TOPICS
TOPICS IN COGNITIVE SCIENCE

Topics in Cognitive Science 5 (2013) 367–386

ISSN:1756-8757 print / 1756-8765 online
DOI: 10.1111/tops.12014

Numerical Architecture

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Received 22 April 2011; received in revised form 21 January 2012; accepted 19 April 2012

Abstract

The idea that there is a "Number Sense" (Dehaene, 1997) or "Core Knowledge" of number ensconced in a modular processing system (Carey, 2009) has gained popularity as the study of numerical cognition has matured. However, these claims are generally made with little, if any, detailed examination of which modular properties are instantiated in numerical processing. In this article, I aim to rectify this situation by detailing the modular properties on display in numerical cognitive processing. In the process, I review literature from across the cognitive sciences and describe how the evidence reported in these works supports the hypothesis that numerical cognitive processing is modular. I outline the properties that would suffice for deeming a certain processing system a modular processing system. Subsequently, I use behavioral, neuropsychological, philosophical, and anthropological evidence to show that the number module is domain specific, informationally encapsulated, neurally localizable, subject to specific pathological breakdowns, mandatory, fast, and inaccessible at the person level; in other words, I use the evidence to demonstrate that some of our numerical capacity is housed in modular casing.

Keywords: Modularity; Numerical cognition; Cognitive architecture; Nativism; Analog magnitudes; Mental processes; Automaticity

1. Introduction

Traditional models of modularity attempted to individuate mental modules by focusing on the five sense organs plus language (Fodor, 1983). These models then ballooned, widening their scope from just (e.g.,) vision, audition, and language, to include proposals for a postural module (e.g., Massion & Duffose, 1988), a face recognition module (e.g., Kanwisher, McDermott, & Chun, 1997), a letter recognition module (e.g., Polk et al., 2002), a theory of mind module (e.g., Baron-Cohen, 1995; Scholl & Leslie, 1999), a cheater detection module (e.g., Cosmides, Barrett, & Tooby, 2010), and so forth. In this

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article, I propose another candidate for modularity: an innate number system. This type of proposal in itself is not novel; indeed, some eminent theorists in cognitive science have proposed such a model before, most notably Carey (2009), Dehaene (1997), Margolis and Laurence (2008), and Barrett and Kurzban (2006). However, none of these authors have detailed the sense in which numerical cognitive processing approximates a modular style of processing. This article's goal is to fill that lacuna by sketching how our processing of numerical stimuli has the hallmark properties of modules.

Such a project is of both theoretical and experimental import. One sees passing mention of a number module in developmental psychology, cognitive psychology, evolutionary psychology, cognitive neuroscience, and philosophy; in other words, the idea looms large across the cognitive sciences. However, without some detail for how to understand numerical modularity we are left with little guide of what the claim amounts to. Thus, it is important to delve into the details of numerical modularity, for it is near impossible to evaluate the number module hypothesis without a clear sense of the evidence in favor of the module in conjunction with some account of the putative module's abstract properties. Clarifying this matter will in turn dictate what numerical processing tells us about the structure of the mind and can help guide future avenues of experimental exploration.

In speaking of modularity, I intend to follow the strictest use of modularity, using its strongest sense available. The diagnostic features I will rely on follow Fodor's traditional conception of modularity (Fodor, 1983) and not some of the looser conceptions at play in discussions of massive modularity (e.g., Barrett & Kurzban, 2006; Carruthers, 2006; Pinker, 1997). The reasoning for this is plain: If something counts as a Fodorean module, then it also meets the criteria for weaker notions of modularity. In particular, Fodorean modularity entails the modularity at use in evolutionary psychology. In evolutionary psychology (e.g., Barrett & Kurzban, 2006), modules are just understood as functionally individuable subprograms. Fodorean modules have additional attributes over and above the easy-to-come-by functional individuation. The properties of a Fodorean module (henceforth a "module") are specified as follows: the modular systems are the mental processes that are domain specific and mostly inaccessible to conscious thought. They are mandatory in that their processing is automatic like a reflex; the modules must process their proprietary stimuli without the need of endogenous control. Like a reflex, the modules must be fast and informationally encapsulated. Modules are assumed to be innate and thus have a regular maturational growth and decay. Lastly, the modules should be associated with some type of fixed neural architecture.[1,2] In what follows, I will argue that the innate number system is instantiated in its own site-specific modular mental process, one which has all of the aforementioned properties.

2. The accumulator

The model for numerical cognition that I will be working with is the "accumulator" model (Gallistel & Gelman, 1992; Meck & Church, 1983). This model has had substantial success in predicting and explaining behavior and is widely—though by no means universally (a topic I will return to in section 4.1)—accepted as a psychological information-processing model of how the mind computes numerical quantities. Importantly, the accumulator model is what the aforementioned proponents of numerical modularity have in mind when they discuss the existence of a number module. The accumulator model is dissociable from, and ontogenetically and phylogenetically prior to, the more familiar language-based discrete number system that we learn in school (Gallistel & Gelman, 2000). The accumulator system is a system that grounds our numerical competence; it is what allows us to understand and process numerical quantities as numerical quantities and not just as words in a language.[3]

The accumulator system contains a "pacemaker" that releases one pulse per number counted. The pulses "get filled" into an "accumulator," a metaphorical graduated cylinder (Meck & Church, 1983; Gallistel & Gelman, 1992).[4] The accumulator's value is then automatically transferred to working memory and compared to other past accumulator values that are stored in long-term memory. A comparator mechanism then decides what accumulator value in long-term memory is closest to the current working memory accumulator value. If the accumulator value is positively reinforced, then that particular accumulator value gets stored in long-term memory to make bi-directional mappings between the long-term accumulator value and the transient working memory accumulator value more accurate.[5] More formally speaking, the accumulator model is frequently understood as a logarithmically compressed subjective number scale, one where (e.g.,) the representations of 19 and 20 are closer together on our mental continuum than 9 and 10. The closer the representations are to each other, the more difficult it is to determine their ordering, thus making higher numbers harder to process (i.e., longer reaction times, more errors) than lower numbers.[6]

The accumulator model can predict and explain a broad range of data. Most notably, it can predict and explain the size and distance effects. The "size effect" denotes the following phenomenon: the greater the numerical size between a pair of numbers, the longer it takes to distinguish the numbers (assuming the distance between the two numbers is kept the same).[7] For example, if one's task is to pick the greater of two quantities, it is quicker for one to distinguish 5 items from 7 items than it is to distinguish 7 items from 9 items, which itself is an easier pair to distinguish than 11 items from 13 items (see, e.g., Gallistel & Gelman, 2000; Moyer & Landauer, 1967). Thus, as we keep the distance between two sets the same, the greater the size of the items in the sets, the longer it takes to distinguish the quantities. The "distance effect" names the phenomenon whereby two numbers are more quickly distinguishable as the distance between the two numbers increases. Thus, in a task where one needs to pick the larger of two numbers, one will be quicker at making the judgment that 9 is greater than 5 than one is when judging that 9 is greater than 8.

When one posits that analog magnitude representations are the representational medium of numerical thought, it becomes clear why the size and distance effects arise. Analog magnitude representations naturally lead to well-known Weber/Fechner effects, consequences of Weber/Fechner's law. Weber/Fechner's law states that the discriminability of two perceived magnitudes is fixed by the ratio of the two objective magnitudes.

What follows from the law is that the greater the ratio between two magnitudes, the easier they are to distinguish, which in essence is the distance effect. Weber/Fechner's Law also implies that in order to create a "just noticeable difference," larger differences are required for greater intensities, which in essence is just the size effect. Since the accumulator system is the representational system that causes the size and distance effects, which is just the instantiation of Weber/Fechner's law in the numerical case, whenever we see numerical size and distance effects we are justifiably licensed to infer that the accumulator system has been active (though, of course, such an inference still accrues some epistemic risk).

A fairly astonishing finding is that the size and distance effects hold not only when one is perceptually distinguishing numerosities but also when one is distinguishing Arabic numerals from one another (Moyer & Landauer, 1967, ibid). This surprising finding is quite robustly reproducible (see, e.g., Dehaene et al., 1998; Whalen, Gelman, & Gallistel, 1999). Moreover, the size and distance effects are relatively species-inspecific: They are observable not only in adult and infant humans but also in a varied array of other creatures such as pigeons, rats, and monkeys.[8] If we posit the accumulator model as part of our numerical processing system, then we can explain both the size and distance effects, for the analog magnitude accumulator system would be active whenever we process numerical stimuli regardless of the stimuli's format (such as Arabic numerals).

In addition, the accumulator model finds evidential support even outside of the deluge of behavioral evidence; for example, there are studies of cultures, the Piraha and the Munduruku, that have no number words or any exact numerical representational system, but whose numerical behavior displays the size and distance effects (Gordon, 2004; Pica, Lemer, Izard, & Dehaene, 2004). There are also well-known double dissociations between people who lose their memory of language-based "numerical" knowledge (e.g., remembering that 5×9 is 45) yet still retain what one might consider the core of numerical competence[9] and thus display the size and distance effect in their numerical judgments (Lemer, Dehaene, Spelke, & Cohen, 2003, see also the end of section 3).

In summary, we have strong evidence in favor of (a) the accumulator model being phylogenetically ancient and thus (probably)[10] innate in us, and (b) the model being a model of at least some part of our numerical competence. I will henceforth assume the validity of the accumulator model as (part of) our numerical processing system. Next, I will explain how the accumulator model can be used to defend the claim that numerical perception is modular.

3. Modular properties of numerical processing

We are now in a position to see how numerical cognitive processing can be seen as modular. In what follows, I will assume that the input for the numerical module can come from any sense modality.[11] This situation is similar to the language module hypothesis that allows the language module to cross-classify sense organs and thus allow for effects like the McGurk effect (McGurk & Macdonald, 1976). Because, ceteris paribus, one can ascertain language by reading or writing, the input can either be auditory, visual, or tactile; likewise, because one can ascertain numerical stimuli from either hearing sounds or looking at numerosities or Arabic numerals, we can infer that the number module cross-classifies sense organs. I will proceed by assuming that the input for the accumulator system (or "number module") can come from any sense modality and that the output is a representation whose referent is a (perhaps fuzzy) numerical quantity.[12] Let us now turn our attention to the modular properties (domain specificity, mandatoriness, informational encapsulation, inaccessibility, quick processing, common developmental maturation and breakdowns, and neural localizability) inherent in numerical processing.

First, the number module appears to be innate. It is phylogenetically ancient—as previously mentioned, one can infer its existence in locations up and down vast swaths of the phylogenetic tree—and it is ontogenetically primitive, for it is up and running in astonishingly young infants. Even four-day-old infants are able to make numerical discriminations (Bijeljac-Babic, Bertoncini, & Mehler, 1993), and five-month-old infants have been shown to have implicit knowledge of addition (Wynn, 1992). Consequently, the ontogenetic development of the accumulator system should not be in question: Since infants have the system available for use soon after leaving the womb, the effects of the accumulator system appear to be unlearned.

Thus, the innateness criterion appears to be attainable (though see section 4.1 for some dissenting views). Note that the innateness claim does not simply hinge on early discrimination abilities (of course, many innate traits do not appear early—for example, secondary sexual characteristics); instead, it is strongly buttressed from the comparative evidence. For example, the analog magnitude-based behavioral results we see in human infants and adults are also found in a wide array of other species (pigeons, rats, macaques, apes, etc.).

The number module is domain specific. Originally, the accumulator model was proposed to do double duty (Meck & Church, 1983): It was hypothesized to process both time and number, for it fits both the numerical and temporal data quite well. Yet, even though the model is predictive for both time and number, one need not infer that the very same (token identical) model works on both categories. For instance, Dormal, Seron, and Pasenti (2006) found that temporal cues do not interfere with numerosity processing in a numerical Stroop task, showing that numerical processing is independent of temporal processing. Thus, both systems are functionally distinguishable, even if their processing is quite similar.[13] Another datum in favor of separating the accumulator system for number and from that of time is that animals and humans can simultaneously keep track of time and number, implying that they are dissociable systems that can work in parallel (Meck & Church, 1983). Though we may wish to conclude that there is an accumulator-like system for temporal processing, it appears that it is a separable system from the accumulator system for number, which is the candidate modular process currently at issue.

Another consideration in favor of the domain specificity of the number module is that the module works across different modalities. An illuminating example of the language module's domain specificity is the existence of effects like the McGurk effect (McGurk & Macdonald, 1976), which shows that the language module can be activated by linguistic stimuli when presented auditorily or visually. Thus, the language module is domain specific to language and not inherently modality specific. Likewise, analogous inferences can be drawn about the number module. Infants can detect identities of numerical values across domains (Starkey, Spelke, & Gelman, 1983, 1990). Adult perception and judgment of numerical relations of identity and difference also do not differ in any significant way regardless of whether trials are intramodal or intermodal (Barth, Kanwisher, & Spelke, 2003). The cross-modal representational format of numerical representations is evidence that suggests the domain specificity of the number module, for the accumulator becomes activated by tracking its proprietary inputs regardless of the modality that accomplishes the tracking.

The case for the "top-down" access of the number module is not in dispute. Top-down access is meant to imply that one cannot introspect the bottom-up processing of a module; rather, all we, at the person level, have access to is the output of the module. If access were bottom-up, then we could expect people to be able to ascertain the mid-level numerical representations for processing outside of the accumulator domain; that is, we would expect some inferential promiscuity (Stich, 1978).[14] However, the analog magnitude representations that are at play in the accumulator are not representations that are "inferentially promiscuous": There is no extant reason, theoretical or experimental, to believe that the analog representations are used or usable for any other type of mental process or behavioral manipulation.[15] The claim is not that there is not analog processing elsewhere in the mind/brain, for that would beg the question against projects as various as Shepard's mental rotation studies (e.g., Shepard & Metzler, 1971), Kossyln's work in mental-map searches (e.g., Kosslyn & Pomerantz, 1977), and Prinz's iconic theory of concepts (Prinz, 2002). For present purposes, I assume an agnostic stance toward any of these other programs. All I wish to assert is that whatever type of analog processing is being done in these areas, the processing is computationally distinct from the analog mental representations at play in the accumulator.

Moreover, the results in the studies of numerical cognition are *surprising*. One would not be able to introspect one's mental processing and conclude that it would take them longer to distinguish the Arabic numeral 7 from 6 than it would take to distinguish 7 from 4. The accumulator is an unconscious mental processor, one where the computations at hand are startlingly different from the computations one might think one theoretically goes through when judging the size of numbers. If we did have top-down access to the workings of the accumulator then we should not be so shocked at the size and distance results we find when processing Arabic numerals.

Fodor (1983) provides some anecdotal evidence that subjects' access to numerical representation is top-down. He writes,

> The generalization about the relative inaccessibility of intermediate levels of input analysis is pretty rough, but all sorts of anecdotal and experimental considerations suggest that something of the sort is going on. A well-known psychological party trick goes like this:
>
> E: Please look at your watch and tell me the time.
>
> S: (Does so)

E: Now tell me, without looking again, what is the shape of the numerals on your watch face?

S: (Stumped, evinces bafflement and awe.)[16]

> The point is that visual information which specifies the shape of the numerals must be registered when one reads one's watch, but from the point of view of access to later report, that information does not take. One recalls, as it were, pure position with no shape in the position occupied (57).

We now have reason to go even further than Fodor did. Were the experimenter to have then tested the subject on a number discrimination task, the subject's behavior would have obeyed Weber's Law and thus showed the size and distance effects. When processing numerals the subject must have noticed the shape, but the subject loses access to the shape. When asking the subject for the time, the subject remembers what time it is, but not what the shape of the numerals look like. Yet surely the shape is being processed (if it was not how would people see the numeral?) and in its being processed, the numerical information of the Arabic numeral is also processed and automatically calculated in the analog magnitude format.

Wherever access is top-down, we have some reason to believe that there will be informational encapsulation. Fodor writes that modular inaccessibility is

> in effect, the inverse of encapsulation. Just as information that is available to its computations cannot get into a module so the information that is available to its computations is supposed to be proprietary and unable to get out. In particular, it is supposed not to be available for the subject's voluntary report (Fodor, 1998, p. 127).

It is not unreasonable to suppose that where we find inaccessibility, we should also find encapsulation. Unsurprisingly, all the evidence we have points to the fact that the number module is not "cognitively penetrable" (Pylyshyn, 1984). Illusions like the Muller-Lyre are paradigm cases in favor of the cognitive impenetrability of the visual module. Analogously, there are illusions in numerical processing, though curiously these illusions are rarely if ever discussed. For example, the Stroop effect is repeatable in the numerical domain (e.g., Besner & Coltheart, 1979). If a subject is presented with two Arabic numerals, the physical size of the numerals will interact with their semantic values in interesting ways. If we change the physical size of the numerals so that the numeral denoting the larger quantity (e.g., 9) is in a physically smaller font than the numeral denoting the smaller quantity (e.g., 5), then it will take subjects longer to choose the target quantity, regardless of whether the task asks them to choose the smaller or larger number. Although the Stroop effect is not the most paradigmatic illusion the effect does not rely on a false belief), it does pump one's intuitions in the right direction.

An even more paradigmatic numerical illusion occurs when subjects make comparative judgments among large sets of numerosities. If a subject is presented with two sets containing an equally large amount of dots (say 100), then one would expect them to judge the sets as more or less equal. However, this is not the case when the spacing in the two sets is such that one of the sets is evenly spaced and the other set is unevenly spaced. In the "Regular-Random" illusion (Fig. 1; Messenger, 1903; Ginsburg, 1976, 1978, 1980) subjects judge that the unevenly spaced set contains fewer items than the evenly spaced set and the perception of the evenly spaced set as larger will not go away even when one is told that the sets contain an equal amount of dots. The belief that the sets are equal cannot affect the numerical/perceptual processing because the number module is informationally encapsulated. The same holds true for the "Solitaire Illusion" (Fig. 2), an illusion where sets that contain one large cluster will be seen as having more items than sets containing a few small clusters, even though the two sets are equal (Frith & Frith, 1972). The fact that these perceptual effects do not go away when subjects are told of the equivalence between the sets is another datum in favor of the cognitive impenetrability of the number module.

The number module's informational encapsulation allows for extremely fast processing. For example, one study in non-verbal counting in adult humans shows that non-verbal accumulator based counting occurs much faster than verbal counting (Cordes, Gelman, & Gallistel, 2002). Experimenters asked subjects to non-verbally count by continually pressing

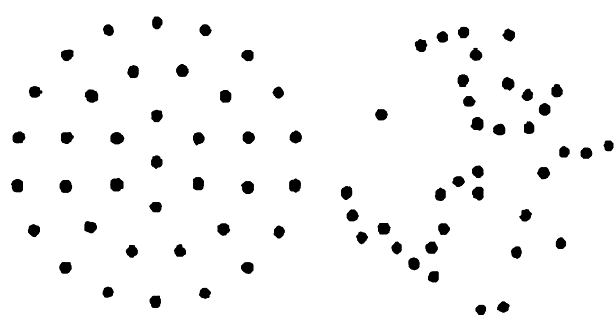

Fig. 1. Regular-Random illusion (from Ginsburg, 1976).

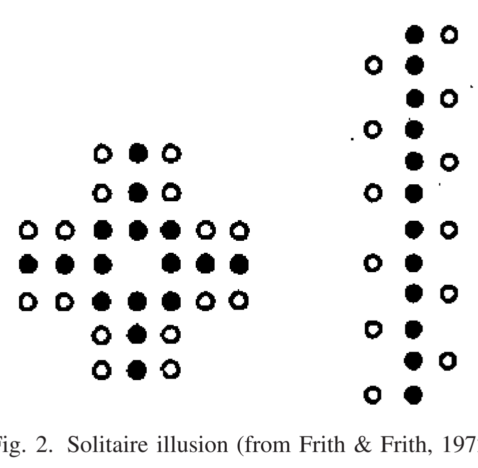

Fig. 2. Solitaire illusion (from Frith & Frith, 1972).

a button until they believed that they had reached a pre-specified value that varied from trial to trial. The subjects were asked to either repeat the word "the" continually or to sing "Mary had a little lamb" while non-verbally counting as fast as possible. The amount of time per accumulator based counting was 125 ms as opposed to 200 ms for verbal counting (derived from Whalen, Gelman, & Gallistel, 1999). In addition, numerical stimuli can cause the probative size and distance effects even in priming situations. The number module is fast enough that even a 43 ms display of a numeral is enough exposure to cause an immediate tokening of an analog numerical representation (Reynvoet & Brysbaert, 2004).

The processing of numerical stimuli is not only very fast but also appears to be mandatory: The merest flicker of numerical information sets off the ballistic processing. The criterion of "mandatoriness" itself contains a systematic ambiguity, one that has yet to be mentioned in the literature. Mandatoriness can be understood as a form of *automaticity*, where the processor has to compute any of its proper inputs when it comes into contact with that input; mandatoriness can also have a *ballistic* connotation, which entails that anytime a module begins processing its input it cannot stop its processing until it delivers an output. These senses of mandatoriness are not only conceptually distinct, but they appear to come apart empirically also. For example, (putative) cheater detection modules (e.g., Cosmides, Barrett, & Tooby, 2010) are ballistic but not mandatory: Merely encountering a "social exchange" conditional is not enough to ensure that someone parses the truth values—for that to happen one must think about the conditional as a conditional. Likewise, many models of language processing see language modules as mandatory (i.e., you cannot help but hear a sentence in your language as a sentence) but not ballistic (processing can crash for garden path sentences). Yet the number module appears to be mandatory in both the automatic and ballistic sense.

When one encounters numerical stimuli of any type, whether numerosity based or Arabic numeral based, the accumulator system shows its typical effects.[17] As discussed above, the size and distance effects can take hold whether one is judging sets of objects or merely symbols denoting numerical values, and they arise with super minimal (prime-level) exposure times. In other words, there is no way to sidestep the processing of the accumulator system—it is set off automatically. Thus, the number module acts like a mental reflex. The processing occurs subconsciously and the subjects need not be aware of the fact that any stimulus was presented to them. But even though the subjects cannot say what the stimulus was, the effects of the primed number are evident through the subjects' response times, showing that the subjects automatically processed the stimulus.[18] Even when one is presented Arabic numerals, one is forced to automatically translate the stimulus into analog mental magnitudes.

The available evidence is consistent with the ballistic reading of mandatoriness, too. The paradigmatic way to shut down a mental process is by depleting attentional and cognitive resources through cognitive load. However, extra load brought on from competing cognitive tasks does not interfere with the number module producing an output and there is no numerical equivalent to a crashing garden path sentence. Lastly, as aforementioned even temporal processing tasks that recruit an isomorphic set of analog magnitude representations do not interfere with the accumulator's numerical processing (Dormal et al. 2006).

All modules are functionally defined and thus computationally isolatable. However, the number module is not just computationally, but also neurally, isolatable. As such, it can be selectively impaired and show the type of characteristic and specific breakdowns that are the hallmark of other putative modules (prosopagnosia, alexia, agraphia, etc.). Acalculia is a type of "number blindness"; it is an acquired syndrome, one which occurs because of trauma to one's parietal cortex.

The accumulator system is thought to be located bilaterally in one's inferior parietal cortex. Specifically, fMRI studies have shown the accumulator system to be present more specifically in the left and right intraparietal sulci extending anteriorly to the postcentral sulcus (Dehaene et al., 1999). Hence, the number module appears to be associated with some fixed neural architecture. When trauma occurs in the inferior parietal cortex, the ensuing deficits of acalculia are expectable (Cohen-Kadosh & Walsh, 2009). The subjects will lose almost all competence at even the most hideously mundane numerical tasks (and other more tacitly numerical tasks, such as understanding natural language quantifiers; Clark & Grossman, 2007).

For a more concrete idea of the deficits associated with losing one's accumulator system, take patient MAR who had a right inferior parietal lesion that caused some very odd numerical deficiencies (from Dehaene, 1997). Patient MAR erred 16.8% of the time on number comparison trials (figuring our whether, e.g., 8 was bigger than 2). MAR also answered incorrectly on 20% of proximity judgment trials (figuring out which of two numbers were closer in proximity to a target value, for example, judging that 5 is closer to 4 than 9 is). A sampling of MAR's specific errors may convey the severity of the impairment. MAR judged that 10 was closer to 5 than to 9, that 8 was closer to 4 than to 9, and that 28 was closer to 21 than 27. Lastly, on number bisection tasks (tasks where one has to pick a number that was the average of two presented numbers, e.g., for 2 and 4 the answer is 3), MAR answered incorrectly 77.4% of the time.[19]

In summary, it appears that numerical processing can be seen as modular. In the next section, I deal with some criticisms of the case for numerical modularity. In particular, I address objections to the accumulator model itself, to nativist approaches to number acquisition, and to the overall utility of a modularity picture.

4. Objections

Many theorists in quite different fields are antecedently convinced of the reality of the accumulator model; some of these theorists even assert that numerical modularity is probably true. To these theorists the discussion so far can be seen as the first detailed attempt to flesh out what numerical modularity amounts to. However, as can be expected in a field as variegated as cognitive science not everyone endorses the accumulator model or even the utility of a modularity approach. In this section, I will attempt to assuage some of the concerns of those who are hostile to either the analog magnitude picture or the modular one.

4.1. Defending accumulator model

Some theorists, often of an empiricist stripe, have qualms about the accumulator model's explanatory power and descriptive adequacy. A complete defense of the accumulator model is beyond the scope of the current project's space limitations (for a fuller defense see Mandelbaum, 2005). Consequently, this discussion will have to be a bit compressed. That said, there are some strong reasons to endorse the accumulator system. For one thing, it has become increasingly hard to doubt that there are in fact analog magnitude effects that need to be accounted for. Infants at 6 months of age can distinguish numerosities that are in a 2:1 ratio and adults show similar effects (see, e.g., Brannon, Abbot, & Lutz, 2004; McCrink & Wynn, 2007; Xu, Spelke, & Goddard, 2005). Moreover, these discriminations occur when other extraneous variables are controlled for (such as total contour length, total filled area, total brightness, item size, array size, and density). No doubt there are still theorists who reject these proposals. However, I find these positions less appealing than one's based around the accumulator model. I will now quickly canvass a few competing positions.

Starkey and Cooper (1980) attempt to advance a subitizing model instead of an analog magnitude model. However, there is good reason one should be skeptical of subitizing models. For one thing, the subitizing system appears to be a completely different mental process than whatever is at play for larger numbers. Subitizing has been hypothesized as an object file, FINST (see Scholl & Pylyshyn, 1999) type system, one that can only operate to track up to approximately 4 objects. Beyond that, the subitizing system stops working. Yet the infant discrimination data deals with numerical distinctions of much larger sets of numbers (e.g., distinguishing 16 from 32; Xu, Spelke, & Goddard, 2005). Moreover, the signature analog magnitude behavioral results are not apparent when dealing with sets of 1–4 (Clearfield & Mix, 1999; Feigenson, Carey, & Spelke, 2002; LeCorre & Carey, 2006), which lends more evidence for the claim that the subitizing system is a wholly separate mental process from whatever underwrites the analog magnitude effects.[20]

Mix et al. (2002) have suggested that infants make their discriminations not from an accumulator-style process but rather based on whatever correlated non-numerical cue is not controlled for in a given trial (although the cited experiments generally control for all of these across trials, no single trial can control for each variable). No doubt this is a coherent position to hold, but it is quite a hard one to prove (especially in light of some of the considerations raised toward the end of the next section). Perhaps, it is best to conclude that although the evidence in favor of the accumulator model is not airtight, the acceptance of arguments that reject that accumulator system are also far from obligatory. I will end just by pointing out that without an accumulator model it is very hard to see what mental process could underwrite numerical modularity, so if one is inclined toward a number module, one should be reasonably optimistic about the accumulator's existence.[21,22]

4.2. Nativism

So far, the discussion has run together nativism and modularity. Doing so, is common enough, as innateness was an original criterion of a modular system. But though running nativism and modularity together is commonplace, it is not mandatory to do so. Modularity is, at its core, a hypothesis about the structure of mental processes. The main claims in modularity theory are contingent on the specific character of a given mental process: for a process to be considered modular it has to be domain specific, mandatory, and informationally encapsulated. In other words, the real crux of modularity is that a given mental process is autonomous and automatic—its workings are sequestered from the rest of higher cognition and it can operate in the background of the mind. Nothing about the core formulation of modularity demands anything like a nativistic approach to mental processes or concepts. Nativism concerns itself with the etiology of certain structures and representations. Since modularity is concerned with current processing and not how those processes got there, one could be a thoroughgoing empiricist and still support the existence of certain modular processes. In fact, there are some such relatively well-known theories of this flavor around; for example, Karmiloff-Smith (1992) defends a view of modularity where the modules themselves are acquired (or perhaps better, assembled) through learning and experience. Likewise, modularity is compatible with the mental representations that the module uses being innate or acquired. That is, one can buy into a modular view of numerical cognition *while still holding that number concepts are themselves acquired*. Failing to make the distinction between modular processing and nativism has caused some theorists (such as Cohen-Kadosh & Walsh, 2009) to incorrectly malign modularity theory while only actually taking aim at nativism.

So why is nativism so often run together with modularity theory if the two are separable empirical hypotheses? I suspect this is the case for two reasons. Historically, very few theorists have been empiricists about core mental processes. Even the classic British Empiricists (Hume, Locke, Berkeley) were nativists about the mental processes they endorsed: They did not think that association (as seen as a mental process) was learned, nor did they think that the sensorium was acquired (likewise, few contemporary connectionists think that the process of association is acquired, though of course experience dictates how the particular associative weights are set). Like the classic empiricists, one could hold that all concepts are acquired while still maintaining that the processes that range over these concepts are innate. Since it is hard—though not necessarily impossible—to see how core mental processes could be learned, it seems reasonable to default to an inference of 'innate mental process' from 'mental process'. Indeed, it seems likely that modularity theory's association with nativism is predicated on this (non-apodictic) inference.

All of that having been said, I am still sympathetic to nativist claims in numerical cognition, though nativism is an additional claim over and above modularity. The main reasons for being pro-nativism are two-fold: (a) the behavioral signatures of analog magnitude numerical representations are isomorphic across a diverse range of species, including human infants and adults, and (b) human adults continue to recruit their analog magnitude representations even after they acquire a more complex, efficient, and exact discrete numerical representational system. (a) Shows that the magnitude system is phylogenetically ancient, so it is sensible to suppose that since we share the same mental process with species much lower on the evolutionary tree, we may inherit it through our genes. As for (b), if the accumulator system were just a learned representational medium it is hard to see why this scaffolded process would not be discarded when a more efficient system is in place. Yet the analog magnitude effects are without fail detectable even when dealing with Arabic numerals. This is not a surprising datum if one assumes that the system's processing is mandatory, and of course if the system is innate, then we cannot just choose to discard it because of its shortcomings. Add to this, other arguments about how soon the analog magnitude system comes on line (which is a hotly debated topic, but there are sane theorists who think it is online almost immediately after birth; at the very least analog magnitude effects are clearly detectable at 6 months, Xu, Spelke, & Goddard, 2005; McCrink & Wynn, 2007) and arguments about what exogenous stimulus and learning situation could possibly underwrite the acquisition of a purely abstract concept of number (arguments as old as Plato), and it becomes reasonable to presume that there is an innate number module that computes over innate, fuzzy numerical concepts. But to reiterate, although I am sympathetic to a nativist account of numerical mental processes and numerical concepts, one need not take a nativist position to endorse modularity theory; the modular processing claims stand or fall on their own.

4.3. What good is modularity for cognitive science anyway?

Prinz (2006; and an anonymous reviewer) raises the question of the value of modular models. Prinz attacks modules along two axes: First, he wonders whether any systems actually contain the properties of modules (e.g., "I think the properties on Fodor's list neither can be used neither jointly nor individually to circumscribe an interesting class of systems," Prinz, 2006, ibid, p. 22), and second, he criticizes whether an abstract characterization of modularity is useful for cognitive science. Prinz's first criticism is an ontological one: He is skeptical whether any actual system is modular. I applaud such skepticism; too many loose claims of modularity are made without detailing what evidence there is in favor of the existence of the module. However, the main aim of this essay is to fill this lacuna with some actual details of the state of the evidence in favor of a numerical module. I agree with Prinz that is incumbent upon the proponent of modularity to sketch a picture that shows how the system has interesting modular properties. Though one might want to argue with the state of the evidence presented here, the blanket argument against modularity theory does not apply since this essay has canvassed how the data can be interpreted to support modularity.

My response to Prinz's second concern is two-fold. First, I think it is a justifiable methodological approach to assume that the utility of any claim is secondary to its truth. If there truly are modular systems, that fact should be of great interest to cognitive science. After all, cognitive science wants to uncover truths about the mind and this is not an easy task. If there are modules, that is a deep fact about the mind and should be embraced regardless of its predictive value, for finding a deep truth is a good in itself (and is not so easy to accomplish). Second, it remains unclear why one would hold that modularity does not guide research and make predictions. A very salient empirical prediction would naturally follow from the number module hypothesis: If there is a number module, then we should be able to find that the size and distance effects are replicable cross-modally. That is, we should be able to (e.g.,) prime a subject auditorily and then have the subject visually identify a number. The subjects' performance should then display the normal number priming identification effects. For example, if the subject is primed with six tone bursts and then visually presented with the Arabic numeral 5, then the subject should be quicker to identify the 5 than if the subject is primed with 8 tone bursts and then visually presented with the Arabic numeral 5.[23] Thus, like any robust theoretical proposal, the number module proposal leads to specific empirical predictions and should be a fruitful area of research for cognitive scientists. Of course, making predictions is different than having a realized computational model, but it is hard to see exactly why these concrete predictions do not constitute real progress.

5. Conclusion

I have argued that the accumulator processing model can be seen as a fast, informationally encapsulated, domain specific, automatic, ballistic, mental process; in other words, a number module. In doing so, I have made the first sustained attempt I know of to show what the number module would amount to. I think we have some strong reasons to expect that there is a number module. From an evolutionary perspective, we can see why such a module would be advantageous: Even without higher cognitive functions, foraging animals would need to keep track of paths that led to more food and distinguish these from paths that were fruitless. A necessary condition on this task would be to have some sort of number processing system. Thus, we can see why the number processing system would be phylogenetically ancient, and we can see why the system could have arisen separate from a central cognitive system that underwrites higher cognitive functioning.[24]

Notes

1. One should note that merely mentioning some fixed neural architecture as a possible property of (psychological) modules does not commit one to Anderson's conception of "anatomical modularity" (Anderson, 2010). Moreover, being associated with fixed neural architecture does *not* entail that the neural regions underlying the module are only used for the module's processing; all that is needed is that there is some local, fixed neural region that is regularly activated when the corresponding psychological process is likewise active. It is non-locality, but not non-selectivity, that can cause trouble for the putative neurological symptom of psychological modularity.
2. The reader may notice that "shallow outputs," which is a feature of Fodor's original conception of modularity does not appear on this list. This is because it is unclear that shallow outputs were anything more than a red herring caused by

overly focusing on the visual case. It is not just very unclear what a "shallow output" would amount to in the numeric case but also in areas of theory of mind, letter recognition, posture, etc. Thus, I propose to follow Fodor (1998) in dropping shallow outputs as an indicative property of modularity.

3. Even those who appear to be suspicious about the accumulator model are not generally suspicious of its existence or its necessary role as part of the process of numerical cognition, but of its ability to generate discrete numerical representations for higher integers (viz., higher than 3 or 4) without linguistic scaffolding. These theorists just deny that the accumulator can be the whole story of adult numerical competence (see Carey, 2009 or Spaepen, Coppola, Spelke, Carey, & Goldin-Meadow, 2011). Of course, some theorists (though not Carey) are inclined to deny its existence completely; we will return to arguments for the existence of the accumulator in section 4.1.
4. The scare quotes around "get filled" are meant to imply that though the filling is metaphorical, it is a functional equivalent of a certain computational mental process. One can think of the analog magnitude "water bursts" as energy potentials (Margolis & Laurence, 2005).
5. "Bi-directional mappings" might be a bit too ambiguous. All that it means in the present context is the ability to compare one's current working memory accumulator value, the one brought about by stimuli in the organism's environment and held in working memory, to the accumulator values that are held in long-term memory.
6. Some theorists prefer to interpret the system as one that is not logarithmic, but instead linear with scalar variability (e.g., Brannon, Wusthoff, Gallistel, & Gibbon, 2001). Both the logarithmic and linear interpretations can explain the size and distance effects, so both are descriptively adequate for our current purposes. Since, the differences between the logarithmic and linear views will not make a difference to the arguments that follow, the issue will be ignored hereafter.
7. Of course, though I speak of numbers, all of the stimuli used in these experiments are, strictly speaking, numerosities, and it is a (fairly untendentious) theoretical inference that what subjects are responding to is the numerical properties instantiated in the numerosities.
8. See, for example, Platt and Johnson (1971) and Mechner (1958) for rats; Roberts, Coughlin, and Roberts (2000) and Brannon et al. (2001) for pigeons; Brannon (2002) for human infants; Cordes, Gelman, and Gallistel (2002) for human adults; and Brannon and Terrace (2002) for rhesus macaque monkeys.
9. Those who lose the language-based system (through, e.g., trauma) still can perform well at inherently numerical tasks such as number bisection tasks (e.g., saying whether 30 is in-between 20 and 40) and numerical comparison tasks (e.g., saying whether 20 is less than 30), showing that they still have some core understanding of number. On the contrary, those who suffer parietal damage and lose access to their accumulator system while maintaining the language-based system cannot perform the basic numerical tasks of number bisection and numerical comparison. For more on this topic see the end of section 3.

10. Of course, even an innate accumulator processing system does not itself entail innate concepts of number. In general, any innate processor can (logically speaking) process representations that are acquired. For more on numerical modularity's relation to innate number concepts, see section 4.2.
11. I thus apparently part ways with Margolis and Laurence (2008), who hypothesize that the inputs of the putative number module are the subitized object files from the object indexing system. However, they explicitly leave open the possibility that there are similar object file systems corresponding to non-visual modalities. As I discuss in the text immediately below, multimodal input for numerical processing is necessary because numerical competence is not inherently visual (for one thing, although we have numerous reports of acaculiacs, none of them have an impairment that is restricted to any modality specific numerical information; Polk et al., 2002). Consequently, I need not disagree with Margolis and Laurence, pending how the rest of their view is spelled out (i.e., whether they can find other object file like systems in non-visual modalities) in their in-progress book, *Think of a Number*. In any case, we will have to specify what the proprietary inputs are; and since numbers are abstracta, they cannot serve as the proximal cause so something else, such as object files, will have to do.
12. In principle, numerical stimuli should be detectable via any modality. This is a natural consequence of the types of things numbers are, viz. inherently abstracta (even for token instances of numbers).
13. Functional distinguishability is all that basic modularity requires, though of course functional distinctness does not necessarily imply computational distinctness. It is lamentable that the difference between the two is often ignored, for many evolutionary psychologists run these together (e.g., Barrett & Kurzban, 2006) when they can clearly come apart, as in the current case.
14. To a first approximation a representation is inferentially promiscuous if it can be used in inferences outside of a single domain. One's belief that the sky is blue thus counts as inferentially promiscuous, whereas the visual system's knowledge that there is one overhead light source does not (for an explanation of the latter datum, see Scholl, 2005).
15. Of course, this is not meant to imply that the accumulator model of timing is not structurally similar to the number module; the claim in the text is just that the token mental representations underwriting numerical processing are not used in temporal processing.
16. For a variation on this theme, see Morton (1967), where a considerable amount of subjects (over 25%) could not even recall the placement of numerals on a telephone (and none of the 151 subjects could remember the placement of letters on the phone).
17. Even theorists that are very opposed to modular interpretations (and the accumulator model) grant this (e.g., Cohen-Kadosh & Walsh, 2009, p. 320).
18. For example, suppose the subjects' task in the number priming paradigm is to identify a number as opposed to pick the larger of two numbers. This difference

causes the "priming distance effect": The closer the prime is to the target, the faster the subject identification of the target. For some examples of the numerical priming paradigm, see Dehaene (1997), Dehaene et al. (1998), and Reynvoet and Brysbaert (2004).

19. These errors were not made because of a misunderstanding of the task at hand. For example, after a trial in which MAR judged that 20 bisected 30 and 40, the experimenters asked him to write down the numbers between 30 and 40 on a piece of paper and point to the middle number. MAR had no problem with this and similar tasks. Moreover, MAR succeeded in non-numerical bisection tasks. MAR had no problem choosing which letter bisected to others (e.g., C is between A and E) nor which day bisected other days (e.g., Wednesday bisects Monday and Friday).
20. That said, there is some recent evidence that analog magnitude effects are detectable in auditory enumeration for small number sets (vanMarle & Wynn, 2009). Of course, this datum is of no help to those who try to deny the accumulator model.
21. This last point makes Carey's position (Carey, 2009; LeCorre & Carey, 2006) all the more puzzling, for Carey both wants to deny that the accumulator plays a particularly important part in the development of number concepts (in particular, they deny that the accumulator plays apart in learning how the counting principles are constructed) and hold that there is a number module. That said, even Carey does not deny the actual existence of the accumulator system (LeCorre & Carey, 2006; p25), and she thinks it is a component of core knowledge (Carey, 2009, p. 188). So Carey appears to accept the accumulator as the basis for numerical modularity; what she denies is its causal efficacy in learning the counting principles.
22. Because of space restrictions, I am unable to deal with other anti-accumulator proposals, such as the mental models proposal of Huttenlocher, Jordan, and Levine (1994), but I have trouble seeing exactly how such models explain why 6 month olds correctly discriminate sets in a 2:1 ratio, but not sets that have smaller ratios.
23. Though cross-notational priming is commonplace (Naccache & Dehaene, 2001), I know of no cross-modal priming studies.
24. The author acknowledge the James Martin School and the ACLS Foundation for their generous support. Institutional support was also provided through fellowships held at Oxford University and Yale University during the writing of the essay. Special thanks to Jesse Prinz, Susanna Siegel, and three anonymous reviewers for their helpful criticisms.

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