractive doxastic logic and a convincing analysis of the preface paradox.

The paper proceeds as follows: Section 2 introduces the preface paradox and generalizes it by deriving a version where Modus Ponens replaces conjunctive aggregation as a rule for reasoning with beliefs. Thus, addressing the paradox requires more than denying that rational beliefs are closed under conjunction. Section 3 constructs a deontic version of the preface paradox based on a set of conflicting obligations. Section 4 discusses possible escape routes and the possibility of rational akrasia whereas Section 5 makes a case for systematically restricting the standard rules for reasoning with obligations. Section 6 transfers this strategy to the epistemic case and applies it to resolving the preface paradox, and the related lottery paradox. While our system of doxastic reasoning does not require reference to probability and/or context, it is compatible with the one of the two directions of the Lockean Thesis: high credence is *sufficient* (but not necessary) for rational belief. Section 7 draws comparisons to rival accounts and Section 8 concludes.

2 The (Epistemic) Preface Paradox

Here is our formulation of the epistemic preface paradox, adapted from Cevolani and Schurz (2017, p. 210):

> Jay, an academic historian, has just published his last book containing the substantive claims $C_1, \ldots, C_n$. Since Jay is a serious scholar, he has carefully checked these claims and he believes that each statement C_i is true. Thus, he infers that also their conjunction $C_1 \wedge \ldots \wedge C_n$ is true. Still, Jay is perfectly conscious of his own fallibility as a writer and researcher. Thus, in the preface of his book, he acknowledges that such an ambitious and long work is bound to contain some error. But this amounts to saying that Jay believes that $\neg C_1 \vee \neg C_2 \ldots \vee \neg C_n$. Thus, Jay seems to entertain two logically incompatible beliefs. How can this be rational?

Since belief expresses an attitude that takes something to be the case, we can hardly believe a proposition and a negation at the same time (or a logical

contradiction). Rational belief should therefore respect the following two principles:

No Contradictory Beliefs Rational belief in A excludes rational belief in $\neg A$: $B(A), B(\neg A) \vdash \bot$.

No Belief in Contradictions Contradictory propositions cannot be rationally believed: $B(\bot) \vdash \bot$.

Jay's epistemic state in the preface paradox clashes with No Contradictory Beliefs, so something must have gone wrong. What exactly? Apparently, Jay's beliefs are justified by a principle like

Sufficient Evidence for Belief A rational agent should believe proposition X whenever the strength of the available evidence for X exceeds a (possibly subjective) threshold t.

This principle sounds plausible: rational belief should track evidence for the truth of a proposition.[1] Unlike the familiar and more specific Lockean Thesis, which we introduce below, the principle makes no assumptions about how belief relates to credence.

For each of the C_i as well as the disjunction $\neg C_1 \vee \neg C_2 \ldots \vee \neg C_n$ ("somewhere in the book, there will be an error") there seems to be sufficient evidence to warrant rational belief, at least when the number of claims n is large enough. This framing of the preface paradox focused on *sufficient* conditions for belief recalls Kyburg's (1961) lottery paradox—and indeed, we will argue for a unified solution to both paradoxes later on.

Moreover, when adding $\bigwedge C_i$ to his belief set, Jay has made use of

Belief Aggregation (Closure of Belief under Conjunction) Rational belief in A and B implies rational belief in $A \wedge B$. Schematically: $B(A), B(B) \vdash B(A \wedge B)$.

This sounds very natural, too: a commitment to the truth of A and B should imply a commitment to the truth of their conjunction. However, Belief Aggregation and Sufficient Evidence for Belief clash with No Belief in Contradictions:

Proposition 1 (Preface Paradox)**.** *Suppose that the strength of the evidence for each proposition in* $S = \{C_1, C_2, \ldots, C_n, \neg C_1 \vee \neg C_2 \ldots \vee \neg C_n\}$ *exceeds threshold*

[1] Compare the Rationality Principle in Priest (2006, p. 109).

t, for a given agent. Then Sufficient Evidence for Belief, Belief Aggregation and No Belief in Contradictions are incompatible.

The proof is simple: From Sufficient Evidence for Belief and repeated Belief Aggregation over the individual claims in S we obtain $B(\bigwedge C_i)$. Since by assumption $B(\neg \bigwedge C_i)$, we can infer $B(\bot)$ and No Belief in Contradictions is violated.

Which of the involved principles should we reject? Some authors claim that contradictions of the form $A \wedge \neg A$ can be rationally believed (e.g., Priest 1998). However, this view is easier to motivate for paradoxical, liar-type propositions or inherently vague propositions than for the historical claims Jay is investigating (e.g., "the Donation of Constantine is a fake"). Thus, giving up No Belief in Contradictions will not resolve the paradox. Provided that Jay has indeed sufficient evidence for each claim he is considering, we must then choose between giving up Sufficient Evidence for Belief and giving up Belief Aggregation. The latter move bars the inference that Jay believes $\bigwedge C_i$ as well as $\neg \bigwedge C_i$ and is advocated, among others, by Kyburg (1961), Foley (1992, 2009), Christensen (2004) and, on the basis of an accuracy-based argument, also Easwaran (2016).[2]

The case for rejecting Belief Aggregation is reinforced by explications of strength of evidence in terms of our credences or graded beliefs (e.g., Carnap 1950; Sprenger and Hartmann 2019, ch. 1). The link between graded and qualitative belief is most naturally expressed by a bridge principle such as

Lockean Thesis (e.g., Foley 1992) Rationally believe proposition X if and only if the credence in X is higher than a threshold $t \in [1/2, 1)$.[3]

We do not need to accept the Lockean Thesis to generate the preface paradox, but it is a particularly popular and intuitively compelling explication

[2]There are more radical ways out of the paradox, too. For example, Jeffrey (1970) denies that there is an epistemically interesting notion of qualitative belief or rational acceptance. All that is required for rational doxastic attitudes is having credences that conform to the laws of probability. Other scholars suggest that rational belief should not be tied to high credence, but to high expected truthlikeness (Cevolani 2017; Cevolani and Schurz 2017). These proposals are discussed in Section 7.

[3]For $t = 1$, a rational agent does not believe anything which is even minimally uncertain, precluding the application of rational belief to ordinary reasoning. For this reason, choosing $t = 1$ is usually not considered as a viable option for connecting credence and rational belief (e.g., Douven 2003, p. 394).

of Sufficient Evidence for Belief. It also allows different agents to apply different standards of evidence (i.e., credence thresholds t). What is more, the Lockean Thesis explains what is wrong with Belief Aggregation: aggregating a lot of propositions, each of which is reasonably believed, we may end up with an improbable proposition for which we have little evidence. Belief Aggregation is therefore too strong.[4]

However, this response is incomplete. The point of having a qualitative notion of belief on top of rational credence is to identify propositions that we regard as true. Therefore, at least *some* laws of classical logic should be valid for reasoning with rational beliefs (for discussion, see Harman 1986; MacFarlane 2004). Whoever rejects Belief Aggregation should therefore identify the inferences that we *can* apply to our rational beliefs. A natural candidate is

Doxastic Modus Ponens Rational belief in both A and the material conditional "if A, then B" implies rational belief in B. Schematically: $B(A), B(A \supset B) \vdash B(B)$.

In support of this rule, we note that Modus Ponens is perhaps the most fundamental inference rule of classical logic. Many axiom systems have it as an inference rule and also in empirical research, it regularly emerges as one of the most strongly endorsed inferences (Evans and Over 2004; Johnson-Laird and Byrne 1991; Oaksford and Chater 2010). Since belief implies commitment to the truth of a proposition, Doxastic Modus Ponens looks like an excellent candidate for valid reasoning with rational beliefs, and indeed, standard doxastic logics such as KD4 accept it as valid (compare Rendsvig, Symons, and Wang 2023; Stalnaker 2006).

However, accepting Doxastic Modus Ponens runs into the same problems as accepting Belief Aggregation:

Proposition 2 (Preface Paradox, version with Modus Ponens)**.** *Suppose that the strength of the evidence for each proposition in* $S = \{C_1, C_2, \ldots, C_n, \neg C_1 \vee \neg C_2 \ldots \vee \neg C_n\}$ *exceeds threshold* t*, for a given agent. Then Sufficient Evidence for Belief, Doxastic Modus Ponens and No Contradictory Beliefs are jointly incompatible.*

[4]For this reason, the preface paradox is sometimes formulated as saying that Belief Aggregation is in tension with the opposite direction of the Lockean Thesis: high credence is *necessary* for belief (e.g., Douven 2003, pp. 390-391). We will get back to this point in Section 6.

Proof. We prove the proposition by induction on the number of claims n. The base case $n = 1$ generates an instance of No Contradictory Beliefs without further assumptions. We now show the inductive step, assuming that the proposition holds for $S_{n-1} = \{C_1, \ldots, C_{n-1}, \neg C_1 \vee \neg C_2 \ldots \vee \neg C_{n-1}\}$. We rewrite the proposition $\neg C_1 \vee \neg C_2 \ldots \vee \neg C_n$ as

$$C_n \supset (C_{n-1} \supset (\ldots \supset \neg C_1))$$

and apply Doxastic Modus Ponens to this proposition and C_n, which are (by Sufficient Evidence for Belief) both in our belief set. This yields

$$B(C_{n-1} \supset (C_{n-2} \supset (\ldots \supset \neg C_1))$$

which is equivalent to

$$B(\neg C_1 \vee \neg C_2 \vee \ldots \vee \neg C_{n-1}).$$

But now we can simply invoke the inductive hypothesis for S_{n-1} and derive a contradiction. Compare the results in Lissia 2022. □

Proposition 2 establishes that it is not the particular power of Belief Aggregation (=closure of belief under conjunction) that creates the paradox.[5] Rather, the paradox emerges if we assume a minimum of inferential power on the system of our beliefs.

We are thus stuck between a rock and a hard place: either we have to give up the eminently plausible principle of Sufficient Evidence for Belief, and its probabilistic explication, the Lockean Thesis. Or we must give up even the most intuitive inference patterns for our beliefs (Belief Aggregation and Doxastic Modus Ponens). But then, which cognitively interesting inference patterns remain for rational belief? We will show a way out of this predicament, but our strategy will benefit from looking at the analogous deontic paradox first.

[5]There is a certain similarity between this result and the objection to Modus Ponens as a rule governing rational belief in McGee 1985. In McGee's alleged counterexample, we rationally believe both $A \rightarrow (B \rightarrow C)$ and A, but we do not believe $B \rightarrow C$. However, the case is different since the natural language indicative conditional does, according to most scholars, not coincide with the material conditional in Doxastic Modus Ponens.

3 The Deontic Preface Paradox

We introduce the deontic version of the preface paradox by means of an example involving standard therapeutic technique for perfectionism, which is very widespread among practitioners of cognitive-behavioural therapy (CBT). In standard definitions, perfectionism is associated with a strong fear of being imperfect or failing to meet some goal or standard. The CBT technique usually goes under the label of "exposure": intentionally allowing oneself to experience an "imperfect" situation which causes anxiety, frustration, or discomfort. The idea is that if the patient lets herself experience, on a regular basis, situations in which she feels anxious and/or frustrated (e.g. not having a perfectly clean apartment, or sending out a document containing a typo), this will progressively reduce her feelings of discomfort. The assumption behind this kind of therapy is that in perfectionism, a huge role is played by the subject's (ill-founded) belief that failing to meet some specific standard will have terrible consequences, e.g., "if there's a typo in my e-mail, everyone will think I'm incompetent". The aim of exposure therapy is to challenge the patient's distorted beliefs, which associate imperfection with catastrophic outcomes.

In a typical exposure assignment, the therapist asks the patient to deliberately fail at some task that she usually carries out perfectionistically. For instance, in the case of a perfectionist high school student, the therapist may recommend that she gets a B instead of an A by handing in her homework late, or, in the case of a manager afraid of making errors, the therapist may suggest that she purposely mispronounces a name during her next presentation. Of course, assignments are always designed so as not to involve excessive risks (e.g., failing an important exam): if an assignment were too risky, exposure could turn out to be counterproductive, since, in addition to the factual damage that the patient may suffer, the incident may provide confirmation for the patient's belief that failure is very dangerous.

Let us now apply this technique to a perfectionist academic: Laura, a historian who has just finished her latest book. She decides to seek help from a professional for dealing with her perfectionist traits, which have a very negative impact on her daily life. Her therapist suggests that she includes just one mistake in the draft of the book she has just finished. The book contains a great many claims that Laura deems scientifically very significant.

Now, Laura has very good evidence that her therapist is highly reliable. In particular, she knows that her therapist has been very successful in treating an impressively high number of people struggling with perfectionism. She has, as a result, strong reasons to follow her therapist's advice: complying with the therapist's task is likely to lead to substantial improvements in her well-being. Moreover, including one single mistake in a book which contains a very high number of (logically independent) claims will be very unlikely to affect its overall scientific value, or her reputation. After all, many important books are known to contain one or—most often—a few errors. In a nutshell, all things considered, not reporting all claims correctly and introducing an error seems the best choice available to Laura. Her reasoning seems to be supported by the following plausible principle:

Perspectivism An agent ought to ϕ if and only if ϕ'ing is her best choice in light of the evidence available to her.

Perspectivism has been defended by a large number of prominent authors (Andrić 2013; Dancy 2000, chapter 3; Gibbons 2010; Kiesewetter 2011, 2017, chapter 8; Lord 2015; Mason 2013; Robertson 2011; Scanlon 2008; Zimmerman 2008, 47–52). The idea is that the best available evidence determines whether or not we should perform an action.

Back to our example. Naturally, Laura begins to wonder where it would be best to include the mistake. She starts by considering claim C_1: given her evidence, C_1 is a significant historical finding. Laura has, as a result, compelling reasons not to present it in a mistaken form. So claim C_1 does not seem the right place for including the mistake; refraining from misreporting C_1 seems, instead, the best choice.

Suppose now that all the claims in Laura's book are scientifically very significant. In this case she will reason in the same way when considering each of the other claims: C_2, C_3, and all the way to C_n. None of them seems to be the right place for introducing an error. In other words: when Laura considers the claims in her book collectively, introducing one error seems the best choice since, given their large number, including one single mistake is unlikely to affect the overall scientific worth of her work. Moreover, complying with the exposure task is very likely to assist in Laura's therapy. In contrast, when she considers each single claim separately from the others, getting one specific claim C_i intentionally wrong does not seem reasonable.

We may assume that in addition to the scholarly value of her book and the advantages for her therapy, Laura also considers the ethical aspects of her actions. However, one single (deliberate) mistake in a volume which includes a large quantity of material does not seem to represent a particularly severe threat to the scientific integrity of the volume. Considerations related to the therapy should prevail. So, by Perspectivism, the claim that Laura ought to introduce an error still seems true. Directed at a single (scientifically important) claim, in contrast, the ethical worry may seem more pressing. In sum, when it comes to any specific claim C_i, reporting it in the form that she regards as correct seems the best option for Laura. Purposely including a mistake only becomes her best option when Laura considers her book as a whole.

Laura's dilemma is that whatever she does, she will do at least *something* she ought not to do. This practical dilemma is inevitable. But can she at least *reason* to her best choice? It seems plausible that practical obligations ("oughts") respect, like beliefs, the following principles:

Obligation Aggregation If we ought to A and also ought to B, then we ought to A *and* B. Formally: $O(A), O(B) \vdash O(A \wedge B)$.

No Impossible Obligations $O(\bot) \vdash \bot$.

The first principle expresses the idea that two different obligations can be expressed as a single obligation, while the second expresses the idea that we cannot be obliged to do something logically impossible. Let R_i denote that Laura reports claim i correctly. $\neg R_1 \vee \ldots \vee \neg R_n$ then denotes that Laura introduces (at least) one error into the book. We can show that given Perspectivism, Laura cannot accept the above two principles:

Proposition 3 (Deontic Preface Paradox)**.** *Suppose that each of the propositions in* $T = \{R_1, R_2, \ldots, R_n, \neg R_1 \vee \neg R_2 \ldots \vee \neg R_n\}$ *is a best choice for an agent in the light of her available evidence. Then Perspectivism, Obligation Aggregation and No Impossible Obligations are jointly incompatible.*

The result is strictly analogous to Proposition 1: iteratively aggregating all the elements of T under the scope of the obligation operator yields $O(\bot)$, since T is a (classically) inconsistent set. But this contradicts No Impossible Obligations. In analogy to the epistemic preface paradox, given the plausibility of Perspectivism and No Impossible Obligations, it seems that Obligation Aggregation has to go.

However, even when we reject Obligation Aggregation, we can construe a similar paradox based on applying Modus Ponens to our obligations. Normal modal logics, including standard deontic logic (SDL), all satisfy the principle that we can apply Modus Ponens under the scope of the strong modal operator:

Deontic Modus Ponens If we ought to A and also ought to B if A, then we also ought to B. Formally: $O(A), O(A \supset B) \vdash O(B)$.

This schema is highly attractive since in analogy to the case of belief, we want *some* classically valid rule for reasoning with the obligations we have. Moreover, suppose we slightly strengthen No Impossible Obligations to the claim that we cannot have contradictory obligations:

No Contradictory Obligations If we ought to A, then we ought not to $\neg A$: $O(A), O(\neg A) \vdash \bot$.

All principles listed so far are valid in SDL (see McNamara and Van De Putte 2022). The following result shows that we do not need to assume Obligation Aggregation: Perspectivism already clashes with Deontic Modus Ponens and No Contradictory Obligations.

Proposition 4 (Deontic Preface Paradox, version with Modus Ponens)**.** *Suppose that each of the propositions in $T = \{R_1, R_2, \ldots, R_n, \neg R_1 \vee \neg R_2 \ldots \vee \neg R_n\}$ is a best choice for an agent in the light of her available evidence. Then Perspectivism, Deontic Modus Ponens and No Contradictory Beliefs are jointly incompatible.*

The proof runs along the lines of the proof of the analogous Proposition 2. Thus, also in the deontic case, the paradox is not exclusively based on aggregating principles: it is a general problem of normal deontic logics. To tackle the paradox, we have to settle for one of the following options: (i) denying that individually best choices can lead to incompatible obligations; (ii) giving up Perspectivism; (iii) weakening SDL as to invalidate one of the premises of the paradox. The following section examines option (i) and (ii).

4 Escape Routes: The Standing of Perspectivism

Perspectivism expresses the idea that the agent's best choices in the light of her available evidence determine her obligations. One might now argue

that such a principle can only provide effective guidance for the agent when conflicting obligations are avoided. This amounts to choosing option (i): best choices must be compatible with each other.

However, this option risks begging the question: one needs a general argument why best choices must be compatible with each other, and why conflicting obligations (which are frequently discussed in the deontic logic and ethics literature) cannot arise. Moreover, one must explain what is wrong with our example, and specify which of Laura's apparent obligations is not a best choice for her. This means that Laura needs to reject one of the implicit assumptions in our description of the scenario, such as:

- A book can be scientifically valid even if it contains an error.
- A single claim cannot be scientifically valid if reported wrongly.
- It can be ethically permissible to falsely report a scientific claim for gaining a therapeutic benefit.
- ...

However, these assumptions look *prima facie* plausible; the burden of proof is on who argues for rejecting one of them.

A second possible way out of the predicament involves replacing Perspectivism with a similar bridge principle. This amounts to choosing option (ii). In fact, Perspectivism is far from being uncontroversial in the literature on obligations and related normative concepts. A common complaint is that although this principle seems to do justice to the sense of "ought" in deliberation, it fails to make sense of how we use "ought" in advice. A famous remark by Judith Thomson (1986, p. 179) expresses this concern:

> On those rare occasions on which someone conceives the idea of asking for my advice on a moral matter, I do not take my field work to be limited to a study of what he believes is the case: I take it to be incumbent on me to find out what is the case.

Along these lines, we could argue that Laura should seek new evidence in order to make a better decision. But our scenario does not hint at any relevant piece of evidence that Laura may be neglecting, and which is in principle accessible to her.

More radically, we could claim that genuine obligations cannot rely on partial evidence or faulty beliefs and replace Perspectivism with the following principle:

Objectivism An agent ought to ϕ if and only if ϕ'ing is the best choice available to her given all the facts, including both those which are accessible to her and those which are not.

This move faces two major problems. First, Objectivism cannot guide agents who need to make decisions under uncertainty. In our scenario, Laura has only limited information; for example she does not know the exact therapeutic effects of misreporting one of the claims. Objectivism may recommend a specific action to her, but she has no way of knowing it. Second, the tension between Laura's obligations does not seem to depend on her epistemic limitations. Even if she knew all relevant facts (therapeutic effect, professional consequences, etc.), she might have reasoned in the same way and concluded that she has, for each individual claim, an obligation to report it correctly, while she also has an obligation to insert an error *somewhere*.

Summing up, Objectivism is not practically useful in deliberation and it does not address the core of the paradox either. Perspectivism may be moot, but rejecting it does not resolve the paradox.

If we find neither option (i) nor (ii) feasible and accept all obligations in T, Laura finds herself in an *akratic* situation. Akrasia is usually defined as acting freely and intentionally against one's best judgment. Indeed, Laura is bound to act akratically, no matter what she does. If she includes an error in her book, then she will fail to act in accordance with one of the $O(R_i)$. If, in contrast, she refrains from including an error in the book, then she will fail to act in accordance with $O(\neg \bigwedge R_i)$.

It can be asked whether this would be that bad, i.e., what is so wrong about akrasia? To be sure, philosophers usually assume that acting akratically entails acting irrationally. In fact, most authors consider akratic action to be the clearest example of practical irrationality. However, certain writers have argued that an akratic action is not necessarily irrational (see, in particular Arpaly 2000; Audi 1990; Frankfurt 1988; McIntyre 2006; Tappolet 2003). The agent's best judgment is not privileged: we may have failed to take all available evidence into account, and our emotional reactions can track reasons which our explicit judgments may overlook. This is why some argue that being moved by these emotions can be rational, even if it leads us to act against our explicit judgments. For example, we might feel that we should not make a certain financial investment, and this feeling might be well-founded, even though our explicit judgment recommends otherwise.

Similarly, Frankfurt (1988) considers the case of an agent who makes a crazy judgment but in the end does not follow her judgment and acts akractically. Although acting akratically is perhaps not fully rational, it may sometimes be more rational than acting in accordance with one's judgment.

Laura's case diverges from cases of rational akrasia in the literature on rational action. No judgmental irrationality applies to Laura's case. Rather, akratic behavior is inevitable because her best judgments lead to a set of obligations full of logical tension. Indeed, Laura's evidence seems to support all of $O(C_i)$ as well as $O(\neg \bigwedge C_i)$.

To sum up, the advocate of Perspectivism who rejects option (i) and (ii) must accept the conclusion that it is sometimes rational to act akratically. This is not necessarily a drawback: our point is simply that Perspectivism's advocates should be aware of this consequence. The next section will pursue this line of thought and advocate that there is a consistent, and not unattractive, way of defending both Perspectivism and rational akrasia. This means to weaken the logic of obligation and to go for option (iii).

5 Escape Routes: Restricting Obligation Aggregation

If one accepts that Laura has conflicting obligations, one needs to reject one of the principles of standard deontic logic used for deriving the paradox. A first candidate is No Contradictory Obligations: conflicting obligations seem to be a regular part of our daily life and it seems to be exactly this conflict that makes moral dilemmas interesting in the first place. Indeed, many deontic logics developed over the last decades allow for conflicting obligations (see, e.g., Beirlaen, Strasser, and Meheus 2013; Goble 2003, 2004; Hilpinen and McNamara 2013; McNamara and Van De Putte 2022).

However, rejecting No Contradictory Obligations alone will not do. In any deontic logic where obligations satisfy Necessitation and are closed under Modus Ponens, No Contradictory Obligations (i.e., $O(A), O(\neg A) \vdash \bot$) is *equivalent* to the principle that there are no impossible obligations (i.e.,

$\vdash \neg O(\bot)$).[6] Since we arguably want to retain No Impossible Obligations, rejecting No Contradictory Obligations will not solve the paradox by itself.[7]

This means that we have to weaken the more substantive principles. To this end, it is helpful to consider the following axiomatization of standard deontic logic (see, e.g. Chellas 1980, p. 191):

$$A \supset B \vdash O(A) \supset O(B) \qquad \text{(Monotonicity)}$$
$$O(A), O(B) \vdash O(A \wedge B) \qquad \text{(Obligation Aggregation)}$$
$$O(\top) \qquad \text{(Necessitation)}$$
$$\neg O(\bot) \qquad \text{(No Impossible Obligations)}$$

Most discussion on modeling conflicting obligations has focused on restricting Monotonicity, Obligation Aggregation or both of them—especially since both Necessitation and No Impossible Obligations seem plausible and not central to deontic dilemmas.[8] Moreover, the combination of Monotonicity, Obligation Aggregation and the scheme *ex falso quodlibet* of classical logic leads to deontic explosion from conflicting obligations, i.e., $O(A), O(\neg A) \vdash O(B)$ (everything is obligatory).[9] Hence, we must not accept Monotonicity and Obligation Aggregation in their current form.

Monotonicity, which we have not discussed so far, is an eminently plausible principle: it says that we must be committed to the logical consequences of our obligations:

> [...] the principle of inheritance of obligations is one of the most fundamental principles of SDL and has strong intuitive appeal. It requires the agent to take moral responsibility for the logical consequences of

[6] Proof: Suppose we have $O(\bot)$, in violation of No Impossible Obligations. Necessitation yields $O(\bot \supset A)$ and $O(\bot \supset \neg A)$ and Modus Ponens then yields $O(A)$ and $O(\neg A)$, in contradiction with No Contradictory Obligations. Conversely, assume $O(A)$ and $O(\neg A)$ for some A. Since $A \supset (\neg A \supset \bot)$ is a truth of classical logic, we infer $O(A \supset (\neg A \supset \bot))$ from Necessitation and then, $O(\neg A \supset \bot)$ using Modus Ponens. Applying Modus Ponens again, with premises $O(\neg A)$ and $O(\neg A \supset \bot)$, yields $O(\bot)$.

[7] Some paraconsistent deontic logics give up No Impossible Obligations (e.g., Costa and Carnielli 1986) along with deontic explosion, but it is not easy to make sense of being obliged to do the logically (as opposed to practically) impossible.

[8] In recent axiomatizations of SDL, such as Hilpinen and McNamara (2013) and McNamara and Van De Putte (2022), Necessitation is presented as a general inference rule along the lines of "if $\vdash A$ then $\vdash O(A)$", from which the axiom in the text can be derived. For reasons of simplicity, we are sticking to Chellas' formulation.

[9] Proof: Obligation Aggregation implies that we can infer $O(A \wedge \neg A)$. Classical logic (ex falso quodlibet) yields $(A \wedge \neg A) \supset B$. Thus we infer by Monotonicity that $O(B)$ for any B.

> what he/she has committed to do. The rejection of the principle, therefore, seems to be contrary to one of our basic moral reasoning patterns. (Nute and Yu 1997, p. 26)

A first proposal is therefore to retain Monotonicity (and Necessitation and No Impossible Obligations) and to reject Obligation Aggregation, as proposed by Lou Goble (2003, 2004) in a series of papers. The resulting logic P is, however, very weak, since we cannot reason by *combining* different obligations we have. We can only reason from each single obligation to its logical implications (e.g., if we ought to A, and A implies B, then we also ought to B). While standard deontic logic may be too strong, this proposal is clearly too weak; it does not provide enough effective guidance.

In our opinion, a more promising proposal consists in the DPM.2 logic developed by Goble (2005). It introduces a dual modal operator expressing permissibility and defines it in the standard way as $P(A) := \neg O(\neg A)$: A is permissible if $\neg A$ is not obligatory. Montonicity and Aggregation are not dropped, but suitably restricted by introducing an additional premise. DPM.2 is axiomatized by

$$
\begin{aligned}
A \equiv B \vdash O(A) \equiv O(B) \quad & \text{(Substitution of Equivalents)} \\
A \supset B, P(A) \vdash O(A) \supset O(B) \quad & \text{(Permitted Monotonicity)} \\
O(A), O(B), P(A \wedge B) \vdash O(A \wedge B) \quad & \text{(Permitted Obligation Aggregation)} \\
O(\top) \quad & \text{(Necessitation)} \\
\neg O(\bot) \quad & \text{(No Impossible Obligations)}
\end{aligned}
$$

The idea is that we can reason monotonically from *permissible* premises, saving the intuitions in favor of Monotonicity and blocking deontic explosion (because $O(\bot)$ does not imply $O(B)$ any more, for any B). Analogously, aggregation is permissible if the aggregate obligation has been permissible in the first place.[10] Moreover, Permitted Obligation Aggregation implies the

[10] Strasser, Beirlaen, and Meheus (2012) suggest to replace Permitted Obligation Aggregation by a principle where each of the conjuncts, but not the conjunction, has to be permissible in the first place. It goes beyond the scope of this paper to review the implications of this choice for deontic logic, but this modification does not yield a resolution of the preface paradox.

following restriction of Deontic Modus Ponens:[11]

$$O(A), O(A \supset B), P(A \wedge B) \vdash O(B) \qquad \text{(Permitted Deontic Modus Ponens)}$$

Applying DPM.2 to the deontic preface paradox yields several interesting observations. First, when obligation implies permission, no proposition in the conjunctive closure of our obligations is optional: either doing it or not doing it is obligatory.

Proposition 5. *Let T be a set of obligations (i.e., $O(X)$ for all $X \in T$), and let $\mathcal{C}(T)$ denote the conjunctive closure of T (i.e., any conjunction of elements of T). Then, for all non-contradictory* DPM.2*-models, i.e., all models such that $O(X)$ implies $P(X)$, either $O(Y)$ or $O(\neg Y)$ for any $Y \in \mathcal{C}(T)$.*

Proof. The proof proceeds by induction on the complexity of the formula. We first note that $P(X)$ for all $X \in T$ because the models are assumed to be non-contradictory. Assume that $Y = \bigwedge_{i=1}^{k} X_i$ is a conjunction of k propositions in T, with $T_k := \{X_1, \ldots, X_k\}$. We now reason by cases.

1. There is a proper subset $T' \subset T_k$ such that $O(\neg(\bigwedge_{X \in T'} X))$. Then also $P(\neg(\bigwedge_{X \in T'} X))$ and so we use Permitted Monotonicity to infer $O(\neg(\bigwedge_{X \in T_k} X))$, i.e., $O(\neg Y)$. Because the model is assumed to be non-contradictory, this implies also $P(\neg Y) = \neg O(\neg Y)$.

2. There is no subset T' as specified in the second case. Then, by the inductive hypothesis, there is a partition $T_k = T' \cup T''$ such that $O(\bigwedge_{X \in T'} X)$ and $O(\bigwedge_{X \in T''} X)$. Now either $\neg P(Y) = O(\neg Y)$ or $P(Y)$. In the second case, we can apply Permitted Obligation Aggregation to $O(\bigwedge_{X \in T'} X)$ and $O(\bigwedge_{X \in T''} X)$ and obtain $O(Y)$. In none of the two cases Y is optional.

Thus, we have either $O(Y)$ nor $O(\neg Y)$ for each element Y of the conjunctive closure of T. □

Second, assume that the obligations described in the deontic preface paradox are *minimal* in the sense that Laura has no obligation to introduce more than one error. This assumption is very plausible: making more than

[11] Proof: Suppose $O(A)$ and $O(A \supset B)$ and $P(A \wedge B)$, then by Permitted Obligation Aggregation, we infer $O(A \wedge (A \supset B)) = O(B)$.

one error would devalue Laura's scholarly work more than her therapy requires. Then we can infer that she should report exactly n-1 claims correctly.

Specifically, Laura can reason "bottom up" until she obtains, for example, the obligations $T' = \{O(R_1 \wedge \ldots \wedge R_{n-1}), O(R_n), O(\neg \bigwedge R_i)\}$. But from here, she cannot go any further. Since by definition $O(\neg \bigwedge R_i) = \neg P(\bigwedge R_i)$, she cannot aggregate $O(R_1 \wedge \ldots \wedge R_{n-1})$ and $O(R_n)$: their conjunction is not permissible and Permitted Obligation Aggregation cannot be applied. For the same reason, she cannot aggregate any other pair of obligations in T'.[12]

To us, this is a rather plausible reconstruction of what happens in the head of a rational deontic reasoner. She infers to more specific obligations as long as she can consistently do so (using the restricted versions of Monotonicity and Obligation Aggregation), until she finally finds herself in a dilemma where she cannot aggregate any further. Laura stops short of accepting inconsistent obligations, but she finds herself in the dilemma of not being able to infer to a practical *decision*. On our reconstruction, rational deontic reasoning exhibits the akratic features that a proponent of Perspectivism (who decides to restrict Obligation Aggregation and Deontic Modus Ponens) should endorse in the first place. For conflicting obligations, deontic reasoning need not decide what an agent should do; in this case it simply highlights the source of the conflict she is in. We now transfer our strategy from the deontic to the epistemic preface paradox.

6 Reconsidering the Preface Paradox

For transferring the above solution proposal to the epistemic preface paradox, we need to define a dual operator for belief: $NR(X) := \neg B(\neg X)$, to be read as "X is not rejected" or "X is a live option" and defined as "$\neg X$ is not believed". This epistemic attitude is analogous to "permissible" in the deontic case and not to be confused with suspension of judgment. The doxastic analogues of the DPM.2 principles then read

$$A \equiv B \vdash B(A) \equiv B(B) \qquad \text{(Substitution of Equivalents)}$$

[12]The second version of the paradox is blocked in a similar way. Due to the minimality of Laura's obligations, we know that $P(\bigwedge_{i \neq k} R_i)$ for any $k \leq n$ and so Laura cannot infer from $O(R_1 \supset (R_2 \supset (\ldots ... \supset \neg R_n)))$ and $O(R_1)$ by means of Permitted Deontic Modus Ponens to $O(R_2 \supset (R_3 \supset (\ldots ... \supset \neg R_n)))$: she would, in this case, end up with a non-permissible obligation.

$$A \supset B, NR(A) \vdash B(A) \supset B(B) \qquad \text{(Permitted Monotonicity)}$$

$$B(A), B(B), NR(A \wedge B) \vdash B(A \wedge B) \qquad \text{(Permitted Belief Aggregation)}$$

$$B(\top) \qquad \text{(Belief in Tautologies)}$$

$$\neg B(\bot) \qquad \text{(No Beliefs in Contradictions)}$$

While the motivation of Permitted Monotonicity and Permitted Belief Aggregation is to deal with beliefs that stand in logical tension, the other three axioms are easily motivated: our epistemic attitudes toward logically equivalent propositions should be identical, we should believe tautologies ($B(\top)$) and not believe contradictions ($\neg B(\bot)$). We call the doxastic logic axiomatized by these five principles and the propositional tautologies CDR, standing for "cautious doxastic reasoning". CDR also restricts Doxastic Modus Ponens in the following way:

$$B(A), B(A \supset B), NR(A \wedge B) \vdash B(B) \qquad \text{(Permitted Doxastic Modus Ponens)}$$

and hence, it is, like DPM.2, a non-normal modal logic.

Since the axioms of CDR and DPM.2 are structurally identical, the result of Proposition 5 transfers to the epistemic case. As before, we consider a CDR-model non-contradictory if $B(X)$ implies $NR(X)$ (i.e., belief implies non-rejection), and we focus on these models only.

Proposition 6. *Let S be a belief set. Then, for any element in the conjunctive closure of S, i.e., any $X \in \mathcal{C}(S)$, either $B(X)$ or $B(\neg X)$ in all non-contradictory* CDR*-models.*

This means that suspension of judgment is not an option for our historian Jay: he must either believe or disbelieve any conjunction of the claims in his book. There are now various CDR-models of Jay's doxastic state, dependent on his confidence in his claims. If he believes that there is at most one error in the book, without being committed to where the error lies, he will believe all conjunctions of n-1 claims in his book and just disbelieve the conjunction of all n claims. But this does not lead to contradiction because from beliefs such as $B(C_1 \wedge \ldots \wedge C_{n-1})$, $B(C_n)$ and $B(\neg(C_1 \wedge \ldots \wedge C_n))$, Jay cannot derive contradictory beliefs because Permitted Belief Aggregation cannot be applied any further. By contrast, if Jay considers the possibility of a second error in the book, he will believe any conjunction of n-2 claims, but not necessarily a conjunction of n-1 claims. Both models express Jay's commitment

to a vast majority of the claims in his book, without recommending him to reject a specific claim. At the same time, Jay has stronger beliefs than the agent in Leitgeb's (2014) solution who does not believe any specific claim in the book, but just the *disjunction* of the conjunctions of a given number of claims. As noted by Cevolani and Schurz (2017), this feature of Leitgeb's account is rather unsatisfactory and our solution avoids it.

Thus, CDR avoids the paradoxical conclusion of the preface paradox and captures the akratic element in Jay's reasoning (i.e., the impossibility of pointing out a specific claim that should be rejected). What about the connection between credences and beliefs? We look at both directions of the Lockean Thesis separately.

First, high credence may impose a *sufficient* condition on rational belief. This means that for any proposition X, if $p(X) > t$ for some threshold $t \in [1/2, 1)$, then $B(X)$. Conversely, $NR(X)$ implies $p(X) \geq 1 - t$. Suppose now that we believe A and B, and we do not reject $A \wedge B$ (and hence, $p(A \wedge B) \geq 1 - t$). Then, we can infer by means of Permitted Belief Aggregation to $B(A \wedge B)$. In particular, we can infer to propositions whose probability is lower than the threshold t as long as they do not fall below a minimum standard of plausibility 1-t.

To our mind, this is a reasonable tradeoff between promoting the logical coherence of our beliefs, maintaining a reasonable degree of inferential power, and avoiding beliefs with poor evidential support. The 1-t threshold is a safeguard against too optimistic reasoning from a set of fallible beliefs.

Second, high credence may impose a *necessary* condition on rational belief. This means that for any proposition X, $B(X)$ implies $p(X) > t$ for some threshold $t \in [1/2, 1)$. Equivalently, $p(X) \geq 1 - t$ implies $NR(X)$. However, this leads into trouble. Suppose we believe A and B and so $p(A), p(B) > t$. Suppose further that $t > p(A \wedge B) > 1 - t$. This is a rather common case of probabilistic dependency between A and B. The Lockean Thesis then yields $NR(A \wedge B)$ and by Permitted Belief Aggregation, we infer $B(A \wedge B)$, which implies $p(A \wedge B) > t$, in contradiction with what we assumed.

Thus, from the standpoint of CDR, high credence can only express a *sufficient* criterion for rational belief. We retain some intuitions behind the necessity claim, too, but they apply to non-rejection instead of belief: not rejecting a proposition requires a probability of at least 1-t, or conversely, we reject (disbelieve) all propositions whose probability is lower than 1-t.

We can illustrate these general observations in another classical challenge for the Lockean Thesis: the *lottery paradox*. Suppose we are facing a lottery with a large number of tickets, say $N = 1000$. Only one of them is winning. W_i denotes the proposition that ticket i wins and $L_i = \neg W_i$ denotes the proposition that ticket i loses. A rational agent will assign credences $p(W_i) = 1/1000$ and $p(L_i) = 999/1000$ for any ticket i. By the Lockean Thesis, a rational agent should believe any of the L_i. However, when aggregating her beliefs, the agent ends up with a contradiction (since $\bigwedge_{1 \leq i \leq N} L_i = \bot$). Again, unrestricted conjunctive aggregation of beliefs leads to disaster.

Since all evidence in the lottery paradox is statistical, the agent has no reason to distinguish between individual tickets. Therefore it makes sense to assume that she should not believe propositions which are less likely than their negation (Worsnip 2016):

Belief-Credence Coherence If $p(X) < 1/2 < p(\neg X)$, then $\neg B(X)$, i.e., the agent does not believe X.

Indeed, for purely statistical evidence, and $\neg X$ being more likely than X, it is hard to see on which grounds the agent should believe X. By itself, Belief-Credence Coherence does not oblige her to believe $\neg X$: she can refrain from believing either proposition. However, assuming Belief-Credence Coherence in CDR yields the following result:

Proposition 7. *Let N be the number of tickets in the lottery paradox. Then, in all non-contradictory* CDR*-models where an agent respects Belief-Credence Coherence and $B(L_i)$ for any $i \leq N$, she believes proposition X if $p(X) > 1/2$, and $\neg X$ if $p(X) < 1/2$.*

Proof. Assume that X is an arbitrary conjunction of the L_i. By Belief-Credence Coherence, if $p(X) < 1/2$, then $\neg B(X)$, and if $p(X) > 1/2$, then $\neg B(\neg X)$. Proposition 6 applied to the belief set $S = \{L_1, L_2, \ldots, L_N\}$ implies that either $B(X)$ or $B(\neg X)$, and so, if $p(X) < 1/2$, then $B(\neg X)$, and if $p(X) > 1/2$, then $B(X)$. □

In other words, an agent who respects Belief-Credence Coherence and reasons according to CDR will align her beliefs with the Lockean Thesis for the threshold $t = 1/2$. She believes all propositions that are more likely than their negations. In the lottery paradox, she will believe of any set of tickets with more than $N/2$ elements that it contains the winning ticket. For the

borderline case of a set T with exactly $N/2$ tickets, the agent can choose: both believing that winning ticket is in T, and that it is in T^C, is compatible with CDR.

This prediction is reasonable: when the evidence is purely statistical, beliefs should be aligned with preponderance of evidence, i.e., whether a proposition is more likely than its negation (for arguments against this view, see Smith 2021). However, this result does not generalize to non-statistical cases: for deciding whether or not to believe a proposition, its practical significance and the overall coherence we wish to give to our beliefs make a contribution over and above our credences.

In a nutshell, our response to the preface paradox, motivated by the analogies to the deontic case, consists in (i) weakening doxastic logic by restricting Belief Aggregation and Modus Ponens, and (ii) maintaining the Lockean Thesis as a *sufficient* condition for qualitative belief. This yields a plausible analysis of doxastic reasoning and the preface paradox that can also be applied to the structurally similar lottery paradox. The following section draws comparisons to alternative solutions.

7 Comparisons

The literature on preface paradox is too vast to discuss in this section, but we would like to highlight the benefits of our account with respect to some major alternatives.

Clearly, our proposal is more ambitious than Jeffrey's (1970) "radical probabilism": i.e., giving up on the concept of qualitative belief, and doing epistemology only in quantitative terms, based on degrees of belief. This approach neglects that the qualitative notion of belief has an important role in our cognitive practices: we often make inferences on the basis of the propositions that we accept or consider to be true, and we need it for making decisions when precise degrees of belief are hard to get by (see also Foley 2009). In fact, the qualitative concept of belief is studied in quite diverse fields such as epistemology, doxastic logic and belief revision theory.

Another radical solution is to give up on the belief-credence connection and to tie rational belief to high expected verisimilitude (Cevolani 2017; Cevolani and Schurz 2017). This proposal deserves to be taken seriously, but it denies that credence is normative for belief. In comparison, our ac-

count preserves the standard view that rational beliefs can be inferred from high credences and it will therefore be more attractive for a majority of the community.

The most common reaction to the preface paradox is to maintain the Lockean Thesis and to declare Belief Aggregation invalid (e.g., Foley 2009; Kyburg 1961). This move, however, prompts the question of why Belief Aggregation often *appears* to be a sound principle: authors who reject it should qualify the circumstances where we can apply it. Typically, these authors make the validity of Belief Aggregation depend on the probabilistic features of the propositions at stake (e.g., Douven 2003; Hawthorne 2009; Kowalewska forthcoming). Such a move is certainly possible, but it gives up the most intriguing aspect of the standard picture: the laws of belief and degree of belief can be formulated separately (in doxastic logic and probability theory, respectively), and the Lockean Thesis connects them in a systematic way. Our account achieves that goal: it formulates the rules for reasoning with beliefs without reference to probability, while maintaining a version of the Lockean Thesis where we sometimes prefer having *coherent* beliefs to aligning them fully with our credential judgments.

Recently, Hannes Leitgeb (2014b, 2017) developed the *stability theory of belief* as a unifying theoretical framework for probabilistic and doxastic reasoning. In Leitgeb's account, Belief Aggregation and Doxastic Modus Ponens are preserved without restrictions, and an agent's beliefs are aligned with her credences satisfying both directions of the Lockean Thesis. Leitgeb achieves this result by requiring that there be a single strongest believed proposition X, and that all other believed propositions be logical consequences of X. However, identifying this proposition (and the threshold in the Lockean thesis) will depend on the context as described by the features of p, and in particular on how the agent partitions the set of possible worlds on which p is defined.

In other words, Leitgeb's preservation of Belief Aggregation and Doxastic Modus Ponens comes at the price of making rational belief depend strongly on context, and on the way the possible worlds are represented. Moreover, he has to deny that Sufficient Evidence for Belief applies to the preface paradox, and specifically to the propositions in the set $S = \{C_1, C_2, \ldots, C_n, \bigvee \neg C_i\}$. Otherwise, Jay could infer to $\bigwedge C_i$ and have con-

tradictory beliefs. Similar remarks apply to Leitgeb's take on the lottery paradox.

The proposal of preserving Belief Aggregation and denying Sufficient Evidence for Belief has also been articulated in the context of a logic of *reasons* by Crupi and Iacona (2023). According to this analysis, if Jay bases all his beliefs on the total available evidence, it is simply not true that he has reason to believe each of the claims in his book *and* the proposition that it contains at least one error. Reasons for belief consist in evidential support, explicated as probabilistic relevance, not in high probability. If somebody wishes to reject the Lockean Thesis for independent reasons, this approach may be useful and promising, but it will not help the epistemologists who consider it a sound criterion for qualitative belief—usually the ones who feel the sting of the preface paradox the most.

Finally, our analysis shares several features with MacFarlane's (2004) discussion of the preface paradox in the context of the more general question of how logic governs rational belief. Like ourselves, MacFarlane rejects the aggregation principle $B(A), B(B) \vdash B(A \wedge B)$ and searches for a weaker belief norm, but his overall diagnosis is different.

First, for MacFarlane, Jay is "not entirely as [he] ought to be" and under an obligation to make his beliefs coherent. While simply giving up one or more beliefs is not the right option, he should collect more evidence in order to convince himself of the falsity of one of the beliefs in S. However, the story can be told in a way that there is simply no more evidence (e.g., historical sources) that Jay could take into account. So we fail to see how this could be a generally valid resolution of the paradox.

Second, again according to MacFarlane, belief norms along the lines of $B(A), B(B) \vdash NR(C)$, for C being any logical consequence of A and B, are *not* refuted by the paradox. This is a position we must reject. It would imply $B(A), B(B) \vdash NR(A \wedge B)$: we would obtain the missing premise for applying Permitted Belief Aggregation and be able to infer $B(A \wedge B)$ from $B(A)$ and $B(B)$. In other words, unrestricted belief aggregation would be valid. Therefore, we must reject sufficiently implausible logical consequences of our beliefs. The Lockean Thesis provides the philosophical motivation for this attitude and points out what may be problematic about MacFarlane's proposed norm for belief. At the same time, we can agree with MacFar-

lane that believing A and B provides a reason for believing $A \wedge B$—but a defeasible reason and not a sufficient one.[13]

8 Conclusions

It is time to wrap up. The standard resolution of the (epistemic) preface paradox consists in rejecting closure under conjunction for reasoning with the propositions we believe—a principle that we have called Belief Aggregation. Alternatively, credence-belief bridge principles like the Lockean Thesis may be rejected.

This paper has outlined a middle way: to maintain the Lockean Thesis as a *sufficient* criterion for qualitative belief, and to *restrict* Belief Aggregation, instead of giving it up. This strategy has been motivated by analyzing an analogous paradox for deontic reasoning. There, giving up the aggregation of obligations is independently motivated and it squares well with what we have identified as an important corollary of the bridge principles between evidence and good decisions: the possibility of rational akrasia.

Both in the epistemic and in the deontic case, Aggregation and Modus Ponens have been restricted to the case where the conjunction of the two premises is not rejected. Specifically, in the epistemic case, one can retain high probability as a sufficient (but not as a necessary) criterion for belief.

In our view, this proposal is a reasonable compromise between being able to reason classically with one's sets of beliefs, and having a bridge principle that connects credence to belief. It bridges the gap between doxastic logicians who have traditionally accepted Belief Aggregation and Modus Ponens in their unrestricted form, and epistemologists who cherish the Lockean Thesis. Specifically, the account preserves the autonomy of doxastic reasoning (i.e., the rules of doxastic logic can be formulated without reference to probability) and is compatible with a systematic credence-belief connection. Further research is needed to investigate the wider implications of our proposal, but we hope to made have a good *prima facie* case.

[13] In the terminology of MacFarlane (2004), this means that we reject the `Wo-` norm and accept the `Wr+` and `Wr-` norms.

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