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4. Atomic energy levels emerge from hyperbolic Fine structure constant spiral

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The Bohr radius for an ionizing electron (H atom) follows a hyperbolic spiral. At specific spiral angles, the angle components cancel returning an integer value for the radius (360°=4*r*, 360+120°=9*r*, 360+180°=16*r*, 360+216°=25*r* ... 720°=∞*r*), and as the orbital radius at these angles (by including wavelengths) matches the principal quantum number *n* energy levels, this spiral can be used to calculate the transition frequencies for each *n*. A gravitational orbital simulation program was modified for atomic orbitals by the addition of an extra fine structure constant alpha term. In this simulation, the orbital (Bohr) radius is treated as analogous to the photon albeit of inverse or reverse phase whereby orbital radius + photon = zero (cancel). The orbital radius rotates, pulling the electron in a circular orbit, during transition the incoming photon attaches to the orbital radius, extending it in discrete steps (the electron itself has a passive role). As the orbital radius continues its rotation during the transition phase, the electron follows this spiral pattern (as it crosses each energy level). As these spiral angles ($0 < \varphi < 720°$) can be used to derive the *n* quantum levels, we can ask if quantization of the atom is a geometrical function of this alpha spiral, and if so, this would indicate a role of alpha within the atom.

1 Introduction

A hyperbolic spiral is a type of spiral with a pitch angle that increases with distance from its center. As this curve widens (radius *r* increases), it approaches an asymptotic line (the y-axis) with the limit set by a scaling factor *a* (as *r* approaches infinity, the y axis approaches *a*).

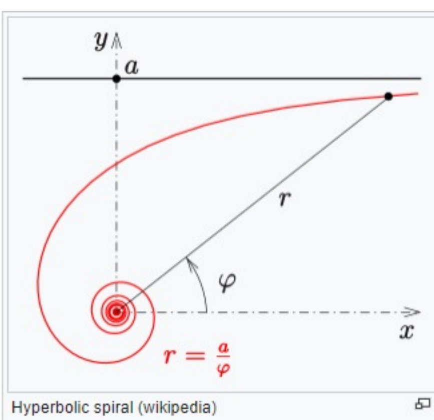

Fig. 1: Hyperbolic spiral (wikipedia)

In its simplest form, a fine structure constant spiral (or alpha spiral) is a specific hyperbolic spiral that appears in electron transitions between atomic orbitals in a Hydrogen atom. It can be represented in Cartesian coordinates by

$$x = a^2 \frac{cos(\varphi)}{\varphi^2},\ y = a^2 \frac{sin(\varphi)}{\varphi^2},\ 0 < \varphi < 4\pi \quad (1)$$

This spiral has only 2 revolutions approaching 4π as the radius approaches infinity. If we set start radius $r = 1$, then at given angles φ, radius *r* will have integer values (the angle components cancel).

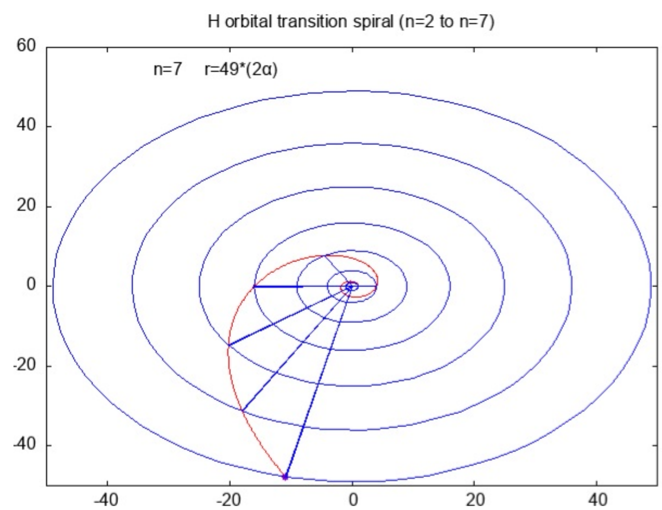

Fig. 2: H orbital transition spiral showing *n* orbits [1]

$$\varphi = (2)\pi,\ r = 4 \quad (2)$$

$$\varphi = (4/3)\pi,\ r = 9 \quad (3)$$

$$\varphi = (1)\pi,\ r = 16 \quad (4)$$

$$\varphi = (4/5)\pi,\ r = 25 \quad (5)$$

$$\varphi = (2/3)\pi,\ r = 36 \quad (6)$$

2 Bohr model

We can map the electron orbit around the orbital as a series of steps with each step the frequency of 1 electron wavelength. The steps are defined according to angle β.

The base radius for each *n* level uses the fine structure constant α. On a 2-D plane;

$$r_{orbital} = 2\alpha n^2 \quad (7)$$

$$v_{orbital} = \frac{1}{2\alpha n} \tag{8}$$

$$\beta = \frac{1}{r_{orbital}\sqrt{r_{orbital}}\sqrt{2\alpha}} \tag{9}$$

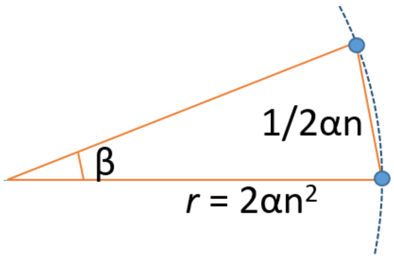

Fig. 3: rotating orbital alpha step

At the n levels this reduces to

$$\beta = \frac{1}{4\alpha^2 n^3} \tag{10}$$

The number of steps (orbital period) for 1 orbit of the electron then becomes

$$t_{orbital} = \frac{2\pi r_{orbital}}{v_{orbital}} = 2\pi 2\alpha n^2 \alpha n \tag{11}$$

A base (reference) orbital (n=1)

$$t_{ref} = 2\pi 4\alpha^2 \tag{12}$$

3 Transition (theory)

In this model, the orbital (Bohr) radius is the origin of the orbital momentum, it is the orbital radius that links the electron and nucleus (proton) and the rotation of this orbital radius results in an electron orbit around the nucleus (proton). This orbital radius is treated as physically analogous to the photon albeit of inverse or reverse phase (orbital radius + photon = zero), and as such it is the orbital radius that absorbs or ejects the photon during transition, in the process the orbital radius is extended or reduced (until the photon is completely emitted).

This absorption/emission process occurs in steps, each step a unit of r_{incr} is exchanged between radius and photon,

$$r_{incr} = \frac{-1}{2\pi 2\alpha} \tag{13}$$

furthermore at each step the orbital radius continues to rotate, the electron, being pulled along by this rotation according to angle β, thereby traces a spiral path (fig.4) as the orbital radius length changes (the electron has a passive role in the transition phase).

This repeats for the wavelength of the photon λ_{photon}, until the photon has been completely absorbed by, or ejected from, the orbital radius (which in turn has extended or contracted accordingly).

$$\lambda_{orbital} = \lambda_{orbital} + \lambda_{photon} \tag{14}$$

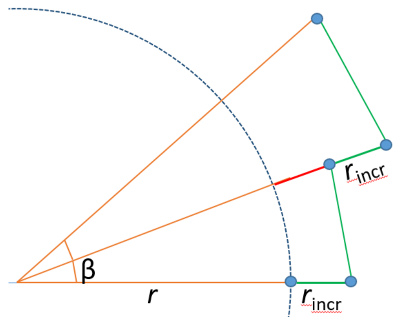

Fig. 4: orbital radius absorbing units from the photon

If the wavelength of $\lambda_{photon} = \lambda_{orbital}$ (the wavelength of the orbital radius), and as these waves are of inverse phase, the orbital radius will be deleted. This is defined by ionization.

$$\lambda_{orbital} + \lambda_{photon} = zero \tag{15}$$

However, an incoming photon separates into 2 photons (initial and final) as per the Rydberg formula.

$$\lambda_{photon} = R\,(\frac{1}{n_i^2} - \frac{1}{n_f^2}) = \frac{R}{n_i^2} - \frac{R}{n_f^2} \tag{16}$$

$$\lambda_{photon} = (\lambda_i) - (\lambda_f)$$

The (λ_i) will subtract from the orbital radius as described above, however the (-λ_f), because of the Rydberg minus term, will have the same phase as the orbital radius and so conversely will increase the orbital radius. Therefore, for the duration of the (λ_i) photon wavelength, the orbital radius does not change, as the +r_{incr} (+λ_i) and -r_{incr} (-λ_f) segments cancel.

$$r_{orbital} = r_{orbital} + (\lambda_i) - (\lambda_f) \tag{17}$$

The (λ_f) has the longer wavelength, and so after (λ_i) has been absorbed by the orbital radius, and for the remaining duration of this (λ_f) photon wavelength, at each transition step the orbital radius will be extended (fig.4). At each step, as the orbital radius increases, the orbital rotation angle β will conversely decrease, and as the velocity of orbital rotation depends on β, the velocity will adjust accordingly.

For example; an n_i=1 ($\lambda_i = 1t_{ref}$) to n_f=2 ($\lambda_f = 4t_{ref}$) orbital transition; the wavelength of λ_i = the wavelength of the radius $\lambda_{orbital}$ and as r_{incr} has a minus value, the 2 wavelengths per step will cancel, however the (λ_f) subtracts r_{incr} units simultaneously,

$$r_{orbital} = r_{orbital} + r_{incr} - r_{incr} \tag{18}$$

the end result being that the electron makes 1 complete orbit, upon completion of which the orbital radius and (λ_i) have canceled, but the (λ_f) still has $3t_{ref}$ segments left, and so transition continues for another $3t_{ref}$ steps until this photon is also absorbed. A n_i=2 to n_f=3 transition would require $t = 4t_{ref}$ + (9-4)t_{ref} steps.

In (figs. 5, 6), the electron begins in the n_i=1 orbital rotating anti-clockwise. A photon $\lambda_{photon} = (\lambda_{1s}) - (\lambda_{2s})$ strikes this orbital raising the electron to the $n = 2$ orbital in discrete steps. A 2nd photon $\lambda_{photon} = (\lambda_{2s}) - (\lambda_{3s})$ then strikes this orbital raising the electron to the $n = 3$ orbital. The spiral occurs because the electron is continuously pulled in an anti-clockwise direction by the orbital radius (which continues rotating during transition). During the transition phase, only the orbital radius changes, the electron itself has a passive role.

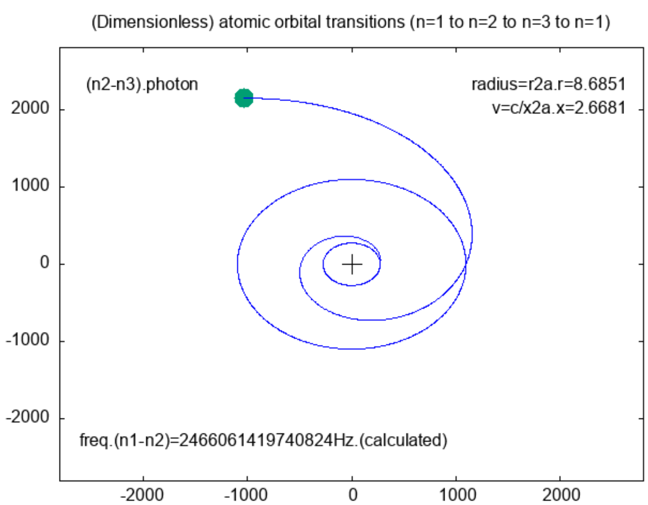

Fig. 5: alpha orbital transition animation 2D [1]

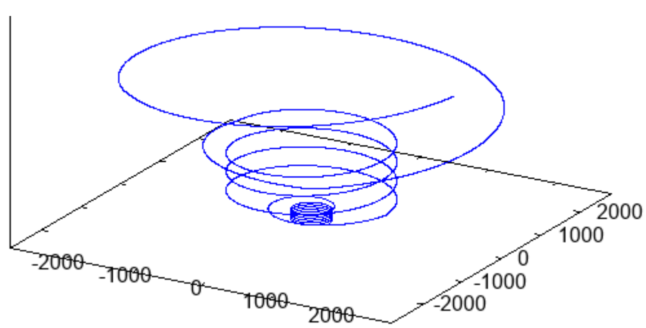

Fig. 6: alpha orbital transition animation 3D [1]

4 Hypersphere

The above is depicted on a 2D plane (in 3D space). If we place the orbital in a 4D expanding hypersphere [2] then we find the orbital is rotating at c in hypersphere co-ordinates [3].

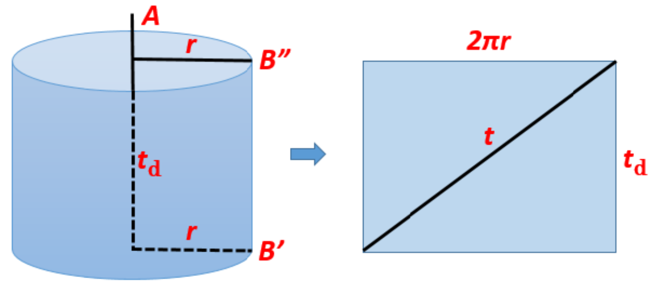

Fig. 7: illustration of B's orbit relative to the A time-line axis

In (fig. 7), while B has a circular orbit period on the 2-axis δ-y plane (horizontal axis as 3-D space) around A (center of mass), it also follows a cylindrical orbit (from B' to B'') around the A (vertical) hyper-sphere time-line expansion axis (the z-axis of the simulation). A moves with the universe expansion (along the time-line axis) at ($v = c$) but is stationary in 3-D space ($v = 0$). B is orbiting A at ($v = c$) but the time-line axis motion is equivalent (and so 'invisible') to both A and B, as a result the orbital period and velocity measures will be defined in terms of 3-D space co-ordinates by observers on A and B giving the familiar formula [4].

$$t_d = t\sqrt{1 - \frac{v^2}{c^2}} \tag{19}$$

5 Alpha spiral

Beginning with $n_i = 1$, $\varphi = 0$, $r = 2\alpha$ for each step during transition;

$$\varphi = \varphi + \beta \tag{20}$$

As β is proportional to the radius, as the radius increases the value of β will reduce correspondingly (likewise reducing the orbital velocity). At discrete angles, the alpha spiral returns an integer value corresponding with the photon wavelength ($0 < \varphi < 4\pi$);

$$\varphi = (2)\pi,\ r = 4(2\alpha) \tag{21}$$

$$\varphi = (8/3)\pi,\ r = 9(2\alpha) \tag{22}$$

$$\varphi = (3)\pi,\ r = 16(2\alpha) \tag{23}$$

$$\varphi = (16/5)\pi,\ r = 25(2\alpha) \tag{24}$$

$$\varphi = (10/3)\pi,\ r = 36(2\alpha) \tag{25}$$

The transition frequency is a combination of the orbital phase and the transition phase.

$$f(n_i\ to\ n_f) = \frac{{n_f}^2 - {n_i}^2}{t_{orbital} + t_{transition}} \tag{26}$$

Including the hypersphere z-axis (note: the orbital phase has a fixed radius, however at the transition phase this needs to be calculated for each discrete step as the orbital velocity depends on the radius).

$$t_{orbital} = t_{ref}\sqrt{1 - \frac{1}{(v_{orbital})^2}} \tag{27}$$

6 H atom

The formula for transition (including the wavelengths of the proton and electron);

$$f(n_i\ to\ n_f) = (\frac{2c}{\lambda_e + \lambda_p})\frac{(n_f{}^2 - n_i{}^2)}{(t_{orbital} + t_{transition})} \tag{28}$$

We can use the spiral to determine when the electron reaches each n level (here n_i=1).

$$\frac{n_f{}^2}{n_f{}^2 - n_i{}^2} = \frac{4\pi}{\varphi} \tag{29}$$

We then take an electron that is being ionized, and calculate the transition frequency f as it reaches each integer radius r on the spiral (fig.2).

7 Notes

I have used $\alpha \sim 137$ as this the commonly recognized value, however officially this is the inverse alpha $\alpha^{-1} \sim 137$.

The simulation was adapted from a gravity simulation program [3] (which derives gravitational orbits from the sum of individual particle-particle orbital pairs at the Planck scale) by adding a transition between orbitals. The program runs an integer loop, incrementing with each additional unit of r_{incr}, and so for convenience a value for alpha was chosen to give an integer value for $3t_{ref}$. The program can be further modified for greater precision, however the purpose of the simulation discussed here was to demonstrate the spiral principle.

$$\alpha = \sqrt{\frac{472129.66667}{8\pi}} = 137.059996197 \tag{30}$$

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