# 3: Programming atomic and gravitational orbitals in a Simulation Hypothesis 

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#### Abstract

This article introduces a method for programming orbitals at the Planck level. Mathematical probability orbitals are replaced with units of 'orbit momentum' with orbital regions derived from geometrical imperatives rather than abstract forces. In this approach the electron does not orbit around a nucleus but rather is maintained within an orbital region by the confines of the geometry of the orbital (this orbit momentum is the orbital). There is no electron transition between orbitals, rather the existing orbital is exchanged for the new orbital by the momentum of the incoming photon. The Rydberg formula describes this 2 -stage process of wave addition followed by wave subtraction. As the electron is physically linked to the nucleus by the orbital an electric force is not required, instead a charged momentum (the sqrt of Planck momentum) is presumed. A gravitational orbit is the sum of individual gravitational orbitals as physical links of orbit momentum, a gravitational force is not required. Mass is replaced by units of Planck mass, with the orbitals linking these Planck mass units. Thus the moon is not orbiting the earth, instead it is propelled by these orbital momenta, its path the sum of this momentum. As orbitals have different momentum densities, movement between orbitals requires a change in momentum, an orbital (momentum) buoyancy. Nuclear binding energy, ionization energy and escape velocity are measures of the momentum required to completely erase the orbitals.


## 1 Introduction

In quantum mechanics an atomic orbital is a mathematical function that can be used to calculate the probability of finding any electron of an atom in any specific region around the atom's nucleus.

This model replaces the (non-physical) probability function and electric force with an orbital as a (physical) link of 'orbit momentum'.

In the atom, the electron is linked to the nucleus by this orbital. It is this orbit momentum which is the origin of electron orbital momentum and it also confers the wave-function (region) within which the electron may be found.

As the particles are physically linked by this orbital, an electric force is not required to maintain the orbit. To change orbits requires a change in orbit momentum.

Atomic orbitals could also be described as anti-photons, being equivalent to photons albeit of inverse phase such that photon + anti-photon cancel. The wavelength of the orbital derives from the relativistic wavelength of the particle (see also article on programming relativity [7]). The rotation (or spin) is a geometrical imperative due to the incompressibility of particle momentum.

A gravitation orbit is treated as the sum of individual discrete orbitals that are analogous to atomic orbitals and may be treated likewise. An object can only change orbits by changing the total orbit momentum, a momentum buoyancy (angular momentum is conserved), thus a gravitational force may be dispensed with. The moon does not orbit the earth, rather the moon is pulled along its orbit path by the momentum of these gravitational orbitals; they are both the track and the locomotive. Consequently, if the orbitals are unaligned, the moon will fall to the earth with a constant acceleration. If they are all perfectly aligned, the moon orbit will be circular at orbital velocity.

Movement between orbitals becomes a function of orbital 'buoyancy', for example, a submarine may travel across the ocean at a fixed depth (i.e. 100 m ) via propeller motion (a motion within the 100 m orbit), but to change from this equilibrium depth in order to rise to the surface or sink further, it must change its mass density (add or eject ballast). And so, while the momentum of the orbital keeps electrons and satellites following their orbits, it is this momentum 'buoyancy' which keeps the satellite from 'floating' off into space or 'falling' to the earth and which keeps the electron within a particular energy level.

To change from 1 orbit to another, whether atomic or gravitational, requires a change of orbital(s), i.e: a change in total orbit momentum. The strong force is equivalent to the momentum of the gravitational orbit or nuclear/atomic orbital whereby the sum orbit(al) momentum is canceled and the particles/objects dissociate.

The formulas cited here can be downloaded in maple format [3]. The relativity model referred to in this article can be found here [7].

## 2 Orbits

2.1. Atomic electron transition is defined as a change of an electron from one energy level to another within an atom, theoretically this should be a discontinuous electron 'jump' from one energy level to another although how this might happen is not clear.
Let us consider the Hydrogen Rydberg formula for transition between and initial $i$ and a final $f$ orbit. The incoming photon $\lambda_{R}$ causes the electron to 'jump' from the $n=i$ to $n=f$ orbit.

$$
\begin{equation*}
\frac{1}{\lambda_{R}}=R \cdot\left(\frac{1}{n_{i}^{2}}-\frac{1}{n_{f}^{2}}\right)=\frac{R}{n_{i}^{2}}-\frac{R}{n_{f}^{2}} \tag{1}
\end{equation*}
$$

The above could be interpreted as referring to 2 photons;

$$
\lambda_{R}=\left(+\lambda_{i}\right)-\left(+\lambda_{f}\right)
$$

Let us suppose a region between a free proton $p^{+}$and a free electron $e^{-}$which we may define as zero. This region then divides into 2 waves of momentum of inverse phase which we may designate as photon $(+\lambda)$ and anti-photon $(-\lambda)$ whereby

$$
(+\lambda)+(-\lambda)=\text { zero }
$$

The photon $(+\lambda)$ leaves (at the speed of light), the anti-photon $(-\lambda)$ however is trapped between the electron and proton and forms a standing wave orbital. Due to the loss of the photon, the energy of $\left(p^{+}+e^{-}+-\lambda\right)<\left(p^{+}+e^{-}+0\right)$ and so stable.

Let us define an $(n=i)$ orbital as $\left(-\lambda_{i}\right)$. The incoming Rydberg photon $\lambda_{R}=\left(+\lambda_{i}\right)-\left(+\lambda_{f}\right)$ arrives in a 2 -step process. First the $\left(+\lambda_{i}\right)$ adds to the existing $\left(-\lambda_{i}\right)$ orbital.

$$
\left(-\lambda_{i}\right)+\left(+\lambda_{i}\right)=\text { zero }
$$

The $\left(-\lambda_{i}\right)$ orbital is canceled and we revert to the free electron and free proton; $p^{+}+e^{-}+0$ (ionization). However we still have the remaining $-\left(+\lambda_{f}\right)$ from the Rydberg formula.

$$
0-\left(+\lambda_{f}\right)=\left(-\lambda_{f}\right)
$$

From this wave addition followed by subtraction we have replaced the $n=i$ orbital with an $n=f$ orbital. The electron has not moved (there was no transition from an $n_{i}$ to $n_{f}$ orbital), however the electron region (boundary) is now determined by the new $n=f$ orbital $\left(-\lambda_{f}\right)$.
2.2. If we take 2 objects $A$ and $B$, where $A$ is a sphere of points with radius $r, \mathrm{~B}$ is a single point and $L=r \omega=$ distance from $B$ to the center of sphere $A$ and then measure the average distance $S$ between B and each point in A then we find at short distances $L<S$. For a sphere of even point distribution we will find this $V$;

$$
\begin{gather*}
V^{2}=\frac{3}{5 \omega^{2}}  \tag{2}\\
L=r \omega=\frac{S}{1+V^{2} / 2}=S \sqrt{1-V^{2}} \tag{3}
\end{gather*}
$$



Fig. 1: $\mathrm{L}=\mathrm{S} \sqrt{\left(1-V^{2}\right)}$

If $S$ is a measure of the momentum between A and B , and if $S>L$, and if momentum cannot be compressed then B will be forced to rotate (spin) around A (fig.1) to compensate. In atomic orbits L is the radial axis (see article on relativity [7]), $S=$ particle wavelength and the $V$ term suggests the geometry of the orbit barycenter.
Positronium [3]: $f_{P}(1 s-2 s)=1233607$ 216.4 MHz [8]

$$
\begin{gather*}
S_{P}=2\left(\lambda_{e}+\frac{\lambda_{p}}{8}\right)  \tag{4}\\
L_{P}=S_{P} \cdot \sqrt{\left(1-\frac{48}{25 \alpha^{3}}\right)}  \tag{5}\\
\frac{3 c}{4} \cdot \frac{1}{4 \pi \alpha^{2} L_{P}}=1233607214.9 \mathrm{MHz} \tag{6}
\end{gather*}
$$

Hydrogen [3]:

$$
f_{H}(1 s-2 s)=2466061413187.035 \mathrm{kHz}[9]
$$

$$
\begin{gather*}
S_{H}=\left(\lambda_{e}+\lambda_{p}\right)  \tag{7}\\
L_{H}=S_{H} \cdot \sqrt{\left(1-\frac{48}{\alpha^{3}}\right)}  \tag{8}\\
\frac{3 c}{4} \cdot \frac{1}{4 \pi \alpha^{2} L_{H}}=2466061422987 \mathrm{kHz} \tag{9}
\end{gather*}
$$

Giving

$$
\begin{equation*}
\frac{L_{P}}{L_{H}} \cdot \frac{2 f_{P}(1 s-2 s)}{f_{H}(1 s-2 s)}=1.000000005 \tag{10}
\end{equation*}
$$

For Deutrium $f_{D}=13.6021343 \mathrm{eV}(13.60213452 \mathrm{eV})$ and Tritium $f_{T}=13.6033658 \mathrm{eV}(13.603 \mathrm{eV})$ we can simplify to the wavelength (from the mass) of the nucleus.

$$
\begin{equation*}
\left(\lambda_{e}+\lambda_{n u c}\right) \cdot \sqrt{\left(1-\frac{56}{\alpha^{3}}\right)} \tag{11}
\end{equation*}
$$

A Helium best fit gives 24.587375 eV ( 24.587387 eV ).

$$
\begin{equation*}
L_{H e}=\left(\frac{5 \lambda_{e}}{9}-\left(2 \lambda_{p}+2 \lambda_{n}\right)\right) \cdot \sqrt{\left(1-\frac{6}{5 \alpha^{2}}\right)} \tag{12}
\end{equation*}
$$

2.3. Objects A and B are divided into even spheres of points where $P \sim 4 \pi r^{3} / 3=$ total number of points (an even mass distribution) and where each point refers to a Planck mass pointstate (see article on relativity [7]) and $L_{g}=$ distance from center of A to center of B. We form an orbital link $l_{i}$ (a unit of momentum) between each point in A with each point in B , the observed 'gravitational' orbit of B around A (mass A >> B) deriving from the average of these individual orbitals $S$.

$$
\begin{equation*}
S=\frac{\left(l_{1}+l_{2}+l_{3} \ldots\right)}{P} \tag{13}
\end{equation*}
$$

The points, although scattered throughout spheres A and B, may be treated as a point mass concentrated at the center of the sphere, thus we can use the Schwarzschild radius $\lambda_{g}$ for A as $\lambda_{g}=P 2 l_{p}$ where $P=M_{A} / m_{P}$ and distance $\mathrm{R}=L_{g} 2 l_{p}$. We
can simplify to a gravity 'Bohr' model albeit our $n$, being an average of the links, is not an integer (for convenience I am using the commonly recognized value $\alpha \sim 137$ ). In planetary orbits typically $L \gg \lambda$, and so we may ignore the V term (only orbits close to a black-hole may resemble atomic orbits where $n$ has a low relative value).

$$
\begin{gather*}
n^{2}=\frac{L_{g}}{\alpha P}  \tag{14}\\
r_{g}=R=\alpha n^{2} \lambda_{g}  \tag{15}\\
v_{g}^{2}=\frac{c^{2} P}{2 L_{g}}=\frac{c^{2}}{2 \alpha n^{2}}  \tag{16}\\
a_{g}=\frac{c^{2}}{2 \alpha^{2} n^{4} \lambda_{g}}  \tag{17}\\
T_{g}^{2}=\frac{8 \pi^{2} \alpha^{3} n^{6} \lambda_{g}^{2}}{c^{2}} \tag{18}
\end{gather*}
$$

Example - Earth surface $\mathrm{R}=6374 \mathrm{~km}(\mathrm{n}=2289.047 \ldots$..)
$\mu_{\text {earth }}=3.986004418(9) \times 10^{14}$ (std grav. parameter)
$\lambda_{\text {earth }}=2 \mu_{\text {earth }} / c^{2}=.00887 \mathrm{~m}$
$M_{\text {earth }}=\mu_{\text {earth }} / G=.597378 \times 10^{25} \mathrm{~kg}$ [3]
$r_{g}=6374.0 \mathrm{~km}$
$a_{g}=9.811 \mathrm{~m} / \mathrm{s}^{2}$
$T_{g}=5064.4125 \mathrm{~s}$
$v_{g}=7907.931 \mathrm{~m} / \mathrm{s}$
Geosynchronous orbit $\mathrm{R}=42164 \mathrm{~km}(\mathrm{n}=5889.66 \ldots)$

$$
\begin{aligned}
& r_{g}=42164.0 \mathrm{~km} \\
& a_{g}=0.2242 \mathrm{~m} / \mathrm{s}^{2} \\
& T_{g}=86163.6 \mathrm{~s} \\
& v_{g}=3074.666 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

2.4. In this example we connect the points between $A=$ earth and $B=1 \mathrm{~kg}$ satellite.

$$
\begin{gather*}
\delta \mu_{G P E}=\frac{G M m}{r_{1}}-\frac{G M m}{r_{2}}  \tag{19}\\
r_{1}=\alpha n_{1} \lambda_{g} \\
r_{2}=\alpha n_{2} \lambda_{g}
\end{gather*}
$$

As Planck units...

$$
\begin{equation*}
\frac{G M m}{r_{g}}=\frac{h c}{2 \pi r_{g}} \cdot \frac{M m}{m_{P}^{2}} \tag{20}
\end{equation*}
$$

Rydberg (gravity)...

$$
\begin{gather*}
R=\frac{1}{2 \pi r_{g}}\left(\frac{1}{n_{1}^{2}}-\frac{1}{n_{2}^{2}}\right)  \tag{21}\\
f=R c\left(\frac{1}{n_{1}^{2}}-\frac{1}{n_{2}^{2}}\right)  \tag{22}\\
N_{\text {orbital }}=\frac{M m}{m_{P}^{2}} \tag{23}
\end{gather*}
$$

$$
\begin{gather*}
E_{\text {orbital }}=h f  \tag{24}\\
E_{\text {tot }}=E_{\text {orbital }} N_{\text {orbital }} \tag{25}
\end{gather*}
$$

Earth mass $M=5.97 \times 10^{24} \mathrm{~kg}$
Satellite mass $m=1 \mathrm{~kg}$
Earth surface $n_{g}=2290(6374 \mathrm{~km})$
Geosynchronous orbit $w=5890(42169 \mathrm{~km})$

$$
\begin{gathered}
f_{\text {orbital }}=n_{2290} \text { orbit }-n_{5890} \text { orbit } \\
f_{\text {orbital }}=7.485-1.132=6.354 \mathrm{~Hz} \\
E_{\text {orbital }}=0.412 \times 10^{-32} \mathrm{~J} \\
N_{\text {orbitals }}=\mathrm{Mm} / m_{P}^{2}=0.126 \times 10^{41} \\
E_{\text {total }}=E_{\text {orbital }} . N_{\text {orbitals }}=53 \mathrm{MJ} / \mathrm{kg}
\end{gathered}
$$

$N_{\text {orbitals }}=\left(M / m_{P}\right) \cdot\left(m / m_{P}\right)=$ total number of links and also corresponds to the gravitational coupling constant $\alpha_{G}=$ $m_{e}^{2} / m_{P}^{2}$ (here treated as the probability of 2 electrons being simultaneously in the point state for any chosen unit of Planck time [2]).

If these $0.126 \times 10^{41}$ orbitals are all unaligned the net result of summing their vectors of motion is that the satellite will appear to fall downwards, the acceleration due to the conservation of angular momentum, if they are all aligned, the satellite will be pulled by them in a circular orbit.
2.5. Time dilation.
2.5.1. Velocity: B (satellite) has a cylindrical orbit around the A (planet) radial axis with orbital period $T_{g} c$ (from $B^{1}$ to $B^{2}$ ) at radius $r_{g}$ and orbital velocity $v_{g}$. The orbital time $t_{g}$ alongside the A axis (fig. 2);

$$
\begin{equation*}
t_{g}=\sqrt{\left(T_{g} c\right)^{2}-\left(2 \pi r_{g}\right)^{2}}=\left(T_{g} c\right) \sqrt{1-\frac{v_{g}^{2}}{c^{2}}} \tag{26}
\end{equation*}
$$



Fig. 2: orbit relative to A timeline axis
2.5.2. Gravitational:

$$
\begin{align*}
& v_{s}=v_{\text {escape }}=\sqrt{2} \cdot v_{g}  \tag{27}\\
& \sqrt{1-\frac{2 G M}{r_{g} c^{2}}}=\sqrt{1-\frac{v_{s}^{2}}{c^{2}}} \tag{28}
\end{align*}
$$

The hole in the Earth (a thought experiment [1]). An apple on a tree-top is in the 6374.005 km orbit. To dissociate that apple
we have to add sufficient momentum to cancel the (gravitational) momentum of the $n_{6374.005}$ orbit. This can be achieved when the apple reaches escape velocity $v_{s}$, hence gravitational time dilation uses $v_{s}$.

If our apple drops from the tree-top orbit and lands on the ground it will transfer momentum to the earth and subsequently change to the lower earth-surface $=6374 \mathrm{~km}$ orbit.

However if there is a hole through the center of the earth then the apple will fall through it to the other side of the earth, then fall back into the hole returning to the tree-top 84 mins later. Having no means to transfer momentum it will remain throughout its journey in the $n_{6374.005}$ tree-top orbit, oscillating back and forth through that hole ad infinitum. The orbital (gravitational) angular momentum is conserved.

I am held on the earth surface by $10^{42}$ orbitals (units of momentum), a momentum buoyancy. I can only change orbits by changing my total orbit momentum.
2.6. The strong force is a momentum equivalent to the total orbit momentum such that the total orbit momentum is canceled and the orbit itself dissociates.

$$
\begin{gather*}
m_{n u c}=m_{p}+m_{n}  \tag{29}\\
\lambda_{s}=\frac{l_{p} m_{P}}{m_{n u c}}  \tag{30}\\
r_{0}=\sqrt{\alpha} \lambda_{s}  \tag{31}\\
R_{s}=\alpha \lambda_{s}  \tag{32}\\
v_{s}^{2}=\frac{c^{2}}{\alpha} \tag{33}
\end{gather*}
$$

The gravitational binding energy $(\mu)$ is the energy required to pull apart an object consisting of loose material and held together only by gravity.

$$
\begin{equation*}
\mu=\frac{3 G m_{n u c}^{2}}{5 R_{s}}=\frac{3 m_{n u c} c^{2}}{5 \alpha}=\frac{3 m_{n u c} v_{s}^{2}}{5} \tag{34}
\end{equation*}
$$

Nuclear binding energy is the energy required to split a nucleus of an atom into its component parts. The electrostatic coulomb constant;

$$
\begin{gather*}
a_{c}=\frac{3 e^{2}}{20 \pi \epsilon r_{0}}  \tag{35}\\
E=\sqrt{( } \alpha) a_{c}=\frac{3 m_{n u c} c^{2}}{5 \alpha}=\frac{3 m_{n u c} v_{s}^{2}}{5} \tag{36}
\end{gather*}
$$

Average binding energy in nucleus $=8.22 \mathrm{MeV} /$ nucleon

### 2.7. Anomalous precession

semi-minor axis: $b=\alpha l^{2} \lambda_{\text {sun }}$
semi-major axis: $a=\alpha n^{2} \lambda_{\text {sun }}$
radius of curvature L

$$
\begin{equation*}
L=\frac{b^{2}}{a}=\frac{a l^{4} \lambda_{s u n}}{n^{2}} \tag{37}
\end{equation*}
$$

$$
\begin{gather*}
\frac{3 \lambda_{\text {sun }}}{2 L}=\frac{3 n^{2}}{2 \alpha l^{4}}  \tag{38}\\
\text { precession }=\frac{3 n^{2}}{2 \alpha l^{4}} \cdot 1296000 .\left(100 T_{\text {earth }} / T_{\text {planet }}\right) \tag{39}
\end{gather*}
$$

| Table 1 [3] | GR [5] |  |
| ---: | :---: | :---: | Observed

2.8. $F_{p}=$ Planck force, $\lambda=$ Schwarzschild radius;

$$
\begin{gather*}
M_{a}=\frac{m_{P} \lambda_{a}}{2 l_{p}}, m_{b}=\frac{m_{P} \lambda_{b}}{2 l_{p}}  \tag{40}\\
F=\frac{M_{a} m_{b} G}{R^{2}}=\frac{\lambda_{a} \lambda_{b} F_{p}}{4 R_{g}^{2}}=\frac{\lambda_{a} \lambda_{b} F_{p}}{4 \alpha^{2} n^{4}\left(\lambda_{a}+\lambda_{b}\right)^{2}} \tag{41}
\end{gather*}
$$

a) If $\lambda_{a}=\lambda_{b}$, the object mass is not required

$$
\begin{equation*}
F=\frac{F_{p}}{\left(4 \alpha n^{2}\right)^{2}} \tag{42}
\end{equation*}
$$

b) If $\lambda_{a} \gg \lambda_{b}$, the relative mass is used

$$
\begin{equation*}
F=\frac{\lambda_{b} F_{p}}{\left(2 \alpha n^{2}\right)^{2} \lambda_{a}} \tag{43}
\end{equation*}
$$

## 3 Sqrt of momentum

In the introduction I used the term 'orbit momentum'. In an article on the virtual electron [2], the sqrt of Planck momentum (denoted $Q$ ) was applied as a separate constant (Planck momentum $=2 \pi Q^{2}$, units $=\mathrm{kgm} / \mathrm{s}=q^{2}$ ) and used to link the mass domain with the charge domain.
$Q$ appears in mass constants as $Q^{2}$ and in charge constants as $Q^{3}$ and $Q^{5}$ and in the electron as $Q^{15}$. As a sqrt, $Q$ can have a plus or a minus solution $\pm Q$, however as mass is $Q^{2}$, mass momentum is always plus. It is this Q that suggested that the electric force could also be replaced with a form of momentum, consequently the designation 'orbit momentum' to differentiate from the common notion of momentum.

$$
\begin{gather*}
Q=1.019113411 \ldots, \text { unit }=q  \tag{44}\\
m_{P}=\frac{2 \pi Q^{2}}{c}, \text { unit }=k g  \tag{45}\\
A=\frac{8 c^{3}}{\alpha Q^{3}}, \text { unit } A=\frac{m^{3}}{q^{3} s^{3}}=\frac{q^{3}}{k g^{3}}  \tag{46}\\
e=A t_{p}=\frac{8 c^{3}}{\alpha Q^{3}} \cdot \frac{2 l_{p}}{c}=\frac{16 l_{p} c^{2}}{\alpha Q^{3}}, \text { units }=\text { A.s }=\frac{q^{3} s}{\mathrm{~kg}^{3}} \tag{47}
\end{gather*}
$$

## 4 Summary

The 3 articles, Part 1 [6], Part 2 [7] and this article as Part 3 propose that by selecting appropriate geometrical forms within an expanding hyper-sphere, physical phenomena may naturally emerge, the laws of physics as descriptions of these phenomena but are not of themselves fundamental properties.

How the physical Planck units for mass, space, time and charge may derive from a mathematical formula for an electron is discussed in [2]. In this the electron geometry encodes the attributes of the electron in terms of magnetic monopoles (ampere-meters) and Planck time. This raises the possibility that space could also be electromagnetic in nature and so can also be measured in ampere meters, and thus the universe itself can be treated as a mathematical particle (a formula that encodes the attributes of the universe) with a frequency $f_{\text {universe }} \sim 10^{123}$. This universe-as-a-particle then becomes the universe pilot wave (the hidden variable).

The geometrical approach also, for example, by attributing a 'fracture-point' to a complex particle geometry, permits us to naturally derive the half-life constant, the analogy being continuously throwing up coffee cups that will only break when they land on their handle. The number of intact cups reduces over time according to $\ln (2)$ [10]. Likewise if nuclear fusion can only occur when 2 such colliding particles are appropriately aligned then particle orientation would be a contributing factor (cold fusion could then be theoretically attainable).

The premise being that by selecting the initial geometrical conditions and an iteration based guiding algorithm, perhaps encoded into the formula for the universe itself, the universe could naturally evolve from the big-bang into the present, these geometries replacing the laws of physics.

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