

Grounds, roots and abysses

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Abstract The aim of this paper is to address the ‘Grounding Grounding Problem’, that is, the question as to what, if anything, grounds facts about grounding. I aim to show that, if a seemingly plausible principle of modal recombination between fundamental facts and the principle customarily called ‘Entailment’ are assumed, it is possible to prove not only that grounding facts featuring fundamental, contingent grounds are derivative but also that either they are (at least) partially grounded in the grounds they feature or they are ‘abysses’ (that is, derivative facts without fundamental grounds and lying at the top of an infinitely descending chain of ground).

Keywords grounding; recombination; Entailment; necessity; fundamentality

1 Introduction

Not every fact is fundamental. Some facts appear to metaphysically depend on other facts and thus, to be *grounded* in them. The notion of *grounding* lies at the centre of a hotly debated discussion in contemporary metaphysics.¹ Among the many questions that are animating the debate, an important issue is represented by the ‘Grounding Grounding Problem’ (henceforth: ‘GGP’).² Letting a ‘grounding fact’ be a fact of the form ‘(the plurality of facts) Γ grounds the fact that p ’, GGP can be presented as the question as to what, if anything, grounds grounding facts. In the literature three main families of theories have been presented in order to provide an answer to GGP:

- (T1) Bennett’s (2011) theory of grounding as a ‘superinternal relation’;³
- (T2) deRossett’s (2013), and Litland’s (forthcoming-a) theories linking grounding to ‘explanatory arguments’;
- (T3) the ‘essentialist’ accounts of Rosen (2010), Fine (2012) and Dasgupta (forthcoming) taking grounding facts to be partially grounded in ‘essential connections’ concerning items involved in the grounding facts themselves.

All these theories appear to agree on the following two theses:

- (G1) Grounding facts are derivative (that is, non-fundamental)

¹ For an introduction to the debate on grounding see, among others: Correia & Schnieder (2012), Trogdon (2013b), Bliss & Trogdon (2014) and Raven (2015).

² For an introduction to GGP see Trogdon (2013b: §7), Bliss & Trogdon (2014: §7) and Raven (2015: §7).

³ A superinternal relation is a relation ‘such that the intrinsic nature of only one of the *relata* [...] guarantees not only that the relation holds, but also that the other *relatum*(*a*) exists and has the intrinsic nature it does’; Bennett, 2011: 32). A different but similar position is defended by Cameron (2014).

- (G2) Grounding facts are at least partially grounded in the grounds they feature (e.g. if the fact that p is grounded in the plurality of facts Γ , then Γ at least partially grounds the fact that Γ grounds the fact that p)⁴

The only *general* arguments that are put forward by (T1-3) for either (G1) and (G2) appear to be the following (the first two are arguments for G1; the third is an argument for G2 which is conditional upon the truth of G1):

The argument from ‘Purity’. According to the principle of ‘Purity’, fundamental facts should feature only fundamental items (see Sider, 2011, 7.2-3). Grounding facts clearly feature non-fundamental items. Therefore, they cannot be fundamental (Bennett, 2011; DeRosset 2013; Dasgupta, 2014; Litland, forthcoming-a)

The argument from modal recombination: Quite plausibly, fundamental facts are open to free modal recombination. Therefore, if grounding facts were fundamental, there should be a ‘metaphysically flat’ world that is exactly like ours, except that nothing grounds anything else. But ‘flatworldism’ is ‘crazy pants’ (Bennett, 2011: 28). Therefore, grounding facts are not fundamental (Bennett, 2011).

The argument from vicious regress: Suppose that grounding facts are derivative and not grounded in the grounds they feature and consider the grounding fact f_3 saying that the fact f_1 is grounded in the fact f_2 . f_2 cannot be the ground for f_3 . Therefore its ground must be a different fact f_4 . In turn, the ground for the fact f_3 that f_4 grounds f_3 must be a further different fact f_6 ... *et sic in infinitum*. Therefore, if grounding facts are derivative, they must be grounded in the grounds they feature. (Bennett 2011).

However, all these arguments appear to be open to some form of criticism:

- (a) The principle of Purity, although indeed highly plausible within Sider’s (2011) theory of joint-carving expressions,⁵ appears to be at least less obvious within a theory of metaphysical grounding that doesn’t endorse Sider’s (2011) views on fundamental facts and entities. Consider a theory that, as I will be assuming in this paper, takes grounding to be a relation between facts and a fact to be fundamental if and only if it is ungrounded.⁶ In this case it seems that one could, for instance, take an entity x to be *ontologically dependent* if and only if there is some entity y , such that the fact that x exists is grounded in some facts about y ,⁷ and then say that an entity x is *fundamental* if and only if x is ontologically *independent* in this specific sense. This, however, would leave open the question as to whether fundamental facts feature only fundamental entities since it would be clearly compatible with the existence of a non-fundamental entity x such that, for some property F , the fact that x is F is fundamental.

⁴ As it is customary I take a fact f to be *fundamental* if and only if there is no plurality of facts Γ grounding it, and *derivative* otherwise (see, for instance, Rosen 2010).

⁵ Even *trivial*: ‘My—somewhat arbitrary—decision for how to define ‘fundamental truth’ will be [...]: a fundamental truth is a truth involving only fundamental terms. Thus understood, purity becomes trivial’ (Sider, 2011: 137).

⁶ See Trogdon (2013b, section 3) for an introduction to the debate on what are the *relata* of the grounding relation.

⁷ Following thus the lead of Correia (2005) and Schnieder (2006).

- (b) As for Bennett’s argument from recombination, if actual grounding facts and ‘normal’ (i.e. non-grounding) facts are freely recombinable, then there is indeed no necessary connection between them and thus, the latter can obtain without the former. However, from this it only follows that there is a world w in which the actual normal facts all obtain without the *actual* grounding facts obtaining, which is compatible with the obtaining in w of grounding facts that are *different* from the actual ones, thus making w a ‘structured’ (that is, ‘non-flat’) world. Some stronger (and arguably less plausible) principle of modal recombination appears thus to be needed in this case.
- (c) Finally, *pace* Bennett (2011), the idea that infinitely descending chains of ground are metaphysically possible has been convincingly defended in the recent literature on grounding.⁸ Notice, however, that even accepting the idea that every fact must be either fundamental or grounded in some fundamental facts (that is, the idea that, as I will say, every derivative fact is ‘rooted’; see below), the ‘regress’ pictured by Bennett doesn’t appear to be vicious. Assume, in fact, that every grounding fact is derivative and that (G2) is false. If g_1 is a fact grounding a derivative fact d_1 , then the grounding fact, d_2 , that g_1 grounds d_1 is itself a derivative fact that isn’t grounded in g_1 but in a different fact g_2 . The corresponding grounding fact d_3 is in turn grounded in a *further* fact g_3 ...*et sic in infinitum*. However, this is compatible with g_1, g_2, g_3, \dots *et cetera* being all either fundamental facts or facts that are grounded in fundamental facts.⁹

On the background of these considerations, the aim of this paper is to present a novel general argument for (a qualified version of) (G1) and (G2) that relies only on two general principles about grounding and its interplay with the notion of metaphysical necessity. As I will show, in fact, if a certain principle of free modal recombination (‘Recombination’) is combined with the principle commonly known in the literature as ‘Entailment’, then it can be proved not only that grounding facts featuring fundamental, contingent grounds are derivative (as G1 requires) but also that either they are (at least) partially grounded in the grounds they feature (as G2 mandates) or they are ‘abysses’, that is, derivative facts having no fundamental ground and lying at the top of an infinitely descending chain of ground.

In this paper I won’t try to directly argue for either Entailment or Recombination. Notice, however, that (as I will stress below) while Entailment is explicitly endorsed by many grounding theorists, Recombination appears to be at least in keeping with other principles of free modal recombination that are widely upheld in the literature. Therefore, even if the argument I present in this paper provides only a partial and conditional answer to GGP, it should be of much interest not only to the specific debate on GGP but also to the more general discussion on the nature of metaphysical grounding and its relation to the notion of metaphysical necessity.

2. Notation and definitions

I will take grounding to be a one-many relation between facts, and facts to be just true propositions ‘individuated by their worldly items and the manner of their combination’ (Rosen, 2010: 124). Following Rosen (2010), I will use ‘ $[p]$ ’ for ‘the fact that p ’, and

⁸ See Bliss (2013), Morganti (2012) and Tahko (2014).

⁹ See Dixon (forthcoming-b) and Rabern and Rabin (forthcoming).

' $[p] \leftarrow \Gamma$ ' for 'the fact that p is grounded in (obtains in virtue of) the plurality of facts Γ . I will take grounding to be irreflexive, anti-symmetric, and transitive.¹⁰

' $<$ ' will stand for 'is one of', so that, for instance, ' $[p] < \Gamma$ ' is to be read 'the fact that p is one of the (facts) Γ '. ' $\wedge \Gamma$ ' will stand for the conjunction of the propositions corresponding to the facts in Γ , and ' $\vee \Gamma$ ' for its disjunction. Therefore, if, for instance, Γ is the plurality of facts $[q_1]$, $[q_2]$, and $[q_3]$, ' $\sim \vee \Gamma$ ' will stand for ' $\sim(q_1 \vee q_2 \vee q_3)$ ' (or, equivalently, for: ' $\sim q_1 \ \& \ \sim q_2 \ \& \ \sim q_3$ '). '**D**', '**F**' and '**C**' are multigrade and distributive predicates standing for 'is a (plurality of) derivative fact(s)', 'is a (plurality of) fundamental fact(s)', 'is a (plurality of) contingent fact(s)':

$$(F) \quad \mathbf{F}([\Gamma]) =_{\text{df}} \wedge \Gamma \ \& \ \forall p(([p] < \Gamma) \rightarrow \sim \exists \Delta([p] \leftarrow \Delta))$$

$$(D) \quad \mathbf{D}([\Gamma]) =_{\text{df}} \wedge \Gamma \ \& \ \forall p(([p] < \Gamma) \rightarrow \exists \Delta([p] \leftarrow \Delta))$$

$$(C) \quad \mathbf{C}([\Gamma]) =_{\text{df}} \wedge \Gamma \ \& \ \forall p(([p] < \Gamma) \rightarrow \diamond \sim p)$$

Furthermore, I will say that a fact is 'rooted' if, and only if, it is grounded in some plurality of fundamental facts:

Roots: A fact is *rooted* if, and only if, it is grounded in some plurality of fundamental facts

$$\mathbf{R}([p]) =_{\text{df}} \exists \Gamma(\mathbf{F}(\Gamma) \ \& \ ([p] \leftarrow \Gamma))$$

Notice that from the fact that grounding is transitive it follows that, if a fact is both derivative and unrooted, then it lies at the top of an infinitely descending chain of ground.¹¹ When this is the case, I will say that the fact in question is an 'abyss':

Abyss: A fact $[p]$ is an *abyss* if, and only if, $[p]$ is both derivative and unrooted

$$\mathbf{A}([p]) =_{\text{df}} \mathbf{D}([p]) \ \& \ \sim \mathbf{R}([p])$$

The idea that there are abysses appears to be incompatible with the idea of metaphysical foundationalism which seems to be in fact definable as the idea that every fact is either fundamental or rooted.^{12,13} For this reason, in this paper I will *not* assume foundationalism.

Finally, Fine (2012: 48-50) has distinguished between a *factive* and a *non-factive* notion of ground. While most authors appears to take the factive notion as a primitive, some (like Litland, forthcoming-a) disagree. I will remain here neutral on this problem and simply focus on the *factive* notion:

¹⁰ The idea that grounding is an irreflexive, anti-symmetric and transitive relation has been challenged by Jenkins (2011), Schaffer (2012) and Rodriguez-Pereyra (2015). For a defence and a discussion see, among others, Raven (2013), Litland (2013), Javier-Castellanos (2014) and Loss (forthcoming).

¹¹ Suppose that a fact A is grounded in B and that B is in turn grounded in the fundamental fact C. If grounding weren't transitive, then A wouldn't have to be necessarily grounded in C. In that case, however, if B was A's only ground, then A wouldn't be *either* rooted *nor* at the top of an infinitely descending chain of ground.

¹² Foundationalism is defended by Cameron (2008), is assumed by Schaffer (2010a), and endorsed by Bennett (2011). While Rosen (2010: 116) and Raven (forthcoming: 8) leave the question open, foundationalism is challenged by Bliss (2013), Morganti (2012) and Tahko (2014).

¹³ Dixon (forthcoming-b) seems to define the very idea that grounding is 'well-founded' in this way.

Factivity: Necessarily, if the fact that p is grounded in the plurality of facts Γ , then it is the case that p and it is the case that $\wedge\Gamma$
 $\Box((p]\leftarrow\Gamma) \rightarrow (p \ \& \ \wedge\Gamma))$

3 Two principles about grounding

My argument relies on two specific principles governing the interplay between the notions of grounding and metaphysical possibility:

Entailment: If the fact that p is grounded in the plurality of facts Γ , then necessarily, if $\wedge\Gamma$ is the case, then p is the case
 $(p]\leftarrow\Gamma) \rightarrow \Box(\wedge\Gamma \rightarrow p)$

Recombination: For any two disjoint pluralities Γ and Δ of contingent, fundamental facts, it is possible for all the facts in Δ to obtain without any fact in Γ obtaining
 $(C(\Gamma) \ \& \ F(\Gamma) \ \& \ C(\Delta) \ \& \ F(\Delta) \ \& \ \sim\exists p([p]\leftarrow\Gamma \ \& \ [p]\leftarrow\Delta)) \rightarrow \Diamond(\wedge\Delta \ \& \ \sim\wedge\Gamma)$

Even if not completely uncontroversial,¹⁴ Entailment is a *prima facie* plausible principle that is explicitly endorsed and defended by many grounding theorists.¹⁵ In the literature on GGP, it is assumed by Rosen (2010: 118), it is taken to be a ‘plausible requirement’ by deRossett (2013: 15), and it is presented by Dasgupta (2014) as a ‘core principle governing ground’. Furthermore, Entailment seems to be entailed by many specific solutions to GGP—such as Bennett’s (2011) theory of grounding as a superinternal relation and the essentialist theories of Rosen (2010), Fine (2012), and Dasgupta (2014).^{16,17}

The general idea that there must be free recombination at the fundamental level of reality appears to be widely endorsed in the literature.¹⁸ Notice, however, that

¹⁴ Its detractors include, among others, Leuenberger (2014), Schaffer (2010b), and Skiles (2015).

¹⁵ Among others: Correia (2005), Rosen (2010), Cameron (2010), Bennett (2011), Audi (2012), Fine (2012), Trogdon (2013a), Loss (forthcoming).

¹⁶ The idea that grounding is a superinternal relation appears to entail the following principle of ‘Superinternality’ (as we might call it):

Superinternality: If the fact that p is grounded in the plurality of facts Γ , then it is necessarily the case that, if $\wedge\Gamma$ is the case, then the fact that p is grounded in the plurality of facts Γ
 $(p]\leftarrow\Gamma) \rightarrow \Box(\wedge\Gamma \rightarrow ([p]\leftarrow\Gamma))$

which, given Factivity, entails Entailment (for a criticism of the weaker principle ‘Internality’ see Litland, forthcoming-b).

¹⁷ Consider, for instance, the grounding fact

(F) The fact that (the event) e includes people acting in way W grounds the fact that e is a conference

Dasgupta (2014) takes the following ‘essential connection’ to be a partial ground for (F):

(*) It is essential to being a conference that if an event contains people acting in way W , then it is a conference

If the *prima facie* plausible principle ‘if it is essential to being an F that p , then it is necessarily the case that p ’ is assumed, then (*) entails

(**) *Necessarily*, if an event contains people acting in way W , then it is a conference and thus the kind of necessitation between ground and ‘groundee’ that is mandated by Entailment:

(***) *Necessarily*, if e contains people acting in way W , then e is a conference

¹⁸ See Bennett (2011: 27), Cameron (2010: 188), Schaffer (2010a: 40) and Ismael and Schaffer (forthcoming: §1.2.3). For a recent criticism of ‘Hume’s Principle(s)’ see Wilson (2010).

Recombination is not a principle concerning the free modal recombination between *wholly distinct entities*.¹⁹ Recombination is a principle of free modal recombination holding between *different facts* that are both *fundamental* and *contingent*, where the notion of fundamentality is understood by means of the notion of grounding which is, in turn, taken to be a notion of metaphysical dependence. Recombination seems to have thus at least the ring of *prima facie* plausibility to it. In fact, even if grounding cannot be reduced to metaphysical necessity, as it is widely accepted, cases of necessitation between contingent facts do strike one as signalling that some form of metaphysical dependence is in play. How could two *contingent* and metaphysically *independent* facts be tied by a necessary connection, and therefore, be such that one *cannot* obtain without the other?²⁰

The general relation between Recombination and the Humean principle of ‘no necessary connection between wholly distinct entities’ is an interesting issue that, alas, lies beyond the limited scope of this paper. However, it might be worth noting that there seem to be at least two possible ways of making sense of the notion of ‘overlap between facts’ according to which Hume’s principle (as applied to facts) entails Recombination:

- (i) The first way is simply to take the notion of overlap between facts to be the classical mereological notion of ‘sharing a part’.²¹ If this is the case, then, if we assume both Hume’s Principle and the idea²² that wholes are grounded in their proper parts, Recombination follows.²³
- (ii) The second way has been recently proposed by Dixon (forthcoming-a) who defines overlap between facts (‘groverlap’) so that, if two facts are different, then they *groverlap* if, and only if, either (a) one partially grounds the other or (b) they share a common ground. Different fundamental facts clearly don’t groverlap. Therefore, if (the properly reformulated version of) Hume’s Principle is accepted, Recombination appears to follow also in this case.

4 The argument

Lemma 1: If Γ is a plurality of fundamental, contingent facts and $[p]$ is grounded in Γ , then the fact that $[p]$ is grounded in Γ is a derivative fact

$$(\mathbf{F}(\Gamma) \ \& \ \mathbf{C}(\Gamma) \ \& \ [p] \leftarrow \Gamma) \rightarrow \mathbf{D}([p] \leftarrow \Gamma)$$

Proof. Consider a plurality of fundamental, contingent facts Γ and a fact $[p]$ such that $[p]$ is grounded in Γ

$$(A1) \quad \mathbf{F}(\Gamma) \ \& \ \mathbf{C}(\Gamma) \ \& \ [p] \leftarrow \Gamma$$

Given Factivity, $[p] \leftarrow \Gamma$ cannot be a necessary fact, since this would entail that also the facts belonging to Γ are necessary. Therefore, $[p] \leftarrow \Gamma$ is a contingent fact.

¹⁹ As upheld, famously, by David Lewis (see, for instance, Lewis 1999: 215 and 2001: 611). For a criticism of the requirement that the entities be wholly distinct, see Cameron (2010: §3).

²⁰ On this see also Bennett (2011: footnote 6)

²¹ See Varzi (2015: §2.2)

²² Plausible, yet not uncontroversial: see Schaffer, 2010a.

²³ In this case, in fact, fundamental facts must be atomic, and thus such that different, fundamental, contingent facts are perforce wholly distinct, and thus (by Hume’s Principle), recombinable.

$$(A2) \quad C([p] \leftarrow \Gamma)$$

Suppose it is also fundamental

$$(A3) \quad F([p] \leftarrow \Gamma)$$

In this case, if $[p] \leftarrow \Gamma$ were one of the facts in Γ , then $[p] \leftarrow \Gamma$ and the *other* facts in Γ ought to be freely recombinable (by Recombination).²⁴ However, by Factivity, $[p] \leftarrow \Gamma$ necessitates Γ , and thus, every fact in Γ . *Contradiction!* Hence, $[p] \leftarrow \Gamma$ isn't one of the facts in Γ

$$(A4) \quad \sim([p] \leftarrow \Gamma) < \Gamma$$

From (A1), (A2), (A3), and (A4) it follows by Recombination that it is metaphysically possible for $[p] \leftarrow \Gamma$ to obtain without any fact in Γ obtaining

$$(A5) \quad \diamond([p] \leftarrow \Gamma) \& \sim \Gamma$$

However, we also have, by Factivity, that $[p] \leftarrow \Gamma$ necessitates Γ .

$$(A6) \quad \Box([p] \leftarrow \Gamma) \rightarrow \wedge \Gamma$$

Contradiction! Hence, $[p] \leftarrow \Gamma$ is not fundamental, but derivative. QED

Lemma 2: If Γ is a plurality of fundamental, contingent facts and $[p]$ is a rooted fact necessitating $\wedge \Gamma$, then $[p]$ is partially grounded in Γ
 $(C(\Gamma) \& F(\Gamma) \& R([p]) \& \Box(p \rightarrow \wedge \Gamma)) \rightarrow ([p] \leftarrow \Gamma)$

Proof. Consider a plurality of fundamental, contingent facts Γ and a fact $[p]$ such that $[p]$ is a rooted fact necessitating Γ .

$$(B1) \quad F(\Gamma) \& C(\Gamma) \& R([p]) \& \Box(p \rightarrow \wedge \Gamma)$$

By Roots, there is a plurality of fundamental facts Δ , such that Δ grounds $[p]$.

$$(B2) \quad [p] \leftarrow \Delta \& F(\Delta)$$

Suppose that $\wedge \Delta$ is necessary

$$(B3) \quad \Box \wedge \Delta$$

By the fact that Δ grounds $[p]$ and Entailment, it follows that $\wedge \Delta$ necessitates p

$$(B4) \quad \Box(\wedge \Delta \rightarrow p)$$

However, we are also assuming that p necessitates $\wedge \Gamma$

²⁴ I am here ignoring the seemingly pathological case in which Γ is the plurality consisting in the very fact $[p] \leftarrow \Gamma$.

$$(B5) \quad \Box(p \rightarrow \wedge \Gamma)$$

From (B4) and (B5) it follows, by the transitivity of necessitation, that $\wedge \Delta$ necessitates $\wedge \Gamma$

$$(B6) \quad \Box(\wedge \Delta \rightarrow \wedge \Gamma)$$

and from (B3) and (B6) it follows, by closure under material implication of metaphysical necessity, that also $\wedge \Gamma$ is necessary

$$(B7) \quad \Box \wedge \Gamma$$

which *contradicts* our assumption about the contingency of Γ . *Therefore*, not every fact in Δ is necessary and some sub-plurality of Δ (' Δ_C ') is a plurality of fundamental, *contingent* facts.

We are assuming that Γ is a plurality of contingent facts. Therefore, if Γ is a sub-plurality of Δ , it must be a sub-plurality of Δ_C (that is, the plurality of *contingent* facts belonging to Δ). Suppose that Γ is *not* a sub-plurality of Δ_C

$$(B8) \quad \sim(\Gamma \subseteq \Delta_C)$$

and let ' Γ/Δ_C ' stand for the plurality of facts that are in Γ but not in Δ_C . Since all the facts in Γ/Δ_C are in Γ , they are all fundamental, contingent facts

$$(B9) \quad C(\Gamma/\Delta_C) \ \& \ F(\Gamma/\Delta_C)$$

Δ_C being fundamental and contingent, it follows by Recombination that it is possible for all the facts in Δ_C to obtain, without any fact in Γ/Δ_C obtaining.

$$(B10) \quad \Diamond(\wedge \Delta_C \ \& \ \sim \vee \Gamma/\Delta_C)$$

However, Γ/Δ_C being a sub-plurality of Γ , it is necessarily the case that if no fact in Γ/Δ_C obtains, then not every fact in Γ obtains

$$(B11) \quad \Box(\sim \vee \Gamma/\Delta_C \rightarrow \sim \wedge \Gamma)$$

It follows thus from (B10) and (B11) that it is possible for Δ_C to obtain without every fact in Γ obtaining

$$(B12) \quad \Diamond(\wedge \Delta_C \ \& \ \sim \wedge \Gamma)$$

Recall now that Δ_C is the plurality of *contingent* facts of Δ , so that every fact in Δ but not in Δ_C (if any) is a necessary fact. This means that from (B12) it also follows that it is possible for Δ to obtain without every fact in Γ obtaining

$$(B13) \quad \Diamond(\wedge \Delta \ \& \ \sim \wedge \Gamma)$$

which contradicts (B6). Therefore, Γ is a sub-plurality of Δ

$$(B14) \quad \Gamma \subseteq \Delta$$

However, by the definition of partial grounding

$$\textbf{Partial grounding: } f \leftarrow \Delta =_{\text{df}} \text{ for some } \Gamma, f \leftarrow \Gamma \text{ and } \Delta \subseteq \Gamma^{25}$$

it follows from (B2) and (B14) that Γ partially grounds $[p]$

$$(B15) \quad [p] \leftarrow \Gamma$$

QED

Theorem: If Γ is a plurality of fundamental, contingent facts grounding $[p]$, then the fact that $[p]$ is grounded in Γ is either partially grounded in Γ or an abyss.
 $(\mathbf{C}(\Gamma) \ \& \ \mathbf{F}(\Gamma) \ \& \ ([p] \leftarrow \Gamma)) \rightarrow ((([p] \leftarrow \Gamma) \leftarrow \Gamma) \vee \mathbf{A}([p] \leftarrow \Gamma))$

Proof. Consider a plurality of fundamental, contingent facts Γ and a fact $[p]$ such that $[p]$ is grounded in Γ

$$(C1) \quad \mathbf{C}(\Gamma) \ \& \ \mathbf{F}(\Gamma) \ \& \ [p] \leftarrow \Gamma$$

By Lemma 1 it follows from (C1) that the fact that Γ grounds $[p]$ is a derivative fact

$$(C2) \quad \mathbf{D}([p] \leftarrow \Gamma)$$

On the other hand, we have, by Factivity, that the fact that Γ grounds $[p]$ necessitates Γ

$$(C3) \quad \square([p] \leftarrow \Gamma) \rightarrow \wedge \Gamma$$

The fact that Γ grounds $[p]$ is thus a derivative fact that necessitates a plurality of fundamental, contingent facts. Suppose that the fact that Γ grounds $[p]$ is rooted.

$$(C4) \quad \mathbf{R}([p] \leftarrow \Gamma)$$

By Lemma 2 it follows from (C1), (C3) and (C4) that the fact that Γ grounds $[p]$ is partially grounded in Γ .

$$(C5) \quad ([p] \leftarrow \Gamma) \leftarrow \Gamma$$

Therefore, if $[p] \leftarrow \Gamma$ isn't partially grounded in Γ , then $[p] \leftarrow \Gamma$ is a derivative and unrooted fact, that is, an *abyss*:

$$(C6) \quad \sim([p] \leftarrow \Gamma) \leftarrow \Gamma \rightarrow \mathbf{A}([p] \leftarrow \Gamma)$$

It follows, thus, that, if Γ is a plurality of fundamental, contingent facts and $[p]$ is a fact grounded in Γ , then either that the fact that Γ grounds $[p]$ is partially grounded in Γ , or it is an abyss. QED

²⁵ Rosen (2010: 115), Fine (2012: 50)

5. Conclusion

Let us take stock. If a grounding fact f featuring a plurality of fundamental, contingent facts Γ grounding a derivative fact $[p]$ were fundamental, then, by Recombination, it could obtain without any of the facts in Γ obtaining. This, however, would contradict Factivity. Therefore, f is derivative. If f is rooted, then it has some fundamental ground by which it is necessitated (by Entailment). Since we already know that, by Factivity, f necessitates Γ , it follows (by the transitivity of necessitation) that the fundamental grounds Δ of f necessitate Γ , and hence, that the totality Δ_C of contingent facts of Δ necessitates each fact in Γ . If Γ weren't a sub-plurality of Δ , then some of the contingent, fundamental facts in Γ would be necessitated by Δ_C , thus contradicting Recombination. Therefore, Γ is a sub-plurality of Δ . By the definition of partial grounding, it follows that, if the fact that Γ grounds $[p]$ is rooted, then it is at least partially grounded in Γ . By Contraposition, if the fact that Γ grounds $[p]$ isn't partially grounded in Γ , then it isn't rooted. Therefore, it is both derivative and unrooted, and thus, an abyss.

We can thus conclude that, if Entailment and Recombination are assumed, every grounding fact f featuring a plurality of fundamental, contingent facts Γ grounding a derivative fact $[p]$ is either at least partially grounded in Γ or an abyss. *Quod Erat Demonstrandum.*

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