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The Existence of Axioms is an Axiom

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Abstract

We introduce the axiom of the existence of axioms and discuss its implications for logical systems and beyond.

Keywords: existence, axioms, loops, decidability, philosophical logic

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Introduction

1. This work emerged from the mathematical insights gained following the study of these books [1–11].

Definitions

2. We adopt the following definitions.

3. `axiom` := a proposition that is impossible to prove mathematically, even with an infinite number of axioms and rules of inference.

4. `theorem` := a proposition proven using axioms and rules of inference.

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Two Mutually Exclusive Axioms

5. $\mathtt{A}_1$: *Axioms exist.*

6. $\mathtt{A}_0$: *Axioms do not exist.*

7. Considering classical logic, either $\mathtt{A}_1$ is true or $\mathtt{A}_0$ is true.

Decomposition

8. Assuming $\mathtt{A}_1$ is true, axioms are atomic, meaning they cannot be broken down into smaller units composed of other axioms.

9. If $\mathtt{A}_0$ is true, then we call something an axiom simply because we do not know how to demonstrate it.

10. Moreover, if $\mathtt{A}_0$ is considered true, it implies that what we call an axiom could potentially be decomposed infinitely into smaller units of theorems.

11. Here, 'decomposed' means that there is an underlying demonstration for what was initially thought to be an axiom.

Axioms and Rules of Inference

12. An axiomatic system is a logical system composed of axioms, as defined in (3), and rules of inference.

Theorems and Rules of Inference

13. A `theorematic` system is a logical system composed exclusively of theorems, as defined in (4), and rules of inference.

14. If $\mathtt{A}_0$ is true, then $\mathtt{A}_0$ becomes a theorem, which can be decomposed, through rules of inference, into infinitely smaller theorems.

Loops

15. ℓ := `loop` := a sequence of applications of rules of inference on axioms or theorems, such that the same object proves itself.

16. $\mathtt{L}$:= loops exist, regardless of whether $\mathtt{A}_0$ or $\mathtt{A}_1$ is true.

17. If $\mathtt{L}$, is it possible to construct an abstract structure $\mathcal{A}$ from ℓ, such that the structure of $\mathcal{A}$ is independent of ℓ?

18. If so, is it possible that the abstractions $\mathcal{A}$ could be used to prove theorems?

19. Does $\mathcal{A}$ contain information about the rules of inference?

Rules of Inference

20. *Can a rule of inference be decomposed into smaller units?*

21. If yes, then are there atomic rules of inference, that is, rules that cannot be decomposed?

22. Following this line of reasoning, we can consider the decomposition of rules of inference as an axiom, as described below.

23. $\mathtt{R}_1$: *Rules of inference can be decomposed into smaller units.*

24. $\mathtt{R}_0$: *Rules of inference cannot be decomposed into smaller units.*

Existence

25. *Is existence a definition, an axiom, a theorem, a rule of inference, or some other type of abstraction?*

Decidability

26. [12–15]

27. Assume that mathematics is consistent.

28. Undecidable propositions, that is, true propositions that cannot be proven within an axiomatic system, differ from the definition of axioms ($\mathtt{A}_1$) as given in (5).

29. This distinction arises because, in the context of undecidability, as stated in Gödel's incompleteness theorem [12], the axiomatic system is considered to be finite.

Final Remarks

30. In a scenario where axioms do not exist fundamentally, mathematics would permanently abandon axiomatic systems and shift to operating exclusively within the framework of theorematic systems.

31. The existence of abstract structures built from logical loops could foster revolutions in mathematics, science, and philosophy.

32. If the rules of inference can be decomposed, whether finitely or infinitely, it will open a path of study for mathematicians and philosophers regarding their axiomatic nature.

33. If the rules of inference can be decomposed, whether finitely or infinitely, it will open new avenues of research for mathematicians and philosophers concerning their axiomatic nature.

Open Invitation

Review, *add* content to, and `co-author` this `white paper` [16, 17].
Join the **Open Philosophy Collaboration**.

Supplementary Files

[18]

How to Cite this White Paper

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Agreement

All authors are in agreement with the guidelines presented in [17].

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