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VAGUENESS & MODALITY—AN ECUMENICAL APPROACH*

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1. Introduction

How does vagueness interact with metaphysical modality and with restrictions of it, such as nomological modality? In particular, how do *definiteness*, *necessity* (understood as restricted in some way or not), and *actuality* interact? This paper proposes a model-theoretic framework for investigating the logic and semantics of that interaction. The framework is put forward in an ecumenical spirit: it is intended to be applicable to all theories of vagueness that express vagueness using a definiteness (or: determinacy) operator. We will show how epistemicists, supervaluationists, and theorists of metaphysical vagueness like Barnes and Williams (2010) can interpret the framework.¹ We will also present a complete axiomatization of the logic we recommend to both epistemicists and local supervaluationists.

2. Preliminaries: Three Views of Vagueness

Before we go on to describe the general framework, we will briefly rehearse how the three views of vagueness we will be concerned with conceive of the definiteness operator.

First, however, a terminological matter: following some but not all of the relevant literature, we will draw a distinction between *vagueness* and *borderlineness*. *Vagueness*, as we are thinking of it, is the phenomenon that manifests itself in

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borderline cases—paradigmatically those found in *sorites* sequences—i.e., cases in which something is neither definitely so nor definitely not so. The definiteness operator is thus definable in terms of the borderlineness operator, and *vice versa*: it is (a) borderline (matter) whether ϕ iff it is neither definite that ϕ nor definite that it is not the case that ϕ ; and it is definite that ϕ iff (ϕ and it is not the case that it is borderline whether ϕ). We will use “borderline” both as a sentential operator (as above) and as a predicate of sentences. The two uses are related in obvious ways: e.g., it is borderline whether ϕ iff the sentence “ ϕ ” is borderline. But one should not assume that this equivalence holds necessarily because (*inter alia*), on certain natural reconstructions of some of the views we will consider, whether “ ϕ ” is definite depends counterfactually on how we use “ ϕ ”, whereas whether it is definite that ϕ normally does not.² Correspondingly, we also use “definite(ly)” as both an operator and a predicate.

(A warning: For brevity and ease of exposition, we will tend to be sloppy about use and mention, and we will make free use of both schemata and generalizations about linguistic expressions, without always indicating which we intend—we trust that the reader can tell. Thus, for example, sometimes Greek letters without quote marks will be used as variables for sentences, and at other times, as above, as schematic sentences, which may occur both within quote marks, which indicate mention—not of the Greek letter but of any sentence that replaces it in instances of the schema—and without.)

Vagueness is more general than borderlineness, among other respects in that borderlineness only pertains to sentences whereas any kind of expression can be vague. For example, some names are vague, but no name is borderline. (It is, of course, another matter whether there are *borderline names*: things that are neither definitely names nor definitely not names.) And while every borderline sentence is vague, not every vague sentence is borderline. Vagueness is something like admitting several precisifications, and borderlineness is something like not having the same truth value on all admissible precisifications. (“Something like” because just what one should say vagueness and borderlineness are depends on which of the three theories to be discussed below one accepts. However, each of the theories has something that occupies roughly the theoretical role that the notion of a precisification does in supervaluationism, and we’ll stick with that term in characterizing vagueness and borderlineness. After reading this section the reader can translate what we say about vagueness and borderlineness using that notion into the language of each of the other three theories.) A sentence may admit more than one precisification—wherefore it will be vague—while being true under every precisification it admits—wherefore it will not be borderline, but definite. For example, the sentence “New College is in Oxford” is vague because it admits various precisifications corresponding to the various admissible ways of drawing a boundary between Oxford and the rest of the world, but it is not a borderline sentence because New College is entirely located within each of those boundaries. At least on some of the views we will consider it is plausible that vagueness for sentences is the possibility of borderlineness, in the schematic sense

that “ ϕ ” is vague iff it is possible that it is borderline whether ϕ , but we will not pursue this theme.³

2.1 Supervaluationism

The idea behind supervaluationism⁴ is that vague expressions may be “precisified” in different ways. For example, the predicate “bald” is vague in that, in some suitable sense of “determine”, our use of it does not determine a unique meaning that divides everything into the predicate’s extension and antiextension; some things are—at least as far as our use of “bald” is concerned—neither in its extension nor in its antiextension. These are the borderline-bald things, or the borderline cases of baldness. However, one can—in some suitable sense of “can”—make “bald” precise in different ways by imposing a cut-off at n hairs, for various numbers n , so that anyone with fewer than n hairs on his or her head is in the extension of “bald” and everything else is in its antiextension. Not only can one precisify a single predicate in this way; one can also simultaneously precisify every vague expression. Speaking loosely for now we think of a precisification as an assignment of a precise *meaning* to each simple non-logical expression of the language. (What exactly meanings are is a question to which we return later.)

The supervaluationist understands the definiteness operator roughly as follows: it is definite that ϕ iff the sentence “ ϕ ” is true under every admissible precisification.

2.2 Epistemicism

The second view we will consider is epistemicism, as developed by Williamson (1994).⁵ Unsurprisingly, epistemicists hold that vagueness is a (merely) epistemic phenomenon. But the Williamsonian epistemicist’s idea is not simply that no one knows where (e.g.) the cut-off for “bald” is. The idea is that borderlineness is a distinctive kind of obstacle to knowledge. According to Williamson, meaning supervenes (*inter alia*) on use, but the meanings of vague expressions depend on use in a particularly fickle way. All vague expressions are *semantically plastic* in that, for each vague expression, there are some very minor changes in global patterns of language use that would result in its having a subtly different meaning than it actually has.⁶ In the case of a sorites-susceptible predicate like “bald”, the subtly different meaning would be associated with a cut-off subtly different from the actual cut-off for baldness. Furthermore, semantic plasticity gives rise to close possibilities of error. Our judgments are not counterfactually sensitive to the slight differences in meaning that slight differences in the use of a vague word would produce. If one judges, “To be bald is to have fewer than n hairs”, even if the cut-off in fact is n , one could easily have judged so even if the use of “bald” had been slightly different and the cut-off for “bald” had been something other than n , in which case one’s judgment would

have been false. According to Williamson's safety-theoretic conception of knowledge, close possibilities of error are incompatible with knowledge, and so one's judgment does not constitute knowledge. The general idea is that, whenever " ϕ " is a borderline sentence, due to the semantic plasticity of " ϕ " there is a close possibility in which " ϕ " is false, as well as a close possibility in which " ϕ " is true, and this precludes knowledge whether ϕ .

Here, then, roughly, is how the epistemicist understands the definiteness operator: for it to be definite that ϕ is for it to be the case that anything we easily could have meant by the sentence " ϕ " is true.⁷

2.3 Metaphysicalism

Whereas the supervaluationist idea is that the meanings of vague expressions are in some sense unsettled, the idea of metaphysical vagueness as developed by Barnes (2010, 2013), Barnes and Williams (2010), and Williams (2008) is that reality itself is in some sense unsettled. There is the concrete world—a mereological fusion of some concrete things, or perhaps just some concrete things—and then there are various perfectly precise ways for reality to be. We will call these *realities*. What is unsettled is which reality is *realized*.⁸ We will think—following Barnes and Williams—of realities as sets of *ersatz* worlds. (Since we want to study the interaction between vagueness and modality, it will not do to take a reality to be just an *ersatz* world: a reality determines not just what is actual, but also what is possible and necessary.)

A defender of metaphysical vagueness—henceforth: a *metaphysicalist*—understands the definiteness operator roughly as follows: is definite that ϕ iff the proposition that ϕ is true in every reality.

The three theories of vagueness discussed above do not, of course, exhaust all of the theoretical options one finds discussed in the literature on vagueness, but they are the ones that have received the most discussion so far. There are several other views which we would have discussed in a more comprehensive exposition of the applications of our model-theoretic framework. One view that we especially regret not being able to discuss is the rich theory of Bacon (forthcoming), in which vagueness is taken to be a property of propositions rather than of the sentences that express them.⁹ We suspect that the correct application of our model-theoretic framework to Bacon's theory and the philosophical gloss that should accompany it would be quite similar to the application to metaphysicalism and the related gloss found in this paper, but only further work will tell.

3. Motivating Questions

Before we dive into the formal framework, it will be useful to consider some issues that turn on the interaction of vagueness and modality.

First, it is standardly thought that the validities of first-order logic are necessarily true; it is also standardly thought that the validities of first-order logic are not borderline, that is, that they are *definitely* true. But once we consider the interaction between vagueness and modality we might think that this is not enough: a first order-validity is definitely necessarily true, necessarily definitely true, definitely necessarily definitely true, and so on. When ϕ is such that a true sentence results from prefixing any finite string of definiteness and necessity operators to it, we say that ϕ is *supernecessary*. One might then be tempted by the following claim:

(Supernecessity) All first-order validities are supernecessary.

Second, in standard logics for the actuality operator A all instances of $\phi \leftrightarrow A\phi$ are valid. But not every instance of $\phi \leftrightarrow A\phi$ is necessarily true—in particular, whenever ϕ is contingent so is $\phi \leftrightarrow A\phi$. But one might think that $\phi \leftrightarrow A\phi$ is like other logical truths in that it is definitely true. If so we should accept the following:

(Actuality) All instances of $\phi \leftrightarrow A\phi$ are definitely true.

(Since an instance of $\phi \leftrightarrow A\phi$ may not even be necessarily true, it is of course not the case that all instances of $\phi \leftrightarrow A\phi$ are supernecessary.)

Third, some philosophers hold¹⁰ that all matters supervene on precise matters, e.g., in the schematic the sense that it is definite that there can be no difference in which things are F without there being some difference in the distribution of precise properties (where “ F ” may be replaced by any one-place predicate). If we want to formulate such theses without explicit quantification over worlds we need modal languages with at least as much expressive power as those with necessity, actuality, and definiteness operators. In such a language one can formulate theses like the following:

(Definite Supervenience) For all x and necessarily for all y : it is definite that if, for all precise properties P (y definitely has P iff x actually definitely has P), then definitely (y has F iff x actually has F)

(Note that for the epistemicist and the supervaluationist all properties are precise. “Precise” here is non-redundant only for the metaphysicalist.)

The actuality operator plays an important role in formulating the supervenience theses, and it must work in a particular way for (Definite Supervenience) to state what we want it to state; we return to this point below.¹¹

Fourth, following Kripke (1980), we might want to say that names are rigid designators. In particular, we might want to accept all instances of the principles of the necessity of identity and of distinctness obtained by instantiating their

variables with names. But we do not want this to rule out that identity statements involving proper names can be borderline. In other words, we should accept:

(Rigid Designation) If “ a ” and “ b ” are two proper names, it is either necessary that $a = b$ or necessary that $a \neq b$. But neither $a = b$ nor $a \neq b$ need be definite

(In appendix A we will see that this leads to some complications in the completeness proof.)

Fifth, there are interesting questions about the interaction of necessity and definiteness; in particular, should we accept:

(Commutativity) Necessarily definitely ϕ iff definitely necessarily ϕ ?

The most interesting direction here is left-to-right: should we accept that if it is necessarily definite that ϕ , then it is definitely necessary that ϕ ? If “necessarily” is itself a source of vagueness we should expect this principle to fail. Taking the supervaluationist as an example, there might then be a precisification v of “necessarily” such that every v -possibility is a definitely- ϕ v -possibility; but it does not follow from this that every precisification of “necessarily” allows only ϕ -possibilities, which is what the consequent requires. We take no stand on this issue here; we only flag it as an issue of interest.

Sixth, there is a sizeable literature on whether “vague existence” makes sense; in particular, there has been a debate over whether vague existence makes sense on “broadly linguistic” views of vagueness.^{12,13} There is an unproblematic type of borderline—therefore vague—existence: if there is nothing which is definitely F , but there is something which is not definitely not F , then it is borderline whether something is F . This kind of borderliness is unremarkable because we can attribute it to the vagueness of the predicate “ F ”.

It is less easy to make sense of the borderline existence of F s when everything is either definitely F or definitely not F . Nevertheless, one might want say that even in cases like this it is possible for it to be a borderline matter whether there are F s. What we need to make room for is that it can be borderline whether actuality contains F s without actuality containing something which is a borderline F . Temporarily allowing ourselves the notion of a “candidate actuality” what we should say is that while all the candidate actualities are without borderline F s, it need not be definite which of the candidate actualities is the real actuality. Once we have necessity, actuality, and definiteness operators in the object-language, we can dispense with the notion of a candidate actuality and state in quasi-logical terms that it is not definite what exists as follows.

- It is possible that there is something x such that it is not definite whether actually there is some y such that $y = x$.

Importantly, this claim is consistent with the claim that everything definitely exists.

We now turn to the development of the framework.

4. The Languages

We study languages of the following kind. Each language has an infinite stock of first-order variables v_0, v_1, \dots ; an infinite stock of individual constants c_0, c_1, \dots ; an infinite stock of first-order predicates $P_0^1, P_1^1, \dots, P_0^2, P_1^2, \dots$, of any arity. Each language also has a distinguished identity predicate “=” and a distinguished existence predicate “ E ”.¹⁴

Each language has the logical constants $\wedge, \neg, \forall, \Box, \Delta$ and A . These express conjunction, negation, universal quantification, metaphysical necessity, definiteness, and metaphysical actuality, in that order.

We let $\vee, \rightarrow, \leftrightarrow, \exists, \Diamond$ be abbreviations in the standard way. We also use ∇ as an abbreviation of $\neg\Delta\neg$.¹⁵

Formulae and sentences are defined in the obvious way. We use ϕ, ψ, \dots , sometimes with subscripts, as (but not only as) metalinguistic variables for sentences or formulae. (In keeping with our carefree ways, we will also sometimes use them as schematic English sentences and as variables for English sentences.)

5. The Model Theory

5.1 From Two-Dimensional to Three-Dimensional Semantics

To give a model theory for a modal logic with an actuality operator we employ *double-indexing*, following Kamp’s (1971) classic work on tense logic.¹⁶ That is to say, we evaluate formulae for truth at pairs of worlds rather than simply worlds. In the standard intuitive picture that goes with double-indexing, the first member of such a pair represents the *context* of utterance and the second what we will call, following Kaplan (1989), the *circumstance* of evaluation: plainly put, a formula ϕ is true at a pair (w, w') iff the proposition ϕ expresses in w is true in w' . $\Box\phi$ is true at (w, w') iff ϕ is true at (w, w'') , for each w'' relevantly accessible from w' . Thus, according to the intuitive picture, $\Box\phi$ is true, as used, in w (w, w) iff the proposition (content) ϕ expresses in w is true in every world relevantly accessible from w . (For maximal generality, we will use an accessibility relation to interpret \Box . When \Box is interpreted as expressing metaphysical necessity, that relation is plausibly taken to be the trivial one that relates every pair of worlds to each other.) A , on the other hand, is intuitively an indexical, in that what it does to the proposition expressed by its operand depends on the context. In particular A rigidifies in that $A\phi$ expresses the necessary proposition in a context in which ϕ expresses a true proposition and expresses the impossible proposition

in a context in which ϕ expresses a false proposition. Thus, $A\phi$ is true at (w, w') iff ϕ is true at (w, w) . As it is often put, \Box “shifts” the circumstance and A “rigidifies” the circumstance by forcing it to be the same as the context.¹⁷

To deal with Δ we need a *triple-indexing*, since Δ must shift a parameter distinct from both the context and the circumstance. We will call the third parameter introduced to deal with Δ the *vagueness parameter*. What exactly the vagueness parameter represents varies between the three different views. For the supervaluationist it is a precisification parameter; for the metaphysicalist, a reality parameter; and for the epistemicist, it is something like a context—a situation in which the language is used.

Intuitively, from the point of view of any of the three theories, $\Delta\phi$ is true at a given value v of the vagueness parameter (holding the other two parameters fixed) iff ϕ is true at every value v' of the vagueness parameter relevantly accessible from v . The relevant accessibility relation will, of course, represent different things from the perspectives of each of the three theories. On the picture we are advocating, then, there are three independent dimensions of semantic evaluation: the vagueness dimension, the contextual dimension, and the modal dimension.

5.2 The Theory Formally

A model is a tuple $\mathcal{M} = (V, W, D, R_{\Box}, R_{\Delta}, [\])$. Here V is a non-empty set of vagueness parameters, W is a non-empty set of modal parameters. D is a function assigning a set of objects to each pair $(v, w) \in V \times W$. Intuitively, $D(v, w)$ is the set of objects that exists in w according to v . Let $\mathbf{D} = \bigcup_{v \in V, w \in W} D(v, w)$; \mathbf{D} is the *outer domain*. The relations $R_{\Box} \subseteq V \times W \times V \times W$ and $R_{\Delta} \subseteq V \times W \times V \times W$ are 4-place accessibility relations— R_{\Box} being the accessibility relation for \Box and R_{Δ} the accessibility relation for Δ .

The most convenient way of thinking of the accessibility relations is as relations between pairs $(v, w) \in V \times W$. We use four-place accessibility relations because we do not want to rule out views according to which which worlds are possible depends on the value of the vagueness parameter, nor, conversely, do we want to rule out views according to which which values the vagueness parameter may take depends on the world (circumstance) of evaluation.

We should impose the following conditions.

- If $(v, w)R_{\Box}(v', w')$, then $v = v'$
- If $(v, w)R_{\Delta}(v', w')$, then $w = w'$

The first condition ensures that in changing the world of evaluation we do not change the value of the vagueness parameter; the second condition ensures that if we change the value of the vagueness parameter we do not change the world of evaluation.¹⁸

We define truth relative to triples (v, c, w) ; here v is the vagueness parameter, c is the context, and w is the world of evaluation. $\llbracket \cdot \rrbracket : \mathcal{L} \times V \times W \times W \rightarrow \mathbf{D}^{<\omega}$ is an interpretation function.

- $\llbracket \cdot \rrbracket$ assigns a function from subsets of \mathbf{D}^n to $\{0, 1\}$ to each (P, v, c, w) where P is an n -ary predicate. We typically write $\llbracket P \rrbracket(v, c, w)$ instead of $\llbracket (P, v, c, w) \rrbracket$.
- For each individual constant d , $\llbracket (d, v, c, w) \rrbracket \in \mathbf{D}$.

We also write $\llbracket d \rrbracket(v, c, w)$ instead of $\llbracket (d, v, c, w) \rrbracket$.

We impose two constraints:

- $(\llbracket E \rrbracket(v, c, w))(a) = 1$ iff $a \in D(v, w)$.
- $\llbracket (v, c, w, d) \rrbracket = \llbracket (v, c, w', d) \rrbracket$ for all w, w' such that $(v, w) R_{\square}(v, w')$.

The first condition ensures that E behaves like an existence predicate. The second condition ensures that the individual constants are rigid with respect to \square .¹⁹

So far we have made no assumptions about how V and W relate to each other. Indeed, the three different theories of vagueness will take different lines on this.

A variable assignment is a function g from the set of variables to \mathbf{D} . Let g be a variable assignment. If d is a constant or a variable we let $\llbracket d \rrbracket_g$ be defined as follows. $\llbracket c \rrbracket_g$ is $\llbracket d \rrbracket$ if c is a constant and $g(c)$ if c is a variable. We define $g(d)(v, c, w)$ to be $g(d)$. We define truth relative to a variable assignment g as follows.

Definition 5.1

- (i) $\mathcal{M}, v, c, w, g \models P(\bar{a})$ iff $(\llbracket P \rrbracket(v, c, w))(\llbracket \bar{a} \rrbracket_g(v, c, w)) = 1$
- (ii) $\mathcal{M}, v, c, w, g \models a = b$ iff $\llbracket a \rrbracket_g(v, c, w) = \llbracket b \rrbracket_g(v, c, w)$
- (iii) $\mathcal{M}, v, c, w, g \models \phi \wedge \psi$ iff $\mathcal{M}, v, c, w, g \models \phi$ and $\mathcal{M}, v, c, w, g \models \psi$
- (iv) $\mathcal{M}, v, c, w, g \models \neg\phi$ iff $\mathcal{M}, v, c, w, g \not\models \phi$
- (v) $\mathcal{M}, v, c, w, g \models \forall x\phi$ iff $\mathcal{M}, v, c, w, g' \models \phi$ for all g' that differs from g at most in that g' assigns a different member of $D(v, c, w)$ to the variable x .
- (vi) $\mathcal{M}, v, c, w, g \models \square\phi$ iff $\mathcal{M}, v, c, w', g \models \phi$ for all (v, w') such that $(v, w) R_{\square}(v, w')$
- (vii) $\mathcal{M}, v, c, w, g \models \Delta\phi$ iff $\mathcal{M}, v', c, w, g \models \phi$ for all v' such that $(v, w) R_{\Delta}(v', w)$
- (viii) $\mathcal{M}, v, c, w, g \models A\phi$ iff $\mathcal{M}, v, c, c, g \models \phi$

As usual a sentence ϕ is true in \mathcal{M} at (v, c, w) iff ϕ is true in \mathcal{M} at (v, c, w) with respect to some (or equivalently: any) variable assignment.

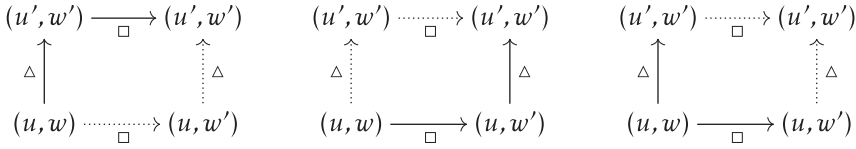


Figure 1. Commutativity and Church-Rosser

6. Products, Fusions, and Relative Products

The models we are working with are known as relativized product models. It is easiest to explain why we chose to work with them by comparing them first to full product models and then to fusion models. A full product model is exactly like our models but instead of having two 4-place relations R_{\square} , R_{Δ} it has two 2-place relations S_{\square} and S_{Δ} . The clauses of the truth-definition for full product models are as before except for the \square and Δ clauses that now read:

- (i) $v, c, w \models \square\phi$ iff $v, c, w' \models \phi$, for all w' such that $wS_{\square}w'$;
- (ii) $v, c, w \models \Delta\phi$ iff $v', c, w \models \phi$ for all v' such that $vS_{\Delta}v'$.

As is well known, product models validate the following principles:

COMMUTATIVITY 1 $\square\Delta\phi \rightarrow \Delta\square\phi$

COMMUTATIVITY 2 $\Delta\square\phi \rightarrow \square\Delta\phi$

CHURCH ROSSER 1 $\diamond\Delta\phi \rightarrow \Delta\diamond\phi$

CHURCH ROSSER 2 $\nabla\square\phi \rightarrow \square\nabla\phi$

We have wanted to leave it open whether which worlds are possible depends on the vagueness parameter; and also whether which values of the vagueness parameter are accessible depends on the world of evaluation. It is for this reason that we opt for relativized product models. It is important to note that this is no loss; as is well known, it is easy to impose conditions on the relativized products that validate the commutativity and Church-Rosser principles, namely:

(Commutativity1) If $(v, w)R_{\square}(v, w')$ and $(v, w')R_{\Delta}(v', w')$ there is v'' such that $(v, w)R_{\Delta}(v'', w)$ and $(v'', w)R_{\square}(v', w')$.

(Commutativity2) If $(m, w)R_{\Delta}(m', w)$ and $(v', w)R_{\square}(v', w')$ there is w'' such that $(v, w)R_{\square}(v, w'')$ and $(v, w'')R_{\Delta}(v', w')$.

(Church-Rosser) If $(v, w)R_{\square}(v_0, w_0)$ and $(v, w)R_{\Delta}(v_1, w_1)$ then there is v_2, w_2 such that $(v_0, w_0)R_{\Delta}(v_2, w_2)$ and $(v_1, w_1)R_{\square}(v_2, w_2)$.

These conditions are depicted in figure 1.

There is a simpler way to avoid validating the Commutativity and Church-Rosser principles: one can use fusion models. In a fusion model one simply works with a set of worlds—not with the product of two sets of worlds. A fusion model

has the form $(W, D, S_{\Delta}, S_{\square}, \llbracket \rrbracket)$. Here S_{\square} and S_{Δ} are two place accessibility relations subject to the condition:

- If $w S_{\square} w'$ and $w S_{\Delta} w'$, then $w = w'$.

The reason we cannot use fusion models is that they do not allow us to introduce the right kind of actuality operator. There is, of course, no problem in extending the two-dimensional approach and giving the clause for an actuality operator as follows.

- $w, w' \models A\phi$ iff $w, w \models \phi$

But this has two bad consequences. First, we cannot guarantee that all instances of $\phi \leftrightarrow A\phi$ come out definitely true. (For suppose that ϕ is true at w but not at w' and that w' is S_{Δ} -accessible from w . Then $\phi \leftrightarrow A\phi$ is not true at (w, w') and so not definitely true at (w, w) .)

Second, and this is perhaps an even more serious objection, such an actuality operator will not allow us to state supervenience theses with the force we want. For consider a borderline red object— a say. We want to say that if a is an actual object and b is any possible object, then if b is exactly like a actually is in all microphysical respects, then it is definite that (b is red iff a is actually red). For this to express what we want, we need compare a in the actual world and b in its world and see if they both count as red—in their respective worlds but with respect to the *same* vagueness parameter. To be able to do that we need some kind of product structure.

A picture should make this clearer. Consider a formula $\square\Delta\square\cdots\Delta A\phi$. In order to evaluate it we have to consider paths through two-dimensional modal space of the form depicted in figure 2. What the actuality operator does is to project down to the Δ axis; it does not simply pull back along the path.

Finally, perhaps the most decisive objection to the use of fusion models is that they would render certain commitments shared by epistemicists and supervaluationists inconsistent. Although we do not deal with second-order quantification in this paper, we want our model-theoretic framework to be capable of being extended to deal with it; otherwise the framework is not appropriately ecumenical. After all, both supervaluationists and epistemicists are committed to there being no vagueness in the world in the sense that for no state of affairs or proposition (we are not fussy about the distinction) p is it borderline whether p .²⁰ This commitment can only be expressed second-order quantification, quantification into sentence position being quantification into the position of a 0-place predicate. On the other hand, no one wants to accept all instances of the schema $\phi \rightarrow \Delta\phi$, which, in effect, says that there is no borderlineness (thus the desired logic will also be a second-order free logic, since it must not validate second-order universal instantiation, which is equivalent to second-order existential generalization). Thirdly, (almost²¹) everyone wants Δ to have a normal modal logic. It turns out that these three desiderata cannot be jointly satisfied using fusion models: see Fritz (2016) for a proof.

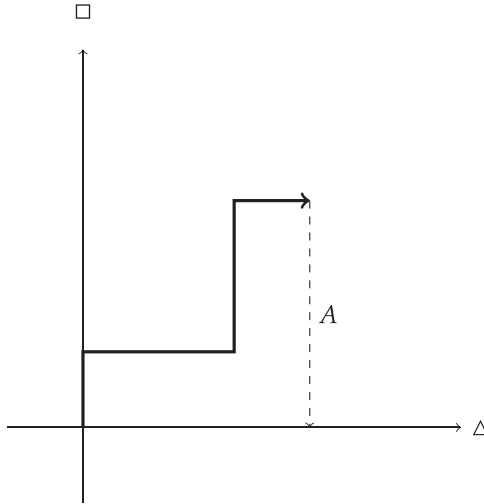


Figure 2. Paths and projections

7. Supervaluationism and Metaphysicalism

Note that what we have so far is an account of when a sentence ϕ is true at a triple of the form (v, c, w) . When, one might wonder, is a sentence *simply true* (in the intended model)? Here the three different accounts of vagueness we look at give different answers. What answer we give will also affect what notions of consequence are appropriate. Before we go on to discuss that, let us see how the supervaluationist and the metaphysicalist interpret the locution “ $v, c, w \models \phi$ ”.

7.1 Supervaluationism

For the supervaluationist the vagueness parameter v together with the context c determines what proposition p is expressed by the sentence ϕ ; that proposition p is evaluated at the world parameter w . c plays a role together with v in determining which proposition is expressed by ϕ . To see this consider a sentence like “It is raining” and its actualization: “Actually, it is raining”. What proposition is expressed by “Actually, it is raining”? To answer this question we cannot simply look to v , we also have to take c into consideration. For if c is a world in which it is raining, then with respect to (v, c) “Actually, it is raining” expresses a necessarily true proposition; if c is a world in which it is not raining, then with respect to (v, c) “Actually, it is raining” expresses a necessarily false proposition.

We propose that what is going on here, from the point of view of the supervaluationist, is that v determines—or perhaps simply is—an assignment of *characters* (in the sense of Kaplan 1989) to all of the simple non-logical

expressions of the language. In our simplified setting we can think of the character of a sentence as a function from worlds (thought of as contexts) to propositions, which in turn we can think of as functions from worlds to truth values. What v contributes is an assignment of a character to ϕ ; what c contributes is the argument to this character.

One interesting consequence of this semantic picture is that characters fail to satisfy the principle of compositionality, as predicted in (Yli-Vakkuri 2013, p. 562, n. 49). It is easy to see that there are models in which, relative to a given value v of the vagueness parameter, some sentences ϕ and ψ have the same character, yet $\Delta\phi$ and $\Delta\psi$ differ in truth value at (v, c, w) , for some c and w , so $\Delta\phi$ and $\Delta\psi$ have different characters relative to v ; the assignment of characters contributed by v , then, is not compositional. This is no mere technical accident. If one thinks of the definiteness operator as a device for, in effect, generalizing over characters its operand could have (in some suitable sense of “could”), as we think the supervaluationist ought to do, then one should expect failures of compositionality for character. The supervaluationist offers some kind of metasemantic story about what vagueness is (which we have not attempted to spell out in any detail); accordingly, for the supervaluationist, the definiteness operator should be sensitive to metasemantic features of its operand, and not only the purely semantic features encoded in the operand’s character.

What we have so far is an account of when a sentence is true with respect to a value of the vagueness parameter—i.e., with respect to a precisification. That is, we have an account of when the proposition expressed by the sentence with respect to the precisification and the context is true. But when is a sentence *simply true*? Whether the supervaluationist has a good answer to this question is unclear.²² But what matters for us here is that combining the supervaluationist outlook with necessity and actuality operators does not raise any new problems. Where we do not have any necessity and actuality operators the supervaluationist seem to have the following options.

- (i) Hold that the only sensible notion of truth is truth at a precisification
- (ii) Hold that truth is *supertruth*—that is, truth at all precisifications
- (iii) Hold that truth is truth at each of some favored subset of precisifications, where the favored subset could be selected by context, and could be a singleton set.²³

These options carry over to the present framework. Note, first, that whatever we do with the vagueness parameter, since we want truth to be truth in the actual world, the context parameter and the evaluation parameter have to be the same the same in the definition of each notion of truth. The three options are then:

- (i) For each world c there is truth of ϕ with respect to (v, c, c) ;—but there is no notion of truth at c simpliciter
- (ii) Given a world c , ϕ is simply true at c iff ϕ is true at (v, c, c) for every v .

- (iii) Given a world c , ϕ is simply true at c iff ϕ is true at (v, c, c) for each $v \in M(c)$, where M is a function from $W \rightarrow \mathcal{P}(V)$.

More important than the notion of truth, perhaps, is the notion of consequence. In the non-modal setting, local consequence is preservation of truth at a precisification; global consequence is preservation of supertruth. Since what we care about in logics with actuality operators is preservation of truth in the actual world, we arrive at the following definitions.

Definition 7.1

- (i) ϕ is a local consequence of Γ (writing: $\Gamma \models_l \phi$) iff, for all models \mathcal{M} , and all v, c , if $\mathcal{M}, v, c, c \models \Gamma$ then $\mathcal{M}, v, c, c \models \phi$
- (ii) ϕ is a global consequence of Γ (writing: $\Gamma \models_g \phi$) iff, for all models \mathcal{M} , and all c , if $\mathcal{M}, v, c, c \models \Gamma$ for all v then $\mathcal{M}, v, c, c \models \phi$, for all v .

Since we are not trying to defend supervenience or even to decide which is the best version of it, we here only note that on either of the definitions of consequence, the following are valid:

- Every validity of (free) first-order logic is such that its supernecessitation is valid
- Every instance of $\phi \leftrightarrow A\phi$ is such that arbitrary definitizations of it are valid
- Every instance of \mathbf{K}_\square , that is, the \mathbf{K} -schema for \square , is such that its supernecessitation is valid; similarly for \mathbf{K}_Δ

We have demanded that as we vary the world of evaluation (but keep the vagueness parameter and the actual world fixed) the individual constants keep the same reference. This allows us to retain the necessity of identity and distinctness. Indeed, we have the following:

- all instances of $a = b \rightarrow \square a = b$ are supernecessary
- all instances of $a \neq b \rightarrow \square a \neq b$ are supernecessary.

Accepting the necessity of identity and distinctness does not force us to accept the definiteness of identity and distinctness. Just as we wanted at the outset.²⁴

The following inference rule, on the other hand, is not valid:²⁵

$$\frac{\phi(a) \quad Ea}{\exists x\phi}$$

An easy counterexample is $\Delta a = a$, where a is vague. For then it is clearly definite that $a = a$, since no matter what vagueness parameter we are looking at the reference of a with respect to that parameter will be identical to itself.

However, since a is vague there is no object x such that a refers to it with respect to every precisification.

The following weakening of the principle is, however, valid.

$$\frac{\phi(a) \quad \exists x \Delta x = a}{\exists x \phi}$$

As in the non-modal case, while global supervaluationism makes every classical validity logically true, certain classical rules of inference are invalid. As is usual, proof by cases and *reductio ad absurdum* as are not valid. This might well be problematic, but it is not *more* problematic in the modal setting than it was in then non-modal setting.²⁶

7.2 Metaphysicalism Again

How should the metaphysicalist understand the locution “ $v, c, w \models \phi$ ”? For the metaphysicalist the role of v is not to determine which proposition is expressed by ϕ : the sentence ϕ has a definite character and, relative to c , there is a proposition that ϕ definitely expresses. Whether the proposition expressed by ϕ is true with respect to the world w depends on what reality is like, and v ranges over realities.

Summing it up, the metaphysicalist understands the locution “ $v, c, w \models \phi$ ” as follows:

- The proposition expressed by ϕ in world c is true at world w relative to reality v

The metaphysicalist has the same options for defining consequence and truth simpliciter as does the supervaluationist. As in the case of supervaluationism what matters for us here is that adding in metaphysical necessity and actuality creates no further problems for the framework.

While the broad formal framework is the same for the two theorists there might be some differences of—important—philosophical detail.

First, the metaphysicalist might have a difference with the supervaluationist over the treatment of identity. It is arguable that vague identity is impossible on a metaphysical view of vagueness (Evans 1978). If that is right, the metaphysicalist should accept not just the necessity of identity and distinctness but also the definiteness of identity and distinctness.²⁷

Second, and more importantly, the supervaluationist holds that there is no “vagueness in reality”. But what is the cash value of this? A natural way of making this precise is to hold that it is never indeterminate whether a given proposition is true or a property is instantiated by an object. The defender of metaphysical indefiniteness, on the other hand, accepts that there are propositions such that it is indefinite whether they are true.

For the supervaluationist propositions are simply true at a world. For the metaphysicalist, on the other hand, a proposition is not simply true at a world: it is only true at a world with respect to a reality. For the metaphysicalist—unlike for the supervaluationist—it can be indefinite whether a *proposition* is true.²⁸

We should be able to express this difference in the modal object-language. Following Williamson (2003) a natural way of doing this is in higher-order modal logic, in particular, with quantification into sentence position. The important difference between the defender of metaphysical indefiniteness and the supervaluationist is then that the former accepts, while the latter rejects the following comprehension principle:

(Definite Comprehension) $\exists p \Delta(p \leftrightarrow \phi)$

Both, of course, accept the following principle:

(Modal Comprehension) $\exists p \Box(p \leftrightarrow \phi)$

It is straightforward enough to extend the present framework to make sense of higher-order quantification, but there will be a range of choice points depending on whether one is a contingentist or a necessitist. We leave the full development of this for a future occasion.²⁹

8. Epistemicism

What does the locution “ $v, c, w \models \phi$ ” mean for the epistemicist? In the case of supervaluationism we claimed that the $v \in V$ should be thought of as being assignments of characters to the simple nonlogical expressions of the language. The simplest suggestion is that, for the epistemicist, the $v \in V$ represent assignments of characters to the simple nonlogical expressions of the language that, *for all we are in a position to know*, are the characters of those expressions. Thus we might take V to represent a set of epistemically possible worlds. Since every metaphysically possible world is an epistemically possible world we should then demand that $W \subseteq V$.

On this simple approach we can take the meaning of “ $v, c, w \models \phi$ ” to be the following:

(Simple Account) The character that, according to the epistemic possibility v , is expressed by ϕ , when applied to c , yields a proposition that is true at w .

On (Simple Account), then, a sentence is definite iff all of the characters it expresses according to all of the accessible epistemic possibilities yield, when

applied to the actual world (taken as a context), propositions that are true in the actual world.

While (Simple Account) has a lot going for it, one might be dissatisfied with it because it does not offer an explanation of what the relevant epistemic possibilities are. Williamson, recall, explains our ignorance of borderline matters by appealing to the semantic plasticity of vague expressions, but (Simple Account) neither says nor entails anything about semantic plasticity. An account of the meaning of “ $v, c, w \models \phi$ ” that is more in line with Williamson’s theory of vagueness can be given, and we will give it below. But before we do, we will consider two epistemicist accounts of the meaning of the definiteness operator that do *not* work. These accounts make no use of the distinction between character and content.

8.1 Two Previous Epistemicist Accounts

One account of definiteness that has figured in the literature is the *other-worldly* account of definiteness or (OWD)³⁰

(OWD) ϕ is definite iff any world w that differs only slightly from the actual world in the global pattern of the use of language is such that the proposition ϕ expresses in w is true *in* w

According to (OWD), to check whether ϕ is definite, we must, so to speak, go to each of the close worlds, check which *proposition* ϕ expresses in that worlds, and check whether that proposition is true in *that very world*.

The fundamental problem with (OWD) is that, according to it, whether it is definite that ϕ turns, in part, on how things are in non-semantic respects in other worlds. Intuitively, this is wrong: whether it is definite that ϕ should turn on how things are only in *semantic* respects in other worlds and on how things are with respect to the subject matter of ϕ in the actual world. As an account of definiteness, (OWD) is bound to produce absurd results.³¹ Nor would it do to amend (OWD) by requiring that the relevantly close worlds differ from the actual world in non-semantic respects as little as the relevant differences in language use allow.³² To see why the amended proposal will not work, consider a sentence $C(n)$ that specifies (in nonmetalinguistic terms) the actual cut-off for some sorites-susceptible predicate P (e.g., “To be bald is to have fewer than n hairs”), and consider a non-vague sentence \mathbf{p} that specifies the actual pattern of use of P in sufficient detail that the material conditional.

(Precision) $\mathbf{p} \rightarrow$ the cut-off for P is n

expresses a necessary truth. $C(n)$ is not definite but \mathbf{p} is definite, so (by the K axiom for Δ) $\mathbf{p} \rightarrow C(n)$ is also not definite. But on any way of understanding (OWD), it would seem that $\mathbf{p} \rightarrow C(n)$ is definite! For go to any w in which \mathbf{p}

expresses a true proposition; because \mathbf{p} is not vague, w will be a world in which the cut-off for P is n , and so $C(n)$ will also express a true proposition in w .³³

The moral of (Precision) is that the epistemicist has to separate variation in semantic fact from variation in (other) matters of fact. To evaluate whether it is definite that ϕ we consider what meanings the sentence ϕ has in close worlds, but we then take those meanings back to the actual world and see whether ϕ , as so interpreted, is true here.

An account of definiteness that does exactly this has been suggested by Caie (2011); we call it *actualistic definiteness*. (We note that Caie himself rejects this account.)

(AD) ϕ is definite iff every world w_0 that differs only slightly from the actual in the pattern of overall use is such that the proposition expressed by ϕ at w_0 is true at the actual world

(AD) avoids the above problems for (OWD); but it runs into a range of other problems, including ones noted by Hawthorne (2006) and Caie (2011). Here we will discuss three, including a version of the problem noticed by Hawthorne and Caie.

The first of these, the *problem of actuality*, was noticed by Yli-Vakkuri (2016, 813–814). We have accepted that any sentence of the form $\phi \leftrightarrow A\phi$ is a logical truth—albeit possibly a contingent one—and is therefore definite. But if ϕ expresses, in some close world w , a proposition whose truth value in w is different from the actual truth value of the proposition ϕ actually expresses, then, in w , $\phi \leftrightarrow A\phi$ expresses a proposition that is actually false, and by (AD) it follows that $\phi \leftrightarrow A\phi$ is not definite, contradicting our assumption.³⁴

The second problem—the *meter problem*—was also noticed by Yli-Vakkuri (2016, 817). As we know, thanks to Kripke (1980, pp. 54f), the sentence

(Meter) The IPM is one meter long,

expresses a contingent truth. The IPM (short for the “International Prototype Meter”) is the Parisian platinum bar whose length is used to fix the reference (and content) of “meter”. The length of the IPM could very easily have been slightly different than it actually is, but because of this very fact, by (AD), (Meter) is not definite—in fact, it is borderline because it is true. For a close world in which the length of the IPM is some length l different from its actual length is a world in which (Meter) expresses the proposition that the length of the IPM is l , a proposition that is actually false. Given that borderlineness precludes knowledge—a non-negotiable assumption for epistemicists³⁵—this has the absurd consequence that one cannot *know* that the IPM is one meter long. But even if one thinks that borderlineness does not preclude knowledge, it seems just wrong to think that it is a borderline matter whether the IPM is one meter long. Similar counterexamples to (AD) can be generated endlessly using suitably chosen examples of reference-fixing descriptions.

The third problem is the *disquotation problem*, which was noticed, in different forms, by Hawthorne (2006, Sec. 13) and Caie (2011, Sec. 5). Here we present Caie's version of it.

Consider disquotational sentences like “Everest refers to Everest” and “‘Bob is bald’ is true iff Bob is bald”. By (AD) some of these sentences would appear to come out borderline. Again, given that borderlineness precludes knowledge, this has the absurd consequence that one cannot know that “Everest” refers to Everest, etc. And again, even if one thinks that borderlineness does not preclude knowledge, it simply seems wrong to think that it could fail to be definite that “Everest” refers to Everest, etc.

To see what the problem is, assume that “Everest” is a vague name in the sense that there is no particular mountain that “Everest” definitely refers to.³⁶ Now consider the sentence

- (1) “Everest” refers to Everest

Caie's argument turns on the idea that, since “Everest” is vague, there is a close possible world in which “Everest” has a referent different from its actual referent—so, a world in which “Everest” does not refer to Everest. Let us grant this (setting aside the worries of footnote 36). Caie assumes that the quotation name “‘Everest’” is not vague. Let us grant this too. Finally, let us grant Caie the following assumption, which he thinks only an extreme sort of semantic holist would deny.

- (2) Amongst the close possible worlds in which “Everest” does not refer to Everest, there is one, call it *u*, in which “refers” has as its semantic value the same intension as it does in the actual world.

On those assumptions the proposition expressed by (1) in *u* is the proposition that “Everest” refers to Everest*, where Everest* is what “Everest” refers to in *u*. The proposition that “Everest” refers to Everest* is false in the actual world, since “Everest” actually refers to Everest and not Everest*, so by (AD) it is not definite that—and since it is true, it is borderline whether—“Everest” refers to Everest.

The critical assumption in this argument is the claim that as used in a world *u* the intension of “refers” is identical with the intension of “refers” as used in the actual world. While this is something one could reject, we will grant it.

There is, of course, nothing special about “Everest”: by the same form of argument, every sentence of the form

- (3) “*n*” refers to *n*

where “*n*” is a vague name, will be borderline. This result is absurd.

Neither (AD) nor (OWD) works. How can we rescue the epistemicist account of vagueness in terms of semantic plasticity?

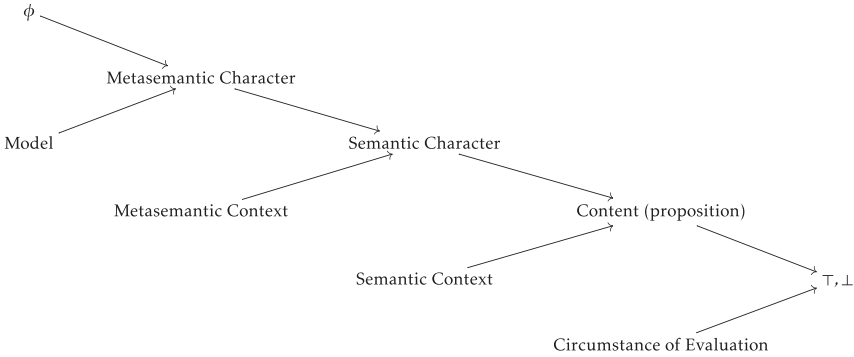


Figure 3. A 3^D-picture

8.2 Two New Epistemicist Accounts

Here is one epistemicist view of the semantics of the definiteness operator. It will turn out to have some controversial consequences, but it is the simplest account we can think of that has any plausibility, and it in fact clearly gets at least a range of paradigm cases right. We think it is a good enough model (in the general scientific sense of “model”) to begin our theorizing with.

The view is based on the elementary observation that the truth value of a sentence (and, more generally, the extension of an expression) is determined in three stages, as depicted in Figure 3. First facts about the use of language (together with other non-semantic facts) determine which character is expressed by a sentence. Then that character is applied to a context of utterance, yielding a proposition (more generally, a content), and finally that proposition is evaluated at a circumstance of evaluation, yielding a truth value. In a loose sense of “context”, we have two contexts here that play a role in determining truth value, and for the purposes of this paper we may think of the contexts as simply metaphysically possible worlds. (This is because the only indexical we are dealing with is *A*, which is sensitive only to the world of the context of utterance.) We will call worlds when they play the first role—determining character—*metasemantic contexts*, when they play the second role—determining content—we will call them *semantic contexts*, and when they play the third role—determining extension—we will follow Kaplan in calling them *circumstances*. On this view, the meaning of “ $v, c, w \models \phi$ ” is: “The character that ϕ has in metasemantic context v , when applied to semantic context c , yields a proposition that is true in w .”

It follows that a sentence is definite iff every character it has in every metasemantic context close to the actual one yields, when applied to the actual semantic context, a proposition that is true in the actual circumstance. The metasemantic contexts that close to a another in the relevant sense are those that are *semantically indiscriminable* from it: roughly, v is accessible from v^* iff the use of the

object language differs between v and v^* only so slightly that, for all the speakers of the object language are in a position to know in v^* , the words they use have the characters they have in v .

On this view, vagueness amounts to a kind of semantic plasticity, but it is plasticity of character rather than content: every borderline sentence expresses, in some close metasemantic context, a character different from its actual character. Vagueness in general amounts to expressing more than one character in some close metasemantic context—representing ways we could easily have used the object language slightly differently.

On this view, then, the points on the vagueness dimension are genuine metaphysical possibilities for the use of language. Each nonlogical expression is associated with a function from metasemantic contexts to characters—we'll call this the expression's *metasemantic character*. Metasemantic characters represent the way an expression's character supervenes on facts about the use of language (and other facts). They are the compositional semantic values of expressions in this picture, if anything is: neither the characters nor the contents expressions have relative to worlds (in their roles as either metasemantic or semantic contexts) obey the principle of compositionality. This may seem radical, but, as in the case of supervaluationism, it is to be expected, for roughly similar reasons: both take the definiteness operator to be sensitive to metasemantic facts about its operand that are not among the purely semantic facts encoded by the operand's character. Less roughly: First note that compositionality fails for both character and content on more familiar epistemicist views too, such as Williamson's, or those discussed in the previous section. These views allow that two sentences ϕ and ψ may express the same proposition while one is borderline and the other is not, because whether a sentence is borderline is not determined by the proposition it actually expresses but rather depends on which propositions the sentence could easily have expressed. Thus $\Delta\phi$ and $\Delta\psi$ may differ in truth value even when ϕ and ψ express the same proposition—a counterexample to the compositionality of content, since content determines truth value—and if neither ϕ nor ψ is an indexical sentence and $\Delta\phi$ and $\Delta\psi$ differ in truth value, $\Delta\phi$ and $\Delta\psi$ will also be a counterexample to the compositionality of character.³⁷ This is because non-indexical sentences have the same character iff they have the same content.

This view makes available a very simple and at least superficially attractive solution to the three problems that plagued the "actualistic" epistemicist semantics for the definiteness operator discussed in §8.1. For consider the following conjecture: all sentences of the form $\phi \leftrightarrow A\phi$, (Meter) and all other similar sentences involving precariously satisfied reference-fixing descriptions, and all disquotational sentences have characters that determine, in every context, a proposition that is true in that context (considered as a circumstance). For brevity, let us say that a character that has this property is *diagonally necessary*. Kaplan (1989) has already taught us that all sentences of the form $\phi \leftrightarrow A\phi$ have diagonally necessary characters, and in fact this is a direct consequence of the three-dimensional model theory on the epistemicist gloss we are now considering. It is not obvious

that the other problematic sentences have diagonally necessary characters, it is reasonably clear that they enjoy a *semantic guarantee of truth* of some kind, and it is tempting to conclude that this semantic guarantee of truth simply amounts to having a diagonally necessary character. Let us suppose that it does, and that we are in a position to know that it does. Because we are in a position to know that the problematic sentences have diagonally necessary characters, no metasemantic context in which they do not have characters is semantically indiscriminable from (close to) the actual one. It follows that each of the problematic sentences has, in every metasemantic context close to the actual one, a character that, when applied to the actual semantic context, yields a proposition that is true in the actual circumstance, and so, on the view we are considering, is definite.

The view solves the problem of actuality, the meter problem, and the disquotation problem all in one fell swoop, but this easy solution comes at a cost: if it is correct, then various expressions that are not generally recognized as indexical are indexical. This is easy to see by comparing the present view with (AD): the two views can only differ on whether a sentence is definite if the character of the sentence is not a constant function, i.e., if the sentence is indexical; a non-indexical sentence is classified as definite by (AD) iff it is classified as definite by the present view. On the present view, then, each of the problematic sentences is indexical, and this can only be so if each of them contains at least one simple indexical constituent. In the case of the problem of actuality this is no cost: *A* is a paradigmatic indexical. But (Meter) and disquotational sentences are less clear cases. If these sentences are indexical, then presumably this is so because *meter* and the reference and truth predicates are indexical, but the indexicality of these expressions is a matter of controversy. This is not an obviously serious problem for the view: some philosophers do think that “meter” is an indexical, and the so-called contextualist solutions to the liar paradox involve attributing indexicality to semantic predicates.³⁸ But some philosophers will prefer an epistemicist view that does not have these controversial consequences. And we have just such a view to offer.

According to the second view, “ $v, c, w \models \phi$ ” is to be read as: “The two-dimensional epistemic intension ϕ has in v (considered as a metasemantic context) determines in c (considered as a *scenario*) a secondary intension that is true in w ”. Here we use the ideology of Chalmers’ (2006) epistemic two-dimensionalism. The two-dimensional epistemic intension associated with a sentence is a function from worlds-*cum*-epistemic possibilities of a certain kind (what Chalmers calls *scenarios*) to functions from worlds-*cum*-metaphysical possibilities to truth values.³⁹ The two-dimensional epistemic intension of a sentence encodes both certain of its epistemic properties—in particular, for Chalmers, those having to do with a *priority*—and its metaphysical modal profile, or what we have been calling the proposition (or content) it expresses, according to each epistemic possibility. For this interpretation of the three-dimensional formal apparatus, we should think of all of the worlds in a model as Chalmers-style centered worlds: triples of a metaphysically possible world, an agent, and a time that represent

certain kinds of epistemic possibilities (scenarios), namely those that cannot be ruled out *a priori*.⁴⁰ Now the *diagonal* of an two-dimensional epistemic intension—or what Chalmers calls its associated *primary intension*, this being the function from scenarios to truth values that assigns truth to w iff the two-dimensional intension assigns truth to (w, w) —is a kind of an epistemic content. For Chalmers, a sentence's primary intension is true at every scenario iff the sentence is *a priori*, but we could just as well take the truth of a primary intension in every scenario to represent some other kind of epistemic necessity. It follows from what has been said, then, that $v, c, c \models \phi$ iff the primary intension ϕ has in v is true in c , and that ϕ is definite iff every primary intension ϕ has in every close world is true in the actual world (scenario).

A solution to the problem of actuality, the meter problem, and the disquotation problem immediately falls out of this view, since Chalmers' system was designed with an eye to ensuring that the sentences implicated in these problems have primary intensions that are true in every scenario. Given that they do, and that they also do in all close worlds, they are all definite on the second view.

Two-dimensional epistemic intensions play the same role in the second view that Kaplanian characters played in the first view. The first view had the surprising—for some, unpalatable—consequence that the problematic sentences were indexical, i.e., had non-constant characters. For analogous reasons the second view has the consequence that the problematic sentences have non-constant two-dimensional epistemic intensions. But this is not at all a surprising consequence—it is a consequence Chalmers intended his theory to have. (And again, analogously with both the first epistemicist view and with our construal of supervenience, we get failures of compositionality for two-dimensional epistemic intensions, which was to be expected.)

One might worry, however, that the second view has other (to some) unpalatable—if not surprising—consequences. For example, and most obviously, it has the consequence that sentences have primary intensions, a consequence that many philosophers deny, at least if “primary intension” is understood as applying to whatever sort of things satisfy the theoretical role Chalmers has carved out in theorizing with that term.⁴¹ Here it is worth noting, however, that we need not assume the truth of everything that Chalmers says about primary intensions to use primary intensions—or something similar enough—to solve the epistemicist's problems. We certainly need not assume, for example, that primary intensions are a kind of narrow content—a claim one of us has argued against at length⁴²—or that they are connected to *a priority* in the way Chalmers claims, as opposed to some other epistemic notion. On a suitably thin conception of primary intensions, they are things whose existence anyone who does semantics for epistemic logic in the standard “possible worlds” way (thus, including Williamson⁴³) acknowledges, and similarly for two-dimensional epistemic intensions and anyone who theorizes about the semantics of the combined logic of knowledge and metaphysical modality (such as, e.g., Rabinowicz and Segerberg 1994).

Finally, we would like to consider a potential objection to both of the epistemicist views about the semantics of the definiteness operator we have surveyed. Recently, Magidor (2016) has argued that (AD) fails as an account of definiteness. One of her objections is that (AD) simply gives the wrong predications about which sentences are borderline. We agree with this, of course: our two proposals are constructed to get around (AD)'s incorrect predictions. However, she makes one objection that may seem to threaten also our two proposals:

Most importantly, the amended proposal completely divorces the notion of borderliness from Williamson's explanation of ignorance that motivated his account in the first place: after all, according to Take-2 [what we call (AD)], having a true belief in a borderline statement does *not* entail that we could have easily had a false belief (the fact that we could have easily had a belief that is *actually* false is neither here nor there as far as epistemic evaluation in *w* goes—for this all that matters is whether one has a belief that is false *in w*). (Magidor 2016, 8–9)

One might think that we are vulnerable to a similar objection. For on either of the proposals considered above, borderliness induces falsehood at a triple of worlds (*v*, @, @) where *v* is close to but not necessarily identical to @. Neither proposal entails that a borderline sentence expresses, in some close world, a proposition that is false in *that* world, which might seem to be required for a safety-theoretic account of ignorance of borderline matters of the kind that Williamson wants to give. In any case, no safety-theoretic account of ignorance of borderline matters falls out of either of the epistemicist views discussed above.

We are not particularly worried by this, for two reasons.

The first reason is that we take it to be clear that the correct safety-theoretic account of ignorance of borderline matters does not entail that every borderline sentence expresses, in some world close, *in the present sense of "close"*, to the actual world, a proposition that is false in that world. We take Yli-Vakkuri's argument against that view (Yli-Vakkuri 2016, 828–829), which Williamson (2016, 847) accepts, to be decisive. Rejecting this false view is compatible with a safety-theoretic account according to which, whenever one accepts a borderline sentence, there is a close world—"close" in the sense that matters to knowledge attributions, as opposed to Δ —in which that sentence (or its counterpart) expresses a proposition that is false in that world. Williamson (2016, 847–850) sketches such an account. It would be a mistake, we maintain, to identify the "closeness" relation the epistemicist uses for interpreting the definiteness operator with the safety theorist's closeness relation. Both are epistemic relations, but they are not the same.

The second reason is that we agree with Williamson that there is no reason to expect the correct safety-theoretic account of ignorance of borderline matters to fall out of the correct semantics for the definiteness operator. An analogy with the standard possible-worlds semantics (which we have assumed) for the metaphysical necessity operator may be useful here. According to that semantics,

$\Box\phi$ is true iff the proposition expressed by ϕ is true in every possible world. This is not a theory, in any substantial sense, of metaphysical necessity. The semantics is fairly uninformative as to the nature of metaphysical necessity, and it cannot be faulted for that since its purpose is not to shed light on the nature of metaphysical necessity. Similarly, the two epistemicist semantics for the definiteness operator considered above should not be expected to shed much light on the nature of vagueness as an epistemic phenomenon. The epistemicist's theory of vagueness can reasonably be expected to explain why we cannot have knowledge of borderline matters, but the epistemicist's semantics for "definitely" cannot. As Williamson himself puts it:

just as a theory of the meaning of the word "light" is not tasked with explaining the underlying nature of light [...] so a theory of the meaning of the word "indefinite" is not tasked with explaining the underlying nature of indefiniteness. [...] Thus, even if safety-theoretic considerations are central to the underlying nature of definiteness, they need not figure in a good semantic account of the word "indefinite" or the Δ operator, just as, even if safety-theoretic considerations are central to the underlying nature of knowledge, they need not figure in a good semantic account of the word "knowledge". (Williamson 2016, 845)

8.3 Truth and Consequence

Whatever else might be said about epistemicism, it has one great advantage over supervaluationism (and arguably also over the version of metaphysicalism discussed here): it makes it perfectly clear what it means for a sentence to be simply true. What it is for the sentence ϕ to be simply true is for the proposition actually expressed by ϕ to be true in the actual world. Or slightly more carefully: it is for the character ϕ actually has to determine, when applied to the actual world, a proposition that is true in the actual world (on the first view) or for the two-dimensional epistemic intension ϕ actually has to determine, when applied to the actual world, a proposition that is true in the actual world (on the second view). In symbols:

- ϕ is true iff $@, @, @ \models \phi$

(This is why it matters, for the epistemicist, that $W \subseteq V$.)

Unlike in the supervaluationist case where there is some unclarity over which notion of consequence is the right one, there is no such worry for the epistemicist. The right notion of consequence here is preservation of simple truth—or, more precisely:

- ϕ is a consequence of Γ iff for all \mathcal{M} and all worlds w , if $\mathcal{M}, w, w, w \models \Gamma$ then $\mathcal{M}, w, w, w \models \phi$.

Appendix A: Completeness

In this appendix we present a proof system for a quantified modal logic with both definiteness, necessity, and actuality operators. The main technical complication in the completeness proof is that individual constants are not rigid with respect to the vagueness parameter.

A.1 Consequence

The models are as in the main text. We need the following notions of consequence.

Definition A.1

- (i) ϕ is a *strong consequence* of Γ ($\Gamma \models_s \phi$) iff for all models \mathcal{M} and all v, c, w such that $\mathcal{M}, v, c, w \models \Gamma$ we have $\mathcal{M}, v, c, w \models \phi$.
- (ii) ϕ is a *consequence* of Γ if for all models \mathcal{M} and all v, w , if $\mathcal{M}, v, w, w \models \Gamma$ then $\mathcal{M}, v, w, w \models \phi$.
- (iii) ϕ is an *epistemicist consequence* of Γ if for all epistemicist models \mathcal{M} and all w , if $\mathcal{M}, w, w, w \models \Gamma$ then $\mathcal{M}, w, w, w \models \phi$.

A sentence ϕ is *strongly valid* if ϕ is a strong consequence of every set of sentences; ϕ is *valid* if ϕ is a consequence of every set of sentences. ϕ is an epistemicist validity if ϕ is an epistemicist consequence of every set of sentences.

A.2 Proof System

We first axiomatize the notion of strong consequence. We have the following axioms and rules for the propositional fragment. We write Γ/ϕ for a rule of proof that allows us to infer ϕ from Γ .

(Taut)	ϕ , for any truth-functional tautology ϕ
(\mathbf{K}_{\Box})	$\Box(\phi \rightarrow \psi) \rightarrow (\Box\phi \rightarrow \Box\psi)$
(\mathbf{K}_{Δ})	$\Delta(\phi \rightarrow \psi) \rightarrow (\Delta\phi \rightarrow \Delta\psi)$
(Modus Ponens)	$\phi, \phi \rightarrow \psi / \psi$
(\Box -Necessitation)	$\phi / \Box\phi$
(Δ -Necessitation)	$\phi / \Delta\phi$

We have the following axioms and rules for quantification. Note the restriction to variables in (UI).

(UI)	$\forall x\phi(x) \rightarrow (Ey \rightarrow \phi(y))$, where y is a <i>variable</i> .
(Actualism)	$\forall xEx$
(Distribution)	$\forall x(\phi \rightarrow \psi) \rightarrow (\forall x\phi \rightarrow \forall x\psi)$
(Vacuous Quantification)	$\phi \rightarrow \forall x\phi$, as long as x is not free in ϕ
(Generalization)	$\phi / \forall x\phi$

We have the following principles governing identity.

(Reflexivity)	$t = t$
(Symmetry)	$s = t \rightarrow t = s$
(Transitivity)	$s = t \rightarrow (t = u \rightarrow s = u)$
(Restricted LL)	$x = y \rightarrow (\phi(x) \rightarrow \phi(y))$.

Note that Leibniz’s law is restricted to variables. This is important, since otherwise all identities would be definite. This restriction means that we cannot derive that identity is an equivalence relation for constants from (LL), so we have to add axioms to this effect separately.

We have the following rigidity axioms

$(\Box(=))$	$c = d \rightarrow \Box(c = d)$
$(\Box(\neq))$	$c \neq d \rightarrow \Box(c \neq d)$
$(\Box\text{-rigidity})$	$Et \rightarrow \exists x\Box(x = t)$
$(\Delta(\neq))$	$x \neq y \rightarrow \Delta(x \neq y)$.

The rule of $(\Delta(\neq))$ is not required if Δ is taken to satisfy **B** $_{\Delta}$. $(\Delta(\neq))$ is required to ensure the definiteness of distinctness.

Since metaphysical necessity should satisfy **S5** and definiteness should satisfy at least **T** we add the following axioms.

(\mathbf{T}_{\Box})	$\Box\phi \rightarrow \phi$
$(\mathbf{5}_{\Box})$	$\Diamond\phi \rightarrow \Box\Diamond\phi$
(\mathbf{T}_{Δ})	$\Delta\phi \rightarrow \phi$

To deal with the actuality operator we introduce axioms to the effect that (i) it is definite that there is an actual world; (ii) that there is only one actual world; (iii) and that the actual world is possible.⁴⁴ Let \mathcal{A} be the following set of sentences of \mathcal{L} : $\{\Delta^n(\Diamond A\phi \rightarrow \phi) : \phi \text{ is a sentence of } \mathcal{L}, n \in \mathbb{N}\}$ We then lay down the following axioms

- (Possible Actuality) $\Delta^n\Diamond(\bigwedge_{0 \leq i \leq m} \phi_i), \alpha_i \in \mathcal{A}$ for each $i \leq m$ and each n .
- (Unique Actuality) $\Delta^n((A\phi \rightarrow A\psi) \rightarrow A(\phi \rightarrow \psi)),$ for each n
- (Actuality Possible) $\Delta^n(A\perp \rightarrow \perp)$

We say that ϕ is a *strict theorem* ($\vdash_s \phi$) if ϕ can be deduced using the above axioms and rules. We say that ϕ is *strictly deducible* from Γ ($\Gamma \vdash_s \phi$) if either $\vdash_s \phi$ or there are $\gamma_0, \gamma_1, \dots, \gamma_n \subseteq \Gamma$ such that $\vdash_s (\gamma_0 \wedge \gamma_1 \wedge \dots \wedge \gamma_n \rightarrow \phi)$.

To axiomatize the validities we do as follows. Say that ϕ deducible ($\vdash \phi$) if $\mathcal{A} \vdash_s \phi$. We say that ϕ is deducible from Γ ($\Gamma \vdash \phi$) if there are $\alpha_0, \dots, \alpha_n \subset \mathcal{A}$ and $\{\gamma_0, \dots, \gamma_m\} \subset \Gamma$ such that $\vdash \bigwedge_{i \leq n, j \leq m} \{\alpha_i, \gamma_j\} \rightarrow \phi$.

Since \Box definitely satisfies **5** and **T** the following are strict theorems.

- (i) $\vdash \neg A\phi \rightarrow A\neg\phi$
- (ii) $A(\phi \rightarrow \psi) \rightarrow (A\phi \rightarrow A\psi)$
- (iii) $A\neg\phi \leftrightarrow \neg A\phi$

If we want to validate the Commutativity and Church-Rosser conditions we add the following axioms.

(Commutativity1) $\Box\Delta\phi \rightarrow \Delta\Box\phi$

(Commutativity2) $\Delta\Box\phi \rightarrow \Box\Delta\phi$

(Church-Rosser) $\Diamond\Delta\phi \rightarrow \Delta\Diamond\phi$

Observation A.2 If $\vdash \phi$ then $\vdash \Delta\phi$.

Proof: Suppose that $\vdash \phi$. Let $\alpha_0, \alpha_1, \dots, \alpha_n$ be such that $\vdash_s \alpha_0 \wedge \alpha_1 \wedge \dots \wedge \alpha_n \rightarrow \phi$. Then since the strict validities are closed under Δ -necessitation, we have $\vdash_s \Delta(\alpha_0 \wedge \dots \wedge \alpha_n \rightarrow \phi)$. So we get $\vdash_s \Delta\alpha_0 \wedge \dots \wedge \Delta\alpha_n \rightarrow \Delta\phi$ by \mathbf{K}_Δ and Modus Ponens. Since $\Delta\alpha_i \in \mathcal{A}$ for each $i \leq n$ it follows that $\vdash \Delta\phi$. \square

Since $\vdash \phi \leftrightarrow A\phi$ (indeed: $\vdash \phi \leftrightarrow \Box A\phi$) it follows that $\vdash \Delta(\phi \leftrightarrow A\phi)$.

Note that while \Box -necessitation is a valid rule of proof for \vdash_s it is not a valid rule of proof for \vdash .

We use the following shorthand. If Π is a sequence $\langle \pi_0, \dots, \pi_n \rangle$ where for each i , \Box_{π_i} is either \Box or Δ we write: $\Box_{\Pi}\phi$ for $\Box_{\pi_0} \Box_{\pi_1} \dots \Box_{\pi_n} \phi$. We write $\Box\phi$ to ambiguously denote $\Box_{\Pi}\phi$ for arbitrary Π .

We use the notation $\odot_{\Pi}\phi$ similarly. Here \odot_{π_i} is either \Diamond or ∇ . We ambiguously write $\odot\phi$, when we mean that $\odot_{\Pi}\phi$ for some Π .

Note that the following are derivable.

($\mathbf{K}_{\text{mixed}}$) $\vdash \Box_{\Pi}(\phi \rightarrow \psi) \rightarrow (\odot_{\Pi}\phi \rightarrow \odot_{\Pi}\psi)$

($\mathbf{T}_{\text{mixed}}$) $\vdash \Box_{\Pi}\phi \rightarrow \phi$

A.3 Soundness

It is routine to establish that all the axioms (except the members of \mathcal{A}) are strongly valid and that \Box -necessitation, Δ -necessitation and Modus Ponens preserve strong validity. This suffices to establish the soundness theorem for \vdash_s .

Proposition A.3 If $\Gamma \vdash_s \phi$ then ϕ is a strong consequence of Γ .

It is also straightforward to establish the soundness theorem for \vdash .

Proposition A.4 If $\Gamma \vdash \phi$, then ϕ is a(n) (epistemicist) consequence of Γ .

Proof: To establish this it suffices to show that each $\alpha \in \mathcal{A}$ is a(n) (epistemicist) validity. Suppose that α is $\Delta^n(\Diamond A\phi \rightarrow \phi)$. Let \mathcal{M} and v, w be given. Let v' be any v' such that v' is reachable from v in n moves along the relation R_Δ . We have to show that $v', w, w \models \Diamond A\phi \rightarrow \phi$. But this is immediate, for

suppose there is w' such that $v', w, w' \models A\phi$, where $(v', w)R_{\square}(v', w')$. Then $v', w, w' \models \phi$ follows by the semantic clause for A . \square

Note that in this result we tacitly use that if $(v, w)R_{\square}(v', w')$ then $v' = v$.

A.4 Diagrams

To prove completeness we extend the method of diagrams from Fine (1978). We begin by establishing completeness for \vdash_s .

One of the complications in our system is that the individual constants are not rigid with respect to Δ . To get around this problem we introduce a special class of constants that are rigid. Let Γ be a (\vdash or \vdash_s) consistent set sentences. Let C be a set of constants none of which occur in Γ . Let \mathcal{L}^+ be the language that results from \mathcal{L} by adding all the constants in C . Let Γ^+ be obtained from Γ by adding the following axioms

- (i) $\square(\forall x\phi \rightarrow (Ec \rightarrow \phi(c)))$, for each ϕ of the extended language and each $c \in C$.
- (ii) $\square(c = d \rightarrow \square c = d)$
- (iii) $\square(c \neq d \rightarrow \square c \neq d)$, for all $c, d \in C$.

It is routine to establish the following.

Proposition A.5 If ϕ is a sentence in \mathcal{L} and $\Gamma^+ \vdash \phi$, then $\Gamma \vdash \phi$.

We call any constant c such that the above principles hold for it a *definite constant*; and we write $D(\mathcal{L})$ for the set of definite constants of \mathcal{L} .

Let V, W be two sets. A *term* (over V, W) is a triple (v, w, ϕ) where $v \in V, w \in W$ and ϕ is sentence in \mathcal{L} . A *diagram* over V, W is a tuple $\mathcal{D} = \langle T, R_{\square}, R_{\Delta} \rangle$. Here T is a set of terms and R_{\square} and R_{Δ} are 4-place relations in $V \times W \times V \times W$ such that:

- If $(v, w)R_{\square}(v', w')$ then $v = v'$
- $(v, w)R_{\Delta}(v', w')$ then $w = w'$.

If $\mathcal{D} = \langle T, R_{\square}, R_{\Delta} \rangle$ we let $T(v, w) = \{\phi : (v, w, \phi) \in T\}$. Note that we do not require that for all $v \in V, w \in W$ there is a term of the form (v, w, ϕ) . If $\mathcal{D} = \langle T, R_{\square}, R_{\Delta} \rangle$ is a diagram, we use $\mathcal{D} + (v, w, \phi)$ to denote the diagram that results from adding (v, w, ϕ) to the set of terms T .

Let \mathcal{D} be a diagram. A *path* in \mathcal{D} is a sequence of pairs $\langle (v_0, w_0), \dots, (v_1, w_1), \dots, (v_k, w_k) \rangle$ where for each i , $(v_i, w_i)R_{\pi_i}(v_{i+1}, w_{i+1})$; here R_{π_i} is either R_{\square} or R_{Δ} . We often write paths in the following way: $(v_0, w_0) \xrightarrow{\pi_0}$

$(v_1, w_1) \xrightarrow{\pi_1} \dots \xrightarrow{\pi_{k-1}} (v_k, w_k)$. If Π is a path in \mathcal{D} the *path description* of Π is the following collection of sentences:

$$D_\Pi = \{\odot_\Pi \bigwedge \Gamma : \Gamma \text{ is finite and } \Gamma \subseteq T(v, w),$$

where (v, w) is the last member of the path $\Pi\}$

Definition A.6 Let \mathcal{D} be a diagram over V, W . We say that \mathcal{D} is *consistent* if for all $v \in V, w \in W$ the following set of sentences is consistent (with respect to \vdash_s).

$$\mathcal{D}(v, w) = T(v, w) \cup \bigcup \{D_\Pi : \Pi \text{ is a path in } \mathcal{D} \text{ such that } \Pi \text{ begins at } (v, w)\}$$

If \mathcal{D} is a diagram, we say that $\mathcal{D}(v, w) \vdash_s \phi$ iff the diagram $\mathcal{D} + (v, w, \neg\phi)$ is inconsistent.

Definition A.7 A diagram $\mathcal{D} = \langle T, R_\square, R_\Delta \rangle$ over V, W is

- (i) \neg -complete if for all $v \in V, w \in W$, either $(v, w, \phi) \in T$ or $(v, w, \neg\phi) \in T$.
- (ii) \diamond -complete if for all v, w if $(v, w, \diamond\phi) \in T$ then there is v, w' such that $(v, w)R_\square(v, w')$ and $(v, w', \phi) \in T$.
- (iii) ∇ -complete if for all v, w if $(v, w, \nabla\phi) \in T$ then there is v', w such that $(v, w)R_\Delta(v', w)$ and $(v', w, \phi) \in T$.
- (iv) \exists -complete if for all $v \in V, w \in W$ if $(v, w, \exists x\phi) \in T$ then there is a definite constant $c \in D(\mathcal{L})$ such that both $(v, w, \phi(c))$ and (v, w, Ec) are in T .
- (v) $R_{\square, \Delta}$ -complete if R_Δ^w is reflexive for all w ; and R_\square^v is an equivalence relation for all v .
- (vi) *nice* if it is consistent, \neg -complete, \diamond -complete, ∇ -complete, \exists -complete, and $R_{\square, \Delta}$ -complete.

Definition A.8 Let $\mathcal{D} = \langle T, R_\square, R_\Delta \rangle$ be a diagram. An *error* in \mathcal{D} is a set of pairs of one of the following forms:

- $\{(v, w)\}$, where $((v, w), (v, w))$ is not in one of R_\square or R_Δ ;
- $\{(v, w), (v, w')\}$ where we have $(v, w)R_\square(v, w')$ but not $(v, w')R_\square(v, w)$
- $\{(v, w), (v, w'), (v, w'')\}$ where we have $(v, w)R_\square(v, w')$ and $(v, w')R_\square(v, w'')$ but not $(v, w)R_\square(v, w'')$.

An *error sequence* is an ordered finite list of errors.

To prove completeness we need the following lemma.

Lemma A.9 Let \mathcal{D} be a consistent, finite diagram, and let $(v_{@}, w_{@})$ be a pair of points in \mathcal{D} . There is a nice diagram \mathcal{D}^+ such that \mathcal{D}^+ extends \mathcal{D} . Moreover, \mathcal{D}^+ has the following features:

- (i) If (v, w) and (v, w') are two distinct indices in \mathcal{D} then $(v, w)R_{\square}(v, w')$
- (ii) If (v, w) and (v', w) are two distinct indices in \mathcal{D} then $(v, w)R_{\Delta}^{\infty}(v', w)$ or $(v', w)R_{\Delta}^{\infty}(v, w)$. (Here R_{Δ}^{∞} is the transitive closure of R_{Δ})
- (iii) Every pair (v, w) is the endpoint of a path starting in $(v_{@}, w_{@})$

Using Lemma A.9 completeness is straightforward.

Theorem A.10 Let Γ is consistent. Then there is a model \mathcal{M} and $v_{@}, w_{@}, w_0$ such that $\mathcal{M}, v_{@}, w_{@}, w_0 \models \Gamma$.

Proof: Consider the following diagram: $\mathcal{D}_0 = \langle T_0, R_{\square}^0, R_{\Delta}^0 \rangle$. Here $T_0 = \{v_{@}\} \times \{w_{@}\} \times \mathcal{A} \cup \{v_{@} \times \{w_0\} \times \Gamma$. We let R_{\square} be the least equivalence relation extending $\{((v_{@}, w_{@}), (v_{@}, w_0))\}$. And we let R_{Δ} be the least reflexive relation. To see that \mathcal{D}_0 is consistent we show that $\mathcal{D}(v_{@}, w_{@})$ is consistent. (The case of $\mathcal{D}(v_{@}, w_0)$ is dealt with similarly.) Suppose then that $\mathcal{D}(v_{@}, w_{@})$ is inconsistent. Then there are $\{\alpha_0, \dots, \alpha_n\} \subset \mathcal{A}$ and $\gamma_0, \dots, \gamma_m$, where each γ_i is a conjunction of formulae in Γ such that

$$\vdash_s \odot_{\Pi_0} \alpha_0 \wedge \dots \wedge \odot_{\Pi_n} \alpha_n \wedge \odot_{\Pi'_0} \gamma_0 \wedge \dots \wedge \odot_{\Pi'_m} \gamma_m \rightarrow \perp$$

Here each Π_i is a path beginning in $(v_{@}, w_{@})$; each Π'_j is path beginning in $(v_{@}, w_{@})$ and ending in $(v_{@}, w_0)$. By $\mathbf{T}_{\text{mixed}}$ and $\mathbf{K}_{\text{mixed}}$ we have that $\vdash_s (\alpha_0 \wedge \alpha_1 \wedge \dots \wedge \alpha_n) \rightarrow \odot_{\Pi_0} \alpha_0 \wedge \dots \wedge \odot_{\Pi_n} \alpha_n$. Since we have $\vdash_s \diamond(\alpha_0 \wedge \dots \wedge \alpha_n)$ it follows by \mathbf{K}_{\square} that $\vdash_s \diamond(\odot_{\Pi_0} \alpha_0 \wedge \dots \odot_{\Pi_n} \alpha_n)$. It then follows by \square -necessitation and \mathbf{K}_{\square} that

$$\vdash_s \diamond(\odot_{\Pi'_0} \gamma_0 \wedge \dots \wedge \odot_{\Pi'_m} \gamma_m \rightarrow \perp)$$

That is

$$\vdash_s \diamond(\square_{\Pi'_0} \neg \gamma_0 \vee \dots \vee \square_{\Pi'_m} \neg \gamma_m)$$

thus

$$\vdash_s \diamond \square_{\Pi'_0} \neg \gamma_0 \vee \dots \vee \diamond \square_{\Pi'_m} \neg \gamma_m$$

Since each of the paths Π'_i begin in $(v_{@}, @)$ and end in $(v_{@}, w_0)$, we know that $\Box_{\Pi'_i}$ has the form $\Delta \cdots \Delta \Box \cdots \Box \gamma_i$. By \mathbf{T}_{Δ} and \mathbf{K}_{\Box} we can get rid of the initial applications of Δ (if any) and get:

$$\vdash_s \Diamond \Box \Box_{\Pi'_0} \neg \gamma_0 \vee \cdots \vee \Diamond \Box \Box_{\Pi'_m} \neg \gamma_m$$

By the 5 axiom for \Box this gives us

$$\vdash_s \Box \Box_{\Pi'_0} \neg \gamma_0 \vee \cdots \vee \Box \Box_{\Pi'_m} \neg \gamma_m$$

But this contradicts that Γ is consistent; for if Γ is consistent, then $\odot_{\Pi}(\gamma_0 \wedge \cdots \wedge \gamma_m)$ is consistent for each path-description Π .

So let \mathcal{D} be a nice diagram extending D_0 where \mathcal{D} has the features claimed in Lemma A.9.

Let $W = W_{(v_{@}, w_{@})} = \{w : (v, w) \text{ occurs on some path starting in } (v_{@}, w_{@})\}$. We let $V = M_{(v_{@}, w_0)} = \{v : (v, w) \text{ occurs on some path starting in } (v_{@}, w_0)\}$.

We define an equivalence relation \approx on the set of definite constants $\mathcal{D}(\mathcal{L})$ as follows. We say that $c \approx d$ iff $(v, w, c = d) \in \mathcal{D}$ for some v, w . It is easy to see that \approx is reflexive and symmetric. To see that it is transitive, suppose that $(v, w, c = d)$ and $(v', w', d = e)$ are both in \mathcal{D} . Then let Π and Π' be paths ending in (v, w) and (v', w') respectively. Then we have $\odot_{\Pi} c = d$ and $\odot_{\Pi'} d = e$ both in $\mathcal{D}(v_{@}, w_{@})$. But by the supernecessity of distinctness for the definite constants we then have $c = d$ and $d = e$ both in $\mathcal{D}(v_{@}, w_{@})$ and so $c = e$ in $\mathcal{D}(v_{@}, w_{@})$. So \approx is transitive.

Let $[c]$ be the equivalence class of c under \approx . We let $\mathbf{D} = \{[c] : c \text{ is a definite constant}\}$. We define the interpretation function $\llbracket \cdot \rrbracket$ as follows.

- If t is a constant we put $\llbracket t \rrbracket(v, w) = [c]$ where $(v, w, c = t) \in \mathcal{D}$.
- If R is an n -place predicate we let $\llbracket R \rrbracket(v, w) = \{([c_0], \dots, [c_{n-1}]) : (v, w, R(c_0, \dots, c_n)) \in \mathcal{D}\}$.

Let $\mathcal{M}_{\mathcal{D}} = \langle V, W, \mathbf{D}, R_{\Box}, R_{\Delta}, \llbracket \cdot \rrbracket \rangle$.

We show that, for all v and w , $\mathcal{M}, v, w_{@}, w \models \phi$ iff $(v, w, \phi) \in \mathcal{D}$ by induction on the complexity of ϕ .

The proof is routine, except for the case of the actuality operator.

Suppose that $v, w_{@}, w \models A\phi$. Then $v, w_{@}, w_{@} \models \phi$. By the induction hypothesis $(v, w_{@}, \phi) \in \mathcal{D}$. Suppose (for contradiction) that $(v, w, \neg A\phi) \in \mathcal{D}$. Then, since A commutes with negation, $(v, w, A\neg\phi) \in \mathcal{D}$. By our assumptions on \mathcal{D} we have that $(v, w_{@})R_{\Box}(v, w)$. So by niceness we have $(v, w_{@}, \Diamond A\neg\phi) \in \mathcal{D}$, otherwise $\mathcal{D}(v, w_{@})$ is inconsistent. By our assumptions we also know that $(v_{@}, w_{@})R_{\Delta}^{\infty}(v, w_{@})$. And so we have $\Diamond A\neg\phi \rightarrow \neg\phi$ a member of $\mathcal{D}(v, w_{@})$. But then $\neg\phi$ is a member of $\mathcal{D}(v, w_{@})$ and so $\mathcal{D}(v, w_{@})$ is inconsistent. We conclude that $(v, w_{@}, A\phi) \in \mathcal{D}$.

Suppose next that $A\phi \in \mathcal{D}(v, w)$. We show first that $\phi \in \mathcal{D}(v, w_{@})$. By our assumptions about \mathcal{D} we know that $(v, w_{@})R_{\square}(v, w)$. Since $\mathcal{D}(v, w_{@})$ is consistent and \neg -complete we have $\diamond A\phi \in \mathcal{D}(v, w_{@})$, and so, since $\mathcal{A} \subset \mathcal{D}(v, w_{@})$ we have ϕ in $\mathcal{D}(v, w_{@})$. The induction hypothesis gives that $v, w_{@}, w_{@} \models \phi$. And so $v, w_{@}, w \models A\phi$.

This concludes the proof of completeness with respect to strong consequence. □

We use same technique to establish completeness with respect to consequence.

Proposition A.11 Suppose $\Gamma \cup \mathcal{A}$ is consistent. There is a model \mathcal{M} and v, w such that $\mathcal{M}, v, w, w \models \Gamma \cup \mathcal{A}$.

Proof: Consider the diagram $\{v_{@}\} \times \{w_{@}\} \times \Gamma \cup \mathcal{A}$ where R_{\square} and R_{Δ} both is the relation $((v, w), (v, w))$. □

We turn towards establishing Lemma A.9.

Proof Lemma A.9: Let $D_0 = \langle T_0, R_{\square}^0, R_{\Delta}^0 \rangle$ be a consistent diagram, where the terms are indexed with pairs in $V_0 \times W_0$. For simplicity assume that V_0 and W_0 are finite and that \mathcal{L} is countable. Let V_1 be countably infinite such that $V_0 \cap V_1 = \emptyset$; and let W_1 be countably infinite such that $W_0 \cap W_1 = \emptyset$. Let $V = V_0 \cup V_1$, $W = W_0 \cup W_1$. Finally, let C be a countably infinite collection of fresh individual constants. Per Observation A.2 the diagram is also consistent in the stronger logic that results from adding the axioms $\square(\forall x\phi \rightarrow (Ec \rightarrow \phi(c)))$, and $c \neq d \rightarrow \square(c \neq d)$ for all new constants c, d . (Recall that these axioms codify that the constants in C are definite constants.)

We construct a sequence of consistent diagrams $\mathcal{D}_0, \mathcal{D}_1, \mathcal{D}_2, \dots$, and a sequence of error sequences E_0, E_1, \dots , as follows. E_0 is the set of errors in \mathcal{D}_0 . (Pick an arbitrary ordering.)

The idea of the construction is that at the even stages $0, 2, 4, \dots$, we may add new terms (v, w, ϕ) to the diagram. At an odd stage $2n + 1$ we pick the least error in E_n and remedy it.

Let \mathcal{L}^+ be the language that results from adding the individual constants C to \mathcal{L} . Let $\eta_0, \eta_2, \eta_4, \dots$, be an enumeration of the terms (v, w, ϕ) such that each term occurs infinitely many times. (Here $v \in V, w \in W$ and $\phi \in \mathcal{L}^+$.)

Assume that $\mathcal{D}_n = \langle T_n, R_{\square}^n, R_{\Delta}^n \rangle$ and E_n have been constructed. We proceed as follows.

- (i) If $n + 1$ is odd, let e be the least error in E_n .
 - If e is of the form $\{(v, w)\}$ add $((v, w), (v, w))$ to R_{\square}^n and R_{Δ}^n ;
 - if e is of the form $\{(v, w), (v, w')\}$ where we have $(v, w)R_{\square}^n(v, w')$ but not $(v, w')R_{\square}^n(v, w)$, add $((v, w'), (v, w))$ to R_{\square}^n .

- if e is of the form $\{(v, w), (v, w'), (v, w'')\}$ where we have $(v, w)R_{\square}^n(v, w')$ and $(v, w')R_{\square}^n(v, w'')$ but not $(v, w)R_{\square}^n(v, w'')$, add $((v, w), (v, w''))$ to R_{\square}^n .
 - This removes the least error in E_n . Order any new errors and add them after the least error in E_n to obtain E_{n+1} .
- (ii) Suppose $n + 1$ is even. Say that $\eta_{n+1} = (v, w, \phi)$. If no term of the form (v, w, ψ) occurs in T_n , put $\mathcal{D}_{n+1} = \mathcal{D}_n$. If a term of the form (v, w, ψ) occurs in T_n we proceed as follows:
- (a) If $\eta_{n+1} \notin T_n$, let $T_{n+1} = T_n \cup \{(v, w, \phi)\}$ or $T_{n+1} = T_n \cup \{(v, w, \neg\phi)\}$, depending on which results in a consistent diagram.
 - (b) If $\eta_{n+1} \in T_n$ and ϕ is not of the form $t = s$, $s \neq t$ (where s, t are old constants), $\exists x\theta$, $\diamond\theta$ or $\nabla\theta$, let $T_{n+1} = T_n$.
 - (c) If ϕ is $s = t$ or $s \neq t$, where s, t are old constants, let c, d be two fresh constants and let $T_{n+1} = T_n \cup \{(v, w, c = s), (v, w, d = t)\}$.
 - (d) If ϕ is $\exists x\theta$ let $c \in C$ be one of the fresh constants that has not been used before and let $T_{n+1} = T_n \cup \{(v, w, \theta(c)), (v, w, Ec)\}$.
 - (e) If ϕ is $\diamond\theta$ let $w' \in W$ be an index that does not occur in T_n and let $T_{n+1} = T_n \cup \{(v, w', \theta)\}$. We extend R_{\square}^n to $R_{\square}^{n+1} = R_{\square}^n \cup \{(v, w), (v, w')\}$.
 - (f) If ϕ is $\nabla\theta$ let $k \in V$ be an index that does not occur in T_n and let $T_{n+1} = T_n \cup \{(k, w, \theta)\}$. We extend R_{Δ}^n to $R_{\Delta}^{n+1} = R_{\Delta}^n \cup \{(v, w), (k, w)\}$.

After having carried out one of these steps we order the new errors in \mathcal{D}_{n+1} and append them to E_n to obtain E_{n+1} .

Claim: \mathcal{D}_{n+1} is consistent if \mathcal{D}_n is consistent. We only consider a few cases. Suppose first that $n + 1$ is even.

We consider the cases where η_{n+1} is $(v, w, \exists x\theta)$ and $\eta_{n+1} = (v, w, \diamond\theta)$.

In the first case we have to show that $\mathcal{D}_n \cup \{(v, w, Ec)\} \cup \{(v, w, \theta(c))\}$ is consistent. Suppose otherwise. Then for some (v', w') we have that $\mathcal{D}^{n+1}(v', w')$ is inconsistent.

Then $\mathcal{D}_n(v', w') \cup \{\odot_{\Pi_0} \wedge \{\Gamma_0, \exists x\theta, \theta(c), Ec\}\} \cup \dots \cup \{\odot_{\Pi_n} \wedge \{\Gamma_n, \exists x\theta, \theta(c), Ec\}\}$ is inconsistent for some v', w' . Here Π_0, \dots, Π_n are some paths beginning in (v', w') and $\Gamma_0, \dots, \Gamma_n$ are some sets of sentences such that $(v, w, \gamma) \in \mathcal{D}_n$, for each $\gamma \in \Gamma_i$, $0 \leq i \leq n$. We then have:

$$\begin{aligned} & \vdash \left(\bigwedge \mathcal{D}_n(v', w') \wedge \odot_{\Pi_0} \left(\bigwedge \Gamma_0 \wedge \exists x\theta \wedge Ec \wedge \theta(c) \right) \wedge \dots \wedge \right. \\ & \left. \odot_{\Pi_n} \left(\bigwedge \Gamma_n \wedge \exists x\theta \wedge (Ec \wedge \theta(c)) \right) \right) \rightarrow \perp \end{aligned}$$

It follows that

$$\begin{aligned} \vdash \bigwedge \mathcal{D}_n(v', w') \rightarrow \Box_{\Pi_0} \left(\bigwedge \Gamma_0 \wedge \exists x\theta \wedge (Ec \rightarrow \neg\theta(c)) \right) \vee \dots \vee \\ \Box_{\Pi_n} \left(\bigwedge \Gamma_n \wedge \exists x\theta \wedge (Ec \rightarrow \neg\theta(c)) \right) \end{aligned}$$

By $\mathbf{T}_{\text{mixed}}$ we get

$$\begin{aligned} \vdash \bigwedge \mathcal{D}_n(v', w') \rightarrow \left(\bigwedge \Gamma_0 \wedge \exists x\theta \wedge (Ec \rightarrow \neg\theta(c)) \right) \\ \vee \dots \vee \left(\bigwedge \Gamma_n \wedge \exists x\theta \wedge (Ec \rightarrow \neg\theta(c)) \right) \end{aligned}$$

By generalization we get:

$$\begin{aligned} \vdash \forall z \left(\bigwedge \mathcal{D}_n(v', w') \rightarrow \left(\bigwedge \Gamma_0 \wedge \exists x\theta \wedge (Ez \rightarrow \neg\theta(z)) \right) \right) \\ \vee \dots \vee \left(\bigwedge \Gamma_n \wedge \exists x\theta \wedge (Ez \rightarrow \neg\theta(z)) \right) \end{aligned}$$

Since c does not occur free in either θ or Γ it follows by (Distribution) and (Vacuous Generalization)

$$\begin{aligned} \vdash \bigwedge \mathcal{D}_n(v', w') \rightarrow \forall z \left(\left(\bigwedge \Gamma_0 \wedge \exists x\theta \wedge (Ez \rightarrow \neg\theta(z)) \right) \right) \\ \vee \dots \vee \left(\bigwedge \Gamma_n \wedge \exists x\theta \wedge (Ez \rightarrow \neg\theta(z)) \right) \end{aligned}$$

Since the consequent is inconsistent we have shown that $\vdash \neg \bigwedge \mathcal{D}_n(v', w')$, contradicting that \mathcal{D}_n was consistent.

We next deal with the case where $\eta_{n+1} = (v, w, \diamond\phi)$. We have to show that adding (v, w', ϕ) and extending R_{\square}^n by adding $(v, w)R_{\square}(v, w')$ leads to a consistent diagram. Suppose otherwise. Then there is some (v_0, w_0) such that $\mathcal{D}_{n+1}(v_0, w_0)$ is inconsistent. That is $\vdash \mathcal{D}_n(v_0, w_0) \wedge \odot_{\Pi_0}\phi_0 \wedge \dots \wedge \odot_{\Pi_n}\phi_n \rightarrow \perp$, where each Π_i is a path in \mathcal{D}_{n+1} . For each Π_i we show that there is path-description Π'_i from \mathcal{D}_n with endformula ψ such that $\odot_{\Pi_i}\phi_i = \odot_{\Pi'_i}\psi$. If we can show this we have shown that \mathcal{D}_n already is inconsistent.

Let Π_i be given. If Π_i is a path in $\mathcal{D}^n(v_0, w_0)$ there is nothing to show. So suppose otherwise. Then Π_i is of the form $\Pi'_i \rightarrow (v, w)R_{\square}(v, w')$, where Π'_i is a \mathcal{D}_n -path. Since (v, w') is fresh, we know that $\phi_i = \phi$. So let $\psi_i = \diamond\phi$. Then $\odot_{\Pi_i}\diamond\phi_i = \odot_{\Pi_i}\phi_i$, which is what we have to show.

Consider next the case where $n+1$ is odd. We consider only the case where the error remedied is a failure of symmetry. Suppose that the least error in E_n is $e = \{(v, w), (v, w')\}$. At stage $n+1$ we then added $((v, w'), (v, w))$ to R_{\square} . Let (v_0, w_0) be an index. We want to show that $\mathcal{D}_n(v_0, w_0) \vdash \mathcal{D}_{n+1}(v_0, w_0)$. Let Π be a path in \mathcal{D}_{n+1} and let $\odot_{\Pi}\phi$ be the corresponding path-description. If Π does not contain a move $(v, w') \rightarrow (v, w)$,

Π is a path in \mathcal{D}_n and there is nothing to do. Assume first that Π contains a single move $(v, w) \xrightarrow{\square} (v, w')$. Then Π has the form

$$(v_0, w_0) \rightarrow \Pi' \rightarrow (v, w') \xrightarrow{\square} (v, w) \rightarrow \Pi''$$

where both Π' and Π'' are paths in \mathcal{D}_n . But then $\odot_{\Pi'}\phi \in \mathcal{D}_n(v, w)$. And so by \mathbf{B}_{\square} we have $\mathcal{D}_n(v, w) \vdash \square\Diamond \odot_{\Pi'}\phi$. It then follows that $\mathcal{D}_n(v, w') \vdash \Diamond \odot_{\Pi'}\phi$ and so we also have $\mathcal{D}_n(v_0, w_0) \vdash \odot_{\Pi'}\Diamond \odot_{\Pi''}\phi_i$, that is, $\mathcal{D}_n(v_0, w_0) \vdash \odot_{\Pi}\phi_i$. If Π contains n moves $(v, w) \xrightarrow{\square}$ we repeat the above procedure n -times, starting with the last move. Since we can do this for each path Π it follows that $\mathcal{D}_n(v_0, w_0) \vdash \mathcal{D}_{n+1}(v_0, w_0)$, and so \mathcal{D}_n is inconsistent if \mathcal{D}_{n+1} is.

Let $T_\omega = \bigcup_{n < \omega} T_n$; let $R_{\square} = \bigcup_{n < \omega} R_{\square}^n$; let $R_{\Delta} = \bigcup_{n < \omega} R_{\Delta}^n$. Let $\mathcal{D} = \langle T_\omega, R_{\square}, R_{\Delta} \rangle$. Clearly, \mathcal{D} is a nice diagram. Moreover, by inspection of the construction we see that the diagram has the other features claimed for it in Lemma A.9. □

Remark A.12 Inspection of the proofs shows that we can arrange that $V \subseteq W$ and that we can set $v_{@} = w_{@}$. This means that we also have established completeness for epistemicist consequence, and indeed that epistemicist consequence coincides with consequence.

By fairly straightforward modifications of the strategy one can obtain the following result. The logic that results from adding the commutativity and Church-Rosser conditions is complete with respect to product models. It is also straightforward to extend this to a completeness proof for constant (increasing/decreasing) domain models by adding the appropriate instances of the Barcan and Converse Barcan formulae.

Notes

1. Independently, Torza (2015) has also proposed a framework for the interaction between necessity and definiteness. But his framework does not show how to deal with an actuality operator.
2. Only normally, because “ ϕ ” may concern how we use “ ϕ ”.
3. Consider the epistemicist, for whom the vagueness of “ ϕ ” roughly speaking consists in the existence of at least two meanings p and q that are indiscriminable from the actual meaning of “ ϕ ”. One could argue as follows. p and q are distinct iff there is a world w in which p and q differ in truth value; so (given further plausible principles) if “ ϕ ” is vague, then there is a world w in which it is borderline whether ϕ , so it is possible that it is borderline whether ϕ . To argue for the converse, we can invoke the plausible (for epistemicists) principle that, if it is possible that it is borderline whether ϕ , then there are actually at last two meanings indiscriminable from the actual meaning of ϕ which possibly differ in truth value.

4. The canonical development is Fine (1975).
5. A similar view has been developed by Sorensen (1988), but we focus on Williamson because Sorensen, unlike Williamson, does not offer an account of ignorance due to vagueness from which any very definite—no pun intended—interpretation of the definiteness operator could be extrapolated.
6. The term “semantic plasticity” is not Williamson’s. It was introduced to the literature by Hawthorne (2006).
7. While our focus will be on Williamson’s version of epistemicism, the formal framework we develop can be taken over by those who are attracted to other versions of epistemicism.
8. Barnes and Williams use the terminology of “actuality” and “actualized”. The present terminology is better, as we will see in §7.2.
9. The reason we do not discuss Bacon’s theory is simply that no draft of his book was available yet at the time when we did the bulk of our work on this paper.
10. E.g., Williamson (1994, Ch. 7, Sec. 4.)
11. Four points. First, one could of course formulate the relevant supervenience theses in a language that explicitly quantifies over possible worlds and individuals. But we assume that some version of modalism is correct, and that “possible world” talk is analyzable in terms of the necessity operator, e.g., as in Fine (1977). (It is in any case of interest to develop a logic in which modalists can express the relevant supervenience theses.) Second, an actuality operator is not enough. Properly to express the supervenience theses of interest we need something like Vlach-operators. The present framework can be extended to incorporate such operators, though it is not entirely straightforward how to do so. Third, one might want to formulate the thesis using propositional (or more generally: higher-order) quantification. While this is of interest we are setting such extensions aside for now. Fourth, and relatedly, if we do have sentential quantification can the values of the sentential variables be vague? The supervaluationist and epistemicist say not; the metaphysicalist says yes. This means that the supervaluationist and epistemicist can only state supervenience schematically.
12. Supervaluationism and epistemicism are both broadly linguistic accounts of vagueness.
13. See, e.g., Bave (2011); Korman (2010); Liebesman and Eklund (2007); Sider (2003, 2009).
14. The latter is just for convenience. The reader can take it to be defined from identity and existential quantification.
15. For us ∇ means “it is not definite *that* not”. In some of the literature on vagueness (going back at least to Evans (1978)) one uses ∇ to mean “it is indefinite *whether*”.
16. Kaplan (1989) is more prominently associated with this idea in the philosophical public mind, no doubt because of the captivating philosophical picture he painted to go along with his doubly indexed semantics for the combined modal-temporal logic LD.
17. Because A is the only indexical we will be dealing with, we can pretend that both contexts and circumstances are simply worlds. To deal with various other indexicals we would have to introduce additional parameters to contexts—*times*, *persons*, *locations* and so on—and if we treat tenses as temporal operators following Montague, Kamp, Kaplan, and many others, we would have to also follow these authors in taking circumstances to be pairs of worlds and times.

18. By imposing these conditions we can look at R_Δ as a function from V to relations in $W \times W$ and we can look at R_\square as a function from W to relations in $V \times V$.
19. We have not demanded that constants be rigid with respect to the context parameter. If we allow them to vary we can model demonstratives and indexicals.
20. See Williamson (2003) and Yli-Vakkuri (2006, 818-819).
21. One exception is Magidor (2006).
22. For the record: we are doubtful.
23. In the terminology of (Asher, Dever, and Pappas 2009) the first view is *local* supervaluationism; the second is *global* supervaluationism. The third view is a hybrid. For more discussion of the options see Asher, Dever, and Pappas (2009) and Varzi (2007).
24. There is a subtlety that the present formulation of the theory does not settle. Is the semantic value of a constant (relative to v, c) a constant individual concept (function from worlds to objects) or is it simply an object? One could reasonably argue that it is only the latter that gives us genuine rigid designation. Since nothing in what follows turns on this and we could implement either account we take no stand on this issue.
25. Since we are working in a free logic if we do not throw in the side premiss *Ea* the scheme is clearly invalid—for reasons having nothing to do with vagueness.
26. As pointed out by Torza (2015, 386), certain modal versions of these principles also fail, e.g., the principle: if $\Gamma, \phi \models_g \perp$, then $\Gamma \models_g \neg\Diamond\phi$. Incidentally, this consequence relation is a counterexample to Barnes and Williams (2010)'s claim that supervaluationist treatments of \square and Δ in which the principle that $\Gamma \models \neg\Diamond\phi$ when $\Gamma, \phi \models \perp$ fails must allow some sentences that are inconsistent in the normal modal logic of \square to be satisfiable. The consequence relation Torza himself defines is also a counterexample.
27. Barnes and Williams (2009) correctly observe that metaphysical vagueness could well give rise to linguistic vagueness, and since there is no problem with vague identity on a linguistic account of vagueness (like supervaluationism) the metaphysicist does not have to rule out that identity can be, in some sense, vague. What the defender of vague identity may have to rule out is that identity can ever be *metaphysically* vague. In our view, if there is both linguistic and metaphysical vagueness one should theorize using languages with *two* distinct definiteness operators.
28. This shows that the criticisms of Barnes and Williams given by Akiba (2015) are mistaken.
29. We should mention that Barnes and Williams develop a formal account of their own. Let us indicate how their account is related to ours. Their basic idea is that each world w is associated with a “halo” of precisificational alternatives to that world. From each halo we then have to “select” a representative. More formally, Barnes and Williams's models are tuples: $\mathcal{M} = (W, \mathcal{U}, D, \Sigma, \mathcal{I})$. Here W is the space of worlds, \mathcal{U} the space of halos, Σ the collection of selection functions, D the domain and \mathcal{I} the interpretation function. They make the (reasonable) demands that for each $\sigma \in \Sigma$ and each $U \in \mathcal{U}$, $\sigma(U) \in U$ and that for each $U \in \mathcal{U}$ and each $w \in U$, there is $\sigma \in \Sigma$ such that $\sigma(U) = w$. They then define the notion of truth relative to halo, selection-function, assignment and model. The relevant semantic clauses are:

- if $P(\bar{a})$ is atomic, $P(\bar{a})$ is true at U, σ, g iff $P(\bar{a})$ is $\sigma(U)$.
- $U, \sigma, g \models D\phi$ iff $U, \sigma', g \models \phi$ for all $\sigma' \in \Sigma$.
- $U, \sigma, g \models \Box\phi$ iff $U', \sigma, g \models \phi$ for all $U \in \mathcal{U}$.

It turns out that the determined by these clauses is exactly the logic determined by the constant domain $S_5 \times S_5$ product logic.

Every model $\mathcal{M} = (W, \mathcal{U}, D, \Sigma, \mathcal{I})$ gives rise to a product model in a natural way. For let the set of worlds of evaluation be \mathcal{U} and let the set of vagueness parameters be Σ . If $P\bar{a}$ is atomic we say that $P\bar{a}$ is true at (σ, U) iff $P(\bar{a})$ is true at $\sigma(U)$. It is a straightforward induction to show that ϕ is true at (σ, U) iff ϕ is true at (σ, U) in their sense.

Conversely, every $S_5 \times S_5$ product model gives rise to one of their models. For let $V \times W$ be the frame of an $S_5 \times S_5$. We let the set of worlds be $V \times W$. Let $\mathcal{U} = \{U_w : U = \{(v, w) : v \in V\}w \in W\}$. For each v let $\sigma_v : \mathcal{U} \rightarrow V \times W$ be defined by $\sigma_v(U) = (v, w)$. Then let $\Sigma = \{\sigma_v : v \in V\}$. If $P(\bar{a})$ is atomic we define $P(\bar{a})$ to be true at $\sigma_v(U)$ iff $P(\bar{a})$ is true at (v, w) . It is a straightforward induction to show that if ϕ is a formula then ϕ is true at (σ, U) in the product logic sense iff ϕ is true at (σ, U) in their sense.

30. See Caie (2011), Magidor (2016), and Yli-Vakkuri (2016). The term “otherworldly” was introduced into the literature by Caie.
31. For example, because there are worlds that differ only slightly from the actual world in the global patterns of language use, but where there were no space flights in 1969, by (OWD) it is not definite that a human walked on the moon in 1969.
32. An account along these lines is developed in (Magidor 2016). Her view is not exactly what is described here, but it is also refuted, in our view, by the argument that follows. Magidor, however, would reject the argument because she rejects the K axiom (what she calls “distribution”) for definiteness. In other work Kearns and Magidor (2012) also rejects one of the premises of the argument: that the semantic supervenes on the non-semantic.
33. This argument is a close variant of Yli-Vakkuri’s (2016, 828-829) argument against a certain kind of semantic plasticity-based explanation of ignorance of borderline matters.
34. This argument assumes that in all close worlds the actuality operator has the rigidifying semantics it is usually taken to actually have. This strikes us as an extremely safe assumption: even if the English “actually” does not work in this way, we can stipulatively introduce an operator that does, in which case no close world will disrespect our stipulation.
35. But not for everyone: Dorr (2003) and Barnett (2011) argue that it is possible to have knowledge of borderline matters.
36. This is not obviously the right way of thinking about names like Everest. For one could think that there is an object that “Everest” definitely refers to; it is just that it is vague what location this object has, what parts it has, and so on. But we can set this problem aside. For even if there is an object such that “Everest” definitely refers to it, one can introduce vague names for which is indefinite which objects they refer to. For let ϕ be any borderline sentence. Following the example of Williamson (1994, 253-254), we can introduce the name “Boris” using the following reference fixing description: “Boris = 0 if ϕ , and otherwise Boris = 1”.

And even if this does not work one can always run the problem using T-sentences, as Hawthorne does.

37. Again, as predicted in Yli-Vakkuri (2013, 562, n. 49).
38. See Putnam (1975, 234) for the classic expression of the view that terms introduced by reference-fixing descriptions are indexicals (there Putnam is discussing natural kind terms, but it is hard to see why his conclusion would not also apply to “meter”); see Parsons (1974) and Glanzberg (2001, 2004) for contextualist solutions to the liar paradox.
39. Or equivalently, as Chalmers has it, functions from pairs of worlds to truth values.
40. There is no harm in having all of the worlds in a triple be centered: the consequence will only be that operators other than Δ are insensitive to the agents and times built into them.
41. For the record: we are skeptical that there are any things that satisfy that role.
42. See Yli-Vakkuri and Hawthorne (forthcoming, ch. 4).
43. See Williamson (2000), Appendix 4.
44. We here extend the presentation of (Hodes 1984).

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