

# GROUNDING AND DEFINING IDENTITY

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## Abstract

I systematically defend a novel account of the grounds for identity and distinctness facts: they are all uniquely zero-grounded. First, the Null Account is shown to avoid a range of problems facing other accounts: a relation satisfying the Null Account would be an excellent candidate for being *the* identity relation. Second, a plenitudinist view of relations suggests that there is such a relation. To flesh out this plenitudinist view I sketch a novel framework for expressing real definitions, use this framework to give a definition of identity, and show how the central features of the identity relation can be deduced from this definition.

## 1 Introduction

Suppose that  $a = a$ : in virtue of what is this the case? Or suppose that  $a \neq b$ : in virtue of what is that the case? This question about the grounds for identity and distinctness facts has seen a fair amount of discussion recently—see e.g., (Burgess, 2012; Fine, 2016; Lo, forthcoming; Shumener, 2017, 2020a, 2020b; Wilhelm, 2020). I have two goals in this paper. First, I want to defend the view that identity facts are *zero-grounded* (or have the null (empty) ground). More specifically:<sup>1</sup>

(Null-Identity) If  $a = b$ , then the unique ground for  $a = b$  is the null ground

(Null-Distinctness) If  $a \neq b$ , then the unique ground for  $a \neq b$  is the null ground

Call this the Null Account and its proponent the Null Theorist. The idea that identity and distinctness facts are zero-grounded has already been suggested by several authors—e.g., Fine (2012, 2016), Shumener (2020b), and Litland (2018)—but none have given it a sustained defense. Developing this defense leads to the second goal. I suggest that one can give a *real definition* of the identity relation as, roughly, that relation the instantiation and non-instantiation of which is grounded in accordance with (Null-Identity) and (Null-Distinctness). However, any such definition appears to be circular since in specifying the conditions under which the empty ground grounds  $a = b$  as opposed to  $a \neq b$  we need to say whether  $a$  is identical to  $b$ . However, this circularity can be overcome by adopting some Wittgensteinian ideas about the role of variables. This leads to the development of

a novel theory of real definition (in general), a real definition of the identity relation (in particular), and, finally, a derivation of the Null Account from the definition of identity.

For the reader’s benefit here is an overview of the paper. The paper begins in § 2 by covering some terminological ground and giving an essentialist formulation of the Null Account. In § 3 I argue for the Null Account by showing how it solves a range of problems. § 4 takes up the problem of Differential Grounding : what explains how the empty ground sometimes grounds  $a = b$  and sometimes grounds  $a \neq b$ ? § 5 explores the plenitudinist idea that *some* relation is defined by being grounded as the Null Account says identity is. The major contributions of the paper are in § 6, which develops the rudiments of a novel account of real definition and gives a definition of identity; the section culminates in § 6.14 which sketches how the account of real definition is related to an account of essence, and indicates how the essentialist formulation of the Null Account can be derived. § 7 concludes.

## 2 Generalized Identity, (Zero-)Ground, and Essence

### 2.1 Generalized identity

The sign for the identity relation is flanked by singular terms, the semantic values of which are *objects*. But singular terms are not the only meaningful expressions; there are monadic (dyadic, triadic, . . . ) predicates, quantifiers, sentential operators, and, of course, there are the sentences themselves. Recently, there has been a lot of interest in *just-is* statements like “for Amy to be taller than Bob just is for Bob to be shorter than Amy” or (more controversially) “for the number of Jupiter’s moons to be four just is for Jupiter to have exactly four moons”. Such *just-is* statements can be understood as generalized identity statements, asserting that analogues of the objectual identity relation hold between the semantic values of predicates, quantifiers, predicates, and sentences.<sup>2</sup>

To express such generalized identities I adopt an extension of the relational type theory of (Dorr, 2016).<sup>3</sup> There is one basic type:  $e$ —the type of objects. There are two kinds of complex types. Whenever  $\tau_0, \dots, \tau_{n-1}$  are some types then  $\langle \tau_0, \dots, \tau_{n-1} \rangle$  is a type—the type of *relations* between items of type  $\tau_0, \dots, \tau_{n-1}$ . As a special case, this yields  $\langle \rangle$ —the type of propositions. Whenever  $\tau$  is a type  $[\tau]$  is a type—the type of *pluralities* of items of type  $\tau$ . As a special case we have  $[\langle \rangle]$  the type of pluralities of propositions. I will not specify a type-theoretic language in full detail, but I will assume that for each type  $\tau$  we have infinitely many variables  $x_0^\tau, x_1^\tau, \dots$  of that type and that for each type  $\tau$  we have a binary identity predicate  $=^{\langle \tau, \tau \rangle}$ .

The Null Account is intended to apply also to generalized identity claims. For reasons of readability I will, however, mainly focus on the case of objectual identity. To that end I use the following abbreviations:  $x, y, z, \dots$ , for first order-variables (i.e., variables of type  $e$ );  $p, q, \dots$  for propositional variables (i.e., variables of type  $\langle \rangle$ );  $P, Q, \dots$ , for monadic predicate variables (variables of type  $\langle e \rangle$ );  $R, S, \dots$  for variables of type  $\langle e, e \rangle$ ;  $xx, yy, zz, \dots$  for plural first order variables (type  $[e]$ );  $pp, qq, \dots$  for plural propositional variables (type  $[\langle \rangle]$ .) I reserve  $=$  for the first-order identity predicate, and use  $\equiv$  ambiguously for various higher-order identity predicates.

## 2.2 Ground and zero-ground

If it is the case that  $\phi$  one may ask what makes it the case that  $\phi$ : the answer gives the grounds for  $\phi$ . Several notions of ground have been distinguished in the literature; the relevant notion here is *strict, full, immediate* ground. It is *full* in that if some facts ground another, then no fact need be added to the former in order to have a complete explanation of the latter. It is *immediate* in that if some facts ground another then they need not do so via grounding some mediating facts.<sup>4</sup> It is *strict* in that if some facts ground another, then the grounded fact does not in turn help (mediately) ground one of the former. In the type-theoretic framework immediate ground is simply a relation of type  $\langle\{\langle\rangle\}, \langle\rangle\rangle$ —a relation between pluralities of propositions and propositions. Throughout the paper I use  $\ll$  for this relation.

I will allow the empty plurality of propositions. (If one is squeamish about this, consider that a plurality is individuated by which members it has. The empty plurality is then the plurality that is individuated by having no members.) I will use  $0$  to denote this empty plurality. This leads us to the following crucial distinction. A proposition  $p$  is *ungrounded* if there are no propositions  $qq$  such that  $qq \ll p$ ; a proposition  $p$  is *zero-grounded* (alternatively: *null-grounded*) if  $0 \ll p$ . Ungrounded propositions are not grounded; zero-grounded ones are.

Zero-grounding is apt to seem perplexing at first—especially in the obscure formulation “grounded, but in nothing”. While this is not the place for an extended argument that the notion—as such—is in good standing, it is worth noting that zero-grounding arises naturally on all broadly “tripartite” views of grounding.<sup>5</sup> Tripartite views hold that the relationship between the grounds and the grounded is *mediated* by what one may call—to use a neutral term—a connection. For examples of tripartite views see (Bader, 2017; Litland, 2017; Schaffer, 2016, 2017a, 2017b; Wilsch, 2015a, 2015b). For Schaffer and Bader the connection is a *law* (or *principle*) specifying which inputs (grounds) generate which outputs (groundeds).<sup>6</sup> Zero-grounding is the special case where the law simply states that given empty input we have an output. For Litland (immediate) ground is connected to explanatory inference in that  $pp$  immediately grounds  $q$  iff there is an explanatory inference from premisses all and only  $pp$  to conclusion  $q$  (cf. deRosset, 2013). One may take the connections to be *rules* of explanatory inference. On this view (immediate) zero-grounding is the special case of an explanatory inference from no (undischarged) premisses.

## 2.3 Essence

The version of the Null Account defended here goes beyond (Null-Identity) and (Null-Distinctness) by endorsing the essentialist claims that it is part of the nature of the identity relation itself that identity and distinctness facts are zero-grounded.<sup>7</sup> These essentialist claims will play an important explanatory role in § 4. To formulate the claims precisely the Null Theorist adopts the notation of (Fine, 1995b). For any item  $a$ ,  $\square_a$  is the sentential operator “it lies in nature of  $a$  that ...”; in particular,  $\square_{=}$  is the sentential operator “it lies in the nature of the identity relation = that ...”.<sup>8</sup> The Null theorist thus accepts:

$$(\text{Null}_{=}) \quad \square_{=}\forall x\forall y(x=y \rightarrow ((0 \ll x=y) \wedge \forall pp(pp \ll x=y \rightarrow 0 \equiv pp)))$$

$(\text{Null}_{\neq}) \sqsubseteq_{=} \forall x \forall y (x \neq y \rightarrow ((0 \ll x \neq y) \wedge \forall pp (pp \ll x \neq y \rightarrow 0 \equiv pp)))$

Note, however, that she accepts neither of the following:

$(\text{Null}_{=}^?) \sqsubseteq_{=} (a = a \rightarrow 0 \ll a = a)$

$(\text{Null}_{\neq}^?) \sqsubseteq_{=} (a \neq b \rightarrow 0 \ll a \neq b)$

For let  $a$  be Socrates and  $b$  the Eifel tower; it is no part of the nature of identity that these two objects are distinct. (Identity “knows nothing” of these particular objects.)

The Null Theorist would, of course, accept that it lies in the nature of identity together with the nature of  $a$  that  $a = a$  is zero-grounded. Thus:

$(\text{Null}_{=}^+) \sqsubseteq_{=,a} (a = a \rightarrow 0 \ll a = a)$

$(\text{Null}_{\neq}^+) \sqsubseteq_{=,a,b} (a \neq b \rightarrow 0 \ll a \neq b)$

I return to how the Null Theorist can accept  $(\text{Null}_{=}^+)$  and  $(\text{Null}_{\neq}^+)$  but reject  $(\text{Null}_{=}^?)$  and  $(\text{Null}_{\neq}^?)$  in § 6.14 below.

### 3 Seven Arguments from Utility and Elegance

First, the Null Account avoids the consequence that every object is fundamental. Several authors—e.g., Fine (2010b), Raven (2016), Litland (2017), deRosset (2013), Sider (2012), and Shumener (2020a, 2020b)—have all accepted that an object is fundamental if the object figures in a fundamental fact. If identity facts are fundamental then every object is fundamental. Since being zero-grounded is a way of being grounded one avoids this consequence.

Second, the Null Account has no problem accounting for identity and distinctness facts involving non-existent objects. More precisely, the account does not have a problem accounting for what *would* account for  $a = a$  even in a situation in which  $a$  does not exist. It might be a truism that all objects exist, but there is still the question about what grounds propositions involving those objects when the objects do not exist. This sets the account part from what one may call the “Existence Account” (Burgess, 2012). This account holds that  $a = a$  is grounded in  $Ea$ , and that  $a \neq b$  is grounded in  $Ea, Eb$  taken together. Since  $a = a$  would still be true when  $a$  does not exist the Existence Account has to be rejected.<sup>9</sup>

Third, the Null Account avoids the problems of what one may call the Leibnizian Account. The Leibnizian holds that the grounds for an identity  $a = b$  is the fact that every property had by  $a$  is had by  $b$  (and *vice versa*). One of  $a$ ’s properties is the property of being identical to  $a$ — $(\lambda x.a = x)$ . There are two main views about how the proposition  $(\lambda x.a = x)a$  relates to  $a = a$ . Fine (2012, pp. 67–71) proposes that  $(\lambda x.a = x)a$  is grounded in  $a = a$  (see also Dixon, 2018). Dorr (2016, pp. 52–66) rejects this view;<sup>10</sup> in its place he proposes that  $(\lambda x.a = x)a$  is in fact identical to  $a = a$ . On either view, we get that  $a = a$  (partly, mediately) grounds  $a = a$ , contradicting the Irreflexivity of ground.<sup>11</sup> Leibnizians can get around this problem by restricting their attention to qualitative properties, but this commits them to the impossibility of qualitatively indiscernible yet

distinct objects. For the reasons made famous by Black (1952) this is implausible. The Null Account avoids such problems.

Fourth—and this is the first in a number of related considerations—the account is *uniform*: all identity-facts are grounded in the same way, as are all distinctness facts. This is unlike recent accounts proposed by Shumener (2020a) and Donaldson (2017). Shumener proposes that identity and distinctness facts for *concrete* objects are grounded in the instantiation of certain quantitative relations; and she is explicit that the account is not meant to apply to abstract objects. In contrast, Donaldson (2017, p. 18)—in the course of exploring the suggestion that we should understand Hume’s Principle as a grounding claim—is led to conclude that true numerical identities like  $0 = 0$  are zero-grounded.<sup>12</sup> But then there is one story about how identities involving concrete objects are grounded and another story about identities involving abstract objects. But it is desirable to have a uniform account of how identities are grounded, an account traceable to the nature of the identity relation itself.

Fifth—building on the above point—the account is *topic neutral*. In specifying how identities are grounded one does not have to mention any particular type of object or property. It is helpful to put the point in terms of *ontological dependence*. Following Fine (1994a, 1994b, 1995a), say that *a rigidly depends on b* iff *b* figures in some proposition that holds in virtue of the nature of *a*. The sense in which the identity relation is topic neutral is that no proposition involving a particular object (or a particular type of object) holds in virtue of the nature of the identity relation.<sup>13</sup>

Sixth, for the account to be uniform and topic neutral it is important that the null ground is the *unique* ground for identity and distinctness facts. This sets the view apart from a view suggested by Fine (2016):

Thus even though we might recognize that it lies in the nature of the notion of identity that any true identity should have a null ground, we might still recognize that there was something about the nature of a particular individual that enabled the identity of that individual to itself to have some other ground. There would then be two routes, so to speak, to the identity of the individual object with itself; and our taking one route should not preclude us from taking the other.

For instance, if *c* is the set  $\{a, b\}$  one might take the identity  $c = c$  both to be zero-grounded and to be grounded in *c*’s having the same members as *c*.

That Fine’s account is not topic neutral can be brought out as follows. A plausible necessary condition for a binary relation *R* to be topic neutral is that the grounding facts involving it are invariant under permutation in the sense that for any permutation  $\pi$  of objects, if  $\Delta(c_0, c_1, \dots) \ll R a b$ , then  $\Delta(\pi(c_0), \pi(c_1), \dots) \ll R \pi(a) \pi(b)$ . Now let  $c = \{a, b\}$  and consider any permutation that maps *c* to the planet Mars while mapping *a* to *a* and *b* to *b*.<sup>14</sup>

While it might be the case that  $\forall x(x \in c \leftrightarrow x \in c)$  grounds  $c = c$  clearly  $\forall x(x \in \text{Mars} \leftrightarrow x \in \text{Mars})$  does not ground  $\text{Mars} = \text{Mars}$ : it would be preposterous to explain why Mars is identical to Mars by making the vacuous claim that all its members are its members.<sup>15</sup>

Seventh, and finally, the account does all this while hewing to the orthodoxy that ground is a relation between pluralities of propositions and propositions. This sets the Null Account apart from the view recently proposed by Wilhelm (2020). Wilhelm follows Schaffer (2009) in holding that ground is a cross-categorical relation, in particular in allowing that objects can ground propositions. Wilhelm argues that it is only by

adopting such a cross-categorical notion of ground that we can give a uniform account of the grounds for identities:  $a$  grounds  $a = a$ , and  $a, b$  grounds  $a \neq b$ . While this is obviously not the place to argue against cross-categorical grounding, any success for the Null Account undercuts this argument for cross-categorical grounding.<sup>16</sup>

## 4 The Problem of Differential Grounding

According to the Null Account identity and distinctness facts have the same—empty—ground; but when and why does the null ground do what? Put in contrastive terms: if one wants to explain why  $p$  rather than  $q$  obtains, one explanation is that  $r$  rather than  $s$  obtains. Since the Null Account holds that identity and distinctness facts have the same ground no such contrastive explanation is in the offing.<sup>17</sup>

The Null Theorist has a natural response: the reason  $0$  sometimes gives rise to an identity fact  $a = a$  and sometimes to a distinctness fact  $a \neq b$  is explained by the natures of the propositions  $a = a$  and  $a \neq b$ . It lies in the nature of the proposition  $a = a$  that  $0 \ll a = a$  and it lies in the nature of the proposition  $a \neq b$  that  $0 \ll a \neq b$ . To explain how the empty ground differentially grounds one must look to the natures of the grounded propositions.

In fact these essentialist claims follow from the essentialist claims (Null<sub>=</sub><sup>+</sup>) and (Null<sub>≠</sub><sup>+</sup>) noted above (§ 2.3). The Chaining axiom from the logic of essence (Fine, 1995b, pp. 248–249) states that if it is essential to the items on which an entity depends that  $q$ , then it is in fact essential to that entity that  $q$ . The proposition  $a \neq b$  depends on  $a, b$ , and  $=$ ; and according to (Null<sub>≠</sub><sup>+</sup>) it is essential to  $a, b, =$  that  $0 \ll a \neq b$ . Thus Chaining delivers that it is essential to the proposition  $a \neq b$  that  $0 \ll a \neq b$ , as desired.

One might worry that this solution just shifts the problem around: for, one might ask, what makes  $a = b$  a proposition where the identity relation is applied to two occurrences of the *same* object as opposed to a proposition where the identity relation is applied to two occurrences of distinct objects? Thus one might think that the proposed explanation of differential grounding is circular: it is only if  $a = b$  is a proposition where the two relata are *identical* that  $0$  grounds it; if instead it is a proposition where the two relata are *distinct*,  $0$  rather grounds its negation.<sup>18</sup>

But there is no circularity here. The problem of grounding identity facts is not to explain when a proposition  $a = b$  is of the form  $a = a$  and when it is not. The problem is just to explain for propositions of the form  $a = a$  what grounds them, and for propositions of the form  $a \neq b$ —where  $a$  and  $b$  are distinct—what grounds them.

## 5 Plenitude and Definition

The Null Account avoids the problem of Differential grounding; and the arguments in § 3 show that a relation satisfying (Null<sub>=</sub>) and (Null<sub>≠</sub>) would be an excellent candidate for playing the role of the identity relation. But why think that there is *any* relation that works in this way? The Null Theorist responds by adopting a plenitudinous view of relations.

It is tempting to think that one can give a real definition of a relation  $R$  by specifying how instantiations of the relation are grounded. (This is the idea behind Rosen’s account of real definition (Rosen, 2015)). But this is too quick: what grounds that a relation is *not* instantiated by some objects must also follow from the nature of the relation (possibly together with the nature of negation). And it is a familiar problem in the theory of ground that one cannot recover the grounds for a negated proposition from the grounds of the proposition itself. I propose to solve this problem by—as is common in work on the logic of ground—adopting a bilateralist framework. First, some terminology: if  $pp$  immediately grounds  $\neg q$  say that  $pp$  immediately *antigrunds*  $q$ . (For more on the notion of antigrund see § 6.6 below.) Here is the revised proposal: to define a binary relation  $R$  one specifies conditions  $\Phi^+, \Phi^-$  such that for any two objects  $a, b$ ,  $\Phi^+(a, b)$  (immediately, non-factively) grounds  $Rab$  and  $\Phi^-(a, b)$  (immediately, non-factively) antigrunds  $Rab$ .

On a plenitudinous view of relations *any* non-paradoxical specification of conditions  $\langle \Phi^+, \Phi^- \rangle$  for ground and antigrund really defines some relation  $R$ . This suggests the following plan for defending the Null Account. One lays down conditions  $\langle E^+, E^- \rangle$  such that the following conditions are met:

- (i) the relation  $=$  defined by  $\langle E^+, E^- \rangle$  satisfies (Null $_=$ ) and (Null $_≠$ );
- (ii)  $=$  satisfies the central features of the identity relation—in particular: Leibniz’s Law; and
- (iii) there is nothing more to the conditions  $\langle E^+, E^- \rangle$  than is required for  $=$  to satisfy the two previous conditions.

The restriction to *non-paradoxical* specifications is not idle. It is well known that plenitudinist views are paradox-prone.<sup>19</sup> For a simple paradox, consider the following grounding condition: *being such that it does not immediately ground itself*. There had better not be a proposition (i.e., zero-adic relation) corresponding to this condition. Obviously the plenitudinist owes us a story about how such paradoxes are to be avoided.<sup>20</sup> Giving such a story is a task for another occasion; here I will simply assume that one can impose some natural restrictions on which propositions exist and that this restriction will not doom the definition of identity proposed below.

I should stress that it is not just Null Theorist who can avail herself of plenitudinism. Someone might, e.g., attempt to define an identity-relation that satisfied the Existence Account. If they could establish the analogues of (i), (ii), and (iii) for their purported definition, a plenitudinous view of relations would show that there is a relation  $=_E$  satisfying the Existence Account. The Null Theorist is happy to grant the existence of a relation like  $=_E$ ; she would just insist that the arguments of § 3 show that the relation  $=_E$  is not the best candidate for being the identity relation.

## 6 Real Definition: Sketch of a Program

To carry out the definition of identity one needs an account of real definition. Unfortunately, the accounts of real definition that are available in the literature are not suitable for the task—at least as currently developed.

According to the account developed by Correia (2017) real definitions are certain (higher-order) *identities*; this renders the account of no use for defining identity itself.<sup>21</sup> The account proposed by Rosen (2015) is not subject to this problem. As presently developed, however, Rosen’s account will not do. In addition to not accounting for antiground, the account also runs into a problem with defining relations (see appendix A). Since the account is in other ways somewhat restrictive (see appendix A and appendix A) rather than trying to repair it I will rather start fresh and develop a novel account of real definition.

I believe the account is of considerable interest apart from its use in defining identity. The account reverses the usual relationship between definition and essence: instead of trying to define definition in terms of essence, the notion of *full* and *immediate* definition is taken as the basic notion and essence is defined in terms of it (§ 6.14). Moreover, the account provides a natural account of the sense in which logical operations are “formal” (§ 6.5).

I develop the account in stages, with refinements being introduced as shortcomings become apparent. Let me stress that the account I develop here is but the beginning of a theory of real definition; in particular, the approach is entirely proof-theoretic, with no model theory. I hope to give a fuller treatment elsewhere.

## 6.1 The grammar of definition: first pass

The idea is that an item—e.g., an object, a property, relation, operation—is immediately and fully defined by the role it plays. Let  $s$  be the item to be defined, and let  $\phi_0(x), \phi_1(x), \dots$  be some sentences in which the variable  $x$  may occur free. As a first pass,

$$s \frac{}{x} \parallel \phi_0, \phi_1, \dots,$$

means that  $s$  is immediately and fully defined by being such that  $\phi_0([it], \phi_1([it]), \dots$ . In  $s \frac{}{x} \parallel \phi_0, \phi_1, \dots$  the variable  $x$  is bound. It is helpful to think of  $s$  as being defined by all propositions of the form  $\phi_0(s/x), \phi_1(s/x), \dots$  being true.<sup>22</sup> (The degenerate case  $s \frac{}{x} \parallel \emptyset$  is allowed; it expresses that  $s$  does not have a definition.)

Some illustrations are in order.

Suppose the (ordinal) number 2 is fully and immediately defined by being a successor of the number 1. Letting  $S$  be the successor relation so that  $Sxy$  means that  $y$  is a successor of  $x$ , this (putative) definition of 2 is expressed as  $2 \frac{}{x} \parallel S 1x$ .<sup>23</sup>

Or to give an example of a definition of a property, suppose that *being a vixen* is, by definition, to be such that being female and a fox grounds one’s being so, and such that not being both female and a fox antigrounds one’s being so. Then the definition of being a vixen could be put as follows. (The “impure” superscript will be explained in § 6.5 below.)

(Vixen<sup>impure</sup>)

$$V \frac{}{R} \parallel \forall x(\text{Fem}(x) \wedge \text{Fox}(x) \ll R(x), \forall x(\neg(\text{Fem}(x) \wedge \text{Fox}(x)) \ll \neg R(x))$$

Or suppose the conjunction operation  $\wedge$  is defined by being that operation such that the proposition  $p \wedge q$  formed by applying  $\wedge$  to two propositions  $p, q$  is grounded in  $p, q$

(taken together), and antigrounded in either of  $\neg p$  or  $\neg q$ . This definition is expressed as follows:

( $\wedge^{\text{impure}}$ )

$$\wedge \frac{\text{R}}{\text{R}} \parallel \forall p \forall q (p, q \ll Rpq), \forall p \forall q (\neg p \ll \neg Rpq), \forall p \forall q (\neg q \ll \neg Rpq)$$

## 6.2 Factivity, Immediacy, and Individuation

It should be uncontroversial that definitions are *factive*. If 2 is defined by being a successor of 1 then 2 had better be a successor of 1. Similarly, if being a vixen is by definition to be such that one's being so is grounded in one's being a fox and one's being female, then for any given vixen  $a$ , it had better be that  $a$ 's being a vixen is grounded in  $a$ 's being a vixen and  $a$ 's being a fox.

This yields the following principle:

$$\frac{a \frac{\text{R}}{\text{R}} \parallel \phi_0, \phi_1, \dots}{\phi_i(a/x)} \text{Factivity}^-$$

What Factivity<sup>-</sup> says is that if  $a$  is, by definition, such that  $\phi_0([it]), \phi_1([it]), \dots$  then, for any  $a$ , and any  $i = 0, 1, 2, \dots$  we can conclude that  $\phi_i(a)$ .

That one is dealing with *immediate* definition is captured by the absence rather than the presence of a principle. If 2 is by definition the successor of 1, it stands to reason that 1 is by definition the successor of 0. On a mediate notion of definition it would then be part of the definition of 2 that 1 is the successor of 0. But on an immediate conception of definition this is not so.<sup>24</sup>

I take the fullness of definition to mean that definitions are *individuating*: no two items have the same real definition. This yields the following principle:

$$\frac{a \frac{\text{R}}{\text{R}} \parallel \phi_0, \phi_1 \dots \quad b \frac{\text{R}}{\text{R}} \parallel \phi_0, \phi_1, \dots \quad \psi}{\psi(b/a)} \text{Individuation}$$

Here  $\psi(b/a)$  results from  $\psi$  by substituting  $b$  for some occurrences of  $a$  in  $\psi$ .

Note that what Individuation requires is just that if two items have the same *definition* then they are the same item. Individuation allows an item  $s$  to be defined by  $\phi$  while a distinct item  $t$  is also  $\phi$ ; this is permitted as long as  $t$  is not *defined* by being  $\phi$ . In particular, Individuation does not rule out that there is a relation  $R$ , differing from identity, such that for all  $x, y$  with  $x, y$  distinct,  $Rxx$  is immediately zero-grounded and  $Rxy$  is immediately zero-antigrounded.<sup>25</sup>

That definitions individuate what they define is not uncontroversial. Yablo and Rosen (2020, p. 129) present the following potential counterexample.

Consider the positive and negative square roots of  $-1$ ,  $i$  and  $-i$ . It may be that the only thing to be said about the natures of these items is that each is defined by the condition  $x^2 + 1 = 0$  and yet the theory requires that these two items are distinct. And so we should be open to the possibility that there might be two *objects* with the same essence of real definition.

For reasons given in § 6.3 below I am not convinced by this example; for present purposes, however, there is no need to take a stand on whether there are distinct *objects* that have the same definition. The Null Theorist only needs to apply Individuation to the definitions of relations and propositions. And, as Rosen and Yablo go on to note, objects present the only plausible case where distinct items share a definition.

### 6.3 Defining identity and the non-circularity of definition

How can the Null Theorist use this account of real definition to define the identity relation? What she wants to say is that identity is that relation  $R$  such that  $Raa$  is zero-grounded for all  $a$ ; and such that  $Rab$  is zero-antigrounded for all  $a, b$  such that  $a, b$  are distinct. Formally,

(Def<sub>=</sub><sup>circ</sup>)

$$=_{\mathcal{R}} \parallel \forall x(0 \ll Rxx), \forall x \forall y(x \neq y \rightarrow 0 \ll \neg Rxy)$$

The problem is that this definition is *circular*: the relation to be defined itself occurs in the definiens—in  $\forall x \forall y(x \neq y \rightarrow 0 \ll \neg Rxy)$ . But real definitions cannot be circular—or so orthodoxy has it.

In this paper I simply assume this orthodoxy. Even those who reject a general non-circularity requirement should allow that there are some non-circular definitions; and it would thus be of interest that one can give a non-circular definition of identity in particular. But while I will not offer an argument in favor of orthodoxy, I will argue that the non-circularity requirement is less restrictive than it might initially appear.<sup>26</sup>

First, the ban on circular definitions is not a ban on *reflexive* definitions. Narcissus is—one may suppose—partly defined by being a self-lover—that is, letting  $n$  be a name for Narcissus, the definition of Narcissus may be of the form  $n \frac{-}{x} \parallel Lxx, \Gamma$ . There is no circularity here; what would be circular would be a definition of the form  $n \frac{-}{x} \parallel Lxn, \Gamma$ .<sup>27</sup>

Secondly, a ban on circular definitions is not a ban on *simultaneous* definitions. Allowing simultaneous definitions undercuts most alleged examples of symmetric definition. As a case of symmetric definition one might offer the square roots of  $-1$ . One might hold that  $i$  is defined as being that root of  $-1$  that is the additive inverse of  $-i$ ; and that  $-i$  is defined as being that root of  $-1$  that is the additive inverse of  $i$ . A ban on circular definitions rules out this pair of definitions; but it does not rule out simultaneously defining  $i, -i$  as additively inverse roots of  $-1$ . Letting  $S$  be the *square root of* relation, such a definition can be expressed as follows:  $i, -i \frac{-}{x, y} \parallel Sx(-1), Sy(-1), x = -y$ .<sup>28</sup> The possibility of simultaneous definition also undercuts Rosen and Yablo's counterexample to Individuation discussed above (§ 6.2).

Thirdly, Rosen (2015, p. 196) objects that banning circular definitions rules out real definition by recursion. The following, e.g., sounds like a plausible definition of the property of being a natural number:

(N<sub>circ</sub>) To be a natural number is to be either zero or the successor of a natural number.

On the face of it this definition looks circular; however, this recursive definition can be expressed non-circularly as follows (with  $\mathbb{N}$  being the property of being a natural

number, and  $S$  being the successor relation):<sup>29</sup>

$$(NR) \quad \mathbb{N} \frac{-}{X} \parallel X0, \forall x \forall y (Xx \wedge Sxy \rightarrow Xy)$$

Returning now to the definition of identity, one might try to get rid of the circularity by reflexivizing ( $\text{Def}_{=}^{\text{circ}}$ ). This produces:

$$(\text{Def}_{=}^{\text{refl}}) \quad = \frac{-}{R} \parallel \forall x (0 \ll Rxx), \forall x \forall y (\neg Rxy \rightarrow 0 \ll \neg Rxy)$$

Unfortunately, while ( $\text{Def}_{=}^{\text{refl}}$ ) is non-circular, it is also completely uninformative: as far as ( $\text{Def}_{=}^{\text{refl}}$ ) is concerned the extension of the identity relation could be any reflexive collection of pairs. Being told that  $=$  is a relation such that *when* it does not hold its not holding is zero-grounded does not help at all with figuring out when it does not hold.

## 6.4 The Wittgensteinian Variable Convention

The key to an informative and non-circular definition of identity is adopting the Wittgensteinian Variable Convention (the WVC for short); this convention requires that distinct bound variables take distinct values. Wehmeier (2012, 2014) has recently used the WVC to defend identity-eliminativism: the thesis that there is no identity relation. The Null Theorist is a reductionist not an eliminativist; she uses languages governed by the WVC to define, not to eliminate, identity. Apart from the four points in the following note I will therefore have nothing further to say about identity-eliminativism.<sup>30</sup>

Nobody should deny that it is possible to speak a language where bound variables work according to the WVC. What is contentious is whether one can rely on such languages in giving a non-circular definition of identity: what the convention says, after all, is that *distinct* bound variables have to take *distinct* values.

First, it is worth pointing out that since the concern is with the metaphysics of the identity relation and not with what it takes to understand a sign for that relation, it is perhaps no problem that in introducing languages governed by the WVC one relies on one's grasp of identity. Such a dependence on identity in the order of understanding need not imply a dependence on identity in the ontological order.

Secondly, in connection with a related objection Wehmeier (2012, pp. 764–765) observes that one only gets this dependence on identity if one attempts to explain the WVC using a language not governed by it. But—speaking in a meta-language governed by the WVC—the convention just comes down to the—correct—claim that if  $u, v$  are two variables then there are  $x, y$  such that  $x$  is the value of  $u$  and  $y$  is the value of  $v$ .

Thirdly, and relatedly, one can learn to speak a language governed by the WVC by the direct method, by learning the appropriate rules of inference. Wehmeier (2004, 2008) has developed sequent calculi and tableaux systems appropriate for doing quantification theory in a language governed by the WVC. (The rule of Factivity in § 6.7 below shows how the WVC can be accommodated in the present account of definition.) While more can be said both on the technical and the philosophical side, from now on I just assume the WVC.

With the WVC in place a natural first pass at a definition is the following:

$$(\text{Def}_{=}^{\text{impure}}) \quad = \frac{-}{R} \parallel \forall x (0 \ll Rxx), \forall x \forall y (0 \ll \neg Rxy)$$

Unlike  $(\text{Def}_{=}^{\text{refl}})$   $(\text{Def}_{=}^{\text{impure}})$  is not uninformative. The problem with the former was that it did not settle when the relation does not hold, but only settled that *when* the relation does not hold its not holding is zero-grounded. However, since  $(\text{Def}_{=}^{\text{impure}})$  is stated in a language governed by the WVC  $\forall x\forall y(0 \ll \neg Rxy)$  does entail that  $R$  does not hold between any two distinct objects  $a, b$ .

Unfortunately, one cannot leave matters with  $(\text{Def}_{=}^{\text{impure}})$ . First, definitions of the form  $(\text{Def}_{=}^{\text{impure}})$  are incompatible with an attractive view about the formality of the logical operations (§ 6.5). Second,  $(\text{Def}_{=}^{\text{impure}})$  is incomplete (§ 6.8).

## 6.5 Logical Purity

Many philosophers are attracted to the view that the logical operations are in some sense *formal*; they are not “about” or do not “concern” any particular items. A tempting way of cashing this out is by requiring that the only item figuring in the definitions of the logical operations is the grounding relation. This is along the right lines but it is too strict in that it rules out that logical operations can be defined in terms of other logical operations. One should only require that definitions of logical operations ultimately bottom out in definitions of operations in which the only item that figures is the grounding relation. (It might also be too strict in that *negation* might be the special case of a logical operation that is not definable—see below § 6.6.)

Some terminology will be useful. Say that a definition is *immediately pure* if the only item figuring in it is the grounding relation;<sup>31</sup> say that a definition is *immediately  $\neg$ -pure* if the only items figuring in it are the grounding relation and negation. An item is *immediately pure ( $\neg$ -pure)* if its definition is immediately pure ( $\neg$ -pure). An item is *pure ( $\neg$ -pure)* if it is either immediately pure ( $\neg$ -pure) or has a definition from pure ( $\neg$ -pure) items.

Logical ( $\neg$ -)Purity is the thesis that all logical operations are ( $\neg$ -)pure. I propose that Logical ( $\neg$ -)Purity captures the idea that the logical operations are formal.<sup>32</sup>

The definition of conjunction above— $(\wedge^{\text{impure}})$ —and the initial definition of identity— $(\text{Def}_{=}^{\text{impure}})$ —are not immediately pure or even immediately  $\neg$ -pure. (The universal quantifier figures in both of them.) Conjunction and identity could nevertheless be ( $\neg$ -)pure provided that the universal quantifier is ( $\neg$ -)pure.

Unfortunately, on the present account it is impossible to give a non-circular definition of the universal quantifier, let alone a  $\neg$ -pure one. How universally quantified propositions are grounded is, of course, a vexed issue. But almost all treatments agree that for any  $P$  and  $a$ ,  $\neg Pa$  is part of an antigrind for  $\forall xPx$ . A definition of  $\forall$  will then take the form:

$$\forall \frac{\circ}{\circ} \parallel \left\{ \begin{array}{l} \vdots \\ \forall P \forall x (\Gamma, \neg Px \ll \neg O(P)) \end{array} \right.$$

This is patently circular.

Below I show how to modify the account of real definition to allow for Logical Purity or at least Logical  $\neg$ -Purity. Using that purity-friendly account of real definition one can define an immediately pure universal quantifier and an immediately pure identity relation. It is the immediately pure identity relation, I submit, that has the best claim to

being *the* identity relation. (Since the paper is about identity and not quantification, I relegate the definition of the universal quantifier to appendix A.)

Before proceeding, the possible special status of negation should be discussed. The reader who is not interested in these issues should feel free to skip ahead to § 6.7.

## 6.6 Negation and Purity (optional)

If one defines antiground in terms of negation and ground, then Logical Purity is out of the question and Logical  $\neg$ -Purity is the best one can hope for. But if negation is a logical operation that has no definition, what stops someone from saying that, say, the universal quantifier has no definition? If the universal quantifier has no definition, the best one can hope for is  $(\neg, \forall)$ -purity, in which case—modulo the issues about symmetry in § 6.8—why not rest with  $(\text{Def}_{=}^{\text{impure}})$ ?

One might respond by taking antiground as primitive and defining negation in terms of it. Here is one possible definition: negation is that operation  $\neg$  such that for all propositions  $p$ ,  $\neg p$  is immediately grounded in the antigrounds for  $p$ ; and  $\neg p$  is immediately antigrounded in  $p$ . One would then allow the antigrounding relation to figure in the definition of an immediately pure idem. A tentative argument in favor of this line is that it defines negation in such a way that the orthodox grounding principles for negation come out valid.

While I am sympathetic to this line, fully defending it lies well beyond the scope of this paper; let me therefore present a different reason for insisting on Logical  $\neg$ -purity. Suppose one adopts a bipolar view of propositions where a proposition and its negation are just two perspectives on the same item. On such a view, negation is not an operation that adds complexity, rather it merely “toggles” between the two perspectives on the same proposition. This view thus accepts the identification of  $\neg\neg p$  with  $p$ —what Dorr (2016) calls Involution. What is special about negation is that, unlike the other logical operations, negation is involved in the very nature of propositions in general.<sup>33</sup>

## 6.7 The grammar of definition: binding definitions

To make room for Logical Purity I propose to allow the definitional operation  $\dashv\vdash$  itself to generalize and bind variables.<sup>34</sup> The general format of definition becomes:

$$s \frac{x_0, x_1, \dots}{y} \dashv\vdash \phi_0, \phi_1, \dots$$

here  $s$  is of some type,  $y$  is a variable of the same type, and  $x_0, x_1, \dots$  are some variables that occur in the sentences  $\phi_0, \phi_1, \dots$ . One may read this as follows: “ $s$  is, by definition, such that for any  $x_0, x_1, x_2, \dots$  one has  $\phi_0([\text{it}], x_0, x_1, \dots), \phi_1([\text{it}], x_0, x_1, \dots), \dots$ ”.

Here is a definition of conjunction:

( $\wedge$ )

$$\wedge \frac{p, q}{R} \dashv\vdash (p, q \ll Rpq), (p, p \ll Rpp), (\neg p \ll \neg Rpp), (\neg p \ll \neg Rpq), (\neg q \ll \neg Rpq)$$

( $\wedge$ ) may be read as follows. “Conjunction is that operation such that propositions formed by applying it to a proposition and a proposition are grounded in those propositions,

propositions formed by applying it to a proposition and that proposition is grounded in that proposition, ... ” Note that ( $\wedge$ ) is immediately  $\neg$ -pure.

Or to illustrate the format with a definition of *vixen*:

(Vixen)

$$\text{Vixen } \frac{x}{R} \Big\| \text{Fem}(x) \wedge \text{Fox}(x) \ll Rx, \neg(\text{Fem}(x) \wedge \text{Fox}(x)) \ll \neg Rx$$

One might read the above definition as follows. “To be a vixen is, by definition, to be such that one’s being so is grounded in one’s being female and one’s being a fox; and one’s not being so is grounded in its being the case that one is not both a fox and a female.” (Vixen) is not immediately  $\neg$ -pure; and since it highly unlikely that *being a fox* is  $\neg$ -pure, it is not  $\neg$ -pure either.

Now that definitions are allowed to bind and generalize variables the factivity principle must be amended.

$$\frac{s \frac{x_0, x_1, x_2, \dots}{y} \Big\| \phi_0, \phi_1, \dots}{\phi_i(s/y, z_0/x_0, z_1/x_1, z_2/x_2, \dots)} \text{Factivity}$$

here  $\phi_i(s/y, z_0/x_0, z_1/x_1, z_2/x_2, \dots)$  means the sentence that results from  $\phi_i$  by replacing the variable  $y$  with  $s$  and the variables  $x_0, x_1, \dots$  with any pairwise distinct variables  $z_0, z_1, \dots$ . The requirement that the variables  $z_0, z_1, \dots$  be pairwise distinct captures the WVC. One may read this rule as follows. If  $s$  is, by definition, such that for any  $x_0, x_1, x_2, \dots$  one has  $\phi_0([it], x_0, x_1, \dots), \phi_1([it], x_0, x_1, \dots), \dots$  and  $z_0, z_1, \dots$  are any things, then, for each  $i = 0, 1, 2, \dots$ , one may conclude  $\phi_i(s, z_0, z_1, \dots)$ .

Using Factivity one can derive the relevant instances of how facts about vixens are (anti)grounded; and more interestingly, one can derive any facts about how conjunctions are (anti)grounded.

## 6.8 Defining identity: first pass

Given what has been said so far the following looks like a natural definition of identity:<sup>35</sup>

(Def $_{=}$ )

$$= \frac{x, y}{R} \Big\| \begin{cases} 0 \ll Rxx \\ 0 \ll \neg Rxy \end{cases}$$

Or in words: “= is, by definition, that  $R$  such that  $R$ ’s holding between a thing and that thing is zero-grounded; and such that the holding of  $R$  between a thing and another thing is zero-antigrounded.”

By applying Factivity to (Def $_{=}$ ) one can derive  $0 \ll x = x$  for each  $x$ , and  $0 \ll \neg x = y$  for each  $x, y$ . (Remember the WVC.) Moreover, this definition of = does not contain the universal quantifier and so is  $\neg$ -pure. Unfortunately, (Def $_{=}$ ) runs into two problems.

First, (Def $_{=}$ ) does not settle whether the identity relation is strictly symmetric—where a relation  $R$  is *strictly symmetric* if the proposition  $Rab$  is identical to the proposition  $Rba$ .<sup>36</sup> Clearly, the identity relation is strictly symmetric;<sup>37</sup> but how can it follow

from a non-circular definition of = that the results of applying = to two objects in different orders yield *identical* propositions?<sup>38</sup>

Second, (Def<sub>=</sub>) leaves it open that the proposition  $a = a$  (or  $a \neq b$ ) has non-null grounds in addition to the empty ground, contrary to the Null Account.

Both these problems can be overcome.

## 6.9 Defining each identity proposition

Three propositions result from applying = to some distinct objects  $x, y$ :  $x = x$ ,  $y = y$  and  $x = y$ , with the latter being identical to  $y = x$ . Taking for now the identity relation = for granted, how can these propositions be defined?

Let  $\mathcal{A}$  stand for the application relation, that is, the relation such that if  $R$  is an  $n$ -place relation and  $\bar{x}$  is an  $n$ -tuple of objects then  $\mathcal{A}(R\bar{x}p)$  means that  $p$  is a result of applying  $R$  to  $\bar{x}$  (in that order).<sup>39</sup> The Null Theorist holds that if  $p$  is such that  $\mathcal{A}(=xxp)$  then  $p$  is defined by  $p$ 's being a result of applying = to  $x, x$  and by  $p$ 's being zero-grounded. Similarly, if  $p$  results from applying = to  $x, y$  then  $p$  is defined by its being a result of applying = to  $x, y$ , by its being a result of applying = to  $y, x$ , and by its being zero-antigrounded.

Formalizing these claims one ends up with the following:

$$\mathcal{A}(=xxp) \rightarrow p \frac{q}{\parallel} \left\{ \begin{array}{l} \mathcal{A}(=xxq) \\ 0 \ll q \end{array} \right.$$

and

$$\mathcal{A}(=xyp) \rightarrow p \frac{q}{\parallel} \left\{ \begin{array}{l} \mathcal{A}(=xyq) \\ \mathcal{A}(=yxq) \\ 0 \ll q \end{array} \right.$$

This suffices to show that the identity relation is strictly symmetric and satisfies Leibniz's Law.

## 6.10 Leibniz's Law and Strict Symmetry

First one must define a notion of consequence. Say that a plurality  $pp$  of propositions is *closed* iff (i)  $pp$  contains the definition of every item figuring in any proposition in  $p$ ; (ii)  $pp$  is closed under Factivity, Individuation (and the principles of  $\ll$ -Reflection and  $\mathcal{A}$ -Reflection to be introduced below); (iii)  $pp$  contains every proposition that is grounded in some propositions amongst the  $pp$ ; and (iv) any proposition amongst the  $pp$  that has some full grounds, has some full grounds amongst the  $pp$ .  $pp$  is *coherent* closed if it is closed and in addition  $pp$  does not contain both a proposition and its negation. Say that  $q$  is a *consequence* of  $pp$  if  $q$  is in every closed plurality containing  $pp$ ;  $q$  is a *coherent consequence* of  $pp$  iff  $q$  is in every closed coherent plurality of propositions containing  $pp$ .

Fix an assignment  $\alpha$  of values to  $x, y$ . Leibniz's Law holds in the sense that the proposition expressed by  $\phi(y/x)$  under the assignment  $\alpha$  is a coherent consequence of the propositions expressed by  $x = y$  and  $\phi(x)$  under  $\alpha$ . Either  $x$  and  $y$  are the same variable

or not. In the first case, the proposition expressed by  $\phi(y/x)$  is the same proposition as the one expressed by  $\phi(x)$ . So suppose that  $x$  and  $y$  are distinct variables. Then, by the definition of identity, the proposition expressed by  $\neg x = y$  under the assignment  $\alpha$  is zero-grounded and so there is no coherent collection of propositions that contains the proposition expressed by  $x = y$  under the assignment  $\alpha$ . Thus every proposition is a coherent consequence of the proposition expressed by  $x = y$  under  $\alpha$ ; in particular, the proposition expressed by  $\phi(y/x)$  under  $\alpha$  is such a consequence.<sup>40</sup>

Strict symmetry of the identity relation is established as follows. Let  $r_0$  be such that  $\mathcal{A}(= xy r_0)$  and  $r_1$  be such that  $\mathcal{A}(= yx r_1)$ . To show that  $r_0 \equiv r_1$  reason as follows. By using Factivity on the definition of the propositional identity relation  $\equiv$  one gets  $0 \ll r_0 \equiv r_0$ . Thus  $r_0 \equiv r_0$  is valid. The following application of Individuation shows that  $r_0 \equiv r_1$ .

$$\frac{r_0 \frac{\parallel}{q} \left\{ \begin{array}{l} \mathcal{A}(= xyq) \\ \mathcal{A}(= yxq) \\ 0 \ll q \end{array} \right. \quad r_1 \frac{\parallel}{q} \left\{ \begin{array}{l} \mathcal{A}(= xyq) \\ \mathcal{A}(= yxq) \\ 0 \ll q \end{array} \right. \quad r_0 \equiv r_0}{r_0 \equiv r_1} \text{Individuation}$$

To show that  $x = x$  (and  $x \neq y$ ) are uniquely zero-grounded further principles are needed.

## 6.11 Definitional Reflection

What has been said so far leaves it open that  $x = x$  has *other* grounds and that  $x = x$  might be the result of applying  $=$  to other objects, or that  $x = x$  is even the result of applying a different relation to some different objects. Clearly, this is not intended: when one defines a proposition  $p$  in terms of how it is (anti)grounded, and in terms of how it results from applying a relation  $R$  to some relata  $x, y$  it should be a consequence of the definition that what is specified in the definition are the *only* (immediate, non-factive) grounds for the proposition, and that the proposition can only result from applying the relation  $R$  to the relata  $x, y$ .

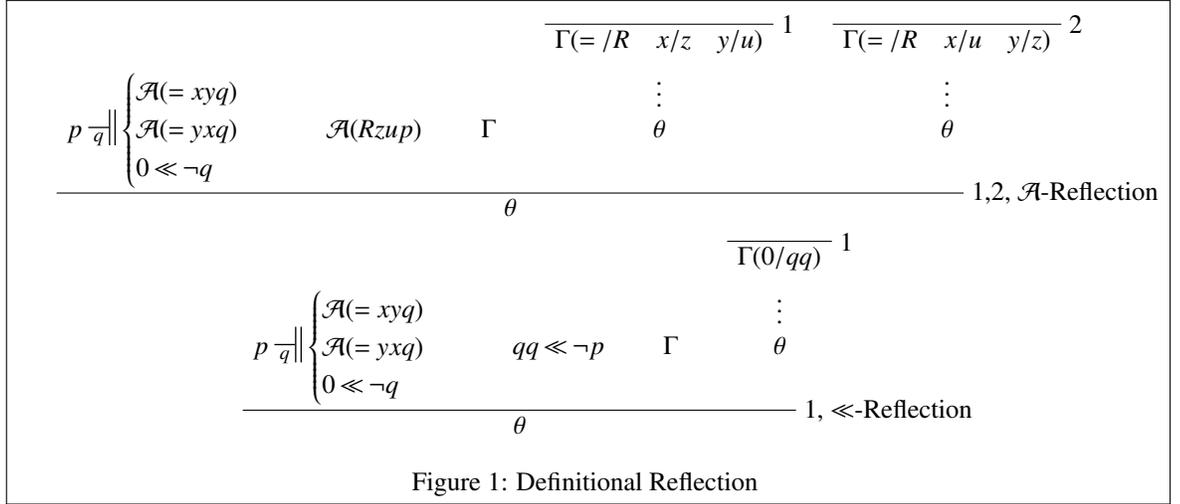
Consider a particular identity-proposition  $p$  with its definition:

$$p \frac{\parallel}{q} \left\{ \begin{array}{l} \mathcal{A}(= xyq) \\ \mathcal{A}(= yxq) \\ 0 \ll \neg q \end{array} \right.$$

It must follow from this definition that if  $\mathcal{A}(Rzup)$  then  $R$  has to be identical to  $=$  and  $z, u$  have to be identical to either  $x, y$  or  $y, x$ ; similarly, if  $qq \ll \neg p$ , it must follow that  $qq$  is identical to  $0$ .

It is possible to state a general principle governing real definition—Definitional Reflection<sup>41</sup>—from which this follows. For present purposes, a fully general statement of this principle is too involved; I therefore only state two special cases that pertain to identity and distinctness in particular.

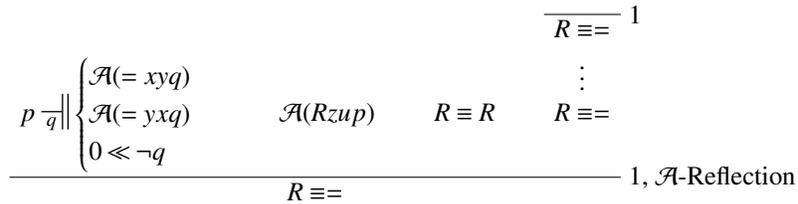
Since the task is to define identity itself, in stating these principles one must avoid the use of identity. Fortunately, this can be done. The cash value of  $R$ 's being identical



to = is that one can replace  $R$  in any proposition in which it occurs with =; the cash value of  $qq$ 's being identical with 0 is that one can replace  $qq$  in any proposition in which they occur with 0. The situation is more complicated with  $z, u$  since one does not know whether they are identical to  $x, y$  or rather to  $y, x$ . However, if  $z, u$  occurs in some proposition  $pp$  then either they are identical to  $x, y$  or  $y, x$ ; thus, whatever follows both from the result of substituting  $x, y$  for  $z, u$  in those propositions  $pp$  as well as from the result of substituting  $y, x$  for  $z, u$  in  $pp$  simply follows from the propositions  $pp$ .

Figure 1 contains the two rules of  $\ll$ -Reflection and  $\mathcal{A}$ -reflection. Here is how the rule of  $\mathcal{A}$ -reflection is to be read. Suppose one has established some sentences  $\Gamma$ . And suppose one has established  $\mathcal{A}(Rzup)$ . Write  $\Gamma(= /R \ x/z \ y/u)$  to mean any collection of sentences that results from  $\Gamma$  by replacing some (possibly zero) occurrences of  $R$  with =, some occurrences of  $z$  with  $x$ , and some occurrences of  $u$  with  $y$ . Now suppose that  $\theta$  follows both from  $\Gamma(= /R \ x/z \ y/u)$  and  $\Gamma(= /R \ y/z \ x/u)$ . Then  $\theta$  follows just from  $\Gamma$  and the definition of  $p$ . (The rule of  $\ll$ -reflection is read similarly.)

It might be instructive to see how, from the assumptions  $\mathcal{A}(Rzup)$  and  $qq \ll \neg p$ , these rules allow us to derive that  $R \equiv =$ ; that  $x = z \wedge y = u \vee x = u \wedge y = z$ ; and that  $0 \equiv qq$ .



Under assumption 1 one substitutes = for *one* of the occurrences of  $R$ .

$$\frac{p \dashv\!\! \dashv \left\{ \begin{array}{l} \mathcal{A}(= xyq) \\ \mathcal{A}(= yxq) \\ 0 \ll \neg q \end{array} \right. \quad \mathcal{A}(Rzup) \quad x = x \wedge y = y \quad \frac{x = z \wedge y = u}{x = z \wedge y = u \vee x = u \wedge y = z} \vee\text{-I} \quad \frac{x = u \wedge y = z}{x = z \wedge y = u \vee x = u \wedge y = z} \vee\text{-I}}{x = z \wedge y = u \vee x = u \wedge y = z} \text{1, } \mathcal{A}\text{-Reflection}$$

Under assumption 1 one substitutes  $z$  for one of the occurrences of  $x$  and  $u$  for one of the occurrences of  $y$ ; under assumption 2 one does the reverse.

$$\frac{p \dashv\!\! \dashv \left\{ \begin{array}{l} \mathcal{A}(= xyq) \\ \mathcal{A}(= yxq) \\ 0 \ll \neg q \end{array} \right. \quad qq \ll \neg p \quad qq \equiv qq \quad \frac{}{qq \equiv 0} \text{1}}{qq \equiv 0} \text{1, } \ll\text{-Reflection}$$

The Reflection rules for the case where  $p$  is of the form  $x = x$  are analogous and are left to the reader.

## 6.12 The grammar of definition: restricting definitions

Finally, one can define the identity relation itself. The thought is that the identity relation  $=$  is that relation  $R$  such that propositions of the form  $Rxx$  and  $Rxy$  are *defined* as above. The natural first attempt is:

$$= \frac{x,y,p}{R} \dashv\!\! \dashv \left\{ \begin{array}{l} \mathcal{A}(Rxxp) \rightarrow p \dashv\!\! \dashv \left\{ \begin{array}{l} \mathcal{A}Rxxq \\ 0 \ll q \end{array} \right. \\ \mathcal{A}(Rxy p) \rightarrow p \dashv\!\! \dashv \left\{ \begin{array}{l} \mathcal{A}Rxyq \\ \mathcal{A}Ryxq \\ 0 \ll \neg q \end{array} \right. \end{array} \right.$$

But this definition is impure on account of the conditional. To avoid this problem one allows the operation  $\dashv\!\! \dashv$  to restrict (or condition) the range of the variables it generalizes.

$$s \frac{x_0 : \psi_0, x_1 : \psi_1, \dots}{x} \dashv\!\! \dashv \left\| \phi_0, \phi_1, \dots$$

means that  $s$  is, by definition, such that if  $x_0$  is any  $\psi_0$ ,  $x_1$  is any  $\psi_1$ , ... one has  $\phi_0([it], x_0, x_1, \dots), \phi_1([it], x_0, x_1, \dots), \dots$ . Note that a variable  $x_i$  restricted by a condition  $\psi_i$  can occur in a later restricting condition  $\psi_j$ .

Instances of Factivity have to be restricted to those entities that meet the restricting conditions. A fully general statement lies beyond the scope of this paper, but for the purposes of defining identity the following will do:

$$\frac{s \frac{y_0 : \psi_0, y_1 : \psi_1, \dots}{x} \dashv\!\! \dashv \left\| \phi_0, \phi_1, \dots \quad \psi_0(z_0/y_0) \quad \psi_1(z_1/y_1) \quad \dots}{\phi_i(s/x, z_0/y_0, z_1/y_1, \dots)} \text{Factivity}^+$$

(Here  $z_0, z_1, \dots$  are pairwise distinct variables.)

Here is what this rule says. Suppose  $s$ , by definition, is such that if  $y_0$  is any  $\psi_0$ ,  $y_1$  is any  $\psi_1$ , ... then one has  $\phi_0([it], y_0, y_1, \dots), \phi_1([it], y_0, y_1, \dots), \dots$ . And suppose one has established that  $z_0$  is  $\psi_0$ ,  $z_1$  is  $\psi_1$ , and so on. Then, for each  $i = 0, 1, \dots$ , one may conclude  $\phi_i(s, z_0, z_1, \dots)$ .

Here is the final definition of identity.

(Def<sub>=</sub>)

$$= \frac{x, y, (p: \mathcal{A}(Rxxp)), (q: \mathcal{A}(Rxyq))}{R} \parallel \left\{ \begin{array}{l} p \text{ } \overline{r} \parallel \left\{ \begin{array}{l} \mathcal{A}(Rxxr) \\ 0 \ll r \end{array} \right. \\ q \text{ } \overline{r} \parallel \left\{ \begin{array}{l} \mathcal{A}(Rxyr) \\ \mathcal{A}(Ryxr) \\ 0 \ll \neg r \end{array} \right. \end{array} \right.$$

One may attempt the following pronunciation: identity is, by definition, that relation  $R$  such that for any  $x, y$ , any proposition  $p$  formed by applying  $R$  to  $x, x$ , and any proposition  $q$  formed by applying  $R$  to  $x, y$ : (i)  $p$  is, by definition, a result of applying  $R$  to  $x, x$  and is in addition zero-grounded; and (ii)  $q$  is, by definition, a result of applying  $R$  to  $x, y$ , also a result of applying  $R$  to  $y, x$ , and is also zero-antigrounded.

### 6.13 Definitions within definitions

Note that a claim about the definition of propositions formed by applying identity is *embedded* in the definition of the identity relation itself. This is crucial to ensuring that the null ground is the *only* ground for identity and distinctness facts.<sup>42</sup> If one had simply defined a relation  $I$  by

(Def<sub>=</sub>)

$$I \frac{x, y, (p: \mathcal{A}(Rxxp)), (q: \mathcal{A}(Rxyq))}{R} \parallel \left\{ \begin{array}{l} \mathcal{A}(Rxxp) \\ 0 \ll p \\ \mathcal{A}(Rxyq) \\ \mathcal{A}(Ryxq) \\ 0 \ll \neg q \end{array} \right.$$

it would follow that all instances  $Ixx$  are zero-grounded and all instances  $Ixy$  are such that  $\neg Ixy$  are zero-grounded. But in defining this relation  $I$  one does not yet have an account of *all* the grounds for propositions of the form  $Ixx$  or  $Ixy$ . For  $Ixx$  might have *other* grounds depending on what the object  $x$  is. This might be one way of accommodating Fine's idea (see § 3 above) that identity-facts may be grounded in several ways.

I have no objections to there being relations like  $I$ ; I would only insist that there is a relation that behaves the way I have claimed identity does—and would not that relation be the most identity-like relation of all?<sup>43</sup>

## 6.14 Essence and definition

Having given the real definition of identity, it is time to bring all this machinery to bear on the original essentialist formulation of the Null Account, that is:

$$(\text{Null}_=) \quad \Box = \forall x \forall y (x = y \rightarrow ((0 \ll x = y) \wedge \forall pp (pp \ll x = y \rightarrow 0 \equiv pp)))$$

$$(\text{Null}_\neq) \quad \Box = \forall x \forall y (x \neq y \rightarrow ((0 \ll x \neq y) \wedge \forall pp (pp \ll x \neq y \rightarrow 0 \equiv pp)))$$

The natural thought is that one can derive  $(\text{Null}_=)$  and  $(\text{Null}_\neq)$  by exploiting the idea that what is essential to some items is what follows from their definitions. Naïvely, one would say that  $p$  follows from definitions of some items  $\mathcal{S}$  iff  $p$  is a (coherent) consequence of the definitions of the items  $\mathcal{S}$ . However, this will massively overgenerate. Given Factivity<sup>+</sup>, the definition of  $=$  given above has every true identity and distinctness claim as a consequence. But as noted in § 2 the defender of the Null Account does not believe that it is essential to identity that Mars is identical to Mars.

There is a natural solution. For the purpose of deriving essentialist claims one further constrains the notion of consequence. Say that  $q$  is an *essential (coherent) consequence* of  $pp$  if  $q$  is a (coherent) consequence of  $pp$  and every item that figures in  $q$  figures in a definition of an item in  $pp$ .

Since the definition of identity contains no unbound singular terms it is not an essential consequence of the definition of identity that  $0 \ll a = a$ , for any particular  $a$ . The Null Theorist thus avoids  $(\text{Null}_=)$ . However, for each  $a$ , it is an essential consequence of the definition of identity together with the definition of  $a$  that  $0 \ll a = a$ , and so the Null Theorist can establish  $(\text{Null}_=)$ .

What about  $(\text{Null}_\neq)$ ? One should not expect

$$(1) \quad \forall x \forall y (x = y \rightarrow ((0 \ll x = y) \wedge \forall pp (pp \ll x = y \rightarrow 0 \equiv pp)))$$

to be an essential consequence of the definition of identity—if it did, identity would depend on the universal quantifier, conjunction and the conditional. What one should expect is that  $(\text{Null}_=)$  essentially follows from the definitions of identity, the universal quantifier, conjunction, and the conditional. To make this out in full one needs definitions of the universal quantifier, conjunction, and the conditional. This is not the place to do this—though see appendix A—but one can be quite confident that those definitions will validate the rules of Conjunction Introduction<sup>44</sup>, Conditional Proof<sup>45</sup>, and Universal Generalization.<sup>46</sup>

One can then reason as follows.<sup>47</sup> Let  $x$  be arbitrary. One first shows that  $(0 \ll x = x) \wedge \forall pp (0 \ll x = x \leftrightarrow pp \equiv 0)$  is coherently valid. That  $0 \ll x = x$  follows by Factivity from the definition of  $=$ . If  $pp \equiv 0$ , then it follows by Leibniz's Law (for  $\equiv$ ) that  $pp \ll x = x$ . So suppose  $pp \ll x = x$ . By the definition of  $x = x$  and  $\ll$ -Reflection we get that  $pp \equiv 0$ . By Conditional Proof we get  $pp \ll x = x \leftrightarrow 0 \equiv pp$ . And by Universal Generalization we get  $\forall pp (pp \ll x = x \leftrightarrow 0 \equiv pp)$ . By Conjunction Introduction  $(0 \ll x = x) \wedge \forall pp (0 \ll x = x \leftrightarrow pp \equiv 0)$ . By Leibniz's Law one then gets  $x = y \rightarrow (0 \ll x = y) \wedge \forall pp (0 \ll x = y \leftrightarrow pp \equiv 0)$ . Finally,  $\forall x \forall y (x = y \rightarrow ((0 \ll x = y) \wedge \forall pp (pp \ll x = y \rightarrow 0 \equiv pp)))$  follows by Universal Generalization. And since the only constituents of  $\forall x \forall y (x = y \rightarrow ((0 \ll x = y) \wedge \forall pp (pp \ll x = y \rightarrow 0 \equiv pp)))$  are  $=, \forall, \rightarrow, \wedge$  we have established that (1) essentially follows from the definitions of

$=, \forall, \wedge, \rightarrow$ , thus establishing (Null<sub>=</sub>).

## 7 Conclusions

In this paper I have put forward and defended the Null Account—the view that identity and distinctness facts are uniquely zero-grounded. First, I argued that the account solves a range of problems and that a relation that behaves the way the Null Account says identity does, is an excellent candidate for being *the* identity relation. Secondly, I used a plenitudinist view of relations to argue that there *is* a relation the instantiations of which are grounded as the Null Account claims identity and distinctness facts are. To develop this plenitudinist view I sketched a novel account of real definition, used this to give a real definition of identity, and showed how the central features of the identity relation could be deduced from its definition.

Much work—both technical and philosophical—remains to be done in fleshing out the above theory of real definition, but I trust that the above application to defining identity shows the work to be worth undertaking.<sup>48</sup>

## A Appendix: Defining the Universal Quantifier

Since the more elaborate parts of the theory of definition are developed in order to make it possible to give a pure definition of the universal quantifier, I would be remiss unless I provided one. The idea, recall, is to define the universal quantifier in terms of how propositions formed by it are (anti)grounded. While there is no universally agreed upon view about how universally quantified propositions are grounded, if there is an orthodox view it is the one proposed by Fine (2012, pp. 58–67). According to this view the grounds for  $\forall xPx$  are  $Pa, Pb, \dots$  together with  $T(a, b, c, \dots)$  where  $T$  is the totality fact that  $a, b, c, \dots$  are all the objects there are; the antigrounds for  $\forall xPx$  are of the form  $\neg Pf, T(a, b, c, \dots, f, \dots)$ . (The puzzles of ground (Fine, 2010a) put pressure on this view, but I will not consider this further here.)

Some preliminaries. First, standardly  $\forall$  both generalizes and binds variables. Here the roles are separated: any variable-binding is done by  $\lambda$ , and  $\forall$  is simply a property of properties. Secondly, the totality property  $T$  is not treated as a variable-arity relation, but simply as a property of pluralities of objects. (For quantifiers over items of higher type  $\tau$ , there will then be totality predicates applying to pluralities of type  $[\tau]$ ; I prescind from discussing these cases.)

Informally, the idea is that the universal quantifier  $\forall$  is, by definition, that property  $O$  of monadic properties such that for any monadic property  $P$ , the proposition  $p$  that results from applying  $O$  to  $P$  is, by definition, such that:

- $p$  is a result of applying  $O$  to  $P$ ;
- for any  $xx$ , and any plurality of propositions  $pp$  containing exactly the propositions of the form  $Py$ , for  $y$  amongst the  $xx$ :
  - $pp, T(xx)$  grounds  $p$ ;

- for any  $xx$ , and any proposition of the form  $\neg Py$  for  $y$  amongst the  $xx$ :
  - $q, T(xx)$  antigrounds  $p$ .

The tricky issue is expressing that  $pp$  is a plurality containing exactly the propositions of the form  $Py$ , for  $y$  amongst the  $xx$ ; the natural way of doing this involves quantification. But one can avoid the use of the quantifiers as follows. A natural thought is that a plurality is defined by the specifying its members. So writing  $y < xx$  for the claim that  $y$  is amongst the  $xx$  one can express that  $pp$  contains exactly the propositions of the form  $Px$ , for  $x$  amongst the  $xx$  as follows:

$$pp \frac{y: y < xx, q: \mathcal{A}(Pyq)}{ss} \parallel q < ss$$

What this says is that  $pp$  is, by definition, that plurality  $ss$  of propositions such that for any  $y$  amongst the  $xx$ , and any  $q$  that results from applying  $P$  to  $y$ ,  $q$  is amongst the  $ss$ . This ensures that each proposition of the form  $Py$ , for  $y$  amongst the  $xx$  is amongst the  $pp$ . Definitional Reflection ensures that that no other propositions are amongst the  $pp$ .

Here, then, is the definition of the universal quantifier.

$$\forall \frac{P, p: \mathcal{A}(OPp)}{O} \parallel p \frac{xx, (z: z < xx), r_0: \mathcal{A}(Pzr_0), r_1: \mathcal{A}(\neg r_0 r_1), (t: \mathcal{A}(Txx t)), (pp: pp \frac{(y: y < xx), (q: \mathcal{A}Pyq)}{ss} \parallel q < ss)}{u} \parallel \left\{ \begin{array}{l} \mathcal{A}(OPu) \\ t, pp \ll u \\ t, r_1 \ll \neg u \end{array} \right.$$

## Notes

1. For more about zero-grounding see § 2 below.
2. An early text is (Correia, 2006). Three important works, listed in order of how fine-grained they allow reality to be are (Rayo, 2013), (Correia & Skiles, 2019), and (Dorr, 2016).
3. In this paper, I leave the exposition somewhat informal. I aim to give a fuller treatment of the theory of definition elsewhere.
4. One obviously also has a notion of mediate ground: this results from the former by closing under Cut.
5. Indeed, the possibility of zero-“construction” was noted by Fine (1991, p. 277) in his tripartite framework for the study of ontology.
6. Wilsch goes even further and tries to define grounding in terms of laws.
7. This is an instance of the widely—though not universally—held view that facts about grounding are mediated by the natures of the grounded. See e.g., (Rosen, 2010, pp. 130–133), (Audi, 2012, pp. 693–696), (Fine, 2012, pp. 74–80), and (Trogon, 2013).
8. This goes beyond Fine in allowing the essentialist operator  $\Box$  to be indexed with items of arbitrary types. See (Ditter, 2022) for a precise formulation of such a “higher-order” logic of essence.
9. I would be remiss if I did not note that there are ways of defending the Existence Account; however, they do all come at a cost. On their own these costs are not decisive reasons to reject the Existence Account; but since the Null Account does not incur any of these costs, it should be favored over the Existence Account.

One option is to reject that  $a = a$  would be true when  $a$  does not exist, thus committing to what is known as “Serious Actualism” (Plantinga, 1983) or the “Being Constraint” (Williamson, 2013). However, Dorr (2016, pp. 55–57) and Goodman (2016, pp. 172–174) argue persuasively that the Being Constraint should be

rejected. Moreover, it is possible to develop a version of the Null Account that abides by the Being Constraint; adopting the Being Constraint thus does not favor the Existence account over the Null Account.

Another option is to modify the Existence Account and hold that the grounds for  $a = a$  is the possibility that  $a$  exists; but this is problematic once one considers the identity of *sets* of impossibilia (Salmon, 1987, 95–97, p. 105n55): an unfertilized egg could have given rise to two distinct persons (depending on which sperm fertilized the egg). Those two people could not co-exist; but the set containing them both would still be identical to itself, the impossibility of its existence notwithstanding.

A final possibility for the defender of the Existence Account is to adopt Williamsonian necessitism, according to which every object exists necessarily. Being forced into necessitism to defend the account is a considerable cost. In any case: the truth of necessitism is compatible with the Null Account, and so does not favor the Existence Account over the Null Account.

10. His main reason for rejecting it is that he takes it to rely on a theory of structured propositions and he takes the Russell-Myhill paradox to show that any theory of structured propositions is inconsistent.
11. I here assume, as is standard, that an instance is a partial ground for a universal generalization and that if  $p$  and  $p \leftrightarrow q$  are true, then  $p$  is a ground for  $p \leftrightarrow q$ .
12. As a grounding principle Hume's Principle says the identity of the number of  $F$ s with the number of  $G$ s is grounded in the existence of a one-to-one correspondence between the  $F$ s and the  $G$ s. Donaldson credits this suggestion to (Rosen, 2010, p. 123) and (Schwartzkopff, 2011, p. 362).
13. Formally,  $a$  rigidly depends on  $b$  iff  $\exists P \Box_a P b$ . The claim that  $=$  depends on no object is:  $\forall x \forall P \neg \Box = P x$ ; and the claim that  $=$  depends on no particular type of object is the claim that  $\forall P (\exists x \exists y (P x \wedge \neg P y) \rightarrow \forall O \neg \Box = O(P))$ .
14. Such a permutation does not respect the facts about set-membership, of course, but a topic neutral notion should not know anything about *sets* in particular.
15. Given the uniformity and topic-neutrality of the account of identity, is there room for substantive criteria of identity? The Null Theorist has to reject that criteria of identity tell us something interesting about the identity relation itself. What she should say, instead, is that identity criteria explain the nature of the things that are identical. For instance, taking the notion of identity for granted, the identity criterion for sets tells us that it is part of the nature of *sets* to stand in the identity relation when their members are the same. Interestingly, a view like this can be found in Frege. Writing about Hume's Principle he says: "We are therefore proposing not to define identity specifically for this case [the case of numbers], but to use the concept of identity, taken as already known, as a means for arriving at that which is to be regarded as being identical [that is, the numbers]" (Frege, 1950, §64).
16. For a compelling criticism of Wilhelm's arguments see (Lo, forthcoming).
17. As Burgess (2012, p. 2) notes a similar worry arises for the existence account: "The puzzle is: how does it determine which? In other words: if identity and distinctness facts have the same ontological bases, why do those bases 'sometimes' give rise to identity facts and other times give rise to distinctness facts?"
18. Shumener (2020a, p. 2092) and Burgess (2012, p. 3) raise a related worry for the Existence account.
19. See, e.g., (Fairchild, 2017) for a paradox based on the Russell-Myhill paradox. For more on the Russell-Myhill paradox see (Deutsch, 2008), (Goodman, 2017), and (Dorr, 2016); and for a predicativist solution to the paradox, see (Walsh, 2016).
20. Or if one is of the dialetheist persuasion: "lived with".
21. The same problem arises if one thinks of the theories of (Dorr, 2016) and (Rayo, 2013) as providing real definitions.
22. Note that this formulation does not *reify* forms or roles. If one were willing to reify forms one could give an "algebraic" account of real definition (cf. Fine, 1994b, p. 55) where the real definition of an item is simply taken to be a plurality of forms. It is not straightforward to develop an account of propositional forms that will work; but there are two accounts that may work. The first is inspired by Fine's work on arbitrary objects (Fine, 1985a, 1985b, 1998, 2017). On this account, *forms* are simply certain (possibly dependent) arbitrary propositions. On the second—less developed—account forms are expressed by the kind of "conditioned"

- or “restricted”  $\lambda$ -expressions discussed in (Fine, 2016, p. 16n9). I have no objection to arbitrary objects (or the restricted properties given by the restricted  $\lambda$ -expressions)—indeed, it would be hard to object to these for a plenitudinist! However, there is a compelling reason to opt for the present more parsimonious account. Arbitrary objects (and restricted properties) are themselves entities that stand in need of definition. It would thus be problematic if every definiens involved arbitrary objects (or restricted properties).
23. For Rosen real definitions are always of properties and relations; it is an advantage of the present account that it naturally deals with the definition of objects.
  24. This means that the logic of real definition will not contain an analogue of the chaining principle from the logic of essence (Fine, 1995b, pp. 248–249).
  25. I should also note that Individuation does not require that an item has a *unique* full definition. Fine (1994b, pp. 66–69) proposes that there are cases where an item has several distinct full immediate definitions. See also (Fine, 2007, p. 61) and (Litland, Forthcoming).
  26. The first two points develop some remarks in (Fine, 1994b, pp. 62–65) within the current account of real definition; the third point is novel.
  27. Of course, in denying that Narcissus is partly defined by loving Narcissus one does not deny that Narcissus in fact loves Narcissus: that much follows from Factivity<sup>-</sup>.
  28. For a general account—in the setting of mathematical structuralism—of how indiscernibilia like  $i, -i$  can be defined, see (Litland, Forthcoming).
  29. That the current account of definition can readily account for simultaneous and recursive definitions gives it an advantage over Rosen’s account.
  30. First, the Null Theorist agrees with Wehmeier that there is no *fundamental* identity relation. But that does not mean that there is no derivative identity relation.
 

Second, to the question: “why is there a derivative identity relation?” the answer is plenitudinism about relations. Paradoxes aside, if there is a definition of it, the relation exists: there is no special pleading on behalf of identity.

Third, it is convenient to have an identity relation: by having one the Null Theorist can take an identity statement “ $a = b$ ”—where “ $a$ ” and “ $b$ ” are names—to express a proposition just about the things named. Wehmeier has to resort to treating such identity-statements as statements of co-reference; the Null Theorist avoids this.

Fourth, the Null Theorist should admit to offering a promissory note. Wehmeier asks for an explanation of what makes identity a (binary) *relation*. His proposed answer is the Wittgensteinian Arity Principle (WAP), according to which “the arity of  $R$  is the maximal number of objects that can possibly be related by  $R$ ” (Wehmeier, 2012, p. 768). Of course, the Null Theorist cannot accept the WAP. In its place she holds that relations are simply *given* with their arity: it is not as if one first has an entity and the question of its arity meaningfully arises. Identity is no exception; it is simply given as a binary relation. Wehmeier (2012, p. 769n16) rightly observes that views of this sort give rise to familiar problems in the metaphysics of relations—in particular those of (Fine, 2000) and (Williamson, 1985). I am confident that these problems can be overcome, but this not the place to go into how this can be done—hence the promissory note. (For some thoughts about how the challenge can be met see (Leo, 2008b, pp. 357–58), (Leo, 2008a, pp. 351–353), and (Gilmore, 2013).)
  31. In § 6.9 I introduce the application relation  $\mathcal{A}$ ; that relation, too, will be allowed in pure definitions.
  32. Logical Purity is, of course, not the only attempt at making precise the idea that the logical operations are formal. The most worked out account of formality is no doubt the Tarski-Sher account (Sher, 1991; Tarski, 1986). The intuition behind this account is that formal notions are those that are indifferent to the identities of the individuals. Tarski and Sher make this precise by classifying an operation as logical if its extension is invariant under permutations of the domain of individuals. (For critical discussion of whether this captures the idea see (Bonnamy, 2006)). For present purposes, the main problem with this account is that there are many operations with the same extension (or even intension). For instance, in addition to the identity-relation = there is the relation—dropping for now the WVC—that holds between two objects  $x, y$  if they are identical

and Trump either is President or not. This relation has the same extension as the identity relation, but it is about Trump (in part) and so is not wholly logical.

Speculatively, I would suggest the following way of extending the Tarski-Sher account to the present framework. A *substitution* is a one-to-one function (strictly speaking: a typed family of functions)  $\sigma$  from items to items that leaves ground and antiground fixed. If  $t$  is an item with definition  $D$ , and  $\sigma$  is a substitution then  $\sigma(D)$  is the definition that results from  $D$  by applying  $\sigma$  to the items figuring in  $D$ . Say that  $\sigma$  *respects definitions* if for all  $t$ , if  $t$  is defined by  $D$  then  $\sigma(t)$  is defined by  $\sigma(D)$ . An *admissible substitution* is a substitution that respects definitions.

I conjecture that the pure items are those that are left invariant under every admissible substitution. These are the items the *natures* of which do not turn on the natures of other items. If Logical Purity holds then one can identify the logical operations with the pure operations.

Some readers may have noticed the similarity to some views recently put forward by Bacon (2018, 2019, 2020). He, too, has proposed characterizing the pure operations as those that are left invariant under substitutions. However, Bacon simply stipulates that admissible substitutions leave the Boolean operations fixed; the present account—assuming Logical Purity—explains this special status of the logical operations by the purity of their definitions.

33. Dorr (2016, pp. 62–63) gives an argument in favor of Involution based on (Ramsey, 1927). For discussion of how principles like Involution interact with grounding principles, see (Krämer, 2019; Litland, 2022; Wilhelm, 2021).
34. The use of variable-binding and generalizing operations in metaphysics is not new here. Rayo (2013) and Correia and Skiles (2019) use such operations to express generalized identities. Dorr (2016) also considers this, though he ultimately favors (largely on technical grounds) the use of  $\lambda$ -abstraction and (ungeneralized) higher-order identity relations. The formalism proposed here owes the most to Glazier’s proposal for expressing laws of metaphysics (Glazier, 2016).
35. For reasons of readability I adopt the following “vertical” notation:  $s \frac{y_0, y_1, \dots}{x} \parallel \left\{ \begin{array}{l} \phi_0 \\ \phi_1 \\ \vdots \end{array} \right.$  is to mean that  $s \frac{y_0, y_1, \dots}{x} \parallel \phi_0, \phi_1, \dots$
36. This is a general issue with defining relations: it must follow from the definition of an  $n$ -ary relation  $R$  whether  $Ra_0 \dots a_{n-1}$  is the same proposition as  $Ra_{\pi(0)} \dots a_{\pi(n-1)}$  (here  $\pi$  is a permutation of  $0, \dots, n-1$ ). Rosen’s theory of definition does not take this into consideration and so is at best incomplete.
37. This is not to deny that there could be a *skew* identity relation for which  $x = y$  was distinct from  $y = x$ . Indeed, it is unclear on what grounds a plenitudinist could reject the existence of such a relation.
38. For the case of the objectual identity relation  $=$  there may be no problem, since what is required is that  $(x = y) \equiv (y = x)$  and one could allow the propositional identity relation  $\equiv$  to figure in the definition of objectual identity. But, of course, one also wants  $(p \equiv q) \equiv (q \equiv p)$ ; and on pain of circularity one cannot ensure this by allowing propositional identity to figure in the definition of propositional identity.
39. Strictly speaking, for each relational type  $\tau$ , there will be a different application relation. For readability, I employ a typically ambiguous notation.
40. However  $\phi(y/x)$  is not a consequence of  $\phi(x)$  and  $x = y$ . For if  $x, y$  are distinct then while there is no coherent closed plurality that contains  $x = y$  there are incoherent ones that do.
41. So named because of its relation to a proof-theoretic principle of the same name (see e.g., Schroeder-Heister, 2013).
42. Definitions that embed definitions are in fact quite common. If one thinks of Hume’s Principle as defining the number-of operation, one would want the number-of operation to be defined in part by the values of the operation’s being, by definition, values of that very operation. Otherwise, it would not follow from the definition of the number-of operation that the numbers (*de re*) are values of that operation.
43. What is at issue here is whether for a proposition of the form  $Rxy$  the grounds for those propositions are

determined solely by the nature of  $R$  or whether the natures of  $x, y$ , too, contribute to determining the grounds for  $Rxy$ . In the case of the identity relation  $=$  the Null Theorist holds that everything is determined by the nature of  $=$ ; but I should stress that not all cases work like this. Consider, e.g., the existence property  $E$ . It is overwhelmingly plausible that the grounds for propositions of the form  $Ex$  is determined by the nature of  $x$  and not (solely) by the nature of  $E$ . For instance, if  $x$  is a set then the grounds for  $Ex$  is the existence of the members of  $x$ ; if  $x$  is a fusion then the grounds for  $Ex$  is the existence of any collection of proper parts of  $x$  that fuse to  $x$ . But one might think that there is nothing in the nature of the existence property itself that knows of sets or fusions.

44. From  $\phi, \psi$  you may infer  $\phi \wedge \psi$ .
45. If you have deduced  $\psi$  from  $\phi$  (together with assumptions  $\Gamma$ ) you may deduce  $\phi \rightarrow \psi$  from  $\Gamma$ .
46. Due to the WVC one has to give this rule in a slightly unfamiliar format (cf Wehmeier, 2004). Suppose you have derived  $\phi(x)$  from  $\Gamma$  and  $x$  is not free in  $\Gamma$ , and that in addition you have derived  $\phi(z_0/x), \phi(z_1/x), \dots$  from  $\Gamma$  where  $z_0, z_1, \dots$  are all the free variables in  $\Gamma$ , then you may infer  $\forall x\phi$  from  $\Gamma$ .
47. The reasoning in the case of (Null $_{\neq}$ ) is similar.
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