# The price of insisting that quantum mechanics is complete

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**Abstract**

The Bare Theory was offered by David Albert as a way of standing by the completeness of quantum mechanics in the face of the measurement problem. This paper surveys objections to the Bare Theory that recur in the literature: what will here be called the oddity objection, the coherence objection, and the context-of-the-universe objection. Critics usually take the Bare Theory to have unacceptably bizarre consequences, but to be free from internal contradiction. Bizarre consequences need not be decisive against the Bare Theory, but a further objection—dubbed here the calibration objection—has been underestimated. This paper argues that the Bare Theory is not only odd but also inconsistent. We can imagine a successor to the Bare Theory—the Stripped Theory—which avoids the objections and fulfills the original promise of the Bare Theory, but at the cost of amplifying the bizarre consequences. The Stripped Theory is either a stunning development in our understanding of the world or a reductio disproving the completeness of quantum mechanics.



## 1 The Bare Theory

We can exemplify the quantum mechanical measurement problem in the usual way by considering a particle like an electron. Note that an electron has two possible spin states in the $x$ direction: up and down. Call the quantum state where the electron is spin up $|x_{up}\rangle$; where it is spin down, $|x_{down}\rangle$. Suppose now that you observe the particle. The relevant features of you involved in observing the electron may be represented crudely as having three possible states: in one you are prepared to make the observation, in another you have observed that the electron is spin up, and in the last you have observed that the electron is spin down. Call the quantum states corresponding to each of the three states $|U_{looking}\rangle$, $|U_{up}\rangle$, and $|U_{down}\rangle$ respectively. Suppose further that you are a meticulous observer. If the particle is in state $|x_{up}\rangle$ and you are in state $|U_{looking}\rangle$, then you will go into state $|U_{up}\rangle$ without fail. If the particle is $|x_{down}\rangle$, then you will go into $|U_{down}\rangle$. We may write these conditions as

$$\begin{aligned} |U_{looking}\rangle|x_{up}\rangle &\rightarrow |U_{up}\rangle|x_{up}\rangle \\ |U_{looking}\rangle|x_{down}\rangle &\rightarrow |U_{down}\rangle|x_{down}\rangle \end{aligned} \quad (1)$$

Now suppose that the electron is neither in the pure state $|x_{up}\rangle$ nor in the pure state $|x_{down}\rangle$, but rather in the arbitrary superposition $(\alpha|x_{up}\rangle + \beta|x_{down}\rangle)$ for some non-zero values of $\alpha$ and $\beta$. From (1), and the fact that the mechanics is linear, we can conclude

$$|U_{looking}\rangle(\alpha|x_{up}\rangle + \beta|x_{down}\rangle) \rightarrow \alpha|U_{up}\rangle|x_{up}\rangle + \beta|U_{down}\rangle|x_{down}\rangle \quad (2)$$

That is, if you look at the electron when it is in a superposition, you will go into a superposition of the pure state associated with your saying that the electron is spin up and the pure state associated with your saying that the electron is spin down. On the usual interpretation (the eigenvalue-eigenstate link) this means that your observation doesn't take on a determinate value at all. Instead, you are in a superposition of saying that it is spin up and saying that it is spin down. This is very puzzling, because it would not seem to you as if you were in a superposition. As a meticulous observer, you would surely make a determinate observation. This paradox captures the crux of the measurement problem.

David Albert ([1992], pp. 116–25) suggests that we consider what would happen if you were asked whether you had made a determinate observation. You wonder to yourself whether you did in fact make a determinate observation. The relevant features of you involved in this act of introspection may be represented crudely as having three possible states: in one you are pondering the matter, in another you have concluded that you did make a determinate observation, and in the last you have concluded that you did not. Call the quantum states corresponding to each of the three states $|V_{wondering}\rangle$, $|V_{yes}\rangle$, and $|V_{no}\rangle$ respectively. Suppose that you are as keen an observer of yourself as you are of electrons. Thus, if the electron is in a pure state you will observe that it is and you will conclude that, yes, you did make a determinate observation. To put this formally:

$$\begin{aligned} |V_{wondering}\rangle|U_{up}\rangle|x_{up}\rangle &\rightarrow |V_{yes}\rangle|U_{up}\rangle|x_{up}\rangle \\ |V_{wondering}\rangle|U_{down}\rangle|x_{down}\rangle &\rightarrow |V_{yes}\rangle|U_{down}\rangle|x_{down}\rangle \end{aligned} \quad (3)$$

It follows from (2) and (3) that

$$|V_{wondering}\rangle|U_{looking}\rangle(\alpha|x_{up}\rangle + \beta|x_{down}\rangle) \rightarrow \\ \alpha|V_{yes}\rangle|U_{up}\rangle|x_{up}\rangle + \beta|V_{yes}\rangle|U_{down}\rangle|x_{down}\rangle \quad (4)$$

We may rewrite the resulting state in (4) as

$$|V_{yes}\rangle(\alpha|U_{up}\rangle|x_{up}\rangle + \beta|U_{down}\rangle|x_{down}\rangle) \quad (5)$$

This contains only a $|V_{yes}\rangle$ term. On the usual interpretation, then, you would be in a determinate state of concluding that you had made a determinate observation. However, you would not be in a determinate state of observing the electron to be spin up or of observing the electron to be spin down![1]

This allows for an odd solution to the measurement problem, one Albert dubs the *Bare Theory*. According to the Bare Theory, the quantum mechanical wave function is a complete and accurate description of the world, the eigenstate-eigenvalue link holds, and wave functions never collapse. As a result, you do not make a determinate observation after you look at an electron that is in a superposition—you will be in a superposition of $|U_{up}\rangle$ and $|U_{down}\rangle$. Yet you will believe that you are in such a determinate state—you will be in the eigenstate $|V_{yes}\rangle$. The paradox is resolved by explaining how it can appear to you that you are in a determinate state while you are, in fact, not in one.

## 2 The Usual Objections

Three objections to the Bare Theory recur in the literature. One is that the Bare Theory is so odd that we should not accept it. Worse than odd, in fact, because the world described by the Bare Theory is manifestly not the world we inhabit. Call this the *oddity objection.* Another objection is that if the Bare Theory were true, then we could not know it to be true. The very evidence we call upon in defense of quantum theory is denied by the Bare Theory. Call this the *coherence objection.* A third objection is that the Bare Theory supposes that the universe is in a determinate state of you being prepared to make an observation. If it were not so—and we have no reason to expect that it would be so—then the Bare Theory cannot explain even your belief that you made a determinate observation. Call this the *context-of-the-universe objection.*

First, the oddity objection. Consider the case of the electron in a superposition and your observation of it. The Bare Theory offers no explanation for why you observe it to be spin up or to be spin down. This seems like a deficiency in the theory, but only if you suppose that you do indeed observe it to be one or the other. This is just what is denied on the Bare Theory. It purports to explain how you can believe that you have a determinate belief, when in fact you have no such thing. Michael Dickson ([1998], p. 47) insists that this is a telling point against the Bare Theory, since it is obviously false in claiming that you would not make a determinate answer if asked whether the electron were spin up or spin down. Bub, et al. ([1998], p. 42) also consider the oddity objection to be telling.

There is something wrong about the oddity objection. Explanations ought to account for as much as possible, to be sure, and should explain why certain things are the case. They may also explain away some things, however, by denying the fact in question and offering an explanation for the appearance. The Bare Theory can be defended against the oddity objection by insisting that the determinate answer about the state of the electron is to be explained away, while the belief that there is a determinate answer is to be accepted and explained. As Jeffrey Barrett observes, "we typically give our theories the chance to explain why things are not as they seem" ([1999], p. 112). We acknowledge this feature of explanation in non-philosophical cases. Suppose a child—terrified of monsters—tells us about the vampire that is lurking outside her window. We are under no obligation to explain why there is a vampire outside the window. We would instead deny that there is a vampire and explain the child's belief that there is a vampire. We deny the alleged supernatural phenomena in explaining the superstition.

This feature of explanation is also perspicuous in the philosophy of mind. There are traditional views of mental life: that beliefs in our minds have a propositional or pseudo-propositional structure, that our conscious lives are transparent to us, that we have incorrigible access to how the world seems to us, and so on. Some philosophers advocate rejecting the received views and adopting a promising account that explains away the alleged features of mental life. For instance, Daniel Dennett ([1991]) tries to explain away many features of consciousness; Paul Churchland aims to explain away propositional structure (passim; for a recent statement, see his [1998], §I). Regardless of how these explanations are ultimately judged, they should not be dismissed on account of oddity alone. There is no formal boundary between what must be explained and what may be explained away, so oddity need not carry logical force.

A second objection turns on the fact that whatever we take to be our evidence for believing in quantum mechanics is denied on the Bare Theory. The data that physicists collect are not determinate observations in abstract, but in each case some particular result: The first observation may be spin up, the second spin down, the third again spin down, and so on. If the Bare Theory is to be believed, such data does not exist even though it is true that for each observation 'The physicist makes a determinate observation' is true. The Bare Theory cannot even recover the relative frequencies in the data, for instance that spin up was observed 49% of the time in a run of trials. Albert ([1992], pp. 122–4) shows that frequencies may be recovered if there are infinitely many observations, but not in a way that generalizes to finite data sets even in the limit of very large numbers of observations.

The objection is not that the Bare Theory is logically incoherent, but rather that it is *empirically incoherent*. The term is due to Barrett ([1996]), but the sentiment is expressed by most authors responding the Bare Theory.[2] Dickson, for instance, objects that the Bare Theory could only be held "on pure faith" since its truth entails that we are mistaken about what we took to be evidence for quantum mechanics. He presses the point further, suggesting that "it is not even clear that the bare theory is susceptible to rational adherence" ([1998], p. 47).

Like the first, this objection needn't be decisive against the Bare Theory. It is always possible to embrace a certain empirical incoherence without abandoning rationality entirely. Sometimes we come to believe something which we reflectively endorse even though we recognize that we initially came to believe it for bad reasons. Perhaps the Bare Theorist will reflectively endorse quantum mechanics even absent the evidence which was critical for its historical acceptance. Perhaps, to use Wittgenstein's metaphor, the evidence is a ladder which must be kicked away once we have climbed to the dizzying height of quantum theory. (The analogy with Wittgenstein is not quite precise. I return to this point below.)

Dickson concludes his brief discussion by remarking, "Probably these consequences of the bare theory are sufficiently bizarre to render it unacceptable to most readers" ([1998], p. 47). Probably they are, but perhaps they are not. What of the reader who has a greater tolerance for the bizarre? In answer to that reader, we would need an objection to the Bare Theory that is more compelling than these.

A third objection, the context-of-the-universe objection, is less easily deflected. As Albert ([1992], pp. 124–5) notes, the universe is unlikely to be in an eigenstate for your existence and hence the initial setup in which you are in the eigenstate $|V_{wondering}\rangle$ is implausible . This means that there will be terms in addition to the $|V_{yes}\rangle$ term, in some of which you do not exist at all. Since the Bare Theory derives its explanatory force from the fact that you will wind up in the eigenstate $|V_{yes}\rangle$, the Bare Theory fails when considered in this wider context. Barrett ([1999], pp. 119–20) considers this to be a decisive objection to the Bare Theory.

# 3 The Calibration Objection

Having considered the usual objections, let's return to the formulation of the Bare Theory above. You are by assumption able to accurately observe whether or not you have made a determinate observation of the electron's spin. Consider what this means: If you are a reliable judge on some matter, we will expect you to assent when the matter obtains and dissent when it fails to obtain. The former condition was expressed in (3); the latter condition may be represented by

$$|V_{wondering}\rangle(\alpha|U_{up}\rangle|x_{up}\rangle + \beta|U_{down}\rangle|x_{down}\rangle) \rightarrow \\ |V_{no}\rangle(\alpha|U_{up}\rangle|x_{up}\rangle + \beta|U_{down}\rangle|x_{down}\rangle) \quad (6)$$

for non-zero coefficients $\alpha$ and $\beta$. That is, if you are in some superposition of $|U_{up}\rangle$ and $|U_{down}\rangle$, you will answer 'No' if asked whether or not you have made a determinate observation.

Considering a superposition for appropriate values of $\alpha$ and $\beta$,[3] it follows from (6) that

$$|V_{wondering}\rangle|U_{up}\rangle|x_{up}\rangle \rightarrow |V_{no}\rangle|U_{up}\rangle|x_{up}\rangle \tag{7}$$

Hence, you would answer 'No' if you are in eigenstate $|U_{up}\rangle$ even though $|U_{up}\rangle$ represents a determinate observation! For the Bare Theory even to have its tenuous hold on explanatory force, your self examination must be of the sort described in (3) rather than of the sort described in (6). However, there is no obvious reason why accuracy in your own self reports is better formalized in terms of your 'Yes' answers (and hence represented by (3)) rather than in terms of your 'No' answers (and hence represented by (6)).

The situation for the Bare Theory is actually worse than that. Since both conditions must hold for your ability to be reliable, it seems plausible to say that if you can accurately answer whether or not your observation of the electron had a determinate result then you should be treated as a system satisfying both (3) and (6). Yet these taken together are inconsistent given quantum mechanics and indeed given any linear dynamics.[4] 'This is a determinate state' is not itself a quantum-mechanical observable, so the assumption that you are accurately able to judge whether your first observation yielded a determinate result is incompatible with the assumption that you are only a quantum system. Call this the *calibration objection*, since it presumes that reliable self reports must be calibrated so as to satisfy both (3) and (6).

Barrett ([1998], p. 324) and Bub, et al. ([1998], pp. 38–9) insist that this objection should not be decisive. The Bare Theory, as a general account, need only explain how our experience is nomologically *possible* not that our experience is *necessary*. The advocate of the Bare Theory might insist: The fact that we think we have determinate experience shows that (3) alone is the correct calibration condition for us as cognitive agents. We have reason to expect our brains to accord with conditions like (3), but what we know about brains makes it unlikely that they can make the interference measurements that would be involved in a condition like (6).

Despite the plausibility of this reply, it will not do. If the semantics of a state is constituted by its inferential force—and since the inferential force of 'I made a determinate observation' is characterized by (3) *and* (6)—then the state $|V_{yes}\rangle$ as represented in the Bare Theory cannot mean 'I made a determinate observation.' One need not accept a fully inferentialist semantic theory in order to see the force of this objection. One need only feel that the meaning of certain claims is bound up in *some* important way with defeasibility conditions. If it were not, why would the state corresponding to 'I made a determinate observation' need to accord with *either* of (3) *or* (6)?

According to the Bare Theory, you are a quantum system and you believe determinately that you made a determinate observation. Yet, by the argument given above, these conditions entail both (3) and (6). Since these are inconsistent, the Bare Theory is logically incoherent. Not only is it unacceptable on assumptions that most people would accept as reasonable, it is unacceptable on its own terms. This is a stronger objection than the usual ones, because it would force the adherent of the Bare Theory to give up not only arguably important beliefs but pieces of the Bare Theory itself.

# 4 Beyond the Bare Theory

Someone enamoured of the Bare Theory may still respond to this objection, but only by making some change in the Bare Theory itself. We might represent the Bare Theory as the conjunction of several claims: (B1) The quantum mechanical wave function is a complete and accurate description of the world. (B2) The eigenstate-eigenvalue link holds. (B3) Wave functions never collapse. (B4) Our belief that we make determinate observations is to be explained by the fact that the observable 'We made a determinate observation' typically returns a 'Yes' answer. Which of the four must be changed?

Barrett ([1998]) and Bub, et al. ([1998]) recommend revisions to the Bare Theory that add hidden variables of some sort or another. This explicitly eschews B1, but also rejects B2 and B4 as an immediate consequence. Of the elements that define the Bare Theory, only B3 would remain. The notion that hidden variables could 'fix the Bare Theory' is rather odd. Would any no-collapse theory, any theory that satisfies B3, count as a variant of the Bare Theory? Suppose instead that our Bare Theorist wants to keep more than that. As we saw in the previous section, the assumption that you are able to accurately observe whether or not your initial observation ended in a determinate result is incompatible with the assumption that your observations are quantum records. That is, B4 is incompatible with the rest. Suppose the Bare Theorist gives up B4 and says that you are *not* able to accurately observe whether or not your initial observation ended in a determinate result. There is nothing special about this self report among others, though, so he must further deny that you are ever able to accurately judge whether a record is in a determinate state. If beliefs are instantiated in quantum systems—and they must be if quantum mechanics is complete—then you will not be able to judge whether or not any observable has a determinate value. You may imagine you make such judgments, the Bare Theorist may insist, but you are in fact misdescribing whatever judgments you actually do make. This successor to the Bare Theory, which we might call the *Stripped Theory*, explains away your determinate beliefs not only at the level of direct observation but also at every level of reflection. We may represent the Stripped Theory as the conjunction of B1–B3 and: (S4) For all observables $O$, it is nomologically impossible to have a belief with the content '$O$ has a determinate value.'

The measurement problem alleges that quantum mechanics could not be complete, since you never find yourself in a superposition. The Bare Theorist aimed to solve the measurement problem by showing that, since quantum mechanics is complete, you would judge yourself to be in a determinate state even if you were in a superposition. Thus, your insistence that you are never in a superposition is consistent with you actually being in a superposition. The Bare Theory fails because, if quantum mechanics is complete, you could not make a judgment that would count as a judgment that you are in a determinate state. (This is the calibration objection.) So the Stripped Theorist responds to the measurement problem by insisting that, since quantum mechanics is complete, you could never judge yourself contentfully to be in a determinate or indeterminate state. Thus, your insistence that you are never in a superposition is vacuous. The Stripped Theory is a natural synthesis in this dialectic, constructed so as to avoid the calibration objection; how does it fair against the other three objections?

One may worry that the Stripped Theory is infinitely more vulnerable to the oddity objection. In considering the oddity objection, however, we saw that explaining away phenomena is sometimes a permissible way of accounting for them. There is no formal restraint on this permissibility, so the objection lacks absolute compulsion here just as it did for the Bare Theory. The same holds for the coherence objection. The reader may find the Stripped Theory too odd or outré, but it's unclear what probative force that should have.

Yet one might press the point: The measurement problem arises because there is a contradiction between the usual interpretation of quantum mechanics (according to which we are superpositions) and our ordinary experience (according to which we are always in determinate states). Merely denying the latter isn't an answer to the measurement problem so much as a refusal to take the problem seriously. We could just as easily deny the former, if our only aim is merely to avoid contradiction. Yet the Stripped Theorist might echo Wittgenstein, who suggests that literal language is only the language of science, that philosophy should "say nothing except what can be said, *i.e.* the propositions of natural science" ([1922], §6.53). The only way to answer someone who wants to say more, Wittgenstein thinks, is to point out that anything more would be meaningless. The Bare and Stripped Theories presume that quantum mechanics is the language of science. The Stripped Theorist's insistence that noone can meaningfully claim to have a determinate belief should be seen as the natural consequence of that presumption, rather than as an *ad hoc* answer to the measurement problem.

Wittgenstein recognized that his account was, by his own standards, meaningless. He explains, "My propositions are elucidatory in this way: he who understands me finally recognizes them as senseless, when he has climbed out through them, on them, over them. (He must so to speak throw away the ladder, after he has climbed up on it)" ([1922], §6.54). Whereas with the Bare Theory physical evidence leads us to an account which describes that evidence as meaningful but literally false, for Wittgenstein the *Tractatus* leads us to an account which describes the *Tractatus* as nonsense. Similarly, with the Stripped Theory physical evidence leads us to an account that describes that evidence as nonsense. The Stripped Theory must be nonsense, too, since there is no way that a quantum system can record the fact that determinateness is not a quantum observable. The Stripped Theory, then, is not a *theory* after all. It merely points to a resolution of the measurement problem. This is odd, certainly, but is it odd enough to be a decisive objection?

To its credit, the Stripped Theory deflects the context-of-the-universe objection. The universe may be in a superposition of your existing and not existing, but how can you say this is not the case? You could only show otherwise if you could tell whether your existence were determinate, but the Stripped Theory holds that we are unable to judge whether the observable 'You exist' has a determinate value. Such a judgment, if it were to accurately capture both positive and negative cases, could never be instantiated in a quantum mechanical record. Our own brains are quantum mechanical records, so your insistence that you exist is to be explained away rather than explained.

To tally the score: The Bare Theory and the Stripped Theory are both vulnerable to the oddity and coherence objections. The context-of-the-universe and calibration objections, which are decisive against the Bare Theory, have no hold on the Stripped Theory. The latter is a worse affront to our intuitions than even the former, but there is a difference between reduction to affront and reductio ad absurdum. It is hard to see how the Stripped Theory could face a logical contradiction when considered on its own terms. In this, it has an advantage over the Bare Theory. It is for us to decide if we have reached a secure enough place that we are prepared to kick away the ladder of evidence from underneath us.

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## Notes

[1]Provided you take my disjunction in the classical and not the quantum logical sense. Similar provisos may be added to logical connectives throughout the paper.

[2]The objection is made in a cursory fashion in Albert ([1992], p. 124) and

Dickson ([1998], p. 47). It is developed with the most care and at the greatest length by Barrett ([1996], [1999], pp. 116–7). A theory is empirically incoherent, Barrett stipulates, if it "fails to predict the existence of reliable records of an observer's measurement results to which the observer has epistemic access" ([1996], p. 50).

[3] Let $A$ be vector produced by assigning $1/\sqrt{2}$ to $\alpha$ and $\beta$; let $B$ be vector produced by assigning $1/\sqrt{2}$ to $\alpha$ and $-1/\sqrt{2}$ to $\beta$. It follows from (6) and the linearity of the dynamics that $(|V_{wondering}\rangle A + |V_{wondering}\rangle B) \rightarrow (|V_{no}\rangle A + |V_{no}\rangle B)$. This may be simplified and renormalized so as to yield (7).

[4]They entail (5) and (7), yet you cannot simultaneously be in the state $|V_{yes}\rangle$ and $|V_{no}\rangle$. GRW avoids this problem, since it posits non-linear evolution of the wave function; there is a sense in which GRW says that quantum mechanics complete (it presumes that the wave function is a complete and accurate description of the world) and a sense in which it does not (it replaces the usual linear dynamics and must deny a strict eigenstate-eigenvalue link).

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