

Mathematical electron model and the SI unit 2017 Special Adjustment

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Following the 26th General Conference on Weights and Measures are fixed the numerical values of the 4 physical constants (h, c, e, k_B). This is premised on the independence of these constants. This article discusses a model of a mathematical electron from which can be defined the Planck units as geometrical objects (mass $M=1$, time $T=2\pi \dots$). In this model these objects are interrelated via this electron geometry such that once we have assigned values to 2 Planck units then we have fixed the values for all Planck units. As all constants can then be defined using geometrical forms (in terms of 2 fixed mathematical constants, 2 unit-specific scalars and a defined relationship between the units kg, m, s, A), the least precise CODATA 2014 constants ($G, h, e, m_e, k_B \dots$) can then be solved via the most precise ($c, \mu_0, \alpha, R_\infty$), with numerical precision limited by the precision of the fine structure constant α . In terms of this model we now for example have 2 separate values for elementary charge, calculated from (c, α, R_∞) and the 2017 revision.

Table 1	Calculated using ($c^*, \mu_0^*, \alpha^*, R_\infty^*$)	CODATA 2014 values
Fine structure constant	$\alpha^* = 137.035999139$	$\alpha = 137.035999139(31)$
Speed of light	$c^* = 299792458 u^{17}$	$c = 299792458$
Permeability	$\mu_0^* = 4\pi/10^7 u^{36}$	$\mu_0 = 4\pi/10^7$
Rydberg constant	$R_\infty^* = 10973731.568 508 u^{13}$	$R_\infty = 10973731.568 508(65)$
Planck constant	$h^* = 6.626 069 134 e-34 u^{19}$	$h = 6.626 070 040(81) e-34$
Elementary charge	$e^* = 1.602 176 511 30 e-19 u^{-27}$	$e = 1.602 176 6208(98) e-19$
von Klitzing (h/e^2)	$R_K^* = 25812.807 45559 u^{73}$	$R_K = 25812.807 4555(59)$
Electron mass	$m_e^* = 9.109 382 312 56 e-31 u^{15}$	$m_e = 9.109 383 56(11) e-31$
Electron wavelength	$\lambda_e^* = 2.426 310 2366 e-12 u^{-13}$	$\lambda_e = 2.426 310 2367(11) e-12$
Boltzmann's constant	$k_B^* = 1.379 510 147 52 e-23 u^{29}$	$k_B = 1.380 648 52(79) e-23$
Gravitation constant	$G^* = 6.672 497 192 29 e-11 u^6$	$G = 6.674 08(31) e-11$
Planck length	$l_p^* = .161 603 660 096 e-34 u^{-13}$	$l_p = .161 6229(38) e-34$
Planck mass	$m_p^* = .217 672 817 580 e-7 u^{15}$	$m_p = .217 6470(51) e-7$
Gyromagnetic ratio	$\gamma_e/2\pi^* = 28024.953 55 u^{-42}$	$\gamma_e/2\pi = 28024.951 64(17)e-7$

1 Background

The mathematical electron model (see [1]) uses a unit-less (units = 1) formula for an electron f_e (eqs. 1, 20) to construct the Planck units as geometrical objects. In this model MLTA are interrelated (see units, eq.1), thus we need only assign numerical values to any 2 Planck units, for example $M = m_p$ and $T = t_p$, in order to fix the numerical values for all Planck units and from these the dimensioned constants ($G, h, c, e, m_e, k_B \dots$) where α (fine structure constant) and $\Omega = 2.0071349496 \dots$ are dimensionless mathematical constants.

$$f_e = 4\pi^2(2^6 3\pi^2 \alpha \Omega^5)^3 = .23895453 \dots \times 10^{23} \quad (1)$$

$$\text{units} = \frac{(AL)^3}{T} = \sqrt{\frac{L^{15}}{M^9 T^{11}}} = 1$$

We find this set of geometries for MLTA;

$$M = (1) \quad (2)$$

$$T = (2\pi) \quad (3)$$

$$L = (2\pi^2 \Omega^2) \quad (4)$$

$$A = \left(\frac{2^6 \pi^3 \Omega^3}{\alpha} \right) \quad (5)$$

To convert to CODATA 2014 values requires 2 dimensioned scalars. In this example I use a mass scalar k and time scalar t with associated units $u^{15} = \text{mass}$, $u^{-30} = \text{time}$. Assigning appropriate numerical values to (k, t)

$$k = m_p = .217672817 \dots \times 10^{-7}, \text{ unit} = u^{15} \text{ (kg)}$$

$$t = t_p/2\pi = .171585512 \dots \times 10^{-43}, \text{ unit} = u^{-30} \text{ (s)}$$

$$M = m_p = (1)k, \text{ unit} = u^{15} \text{ (kg)} \quad (6)$$

$$T = t_p = (2\pi)t, \text{ unit} = u^{-30} \text{ (s)} \quad (7)$$

We can now calculate scalars (l, a) and the Planck units LA

$$l = k^{9/15} t^{11/15}, \text{ unit} = u^{9/15 \cdot 15 + 11/15 \cdot (-30) = -13} \text{ (eq.1)} \quad (8)$$

$$L = l_p = (2\pi^2 \Omega^2)l, \text{ unit} = u^{-13} \text{ (m)} \quad (9)$$

$$\frac{l^{15}}{k^9 t^{11}} = \frac{(.203 \dots \times 10^{-36})^{15}}{(.217 \dots \times 10^{-7})^9 (.171 \dots \times 10^{-43})^{11}} \cdot \frac{u^{-13 \cdot 15}}{u^{15 \cdot 9} u^{-30 \cdot 11}} = 1 \quad (10)$$

$$a = \frac{1}{k^{9/15} t^{6/15}}, \text{ unit} = u^{9/15 \cdot (-15) + 6/15 \cdot 30 = 3} \quad (11)$$

$$A = \left(\frac{64\pi^3\Omega^3}{\alpha} \right) a, \text{ unit} = u^3 \text{ (ampere)} \quad (12)$$

$$\frac{a^3 l^3}{t} = \frac{(.126...x10^{23})^3 (.203...x10^{-36})^3 \cdot u^{3*3} u^{-13*3}}{(.171...x10^{-43})} \cdot \frac{u^{3*3} u^{-13*3}}{u^{-30}} = 1 \quad (13)$$

The physical constants in terms of MLTA ($V = 2L/T$);

$$c^* = V = (2\pi\Omega^2) \frac{k^{3/5}}{t^{4/15}}, u^{9/15*(15)+8/30*(-30)=17} \quad (14)$$

$$G^* = \frac{V^2 L}{M} = (8\pi^4 \Omega^6) k^{4/5} t^{1/5}, u^{12-6=6} \quad (15)$$

$$e^* = AT = \left(\frac{128\pi^4 \Omega^3}{\alpha} \right) \frac{t^{3/5}}{k^{3/5}}, u^{-18-9=-27} \quad (16)$$

$$h^* = 2\pi LVM = (8\pi^4 \Omega^4) k^{11/5} t^{7/15}, u^{33-14=19} \quad (17)$$

$$k_B^* = \frac{\pi VM}{A} = \left(\frac{\alpha}{32\pi\Omega} \right) k^{11/5} t^{2/15}, u^{33-4=29} \quad (18)$$

$$\sigma_e = \frac{3\alpha^2 AL}{\pi^2} = 2^7 3\pi^3 \alpha \Omega^5 t^{1/3}, u^{-10} \quad (19)$$

$$f_e = \frac{\sigma_e^3}{T} = \frac{(2^7 3\pi^3 \alpha \Omega^5 t^{1/3})^3}{2\pi t}, \text{ units} = \frac{(u^{-10})^3}{u^{-30}} = 1 \quad (20)$$

$$m_e^* = \frac{M}{f_e}, \text{ units} = u^{15} \quad (21)$$

$$\lambda_e^* = 2\pi L f_e, \text{ units} = u^{-13} \quad (22)$$

$$\mu_0^* = \frac{\pi V^2 M}{\alpha L A^2} = \left(\frac{\alpha}{2^{11} \pi^5 \Omega^4} \right) \frac{k^{14/5}}{t^{7/15}}, u^{42+14=56} \quad (23)$$

$$R_\infty^* = \frac{m_e}{4\pi l_p \alpha^2 m_p} = \frac{1}{(2^{23} 3^3 \pi^{11} \alpha^5 \Omega^{17})} \frac{1}{k^{3/5} t^{11/15}}, u^{13} \quad (24)$$

For example this common equation for alpha reduces to;

$$\alpha = \frac{2h^*}{\mu_0^* (e^*)^2 c^*} = \alpha, u^{19-56+54-17=0} \quad (25)$$

Inserting the values for (α, Ω, k, t) in the above will give the calculated results listed in table 1.

2 CODATA 2014

By defining all the dimensioned constants in terms of (α, Ω) , 2 scalars and the unit rule set u , we can define constants in terms of each other.

In the following the geometries of the least precise constants $(G^*, h^*, e^*, m_e^*, k_B^*)$ can be constructed using the geometries of the most precise $(c^*, \mu_0^*, R_\infty^*, \alpha^*)$;

$$(h^*)^3 = (8\pi^4 \Omega^4 k^{11/5} t^{7/15} u^{19})^3 = \frac{2\pi^{10} (\mu_0^*)^3}{3^6 (c^*)^5 \alpha^{13} (R_\infty^*)^2}, \text{ unit} = u^{57} \quad (26)$$

$$(e^*)^3 = \frac{4\pi^5}{3^3 (c^*)^4 \alpha^8 (R_\infty^*)}, \text{ unit} = u^{-81} \quad (27)$$

$$(k_B^*)^3 = \frac{\pi^5 (\mu_0^*)^3}{3^3 2 (c^*)^4 \alpha^5 (R_\infty^*)}, \text{ unit} = u^{87} \quad (28)$$

$$(G^*)^5 = \frac{\pi^3 (\mu_0^*)}{2^{20} 3^6 \alpha^{11} (R_\infty^*)^2}, \text{ unit} = u^{30} \quad (29)$$

$$(m_e^*)^3 = \frac{16\pi^{10} (R_\infty^*) (\mu_0^*)^3}{3^6 (c^*)^8 \alpha^7}, \text{ unit} = u^{45} \quad (30)$$

$$(l_p^*)^{15} = \frac{\pi^{22} (\mu_0^*)^9}{2^{35} 3^{24} \alpha^{49} (c^*)^{35} (R_\infty^*)^8}, \text{ unit} = (u^{-13})^{15} \quad (31)$$

$$(m_p^*)^{15} = \frac{2^{25} \pi^{13} (\mu_0^*)^6}{3^6 (c^*)^5 \alpha^{16} (R_\infty^*)^2}, \text{ unit} = (u^{15})^{15} \quad (32)$$

$$\gamma_e / 2\pi = \frac{g l_p^* m_p^*}{2 k_B^* m_e^*}, \text{ unit} = u^{-13-29=3-30-15=-42} \quad (33)$$

$$(\gamma_e / 2\pi)^3 = \frac{g^3 3^3 (c^*)^4}{2^8 \pi^8 \alpha (\mu_0^*)^3 (R_\infty^*)^2} \quad (34)$$

Inserting the above in the alpha formula

$$\alpha^3 = \frac{8(h^*)^3}{(\mu_0^*)^3 (e^*)^6 (c^*)^3} = \alpha^3, \text{ units} = 1 \quad (35)$$

We can then replace $(c^*, \mu_0^*, R_\infty^*, \alpha^*)$ with the numerical CODATA 2014 values for $(c, \mu_0, \alpha, R_\infty)$ giving the calculated results listed in table 1.

3 Revision 2017

As eq.27 uses only $(c^*, R_\infty^*, \alpha)$ and as $e = 1.602176634 \times 10^{-19}$ is the most precise of the revised values I shall discuss it here in terms of this model.

$$\alpha^8 = \frac{4\pi^5}{3^3 (c^*)^4 (e)^3 (R_\infty^*)}, \text{ unit} = 1 \quad (36)$$

Using the CODATA 2014 R_∞ gives $\alpha = 137.03599520$

$$R_\infty^* = \frac{4\pi^5}{3^3 (c^*)^4 (e)^3 (\alpha^8)}, \text{ unit} = 1 \quad (37)$$

Using the CODATA 2014 α gives $R_\infty = 10973729.04723$. In other words we now have 2 separate values for elementary charge, from $(c^*, R_\infty^*, \alpha)$ and from the revision.

Notes:

k_B does not agree with CODATA, however it can be used in eq.33 to solve the gyro-magnetic ratio.

G agrees with Rosi et al $G = 6.67191(77)(62) \times 10^{-11}$ [3].

A complete discussion of this model can be found here [1], the formulas used in this article downloaded in maple format [2], for convenience I use the commonly recognized value for alpha as $\alpha \sim 137$;

References

1. Macleod, Malcolm J. "*Programming Planck units from a virtual electron; a Simulation Hypothesis*" Eur. Phys. J. Plus (2018) 133: 278
 2. planckmomentum.com/rev2017.zip
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 3. Rosi, G.; Sorrentino, F.; Cacciapuoti, L.; Prevedelli, M.; Tino, G. M. (26 June 2014).
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