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Niels Öffenberger und Alejandro G. Vigo

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ANALOGICAL DEDUCTION VIA A CALCULUS OF PREDICABLES

JOSEPH P. LI VECCHI

1. Introduction

Medieval and modern logicians typically agree with Aristotle that a term is univocal if in different cases it corresponds to the same rational content, and equivocal if in different cases its corresponds to different rational content.[1] They agree also that a syllogism is valid if and only if premises entail conclusion with the force of logical necessity, and that such entailment requires a non-equivocal middle term.[2] However, these logical traditions differ about whether the middle term of a valid syllogism must be univocal. Thomas de Vio Gaetano (1469-1534), known as Cajetan to the Anglophone world, holds with Aristotle and Aquinas that a valid syllogism may alternatively employ an analogous middle term. For Cajetan a term is analogous if the rational content to which it corresponds in one case is different but logically related to the rational content to which it corresponds in another case so that it may function as a syllogistic middle.[3] A term is equivocal if in different cases it corresponds to rational content that is different but not so related. By contrast, David Hume (1711-1776), in the spirit of John Duns Scotus, makes no such contrast between analogous and equivocal terms.[4] Hume judges that syl-

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[1] Aristotle, *Categories*, 1a1-13; Aquinas, Thomas, *Commentary on the Metaphysics of Aristotle*, lib. 4, lect. 1 n. 7; Mill, John Stuart, *A System of Logic: Ratiocinative and Inductive*, Chapter II "Of Names", sect. 8 "Univocal and Aequivocal".

[2] Aristotle, *Prior Analytics*, I, 1, 24b18-30; *Posterior Analytics*, I, 11, 77a5–9.

[3] Cajetan, Thomas de Vio, *De nominum analogia, Scripta philosophica*, ed. Zammit, P. N., (Rome, Institutum Angelicum, 1934, revised by Hering, H., 1951), sects. 106, 109. Aristotle, *Posterior Analytics*, II, 14, 98a20-23; 99a16; Aquinas, Thomas, *Summa theologiae*, Ia, q. 13, a. 5, co.

[4] Scotus, John Duns, *Ordinatio*, I, d. 3, qq. 1, 2, responsio 2, *Opera omnia*, ed. K. Balić (Rome, Editio Vaticana, 1954). Hume does not employ the distinction, traceable among Latin philosophers to Boethius, between chance equivocation (*a casu*) and deliberate equivocation (*a consilio*), according to which analogy is identified with a subdivision of the latter. An investigation into the tradition of divisions of equivocation and analogy would profitably begin with the

logisms with analogous middles can be inductively cogent at best since, in his view, the different rational content corresponding to the analogues is related sufficiently to ensure only the probable entailment of conclusion from premises.[5] The present article endeavors to employ the technical apparatus of standard symbolic logic to support Cajetan's claim.[6]

2. Ancient and Scholastic Doctrines of Analogy

Scholastic doctrine on analogy and inference draws its principle inspiration from the logical works of Aristotle, who makes three fundamental assertions pertaining to syllogistic inference and analogous middle terms. First, Aristotle recognizes that only non-equivocal middle terms possess the logical unity required for scientific demonstration.

"Demonstration necessarily implies the possibility of truly predicating the same term of many individuals. Without this possibility we have no universal, and without a universal we have no middle term, and so demonstration

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work of E. Jennifer Ashworth, particularly her "Medieval Theories of Analogy", in: *Stanford Encyclopedia of Philosophy*, 1999-2009, ed. Edward N. Zalta.

[5] Hume, David, *Dialogues Concerning Natural Religion*, Part II, p. 144, ed. Norman Kemp Smith (London, Thomas Nelson & Sons, 1947).

[6] Previous attempts to formally express the logic of analogy include Bochenski, Joseph, "On analogy", *The Thomist* 11 (1948) 474–497, reprinted in: Menne, Albert, ed., *Logico-Philosophical Studies* (Reidel, Dortrecht, 1962); Dorrough, D. C., "A Logical Calculus of Analogy Involving Functions of Order 2", *Notre Dame Journal of Formal Logic*, XI, no. 3 (1970), 321–336; Krause, Andrej, *Zur Analogie bei Cajetan und Thomas von Aquin* (Studien & Vorträger am Philosophischen Institut Halle, Hallescher Verlage, 1999). For an exposition and discussion of Cajetan's doctrine on analogy see Hochschild, J., in: *The Semantics of Analogy According to Thomas De Vio Cajetan's De nominum analogia* (University of Notre Dame, dissertation, 2001). Also of interest is the late Ralph McInerny's criticism of *De nominum analogia* as an inaccurate exposition of Aquinas' doctrine, for example, "Where Cajetan Went Wrong", in: *Aquinas and Analogy* (The Catholic University of America Press, Washington, DC, 1996), 3–29. Hochschild argues that Cajetan's aim is not so much to represent Aquinas as to elucidate the nature of analogy. Juthe, A., "Argument by Analogy", *Argumentation*, 19 (2005), 1–27, discusses thelogical structure of analogical argument, though with little mention of Aquinas and none of Cajetan.

becomes impossible. We conclude, then, that demonstration requires a single identical term unequivocally predicable of a number of individuals."[7]

The rational content that an equivocal middle term associates with the major term differs from the rational content that it associates with the minor so that the middle does not logically unite major and minor under a single general concept, and the conclusion does not follow with deductive force. It should be noted, however, that Aristotle stipulates only that a middle term should be non-equivocal (mè homónymon). He does not, for example, go so far as to require it to be univocal (synónymon), a form of predication that he discusses elsewhere without hesitation.[8]

Second, Aristotle claims that in addition to univocal and equivocal terms a third type must be recognized since sometimes the essence of a non-empirical thing can be understood by comparison to the empirical.

"We must not seek a definition of everything but sometimes be content to grasp things by analogy, as we grasp what a builder is from a building, and what sleeping is from waking, and what the ability to see is, even when eyes are shut, from the act of seeing, and the notion of 'unshaped material', from what has shape, and the notion of 'the unworked' from that of a finished product."[9]

Aristotle's examples here raise many questions worthy of consideration. For example, the notion of 'unshaped material' on the face of it seems to be paradoxical. Strictly speaking, any material thing has shape. Aristotle's point here is that we may speak of a material thing as being unshaped with respect to some shape that it potentially possesses. While the "unshaped" is not empirically verifiable, it can be understood by analogy with what is empirically verifiable.

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[7] Aristotle, *Posterior Analytics*, I, 11, 77a5–9. Translations of all non-English texts throughout this essay are mine unless otherwise noted.

[8] Aristotle, *Categories*, 1a6.

[9] Aristotle, *Metaphysics*, X, 6, 1048a36-1048b4.

Finally, speaking directly to the question of analogous terms in logical inference, Aristotle explicitly asserts that an analogous middle term is a legitimate means of uniting the major and minor terms of a syllogism.

"A further method of selecting [a middle term to establish entailment] is by analogy: for we cannot find a single identical name for a squid's pounce, a fish's spine, and an animal's bone, although these possess common properties as if there were a single osseous nature... Analogical middle terms can be used to prove that things are identical by analogy."[10]

Here Aristotle's remark addresses the conventional nature of language and the theory of natural kinds. Given that common linguistic usage does not exhaustively reflect the real similarities among things, a term properly applicable to one may be applicable also to a similar thing by analogy in view of some common rational characteristic. When such a term is employed as a syllogistic middle it possesses sufficient logical unity to unite extremes, and thereby to engender scientific demonstration. In the absence of a deeper dialectical analysis or a more precise technical apparatus, however, Aristotle's remarks remain inchoate.

Thomas Aquinas, promoting the scholastic agenda of natural theology, famously applies Aristotle's doctrines on analogical predication and inference to the problem of establishing the rationality of discourse about the divine attributes.

"It is impossible to predicate something of God and creature univocally. This is so because every effect not equal to the power of the agent cause receives likeness of the agent not according to the same ratio, but deficiently, so that what is divided and many in the effects, in the cause is simply and in one manner... But neither is any predicated purely equivocally, as some have said, for otherwise nothing could be known or demonstrated about God from creatures without always encountering the fallacy of equivocation... Thus, it must be admitted that such names are said of God and creatures by analogy, which is to say, by proportion."[11]

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[10] Aristotle, *Posterior Analytics*, II, 14, 98a20-23; 99a16.

[11] Aquinas, Thomas, *Summa theologiae* Ia, q. 13, a. 5, co. Cf. *De potentia Dei*, q. 7, a. 7.

On the one hand, the same rational content cannot be predicated of God and creature univocally on pain of metaphysically assimilating God the cause of all perfections, to the creatures in which these perfections are divided among many. On the other hand, if rational content could be predicated of God and creature only equivocally, then there would be no science of the divine attributes.

"With his doctrine on analogical predication Aquinas aims to bridge the epistemic gap between God and creature, while preserving their metaphysical difference. In one kind of predication, traditionally called "analogy of inequality", the same term relates different things to rational content that is the same inasmuch as it ascribes membership of many things to a common genus, although in different degrees in each case."[12]

Cajetan agrees with Aquinas that such predication is univocal with respect to the rational content predicated of different subjects, and equivocal with respect to the being this rational content assumes in subjects of predication.

"This analogy St. Thomas in his Commentary on the Sentences I, dist. 19 calls "analogy according only to being" since the analogues are comparable in the ratio signified by the common name, but are not comparable in the being of that ratio. For it has more perfect being in one case than in the other... This sort of analogy the logician calls "univocal", but the philosopher "equivocal" since the one considers the intentions of the word, and the other considers natures."[13]

Whether considered logically with respect to the intentions predicated, or metaphysically with respect to the being of what is predicated, this kind of predication is not appropriate for God and creature since considered as uni-

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[12] Aquinas, *Scriptum super libros sententiarum*, lib. I, dist. 19, q. 5 a. 2 ad 1: Analogy can be said "according to being and not according to intention, and that happens when many things are similar in the intention of something common, but that common thing does not have the being of one intention in all of them, as when all bodies are similar in the intention of corporality".

[13] Cajetan, op. cit., sects. 5-6.

vocation it metaphysically assimilates God and creature, and considered as equivocation it fosters no science of the divine attributes.

In another kind of predication, traditionally called "analogy of attribution", there is a shared unity of meaning derivable from the proportional relation of analogues to some single thing. Here the same rational content is predicated of many things, but has being only in one of them to which the others are related.[14] This is the "*pròs hén*" analogy of Aristotle's *Metaphysics*,[15] call analogia proportionis by Aquinas and analogia attributionis by Cajetan, and often divided into analogy of attribution by intrinsic or extrinsic denomination.[16]

Aquinas notes that the unity of meaning in this kind of predication is entirely lacking among pure equivocals, and is different only in proportion from the unity of meaning among univocals.

"In what is said analogically, the ratio is neither one, as it is in univocals, nor totally different, as in equivocals. Rather, an analogous term signifies different proportions with respect to some one thing, as 'healthy' when said of urine signifies a sign of an animal's health, and when said of medicine signifies the cause of that health."[17]

Commenting on Aquinas' doctrine Cajetan notes that such predication is a form of equivocation.

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[14] Aquinas, *Scriptum super libros sententiarum*, lib. I, dist. 19, q. 5 a. 2 ad 1: Analogy can be said "according to intention alone, and not according to being, and this is when an intention refers to many in a determinate order, of which it does not have being except in one, as the intention of health refers to animal, urine, and diet in different ways, in a determinate order, not according to different being, since the being of health is only in an animal".

[15] Aristotle, *Metaphysics*, IV, 2: "Being is said variously, but related to one (*pròs hén*), and of the same nature and not just the same name, as the healthy is related to health, to preserve it, to produce it, to signify being healthy, and the reason why it can be".

[16] Aquinas, *Summa theologiae*, loc. cit.; Cajetan, op. cit., sects. 10-11.

[17] Aquinas, *Summa theologiae*, loc. cit.: "Neque enim in his quae analogice dicuntur, est una ratio, sicut est in univocis; nec totaliter diversa, sicut in aequivocis; sed nomen quod sic multipliciter dicitur, significat diversas proportiones ad aliquid unum; sicut sanum, de urina dictum, significat signum sanitatis animalis, de medicina vero dictum, significat causam eiusdem sanitatis".

"These analogues are called "equivocal" by the logician, as is clear in the beginning of the Categories, where animal is said equivocally of a real animal and of a picture of an animal. An animal picture is called "animal", not by pure equivocation, but by attribution to a real animal, and in its ratio inasmuch as it manifestly shows an animal, a real animal is mean... These are called analogous among the Latins because diverse proportions are said to relate to one... Nevertheless this is a misuse of the term, though much less so than the first case, [the so-called "analogy of inequality"]."[18]

Thus, for Cajetan, the analogy of attribution, like the analogy of inequality, cannot foster the science of the divine names.

In a third kind of predication, called "analogy of proper proportionality" by Cajetan,[19] unity of meaning derives neither from common rational content nor from the relation of analogues to some single thing, but from the identity of the proportion exhibited by the rational content of each analogue.[20] Aquinas provides one example from mathematics and another from psychology.

"Agreement is occasionally noted, not between two things that have a proportion between them, but between two related proportions. For example, six has something in common with four because six is two times three, just as four is two times two. The agreement between healthy urine and healthy medicine is one of proportion. That between six and four is one of proportionality. We find something predicated analogously of two realities according to the first type of agreement when one of them has a relation to the other... Sometimes, however, a thing is predicated analogously according to the second type of agreement, as when sight is predicated of bodily sight and of the intellect because understanding is in the mind as sight is in the eye."[21]

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[18] Cajetan, op. cit., sects. 19, 21.

[19] Cajetan, op. cit., distinguishes proper proportionality from metaphorical proportionality in sect. 25.

[20] Cf. Aquinas, *Scriptum super libros sententiarum*, lib. I, dist. 19, q. 5 a. 2 ad 1: Analogy can be said "according to intention and to being, as when it is similar neither in a common intention, nor in being, as 'being' (*ens*) is said of substance and of accident. And of such kind the common nature must have some being in each of the analogues of which it is said, but differing according to a ratio of greater or lesser perfection".

[21] Aquinas, Thomas, *Quaestiones disputatae de veritate*, q. 2, a. 11.

While Aquinas does recognize this as the type of analogy suitable for predication of God and of creature,[22] he does not explain the logical superiority of this type over the first two with respect to the capacity to foster valid deduction.[23] Thus his logical doctrine on analogy as well as his natural theology of the divine attributes is left open to the criticism of John Duns Scotus.

Scotus focuses his analysis of valid deduction on the capacity of terms to engender contradiction when they are simultaneously affirmed and denied of the same subject. It is clear to Scotus that univocal terms possess the logical unity required to beget such contradiction:

"Let there be no disagreement concerning the word 'univocation'. I call a concept 'univocal' which is one in the sense that its unity suffices for contradiction when affirming and denying it of the same thing."

Lacking a comprehensive logical analysis of analogous terms, however, Scotus concludes that only a univocal term has sufficient logical unity of meaning to function "as a syllogistic middle so that the extremes united by the middle can be judged to be united as one without the fallacy of equivocation".[24] Therefore, in opposition to his Thomistic contemporaries, Scotus limits what can be known scientifically about God's existence or nature to what can be deduced by syllogisms employing exclusively univocal middle terms. Given the theological requirement of metaphysical difference between God and creature, rational theology under Scotus' regimen becomes a largely impossible project. For Scotus only a univocal middle term is sufficient to ensure that a syllogism's conclusion follows with logical necessity from its premises. Syllogisms with analogous middle terms commit the fallacy of equivocation.

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[22] Aquinas, Thomas, *Scriptum super libros sententiarum*, lib. I, dist. 19, q. 5 a. 2 ad 1: "Et similiter dico, quod veritas et bonitas et omnia hujusmodi dicuntur analogice de Deo et creaturis".

[23] While this is Aquinas' text that best supports Cajetan's claims about the deductive capacity of analogues by proper proportionality, it is commonly treated as a deviation from Aquinas' usual teaching following Bernard Montagnes' commentary and evaluation in *La doctrine de l'analogie de l'être d'après Saint Thomas d'Aquin* (Paris, Publications Universitaires, Louvain, Béatrice-Nouwelaerts, 1963).

[24] John Duns Scotus, *Ordinatio*, I, d. 3, qq. 1, 2, responsio 2. Scotus appears to be the first to define univocals in terms of their ability to mediate syllogistic inference.

Cajetan explicitly rejects Scotus' conclusion that only univocal middle terms suffice to avoid the fallacy of equivocation. Terms related by the analogy of proper proportionality possess sufficient logical unity to unite the major and minor terms of a syllogism.

"They are deceived who follow Scotus... Seeing in the analogue the diversity of logical features they do not consider its concealed unity and identity. For logical features can be accepted in two ways: one, in themselves, as they are distinguished from each other and those which agree with them as such, and another, as they are proportionally the same. Used in the first way they lead to equivocation. Used in the second way they do not because whatever agrees with one case agrees with the other case proportionally, and whatever is denied of one, is denied of the other proportionally. Whatever agrees with a similar, insofar as it is similar, agrees also with that to which it is similar, while always saving the proportionality."[25]

The meaning of an analogue can be ascertained because its rational content exhibits the same proportion exhibited by the rational content of another analogue whose meaning is already known. A syllogism whose middle term has rational content that is different in each premise but that is identical in virtue of a shared proportion avoids the fallacy of equivocation. On this basis Cajetan judges analogous middles to be suitable for use in deductive reasoning about the divine attributes. "With the proportionality saved, there is science of the analogue."[26]

On Cajetan's view terms related by the analogy of proper proportionality are unified by a concept that is superior, or more inclusive in extension than their concepts, namely, the concept of a proportion exhibited by a term's rational content. Univocal terms by contrast share a superior concept founded on particular shared rational content, not on a proportion exhibited by this content.

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[25] Cajetan, op. cit., sect. 106.

[26] Ibid., sect. 109: "Proportionalitate autem servata, de analogis scientiam esse".

"The superiority is based on the identity of the ratio of what is signified, that is, on the fact that what is signified is found not only in this [thing], but that same feature, same in ratio though not in number, is found in another [thing]. Univocation, however, is founded on every kind of identity, namely on the identity of the ratio of the thing signified, that is, on the fact that the ratio of the thing signified in this [thing] and in that are wholly the same... The analogue is proportionally superior, because it is founded on the identity of the proportions of the ratio of the things signified. The superiority of the univocal, however, is simple and precise, since it is founded on every kind of identity of the ratio of the thing signified."[27]

The concept of the proportion exhibited by the rational content of analogues is correspondingly imperfect in that its intention excludes the differentiae of the analogues. This superior and imperfect concept is not a common genus, but the concept of the identity of the proportion exhibited by the rational content of each analogue.[28] Thus, the foundation of sameness of analogous terms is not that they fall under an identical superior and imperfect concept whose rational contents are exhibited by both analogues.

If such a concept did constitute a common genus, for Cajetan it could not be used in predication of God and creature. In analogous predication of God and creature no rational content is taken univocally but its proportional identity is shared.

"The ratio of wisdom is accepted and aspects of it that are imperfections are taken away from it by the intellect. From the fact that that which is formally proper to it, includes perfection without imperfection, it is concluded thus that in God the ratio of wisdom is not wholly other nor wholly the same, but the same proportionally, because sameness between God and creature is not univocal, but analogous."[29]

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[27] Ibid., sect. 67-68.

[28] Ibid., sect. 70. Since this analogy indicates identity, formally and simply speaking the analogue is conceded to be an inconvertible and more common predicate. Nevertheless, as a universal it is not a genus, species, proper, definition, difference, or accident.

[29] Ibid., sect. 110.

Cajetan gives no indication of how the validity of syllogisms with analogous middles may be formally expressed. An explicit formalized account is given below of the proportional identity of rational content of syllogistic middles. This account, however, differs from Cajetan's by requiring that analogous predicates belong to a common genus, which each predicate instantiates according to its own differentia. Since this requirement concerns the analogues predicated and not their subjects of predication, it does not present an obstacle to analogous predication of God and creature, which cannot belong to a common genus. Though the point would require its own dedicated treatment, it may be noted briefly that Aquinas addresses this problem by distinguishing between the subject of predication according to its entity (*secundum rem*) and according to the rational content of its concept (*secundum rationem*).

"Plurality of attributes [predicated of God] in no way prejudges the highest unity, since those which in others are plural in him are one, and the plurality remains such secundum rationem, which is not opposed to the highest unity in re."[30]

Theology demands that God and creature share no common genus, but terms predicated of them may possess common rational content.

God and creature in no way have the same nature univocally speaking, but the rational content used to refer to them may have a common proportion. This proportion may be expressed in terms of the rational content of a genus and its differentiae. Adequate logical formalization of analogues by proper proportionality must express the relation of the identical proportion of rational content exhibited by each analogue to a common genus so that a common term may be used to indicate the different but proportionally identical rational content of each analogue without equivocation. Instantiation of the common genus occurs according to the identical proportion exhibited by the generic differentiae of each analogue.

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[30] Aquinas, *Scriptum super libros sententiarum*, lib. I, distinctio 2, questio 1, a. 2, ad 1. Aquinas treats the distinction in *Scriptum super libros sententiarum*, lib. I, d. 2, q. 1, aa. 2-3, *In De hebdomadibus*, lectio 2, nn. 32, 33, Summa theologica I, 11, 12, 13; and *Summa contra gentiles* 35, 6, 7.

The validity of a syllogism with an analogous middle does not depend upon the shared rational content corresponding to the common genus of the analogues, but on the common proportion exhibited by the differentia of a common genus. The common genus is the origin of a common term, which is used in different cases according to various differentiae, but which corresponds in each case to an identical proportion, thereby avoiding equivocation. Thus, the analogy of proper proportionality is established by more than structural isomorphism. In analogy structural isomorphism must be expressed with respect to rational content corresponding to a common genus. Formal analysis of the inferential force of syllogisms with analogous middles must express both the proportional isomorphism and the genus common to the analogous predicates in order to make formally manifest that the common proportion exhibited by the various generic differentiae is referred to by the analogous term.[31]

3. The Formalization of Analogical Deduction

Cajetan's observations about analogical deduction may be developed and supplemented by examining a natural language argument whose middle term is analogous by proper proportionality. If the term 'clear-sighted' in the fol-

[31] Following Cajetan, previous attempts to formalize the logical structure of analogy of proper proportionality do not recognize the role of the common genus in expressing that equivocation has been avoided. Bochenski, op. cit., (114–115) is concerned with expressing that cases of analogy are isomorphic relations of particular kinds: "These formal properties are different in each case of couples of isomorphic relations... E.g., in some cases both relations will be included in diversity and will be transitive; in other cases they will be intransitive and asymmetric etc.". To the contrary, in order to formalize the logical structure of analogy, rather than "introducing into the system the name of a new relation [for each case of analogy]", one need only to express the unique rational content of each analogue within a standard isomorphic logical structure, as will be illustrated below. A summary and partial critique of Bochenski's article may be found in Wilder, Alfred, "Bochenski and the Problem of Analogy", *Angelicum* 80 (2003) 35–52. Similarly, to the extent that Krause, op. cit. follows Bochenski's formal analysis in interpreting analogical predication, his interpretation of Cajetan's fundamenta similitudinis in analogues (Krause, 81) does not identify all features of the common isomorphic logical structure of analogy. The same can be said of Dorrough, op. cit., which examines a set of cases whose analogical relations depend on shared rational content (323–324), and to that extent, do not shed light on Cajetan's theological cases involving analogy between God and creature.

lowing argument were interpreted as being equivocal, meaning 'has accurate sensory perception' (S) in premise 1, and 'has accurate rational perception' (R) in premise 2, formalization in standard first-order predicate logic would show that the predicate 'has true beliefs' (B) in the consequent of premise 1 cannot be affirmed of Tiresias (t) with the force of deductive necessity:

1) If Tiresias is clear-sighted, then he has true beliefs
2) Tiresias is clear-sighted
3) Therefore, Tiresias has true beliefs.

1) $St \supset Bt$
2) Rt
3) Bt not justified by *modus ponens*.

However, standard first-order predicate logic ignores three features of this argument that ought to be expressed symbolically if the inferential link between premises and conclusion is to be made formally explicit.

First, the common genus of the analogues must be expressed. The two senses of 'clear-sighted', namely 'has accurate perception in vision' and 'has accurate perception in understanding' are both members of the genus 'has accurate perception' (P). As mentioned above, Cajetan recognizes only that terms related by the analogy of proper proportionality are related by the concept of their proportional identity.[32] He does not recognize that the proportional identity exhibited by the rational content of the analogues is expressed by differentiae of a common genus. It should be noted, however, that in agreement with Cajetan, the inferential capacity of analogous terms does not derive from the shared rational content attributable to this common genus.

The common genus of analogues is not only a broader concept, but also a comparatively imperfect one since it excludes the respective differentiae of analogous terms.[33] The second feature of an adequate symbolization of the analogy of proper proportionality that is overlooked by standard first-order predicate logic is the expression of these differentiae.

[32] Cajetan, op. cit., sects. 36, 40.

[33] Loc. cit.

While Cajetan overlooks the meaning of the common genus in expressing the common proportionality of analogues, he does recognize that analyzing terms "as they are proportionally the same" avoids equivocation.[34] Expression of this proportional similarity constitutes the third feature of an adequate symbolization of the analogy of proper proportionality that is overlooked by standard first-order predicate logic.

Given these three features of analogical predication the following technical apparatus permits evaluation of the deductive validity of the cited argument via what may be called a "calculus of predicables":

Let 'P' be a constant for 'perception'. Let 'B' be a constant for 'belief'.
Let 'V' be constant for 'vision'. Let 'E' be constant for 'eye'.
Let 'U' be constant for 'understanding'. Let 'M' be constant for 'mind'.

A term's generic difference can be expressed as a proportion in two different ways by filling the blank spaces of a proportion function '···/...'. The first of these ways expresses a proportion between the difference actually predicated and a range of possible differences for that genus. In this case the proportion function '···/...' is filled with a constant for the actual difference in the first blank, and the range of differences to which the actual difference belongs in the second blank. This can be expressed by expanding the symbolization key given above as follows:

Let 'S' be constant for 'sensation'.
Let 'O' be constant for 'olfaction'.
Let 'T' be constant for 'touch'.
Let 'A' be constant for 'audition'.
Let 'G' be constant for 'gustation'.

The difference symbol '$^{S}/_{S}$', expresses the proportion between the actual difference 'sensation' and a range of differences including only itself. Similarly, the difference symbol '$^{VOTAG}/_{VOTAG}$' expresses the proportion obtaining between the full actualization of the possible range of differences and that range. By contrast, the difference symbol '$^{OTG}/_{VOTAG}$' expresses the pro-

[34] Op. cit., sect. 106.

portion between the actual differences 'olfaction', 'touch' and 'gustation', and the complete range of possible differences for that genus.

An analogous term may be symbolized by filling the first blank space of an analogue function '…···/…', with a genus constant in the first blank, and a proportion function '···/…' as described above. For example, with respect to the analogues of the term 'clear-sighted' the analogue symbol '($P^{S}/_{S}$)' signifies that the genus corresponding to the predicate 'has accurate perception' is predicated in relation to the difference 'sensory' in the proportion of $^{S}/_{S}$. Similarly, the analogue symbol '$P^{T}/_{VOTAG}$' signifies that the same genus is predicated in relation to the difference 'tactile' in the proportion of $^{T}/_{VOTAG}$. Such symbolic formalization of the "predicamental structure" of analogous terms, the relation of their differences, species, genera etc., permits the proportional identity of these terms to be formally manifest.

The analogy of inequality can be formalized by means of this first mode of expressing a term as a proportion. This can be seen by formalizing the following natural language argument in terms of the apparatus and symbolization key articulated above:

1) If Helen Keller is clear-sighted, then she has true beliefs
2) Helen Keller is clear-sighted

3) Therefore, Helen Keller has true beliefs.

1) $(P\ ^{VOTAG}/_{VOTAG})h \supset (B\ ^{VOTAG}/_{VOTAG})h$
2) $(P\ ^{OTG}/_{VOTAG})h$

3) $(B\ ^{VOTAG}/_{VOTAG})h$

In this argument the instances of the middle 'clear-sighted' are similar in the intention of something common, but that common thing does not have the being of one intention in them, so the middle is equivocal. The argument would become valid only if the equivocal middle were replaced in each case by a common predicate, thus forging the inferential link via a univocal term, as the following formalization shows:

1) $(P\ ^{S}/_{S})h \supset (B\ ^{S}/_{S})h$
2) $(P\ ^{S}/_{S})h$
3) $(B\ ^{S}/_{S})h$

A term's generic difference can be expressed as a proportion in a second way. This can be seen where an analogous term is symbolized via an analogue function '$_{...}{}^{\cdots}/_{...}$' in which the first blank space is filled with a genus constant, and a proportion function '$^{\cdots}/_{...}$' is filled with a constant for the generic difference in the first space, and a constant for something to which this difference is being related proportionally in the second instance. For example, returning to the argument about Tiresias in which 'clear-sighted' is interpreted as 'has accurate perception in vision' (V) in premise 1, and 'has accurate perception in understanding' (U) in premise 2, the proportion function '$^{\cdots}/_{...}$' may be filled with a constant for the actual generic difference 'vision' in the first instance, and a constant for the eye, the organ by which that difference is actualized in the second instance. The analogue function '$P\ ^{V}/_{E}$' may thus signify that the genus 'perception' is predicated in relation to the actual difference 'vision',and that the actual difference is expressed in relation to the organ of vision, the eye, as the proportion '$^{V}/_{E}$'.

The analogue function '$P\ ^{V}/_{E}$' may also be set in an equation whose second term is a numerical proportion. For example, the expression '$P\ ^{V}/_{E} = {}^{1}/_{1}$' signifies that the actual generic difference 'vision' of the genus 'perception' actualizes its organ, the eye, completely, or in a proportion of $^{1}/_{1}$. The expression '$P\ ^{V}/_{E} = ½$' signifies that the actual generic difference 'vision' of the genus 'perception' actualizes its organ, the eye, half way, or in a proportion of $^{1}/_{2}$. The expression '$B\ ^{V}/_{E} = {}^{1}/_{1}$' signifies that a belief based on the actual generic difference 'vision' actualizes its organ the eye completely. The expression '$P\ ^{U}/_{M} = {}^{1}/_{1}$' signifies that the actual generic difference 'understanding' of the genus 'perception' actualizes its organ the mind completely; and the expression '$B\ ^{U}/_{M} = {}^{1}/_{1}$' signifies that the belief based on the actual generic difference 'understanding' actualizes its organ the mind completely.

From the expressions '$P\ ^{V}/_{E} = {}^{1}/_{1}$' and '$P\ ^{U}/_{M} = {}^{1}/_{1}$' a superior concept may be abstracted that represents the common proportional identity, '$P\ ^{1}/_{1}$', meaning that in each case there is complete perception according to some unspecified generic difference and means. Similarly from '$B\ ^{V}/_{E} = {}^{1}/1$' and '$B\ ^{U}/_{M} = {}^{1}/_{1}$' a superior concept may be abstracted that represents the common

proportional identity, '$B\ {}^{1}/_{1}$', meaning that in each case there is proportionally identical true belief according to some unspecified generic difference and means.

By expressing the proportional identity of the analogues but not their generic differentiae and means the following argument may be formalized in first-order predicate logic as indicated, and thus, recognized as a valid instance of *modus ponens*:

1) If Tiresias has perception in a proportion of $^{1}/_{1}$ with respect to some unspecified generic difference and means, then he has true belief in a proportion of $^{1}/_{1}$ with respect to some unspecified generic difference and means
2) Tiresias has perception in a proportion of $^{1}/_{1}$ with respect to some unspecified generic difference and means

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3) Therefore, Tiresias has true belief in a proportion of $^{1}/_{1}$ with respect to some unspecified generic difference and means.

1) $(P\ {}^{1}/_{1})t \supset (B\ {}^{1}/_{1})t$
2) $(P\ {}^{1}/_{1})t$

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3) $(B\ {}^{1}/_{1})t$ 1, 2 *modus ponens*.

On the other hand, by formalizing not only the proportional identity of the analogues but also their generic differentiae and means, the analogical inferential link of the following argument may be formalized as indicated:

1) If Tiresias has clear perception in the sense of having full ocular vision, then he has beliefs substantiated by full ocular vision
2)Tiresias has clear perception in the sense of having full mental understanding

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3) Therefore, Tiresias has beliefs substantiated by full mental understanding.

1) $(P\ {}^{1}/_{1}\ ({}^{V}/_{E} = {}^{1}/_{1}))t \supset (B\ {}^{1}/_{1}\ ({}^{V}/_{E} = {}^{1}/_{1}))t$
2) $(P\ {}^{1}/1\ ({}^{U}/_{M} = {}^{1}/_{1}))t$

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3) $(B\ {}^{1}/_{1}\ ({}^{U}/_{M} = {}^{1}/_{1}))t$ 1, 2 *modus ponens analogice*.

The rule of inference justifying this entailment functions much like the familiar modus ponens rule, with the significant difference that the inferential link is constituted not by a univocal term, but by an analogous one in virtue of the proportional identity of its instances. Accordingly, this rule may be called modus ponens analogice. The legitimacy of this rule and the calculus of predicables presented here ought to be judged on the same basis as that of the traditional modus ponens rule, namely, immediate logical intuition of its truth-preserving capacity. The present article has endeavored to facilitate this logical intuition by making explicit the relevant logical structures employed in inference by the analogy of proportionality.[35]

Prof. Dr. Joseph P. Li Vecchi
Department of Philosophy
The University of Akron
Akron, Ohio, USA

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