Multimodal Abduction in Knowledge Development

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Abstract

From the perspective of distributed cognition I will stress how abduction is essentially *multimodal*, in that both data and hypotheses can have a full range of verbal and sensory representations, involving words, sights, images, smells, etc., but also kinesthetic and motor experiences and other feelings such as pain, and thus all sensory modalities. The presence of kinesthetic and motor aspects plainly demonstrates that abductive reasoning is basically manipulative. We can also see, in this regard, how implicit factors take part in the abductive procedure, which consequently acquires the character of a kind of "thinking through doing". This paper further describes 1) the fact that hypotheses in science can be built through different cognitive mediators and so they can also model the same cognitive aspect in different ways; how they can be carriers/producers of knowledge in a *multimodal* way; 2) the problem of the possible *non-explanatory* and *instrumental* nature of abductive reasoning ¡and the analysis of the consequences for induction; 3) the role of *manipulative* abduction in building new evidence/experiments and how they trigger smart inductive inferences.

Multimodal Abduction

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Logical models and computational automation of abductive and inductive cognition are certainly compelled to neglect "[...] important philosophical concerns relating to causality and creativity" (Ray 2007). Similarly, philosophical and cognitive research certainly lack the rigor of logical models and the applicative chances of the computational ones. However, I think that the dialogue can still be fruitful. To this aim I propose in this paper some considerations that deal with the problem of the *multimodality* of data and hypotheses and the distributed and dynamic character of *evidence/experiment* in abductive reasoning.

Peirce considers inferential any cognitive activity whatever, not only conscious abstract thought; he also includes perceptual knowledge and subconscious cognitive activity. For instance in subconscious mental activities visual representations play an immediate role. Many commentators criticized this Peircean ambiguity in treating abduction at the same time as inference and perception. It is important to clarify this problem, because perception and imagery are kinds of that model-based cognition which I am exploiting to explain abduction: I contend that we can render consistent the two views (Magnani 2006), beyond Peirce, but perhaps also within the Peircean texts, partially taking advantage of the concept of *multimodal abduction*, which depicts hybrid aspects of abductive reasoning. (Thagard 2005; 2007) observes, that abductive inference can be visual as well as verbal, and consequently acknowledges the sentential, model-based, and manipulative nature of abduction I stressed in my previous research on th subject. For example, both data and hypotheses can be visually represented:

> For example, when I see a scratch along the side of my car, I can generate the mental image of a grocery cart sliding into the car and producing the scratch. In this case both the target (the scratch) and the hypothesis (the collision) are visually represented. [...] It is an interesting question whether hypotheses can be represented using all sensory modalities. For vision the answer is obvious, as images and diagrams can clearly be used to represent events and structures that have causal effects.

Indeed hypotheses can be also represented using other sensory modalities:

> I may recoil because something I touch feels slimy, or jump because of a loud noise, or frown because of a rotten smell, or gag because something tastes too salty. Hence in explaining my own behavior my mental image of the full range of examples of sensory experiences may have causal significance. Applying such explanations of the behavior of others requires projecting onto them the possession of sensory experiences that I think are like the ones that I have in similar situations. [...] Empathy works the same way, when I explain people's behavior in a particular situation by inferring that they are having the same kind of emotional experience that I have in similar situations (Thagard 2007).

Ignorance Preserving Reasoning and Non-Explanatory Abduction

(Gabbay & Woods 2005) contend that abduction presents an *ignorance preserving* (but also *ignorance mitigating*) character. Abductive reasoning is a *response* to an ignorance-problem: "One has an ignorance-problem when one has a cognitive target that cannot be attained on the basis of what one currently knows. Ignorance problems trigger one or other of three responses. In the one case, one overcomes one's ignorance by attaining some additional knowledge. In the second instance, one yields to one's ignorance (at least for the time being). In the third instance, one abduces" (Gabbay & Woods 2009, chapter five).

In this perspective the general form of an abductive inference can be rendered as follows, putting T for the agent's target at a time, K for his (or its) knowledge-base at that time, K^* for an accessible successor-base of K,[1] R as the attainment relation for T, H as the agent's hypothesis; $K(H)$ as K's adaptation of H, that is the revision of K upon the addition of H and R^{pres} as the relation of presumptive attainment relative to T. The general structure can be illustrated as follows:

1. $T!$	[setting of T as target]
2. $\neg(R(K,T))$	[fact]
3. $\neg(R(K^*,T))$	[fact]
4. $H \notin K$	[fact]
5. $H \notin K^*$	[fact]
6. $\neg R(H,T)$	[fact]
7. $R^{pres}(K(H),T)$	[fact]
8. H meets further conditions $S_1, ...S_n$	[fact].
9. Therefore, $C(H)$	[sub-conclusion, 1-7]
10. Therefore, H^c	[conclusion, 1-8] (cf. (Woods 2009) and (Gabbay & Woods 2005, pp. 47–48)).

[Note: Basically, line 8. indicates that H has no more plausible or relevant rival constituting a greater degree of subjunctive attainment. $C(H)$ is read "It is justified (or reasonable) to conjecture that H" and H^c its activation."

In sum, T cannot be attained on the basis of K. Neither can it be attained on the basis of any successor K^* of K that the agent knows then and there how to construct. H is not in K: H is a hypothesis that when reconciled to K produces an updated $K(H)$. H is such that if it were true, then $K(H)$ would attain T. The problem is that H is only hypothesized, so that the truth is not assured. Accordingly Gabbay and Woods contend that $K(H)$ presumptively attains T. That is, having hypothesized that H, the agent just "presumes" that his target is now attained. Given the fact that presumptive attainment is not attainment, the agent's abduction must be considered as preserving the ignorance that already gave rise to her (or its, in the case for example of a machine) initial ignorance-problem. Accordingly, abduction does not have to be considered the "solution" of an ignorance problem, but rather a response to it, in which the agent reaches presumptive attainment rather than actual attainment. $C(H)$ expresses the conclusion that it follows from the facts of the schema that H is a worthy object of conjecture. In order to solve a problem it is not necessary that an agent actually conjectures a hypothesis, but it is necessary that she states that the hypothesis is worthy of conjecture.

The superscript in H^c is a label. It reminds us, Gabbay and Woods say, that H "[...] has been let loose on sufferance" (cf. (Gabbay & Woods 2009)). Through abduction the basic ignorance – that does not have to be considered a total "ignorance" – is neither solved nor left intact: it is an ignorance-preserving accommodation of the problem at hand. As I have already stressed, even though in a defeasible way, further action can be triggered either to find further abductions or to solve the ignorance problem, possibly leading to what it is called in the literature the inference to the best explanation. It is clear that in this framework the inference to the best explanation – if considered as a truth conferring achievement – cannot be a case of abduction, because abductive inference is constitutively ignorance preserving. In this perspective the inference to the best explanation also involves the role of *induction*. Of course it can be said that the requests of originary thinking are related to the depth of the abducer's ignorance.

Non-Explanatory and Instrumental Abduction

(Gabbay & Woods 2005) also contend – and I agree with them – that abduction *is not intrinsically explanationist*, like for example its description in terms of inference to the best explanation would suggest. Not only that, abduction can also be merely *instrumental*. This conviction constitutes the main reason for proposing the GW-schema (Gabbay-Woods), which offers a representation of abductive cases not captured by that of the AKM (Aliseda-Kuipers-Magnani-Meheus), restricted to the explanatory cases. In my previous book on abduction (Magnani 2001) I made some examples of abductive reasoning that basically are non-explanatory and/or instrumentalist without clearly acknowledging it. Gabbay and Woods's distinction between explanatory, non-explanatory and instrumental abduction is orthogonal to mine in terms of the theoretical and manipulative (including the subclasses of sentential and model-based) and further allows us to explore fundamental features of abductive cognition. Hence, if we maintain that E explains E' *only if* the first implies the second, certainly the reverse does not hold. This means that various cases of abduction are consequentialist but not explanationist. Other cases are neither consequentialist nor explanationist.

Non-explanatory modes of abduction are clearly exploited in the "reverse mathematics" pioneered by Harvey Friedman and his colleagues, e.g., (Friedman & Simpson 2000), where propositions can be taken as axioms because they support the axiomatic proofs of target theorems. The target of reverse mathematics is to answer this fundamental question: What are the appropriate axioms for mathematics? The problems is to discover which are the appropriate axioms for proving particular theorems in central mathematical areas such as algebra, analysis, and topology (cf. (Simpson 1999)). The idea of reverse mathematics origi-

[1]"K^* is an accessible successor to K to the degree that an agent has the know-how to construct it in a timely way; i.e., in ways that are of service in the attainment of targets linked to K" (Gabbay & Woods 2009, chapter five, footnote 20).

nates with Russell's notion of the regressive method in mathematics (Russell 1973), and is also present in some remarks of Gödel.[2] (Gabbay & Woods 2005, p. 128) conclude, following Russell, that regressive abduction is both instrumental and non-explanatory.

Furthermore, often in physics the target is the discovery of physical dependencies which (Gabbay & Woods 2005, pp. 122–123) consider explanatorily undetermined. In this case abduction can exhibit an *instrumental* aspect. I have contended in (Magnani Forthcoming, chapter two) that this character is sometimes related to the conventional nature of the involved hypotheses. Moreover, also in many AI approaches based on logic programming and belief revision explanationism tends to disappear and abduction is mainly considered as proof theoretic and algorithmic: "On this view, an H is legitimately dischargeable to the extent to which it makes it possible to prove (or compute) from a database a formula not provable (or computable) from it as it is currently structured. This makes it natural to think of AI-abduction in terms of belief-revision theory, of which belief-revision according to explanatory force is only a part" (Gabbay & Woods 2005, p. 88). However, the explanatory character is subsumed in these AI approaches as a philosophical conception.

In sum, Gabbay and Woods maintain we can face a kind of abduction that, basically,

- is not plausibilist

at least in the sense we consider it on the explanatory framework.

They say: "It is not uncommon for philosophers to speak of the contribution made by the hypothesis of action-at-a-distance as one of explaining otherwise unexplainable observational data. [...] Like numerous instances of D-N explanation, Newtonian explanations need convey no elucidation of their explicanda. They need confer no jot of further intelligibility to them. The action at-a-distance equation serves Newton's theory in a wholly instrumental sense. It allows the gravitational theory to predict observations that it would not otherwise be able to predict" (Gabbay & Woods 2005, pp. 118-119). In this case Newtonian explanations are seen as epistemically agnostic conjectures, that is they lack epistemic virtues. These abductions are secured by instrumental considerations and accepted because doing so enables one's target to be hit. They cannot be discharged because of their possible implausibility, for example on the basis of empirical disconfirmation.

Abduction: Multimodal Hypotheses and Heuristics

Multimodal Hypotheses through Different Cognitive Mediators

Also in scientific reasoning multimodal abduction is at work and different hypotheses can be built through different cognitive mediators so that they can model the same aspect in different ways. (Flach, Kakas, & Ray 2006, p. 21), dealing with the problem of logical modeling and automated computation of the dyad abduction/induction, contend that "Modelling a scientific domain is a continuous process of observing and understanding phenomena according to some currently available model, and using this understanding to improve the original domain model. In this process one starts with a relatively simple model which gets further improved and expanded as the process is iterated. At any given stage of its development, the current model is very likely to be *incomplete* ".

Let us start considering the abductive side of this process. The abductive construction of hypotheses and theories is certainly driven by experimental observation to improve, refine, and complete the model. However, as I have anticipated above, already at the level of abductively making hypotheses, the relationship of the cognitive agent with the "experimental" observation is first of all occurring in a continuous interplay where the cognitive process is that kind of "thinking through doing", which I have described in terms of *manipulative abduction*. It involves the repeated production of new evidence, external to the cognitive subject, which provides new fundamental data to further fuel the reasoning process.

An example which deals with a simple and modestly creative way of guessing general mathematical hypotheses in actual humans can be of help. The research has been built in the pedagogical framework concerning the need of increasing knowledge on the ways in which learners in the area of school algebra develop their abilities. (Rivera & Rossi Becker 2007) illustrate the case of different subjects [elementary majors] who are given sequences of figural and numerical cues which taken together comprise classes of abstract objects such as even and odd numbers and related diagrams: "The accompanying questions oftentimes involve a twin calculation-encapsulation process, that is, from determining specific output values to abductively forming a viable general expression which can generate any element in the class. [...] Thus, the central purpose of generalizing tasks at the elementary level is to help learners develop an ability to generalize from particular instances and be able to express the generalization in ways that are both meaningful to them and valid from the standpoint of institutional practice" (p. 141). The task to be performed is very useful to illustrate a case of multimodal abduction at work (limited to the figural and numerical case), an abductive procedure which is looking for inductively produced generalized hypotheses as closed formulas.

The observational examples O (symbols and diagrams) presented to the subjects are *limited* and *incomplete*, like it is occurring in the case of other usual scientists' creative tasks. The subjects possess some knowledge about mathematics (we would say some theories T, in logical terms) more or less accurate, describing the model of the domain that is under investigation, and the "multimodal" information contained in the examples have to be "multimodally" managed through those available theories. The subjects have to produce new hypothetical knowledge, H, which extends their own preexistent theories such that the observations can

[2] For more details about this, see (Irvine 1989), who also compares Russell's regressive method to Peirce's abduction.

be first of all deduced by the new abductively enriched theories. It is in the abductive process that new "experiments" providing new data are often repeatedly realized.

To make a simple example

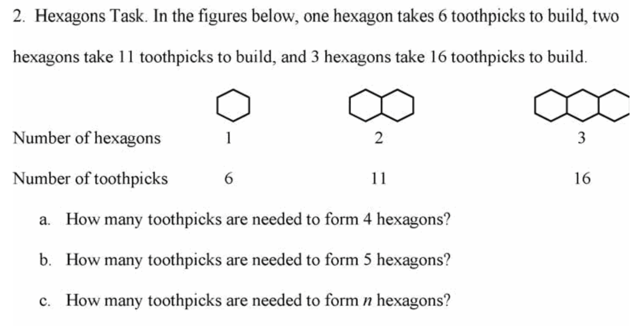
> For instance, the sequence $\{2, 4, 6, 8, \dots\}$ is a class and the closed formula $2n$ is one way of describing the overall structure of each number in the sequence. Also, it is perceptually apparent that evenness is one characteristic that is common to the numbers in the sequence. The Fibonacci sequence $\{1, 1, 2, 3, 5, 8, 13, \dots\}$ is another example of a class that can be generally described by the recursive relation $a_{n+1} = a_{n-1} + a_n$ (where $a_1 = a_2 = 1$). Its closed formula is $(1/\sqrt{5})[((1+\sqrt{5})/2)n - ((1-\sqrt{5})/2)^n]$ with the additional assumption that the numerical cues would obey the stated recursive form. The arithmetic class $\{3, 8, 13, 18, \dots\}$ can be generalized by the direct expression $5n - 2$ under the condition that the class is an increasing sequence and where $n \geqslant 1$. Resemblance encompasses implicit (deep) and explicit (surface) properties that cues within a class have in common, and these properties are not inherently a priori (p. 142).

From the cognitive-psychological point of view we can say the subjects abduce new properties of the objects at play by projecting them onto individual elements of the class being tested: for instance employing *numerical heuristics* (for instance the "finite difference method") "[...] in order to surface properties that are or are not directly knowable due to the incompleteness of the cues presented to the learners" (*ibid.*)[3] It is typical of abduction to be able to increase knowledge (it is an ampliative reasoning) about a class of objects even if they are presented in a very incomplete way: in abductive reasoning ignorance is preserved, but weakened, as illustrated above.

Externalization as an Abductive Experimental Step

We have said that evidence presented to the subjects consists of limited and incomplete *multimodal* cues, such as the ones illustrated in the example of Figure 1, which is very useful to depict the multimodal character of abduction. The subjects abductively work on the available cues: some of them adopt a merely *numerical modality* (and related inferential routines), other a *figural* one (also a hybrid combinations of both is sometimes exploited). To perform the cognitive task they are required to draw or compute two additional cases and they do this abductively by exploiting either *figural* or *numerical hypotheses* thanks to fact they were able to perceive relations possibly leading to the abductive generalization in very different ways. They do this – and this is the main point I want to stress – with the help of a suitably built *new evidence*, that is by reframing the problem in a new external mediator (in our example a sheet of paper is exploited

[3]Subject can mobilize different inferential routines such as, to make an example, *guess and check* and *trial and error*, as Lakatos wonderfully analyzed in his famous book about mathematical reasoning and problem solving (Lakatos 1976). AI computational programs that took advantage of this Lakatosian perspective are illustrated in(Pease *et al.* 2005).


2. Hexagons Task. In the figures below, one hexagon takes 6 toothpicks to build, two hexagons take 11 toothpicks to build, and 3 hexagons take 16 toothpicks to build.

Number of hexagons	1	2	3
Number of toothpicks	6	11	16

a. How many toothpicks are needed to form 4 hexagons?

b. How many toothpicks are needed to form 5 hexagons?

c. How many toothpicks are needed to form *n* hexagons?

Figure 1: Hexagons Task (in (Rivera & Rossi Becker 2007)).

- an example is given in Figure 2) performed thanks to their available inner 1) mathematical knowledge and inferential procedures, other more or less simple rational reasoning devices such as guess and check, trial and error, and various capacity to draw *new* evidence and to execute visual comparison of forms, based on their stored perceptual knowledge gained through the observation of the initial cues, etc. The mediator plays the role, so to say, of a further appropriately built "experimental data", which offer chances for further knowledge.

Various intertwined *representational agents* are at play, in terms of 1) inner knowledge of humans in terms of data, theories, and inferential procedures, 2) knowledge embedded in the evidence provided at the start, 3) evidence (data) that can be picked up in the subsequent rebuilt new "experimental" evidence, to which other aspects of inner available mathematical knowledge and inferential procedures can be applied.[4] For example some subjects that used the *numerical modality* perceived relationships among the elements in the same class rather differently than the ones that used the figural one, and some of them, because of the inefficiency of the adopted abductive process, were not able to reach the final correct generalization; others instead reached a bad one.[5] Most of the 14 subjects that used the *figural modality* (performing what I have called *model-based abduction* (Magnani 1999)) easily reached the solution of the problem at hand. They mainly based the performance on their cognitive inner capacity to draw *new* evidence on their stored perceptual knowledge gained through the observation of the initial cues.

In summary, what happens in these highly *unstable* abductive subprocesses (both from the numerical and figural perspective) is related to what I call *manipulative abduction* (Magnani 2001): the new evidence provided is *practical* and *situational* – that is just *ignorance mitigating* – and in this performance the subjects usually do not benefit of an ex-

[4]Details are illustrated in (Rivera & Rossi Becker 2007).

[5]It is amazing to see that in the solution of this simple task, some subjects engaged complicated abductive processes without being able to reach the correct hypothesis and to trigger the subsequent induction. One subject did not guess any abductive hypothesis.

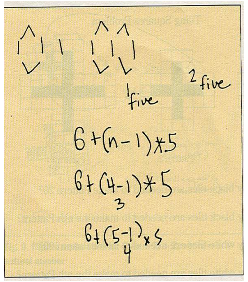


Figure 2: Written solution of the hexagons task (in (Rivera & Rossi Becker 2007)).

plicit conceptual explanation (I have said that it is a kind of "thinking through doing"). As I have already illustrated above the results are produced through heuristics which resort to automatized well-known (and available to the subjects) mathematical procedures but also to general cognitive modalities of depicting symbols or drawings able to the aim of finding features that can possibly be further picked up and stored in the internal memory.[6] These subprocesses are constitutively cyclic: 1) they can suggest new abductive steps where new evidence is built to make available further *situated* abductive generalizations; each of these abductive *situated* generalizations can in turn generate 2) further *universal* inductive generalizations possibly to be withdrawn because of disconfirmation; in this last case a 3) further cyclic abductive-inductive process can restart. The "specificity" of abductive hypotheses is related to their ignorance-preserving character; the "generality" of inductive hypotheses is related to their truth-conferring/probability-enhancing character, at the same time endowed with an evaluative function.

Non-Explanatory Abduction and Induction

In our example, even if, so to say, some abductive steps produce new generated evidence able to transform knowledge from its tacit (in the first cues) to its explicit form, and consequently human agents are able to generate and justify them in an *explanatory* perspective, the final abduced provisional hypothesis $5n + 1$ *does not* really *explain* the data, but it is a fruit of that non-explanatory abduction I have illustrated above. The inductive further step strengthens this aspect, as we will see in the following subsection.

Such explanations – made possible by the construction of new "experimental devices" (through drawings, calculations, sketches), such the one of Figure 2– generate knowledge that is always *specific* to the particular scene of the world concerning the observations explained and the given *multimodal* knowledge and routines available to the human agents. However, they allow them to predict further observable information. Building these new observations/experimental devices through manipulative abduction is central to make possible induction able to generate general knowledge, new and not reachable through abduction. In these perspective they become smart new empirical *samples* able to trigger interesting inductions.

What is the subsequent role of induction in the example illustrated in the previous section? In performing the abductive task to the general form the subjects referred to the fact they immediately saw a relationship among the drawn cues in terms of relational similarity "[...] within classes in which the focus was *not* on the individual clues in a class *per se* but on a possible invariant relational structure that was perceived between and, thus, projected onto the cues" (Rivera & Rossi Becker 2007, p. 151). Through the follow-up inductive stage of generalizations the subjects tested the hypotheses just examining *extensions* (new particular cases beyond what was available at the beginning of the reasoning process). This process was also able to show subjects's disconfirmation capacities: the acknowledged their mistakes in generating bad induction, which had to be abandoned, in so far as they were checked as insufficient in fully capturing in symbolic terms a general attribute that would yield the total number of toothpicks in new generated cues.

Manipulative Abduction: How to Build Evidence Inductively Relevant

We have seen in the mathematical example of the previous sections that abductive processes that are at play can be considered manipulative. I have introduced the concept of *manipulative abduction* - contrasted with theoretical abduction (Magnani 2001) - to illustrate situations where we are thinking through doing and not only, in a pragmatic sense, about doing. So the idea of manipulative abduction goes beyond the well-known role of experiments as capable of forming new scientific laws by means of the results (nature's answers to the investigator's question) they present, or of merely playing a predictive role (in confirmation and in falsification). Manipulative abduction refers to an extra-theoretical behavior that aims at creating communicable accounts of new experiences to integrate them into previously existing systems of experimental and linguistic (theoretical) practices. The abductive construction of new evidence ((Gooding 1990) call these external representational/experimental devices *construals*) in the mathematical case I have illustrated above presents an extra-theoretical behavior of this type.

I think that a better understanding of manipulative abduction at the level of scientific experiment could improve our knowledge of induction, and its distinction from abduction: manipulative abduction can be considered as a kind of basis for further abductive steps but also for possible meaningful inductive generalizations. For example different generated construals can give rise to different inductive generalizations. It is difficult to grasp this distinction through present logical models of the induction/abduction puzzle.

(Josephson 2000) maintains that "An inductive generalization is an inference that goes from the characteristics of

[6]Figure 2 illustrates a further evidence built by the subject engaged in a variable-oriented abduction. In such new expressly built evidence he "experienced" "[...] the use of a variable which substitutes as a general placeholder or expression for a sequence that he perceived to continuosly grow indefinitely" (Rivera & Rossi Becker 2007, p. 151).

some observed sample of individuals to a conclusion about the distribution of those characteristics in some larger populations" (p. 40).

I contend manipulative abduction is the correct way for describing the features of what can trigger "smart inductive generalizations", as contrasted to the trivial ones. For example, in science construals and new built evidence can shed light on this process of sample "production" and "appraisal": through construals, manipulative abduction generates abstract "specific" and "ignorance mitigating" hypotheses, which in the meantime can originate possible bases for further meaningful inductive generalizations through the identification of new samples (or of new features of already available samples). Different generated construals can give rise to different plausible inductive generalizations.

Conclusion

In this paper I have illustrated how abduction is essentially *multimodal*. I think the issue has some consequences for logical models and computational automation of the cognitive dyad abduction-induction in scientific reasoning. First of all scientific hypotheses expressed through different modalities can be logically and computationally taken into account and suitably represented, in so far as they model the same cognitive aspect in different ways and provide different abductive inferential chances: they are different multimodal *knowledge carriers*. Second, the problem of the *non-explanatory* and *instrumental* nature of abductive reasoning/hypotheses reveals that good inductions are favored by abductive steps, which often present *explanatory* features, but that at the end in some cases resort to an inductive *non-explanatory* result, like in the case of mathematics. Third, I have stressed that abductive steps are often occurring in a continuous interplay among the reasoner and its external cognitive environment in which new experimental data (new evidence) are built and at the same time re-offered to the reasoner in a process of *manipulative abduction*: this also refers to the role of this process of building new experimental data (new evidence) in triggering smart inductive inferences.

References

Flach, P. A.; Kakas, A. C.; and Ray, O. 2006. Abduction, induction, and the logic of knowledge development. In Flach, P. A.; Kakas, A. C.; Magnani, L.; and Ray, O., eds., *Workshop on Abduction and Induction in AI and Scientific Modeling*. 21–24.

Friedman, H., and Simpson, S. 2000. Issues and problems in reverse mathematics. *Computability Theory and its Applications: Contemporary Mathematics* 257:127–144.

Gabbay, D. M., and Woods, J. 2005. *The Reach of Abduction*. Amsterdam: North-Holland. Volume 2 of *A Practical Logic of Cognitive Systems*.

Gabbay, D. M., and Woods, J. 2009. *Seductions and Shortcuts: Fallacies in the Cognitive Economy*. New York: Elsevier/North Holland. A Practical Logic of Cognitive Systems, vol. 3. Fortchoming.

Gooding, D. 1990. *Experiment and the Making of Meaning*. Dordrecht: Kluwer.

Irvine, A. 1989. Epistemic logicism and Russell's regressive method. *Philosophical Studies* 55:303–327.

Josephson, J. 2000. Smart inductive generalizations are abductions. In Flach, . P., and Kakas, A., eds., *Abduction and Induction*. Dordrecht: Kluwer Academic. 31–44.

Lakatos, I. 1976. *Proofs and Refutations. The Logic of Mathematical Discovery*. Cambridge: Cambridge University Press.

Magnani, L. 1999. Model-based creative abduction. In Magnani, L.; Nersessian, N. J.; and Thagard, P., eds., *Model-based Reasoning in Scientific Discovery*. New York: Academic/Plenum Publishers. 219–238.

Magnani, L. 2001. *Abduction, Reason, and Science. Processes of Discovery and Explanation*. New York: Kluwer Academic/Plenum Publishers.

Magnani, L. 2006. Mimetic minds. Meaning formation through epistemic mediators and external representations. In Loula, A.; Gudwin, R.; and Queiroz, J., eds., *Artificial Cognition Systems*. Hershey, PA: Idea Group Publishers. 327–357.

Magnani, L. Forthcoming. *Abductive Cognition. The Eco-Cognitive Dimension of Hypothetical Reasoning*.

Pease, A.; Colton, S.; Smaill, A.; and Lee, J. 2005. A model of Lakatos's philosophy of mathematics. In Magnani, L., and Dossena, R., eds., *Computing, Philosophy and Cognition*. London: College Publications. 57–85.

Ray, O. 2007. Automated abduction in scientific discovery. In Magnani, L., and Li, P., eds., *Model-Based Reasoning in Science, Technology, and Medicine*. Berlin: Springer. 103–116.

Rivera, F. D., and Rossi Becker, J. 2007. Abduction-induction (generalization) processes of elementary majors on figural patterns in algebra. *Journal of Mathematical Behavior* 26:140–155.

Russell, B. 1973. The regressive method of discovering the premises of mathematics [1907]. In Lackey, D., ed., *Essays in Analysis*. London: George Allen and Unwin. 45–66.

Simpson, S. G. 1999. *Subsystems of Second Order Arithmetic*. Berlin: Springer. Second edition, forthcoming, Association for Symbolic Logic.

Thagard, P. 2005. How does the brain form hypotheses? Towards a neurologically realistic computational model of explanation. In Thagard, P.; Langley, P.; Magnani, L.; and Shunn, C., eds., *Symposium "Generating explanatory hypotheses: mind, computer, brain, and world"*. Stresa, Italy: Cognitive Science Society, CD-Rom. *Proceedings of the 27th International Cognitive Science Conference*.

Thagard, P. 2007. Abductive inference: from philosophical analysis to neural mechanisms. In Feeney, A., and Heit, E., eds., *Inductive reasoning: Experimental, developmental, and computational approaches* . Cambridge: Cambridge University Press. 226–247.

Woods, J. 2009. Ignorance, inference and proof: abductive logic meets the criminal law. In *Proceedings of the International Conference "Applying Peirce"*. Forthcoming.