Paskian Algebra: A discursive approach to conversational multi-agent systems

Thomas Manning

Department of Education, Practice and Society,
Institute of Education, UCL,
20 Bedford Way,
London, WC1H 0AL,
United Kingdom

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Warning: *Substantial changes have been made to this paper in the published version, and errors and misconceptions persist in this version which have been picked up by peer review. For a more accurate version, please see the published work.*

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Abstract

The purpose of this study is to compile a selection of the various formalisms found in conversation theory to introduce readers to Pask's discursive algebra. In this way, the text demonstrates how concept sharing and concept formation by means of the interaction of two participants may be formalized. The approach taken in this study is to examine the formal notation systems used by Pask and demonstrate how such formalisms may be used to represent concept sharing and concept formation through conversation. The compilation of the discursive algebra using the framework provided by conversation theory could potentially be used as an auxiliary framework to study conversational interactions in multi-agent systems theory.

Keywords— Conversation. Concepts. Semantic drift. Inference. Cyclicity.

Introduction

Scott (2011) argued that cybernetician and educational theorist Gordon Pask pioneered the use of algebraic models in the context of psychology and education. Scott claims that this can be viewed as a formal theory of how humans understand our world as discursive beings (p. 287). His short paper, *Gordon Pask's Algebra of Understanding*, laid a sketch of how such discursive algebra might be compiled. The text aims to develop this account by compiling Pask's books and papers on conversation theory and the interaction of actors' theories (Pask, 1975a, 1975b, 1976b, 1984; Pask and de Zeeuw, 1992; Pask, 1996). In doing so, I seek to provide a cybernetic account of discursive algebra that can be applied in recent conversational approaches in artificial intelligence and multi-agent systems theory (Chaabouni, 2022; Lazaridou *et al*., 2017, 2020; Lu et al., 2020).

In private conversations with myself, Scott has argued that formalism is a way of saying quite a lot with very little; therefore, in my mind, the benefit of formalizing conversational activity is that it creates a heuristic framework for research practitioners to use in the context of conversational approaches to artificial intelligence and multi-agent systems theory. The text does not seek to give a definitive account of the applications of such algebra; instead, the purpose of this paper is to compile what I believe are the base constituents of Pask's discursive algebra so that the algebra may formally illustrate concept sharing and concept forming through the means of conversation.

I begin by explaining the meanings of topics and entailments. I then explain how such constituents involve fuzzy programs and the compilation of a series of fuzzy programs. Subsequently, I will examine the nature of "simple" concepts, which are defined here as a stable compilation of conceptual procedures that produce and reproduce a given topic of interest. I will also examine concepts proper, which involves both conceptual procedures and description giving such that a conversation, if formed, as well as what an algebra of such conversational interactions may look formally. Finally, I end with a brief examination of Pask's no doppelgangers theorem and the implications that may be derived from it by means of the discursive algebra it embodies.

Topics and Entailments

Here, topics are considered as matters and affairs that may be discussed. Topics infer and are inferred by topics; for this reason, a topic may be considered as a proximal *relation* to other topics, that is, *topic relations*. Pask (1975b) argued that a topic relation R_i is derivable from activities such as problem-solving (p. 44). It is assumed here that topics are listed on index i, such that $i = 1,2,3,\ldots,n$ where n is the last number in that sequence of numbers. Such indexed relations i shall be said to correspond to labels, names, or words. Thus, if I had 13 topic relations R_i, I could write the following:

$$\bigcup_{i=1}^{13} R_i = \langle R_1, R_2, \ldots, R_{13} \rangle$$

Such that the union $\cup$ of indexed topics R_i is may be substituted with the set $\langle\rangle$ containing such topics; where the left side of the equation is a simplification of $R_1 \cup R_2 \cup, \ldots, \cup R_{13} = \langle R_1, R_2, \ldots, R_{13} \rangle$. Let a given topic of some index belong to some conversational domain $\mathbb{D}$ where matters are affairs, and may be discussed in conversation. Thus, the expression $R_i \in \mathbb{D}$ means something along the lines of, and there is a relation that is in $\in$ a conversational domain. To illustrate the inter-relatedness of such topics in the domain on pen and paper, clusters of nodes designating topic relations and arcs/arrows $\rightarrow$ designating the entailment of such topics would be used (Pask, 1975b, p. 561).

By *entailment*, Pask (1975b) refers to how one topic relation may legally derive another (p. 553): In other words, entailment relates to the necessary ways and sufficient means by which a topic may be derived. This can be expressed using the expression $R_i \rightarrow R_j$. Such an expression holds if and only if R_i entails another topic R_j (where j is envisaged as a separate index different from i but still belongs to conversational domain $\mathbb{D}$). Instead of writing $R_i \rightarrow R_j$ which implies that the former topic *directly* entails the latter topic, the expression $R_i \vdash R_j$ is preferred (whereby $\vdash$ means yields). The $\vdash$ symbol is sometime preferred over the $\rightarrow$ symbol because it indicates how a given topic may be *indirectly* entailed through being one linkage of many in a sequence of topics.

It is worth giving the reader a practical example of what this process of entailment may look like. The example below is based on online media documenting Paul Pangaro's (2001, 2012) Stanford lectures that took place in the early 2000s. Take a simple entailment, where the two topic relations "Sea" and "Island" derive the topic of "Malta". This can be visualized in the form of ⟨*Sea,Island*⟩ ⊢ *Malta*. This process may be called derivation, where if we create an illustration in which topics are nodes and arrows are arcs, then one is derived from the other (Pask, 1976a, p. 16). This entailment is of a very simple type; however, it cannot be said to permit concept formation because it does not satisfy the condition of cyclicity required in conversation theory. The *principle of cyclicity* holds that: If $R_i \vdash R_j$ then any R_i may be reconstructed from other topic in the domain using the specified relational operators (Pask, 1975b, p. 98). This can be visualized by the following compilation of expressions:

$$\langle Sea, Island \rangle \vdash Malta;\ \langle Sea, Malta \rangle \vdash Island;\ \langle Island, Malta \rangle \vdash Sea$$

The union of such an inference forms an entailment mesh, which can be conceived as a permissive structure that stipulates what may be known (p. 92). In the above example, I may create sentences that express the following knowables: There is an island on the sea called Malta, the sea of Malta has an island, and the island of Malta has a sea. Each statement may then be inferred from others such that the condition of the cyclicity of inference holds

(Pangaro, 2012). This principle acts to prevent semantic drift by imposing the cyclicity of inference as a condition of correctness for discursive agents; thus, *if there is no cyclicity of inference, there are no concepts*.

Programs and Interpretations

The term $PROG\ i$ is generally used to indicate a program or algorithm that yields an R_j (Pask, 1996, p. 354). It is not identical to a concept that requires an interpretation, but it permits the transformations of topic relations. In this text, a program is defined as one that, when applied to a relation or set of relations, may yield some topic relation. It can be viewed in a more human context as an inference, such that a participant infers and endorses the view that $R_i \vdash R_j$. Instead of writing this, however, we can write $R_j = PROG(R_i)$ where $PROG(R_i) = PROG\ i$. Let *PROG* exist as a list of instructions to determine if and how R_i may be transformed into R_j. Programs then correspond to the general theory of algorithms, whereby an algorithm is defined as a set of instructions (prioritized by importance) that determine how input becomes output (Glushkov, 1966; Markov Jr., 1954). Therefore, a program can be interpreted as a calculus that determines how input becomes output through a list of predefined rules (Pask, 1976b, pp. 133-135).

Pask specifically treats programs as existing in the context of calculus operating upon sets of relations that produce fuzzy or approximate values (p. 140). Programs of this type are necessarily related to fuzzy set theory and its applications (Goguen, 1968; Mandami and Assilian, 1975; Zadeh, 1965, 1968). In this subsection, I elaborate on Pask's (1976b) account of how fuzzy set theory can be incorporated into conversation theory and the study of concepts. Pask's understanding of fuzzy set theory is heavily influenced by Goguen's (1968) suggestion of utilizing fuzzy set theoretical approaches as a type of heuristic inference (specifically when dealing with inexact concepts). Utilizing fuzzy set theoretical approaches, it is possible to design a program $PROG$ that produces fuzzy values and manipulates them based on a set of rules. In doing so, it not only infers some R_j from R_i but also assigns *degrees of assuredness* to topic relations (such that we think an inference is either more or less correct) while selecting them based on the program's instructional criteria.

According to Pask (1976b), a program conceived as a series of instructions can be reduced to *conditional imperative* statements and *assignment* statements. Likewise, fuzzy programs can be represented as *fuzzy* or *deterministic* instructions, which may reduce fuzzy conditional imperatives and assignments (p. 140). It is the author's view that Pask (1976b) envisages something akin to Mandami and Assilian's (1975) heater algorithm when speaking of fuzzy programs. Pask (1976b), Mandami and Asslian (1975), and

Gougen (1968) see fuzzy programs as fuzzy *heuristics*; in other words, $PROG$ determines the level of association between two topics based on a single observation (Goguen, 1968, p. 333). However, it is useful to consider the membership function that assigns a participant's degree of assuredness, somewhere in the interval of 0 and 1, during an observation, as it contains a probabilistic function that produces some fuzzy value. From here on, the program may act upon that value using a series of fuzzy or deterministic instructions to produce some result in a single instance of observation. Therefore, it may be beneficial for the reader to envision $PROG$ as a type of heuristic inference if we wish to describe it in more human terms.

Interpretation $INTER$ is a fuzzy program that compiles a series of $PROG$ (Pask, 1976b, p. 141). Compiling such a series of programs produces the universe of interpretation $\mathbb{U}$ (p. 135). The term $INTER$ will then be interpreted as producing $\Rightarrow$ and maintaining a relation R_i that belongs to such a universe $\mathbb{U}$. In other words:

$$INTER \Rightarrow R_i$$

.

Where $R_i \in \mathbb{U}$. By postulating that an interpretation is a compilation of programs, the following expression may be derived:

$$INTER \triangleq PROG(PROG\ j, PROG\ k, \ldots PROG\ n)$$

Where $\triangleq$ means equal by definition. It is assumed that $INTER$ *is* complied with in an L-processor, which is an environment that permits the processing of a conversational language L (typically conceived of as a brain). An interpretation assigns the values that are required for input $PROG$s to be executed, such that it can be viewed as establishing parameters or thresholds needed for its execution (p. 141). $INTER$ then is that which permits us to interpret the topics of syntactical languages, such as first-order predicate calculus (p. 134), as well as more semantically oriented languages, such as L (p. 133).

Simple Concepts

Pask (1975b) did not assume that a concept is a class or category (p. 44). Instead, it is a *reproductive procedure* used to derive a given topic relationship from itself. A procedure in general can be said to consist of a class of programs $PROG$ that derives some topic relation and a class of interpretations $INTER$ for compiling such programs (p. 142). Thus, let there be a class of inferences that are derived during some problem-solving or purposive activity, and a class of compilations of such inferences that attempt to satisfy the problem, goal, or need posed. A standard procedure, $PROC$ is defined as consisting of the set containing the class of programs and the class of interpretation.

$$PROC \triangleq \langle PROG, INTER \rangle$$

where $PROC$ is considered to be an L-procedure that emerges from the interactions of some L-processor. The term L-processor here may be identified with a human brain (although not necessarily so in all cases; Pask, 1976b, p. 136). Such processors are spatiotemporally demarcated on biological and mechanical grounds (Pask, 1976b, p. 7). From here on, it is argued that CON is a $PROC$ if and only if the compilation of the procedure can stabilize the relation (p. 143). This is given by the identity:

$$CON \triangleq PROC(R_i) \Rightarrow R_i$$

Concepts may be compiled through serial, parallel, or concurrent means with processors, yet they must be coherent if they are to be considered concepts (Pask and de Zeeuw, 1992, p. 38). In other words, if the execution of a procedure can yield itself without things such as material contradictions occurring, non-cyclic inference to infinity, and so on. Thus for example, if $PROC(R_i) \Rightarrow R_j$ where $R_j \nvdash R_i$ then a procedure is unable to satisfy the principle of cyclicity. Therefore, such a procedure cannot be classified as a concept within algebra. Anything that goes against the principle of cyclicity is not considered here.

It follows from this point that the topic relation R_H may be substituted for $CON(R_H)$ if it satisfies the principle of cyclicity. Let R_H be the head relation under deliberation such that it is the main topic of interest. We can view this as a thesis posed to us that requires elaboration. It is assumed $R_H \in \mathbb{D}$, such that it exists in the domain of conversation topic. Let a universe of topics $\mathbb{U}$ which is a specific universe of discourse belonging to the conversational domain, be cited by a participant, such that $\mathbb{U} = \langle R_H; R_1, R_2, \ldots, R_n \rangle$, where the semi colon $;$ is used to distinguish the head topic relation from other relations. Let the above expression be simplified to $\mathbb{U} = \langle T, P, Q \rangle$, where $T = R_H$. Suppose the participant makes the claim $PROG(T, Q) \Rightarrow P$ and $PROG(T, P) \Rightarrow Q$; then the compilation of this form would be:

$$INTER(P, Q) = PROG(PROG(T, Q), PROG(T, P))$$

And if:

$$INTER(P, Q) \Rightarrow T$$

It then holds that $R_H = CON(R_H)$, by virtue of $T = R_H$. To satisfy such cyclicity, topic relations can also be viewed as tokens of eigenbehaviors (von Foerster, 2003). Following von Foerster (2003), eigenbehavior may be characterized as a self-generating or self-defining behavior or function *f*. Meanwhile, an eigenvalue is both the input *x* and output *x*. If x satisfies the expression $x = f(x)$ such that $x = f(f(f(\ldots)))$, then it may be considered an eigenvalue (von Foerster, 2003, p. 264). It may, therefore, be conceptualized as a token of an eigenbehavior, which if inserted into itself produces itself. Because $R_i = CON(R_i)$, concepts and topics may be considered eigenforms.

Concepts Proper

Proper concepts require both concepts of topic relations and descriptions of topic relations utilized in speech. Because proper concepts require descriptions of topics, there must be a shared language by which communication between participants may occur. Let this language be designated object language L, which prioritizes semantic or material associations over and above its syntax. Thus, it differs from computational languages, which are syntactical by being interpreted languages that are given a semantic component with respect to the conversational domain (Pask, 1975b, p. 160). Object language has two modes of verbal activity: Discursive activity that occurs at L^1 indicates how a topic relation may be known or yielded in an activity (p. 160), while discursive activity that happens at L^0 indicates what may be done to bring about the topic relations (p. 160). According to Scott (2015, p. 62), this can be conceptualized as a discursive activity occurring at the level of knowing why and the level of knowing how. Recent theoretical developments in conversational approaches in natural language processing have also made a similar distinction, but Pask and associates use predates by approximately half a century (Lazaridou *et al*., 2020, p, 7663)

The conditions for the emergence of concepts proper can at their simplest be illustrated through a conversational skeleton (see Figure 1). Given two participants, A and B, each participant is said to have repertoires of procedures π that may be utilized to produce speech descriptions $D(R)$. Both repertoires of procedures and descriptions are demarcated along L^1 and L^0 lines, such that π^0 consists of $PROC(R_i) = REP(R_i)$ which satisfy a given topic and π^1 consist of $REP(PROC(R_i)) = REP(REP(R_i))$ which reproduce REP the latter (Pask, 1975a, p. 297) For simplicity, let π^0 contain concepts and π^1 contain memories of those concepts. There is also a modeling facility $Mod.Fac.$ This is an environment in which learning and problem solving occur, and where the derivation of topics may be bought about through model building activities (Pask, 1975b, p. 559).

Generating Descriptions and Procedures

Let the object language L utilize the concept proper set $\langle CON, D\rangle$. By designating this set concepts proper set, Pask and de Zeeuw (1992, p. 38) are taken in this text to be implicitly inferring Lev Vygotsky's (1934/1962) notion of the true concept. Concept formation is the result of a complex activity in which all basic functions are involved. The process cannot ... be reduced to association, attention, imagery, inference, or determining tendencies" (Vygotsky, 1934/1962, p. 58). Instead, Vygotsky argues that such discursive activity can only be considered to result in true concept formation through the word (p. 59). For Vygotsky, it is the union of the speech descriptions D and simple concepts CON that results in proper concepts. There have been numerous acknowledgements from both the primary and secondary sources of Vygotsky's (1932/1962) influence on conversation theory (Laurillard, 2002; Pask, 1975b, 1976b; Scott, 2011, 2021; Tilak and Glassman, 2022). For this reason, the set of proper concepts will be designated as Lev set $Л$, such that

$$Л \triangleq \langle CON, D\rangle$$

where CON is short for the concept of topic $CON(T)$, and D is short for a description of topic $D(T)$. Because the $Л$ set by definition requires both concepts and descriptions, it necessarily relates to Pask's (1976b) conception of description and procedure building. Description building for Pask (1976b) is equated to appreciating a topic (p. 102), such that a student who appreciates a topic of inquiry is able to build a description of the topic. Procedure building is a concept building exercise in which a concept is constructed to realize the description (Pask, 1976b, p. 102). The nature of both description and procedure building is only briefly examined in this text to give the reader a sense of how such processes are conceived in conversation theory. Description building DB acts upon topic relations to produce new relation, such that $DB(R_i, R_j) \Rightarrow R_k$. Such topic relations may be viewed as forming a description of topic relations (p. 165). The procedure of building PB also acts upon some applied procedures if a relation is given to produce new procedures (p.165). This can be represented as the following expression: $PB(R_k; PROC^0 i, PROC^0 j) \Rightarrow PROC^0 k$. There is also a third category of procedure combing PC which is the same as the above expression except for the fact it lacks a topic relation (p. 169). However, for simplicity, it is assumed for convenience that PC is in PB. Because description building and procedure building act upon procedures and topic relations, this implies that they also act on the program and interpretation set, respectively (Pask, 1976b, p. 165). Now, let the description building be identified with CON^+ and the procedure building and combining be identified with CON^-. The union of such operations can be simplified into the general expression CON^* which "indicates a productive and, incidentally, reproductive operator of the form Con but peculiar to participant Z" (Pask and de Zeeuw, 1992, pp. 38-39). The operator CON^* is treated as synonymous to CON', where the latter operator is a "second order operator of the form [of a] concept, but a concept that acts upon other concepts to produce and, incidentally reproduce the same or fresh ones" (p. 38). This has the following identity.

$$CON^* \triangleq \langle CON^+, CON^- \rangle$$

Thus, CON^* consists of description and procedure-building processes. Let there be a number of topics PQ that derive topic T as represented by the description and procedure building operators. In this case, it follows a series of descriptions D or procedures $PROC$ that.

$$CON^+(D(P), D(Q)) \Rightarrow D(T)$$

Where any D is treated as equivalent to its topics such that $D(R_i) = R_i$, and:

$$CON^-(D(T); PROC^0(P), PROC^0(Q)) \Rightarrow PROC^0(T)$$

Given a description of T, a participant will attempt to apply a series of established procedures (perhaps in the form of skills or practical knowledge) to generate a procedure that would satisfy that topic. It should be noted, however, that the above two expressions are illustrative and not exhaustive of the type of form description and procedure building may take.

Conversational Interactions

Proper concepts are argued to produce the conditions necessary for the maintenance of stable conceptual and conversational forms. It is possible to formalize a conversational move by one participant to another (or themselves), in such a way that maintains the structural integrity of the $Л$ set. In doing so, not only are simple concepts that correspond to the idea of an eigenform, as well as the conversations themselves. The constituent components that yield the form of the expression must be discussed before this point can be illustrated using algebra.

Let there be an application operator AP that "permits activation in any system, a brain, a society, a Petri Net, [and] as a [limited] case, in a standard computing machine" (Pask and de Zeeuw, 1992, p. 38). The application operator $AP(\ldots)$ is a permissive operator and its sister operator $\&\langle AP(\ldots)\rangle$ is an imperative operator. The term permissive refers to what a participant is *allowed* to do, and imperative denotes what a participant is *obliged* or *necessitated* to do (von Wright, 1951, p. 1).

Following von Wright (1951, p. 2), it is useful to think of such deontic prescriptions or permissions as guiding the acts that a participant might perform in a conversation. Therefore, $AP(Л)$ can be viewed as an application operator permitting acts that are contained within the set. This argument holds as if $AP(Л) = AP(CON, D)$, then it follows that $AP(CON, D) = AP(CON) \cup AP(D)$. Thus, it can be deduced that $AP(CON) \cup AP(D) = AP(CON) \vee AP(D)$, whereby $\vee$ is a logical operator.

Therefore, it holds that:

$$AP(Л) = AP(CON) \vee AP(D)$$

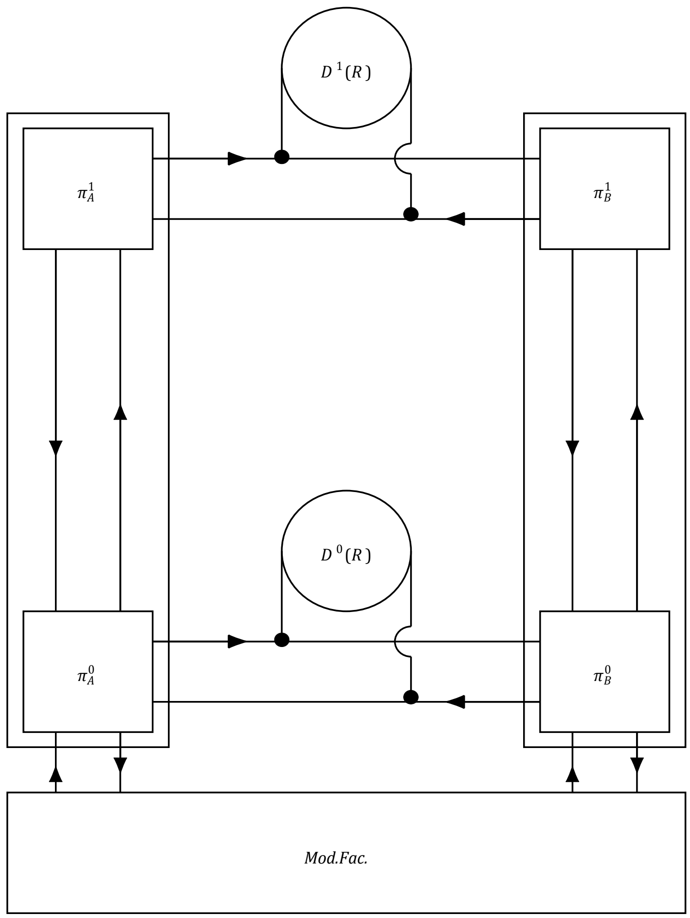
This represents all possible choices that $AP(Л)$ permits, such that a participant can choose/respond to such-and-such by permitting the evocation of a concept or a description. This form of argument also holds for $AP(CON^{*})$, such that $AP(CON^{*}) = AP(CON^{+}) \vee AP(CON^{-})$. In other words, a participant can either engage in description or procedure building given activation via AP.

It is worth reinforcing the idea that descriptions D – technically speaking, are merely specific modes of the concept rather than something separate from it. Both concepts and descriptions convey topics T, and therefore are reducible to them, such that $T = CON(T)$ and $T = D(T)$. By virtue of conveying topics, expressions of form D have the form of $CON(D)$ for a participant receiving D as Scott (2011, p. 287) notes (which can then be reduced to the eigenvalue T). A participant sending D to another participant also has the form $CON(D)$, such that they acknowledge their talk to another participant about a given topic T). Thus, in a sense:

$$Л = \langle CON, CON(D) \rangle$$

It may be considered a better representation of the $Л$ ¸ based on the inherent conceptual nature of the descriptions. This also highlights the phenomenological character of giving and receiving descriptions, as it presupposes a conceptual acknowledgment of a description uttered.

Figure 1. *Example of a conversational skeleton.*



Note. Example of a simplified conversational skeleton, where A and B are represented by two parallel rectangles. Discursive activity that occurs through object language L is divided along L^0, L^1 lines of discourse. Let π indicate a repertoire of procedures and let $D(R)$ indicate a description given during a conversation. Meanwhile, let the modeling facility $Mod.Fac.$ indicates the setting in which learning occurs. A participant's interaction with a modelling facility may be directed along vertical arcs, whereas conversational interactions between participants may be directed along horizontal arcs. A combination of the two is required for concepts arising from conversations between human peers.

Conversational Moves

Having discussed the nature of the application operator AP and certain technicalities involving the $Л$ set, it is worth examining a conversational interaction between two participants (each making an individual conversational move). Figure 1 can be used to illustrate such an interaction, where a repertoire of procedures π is associated with the set $\langle CON^*, CON\rangle$. From here on, it is argued that a conversational move can be expressed in the most general sense as

$$AP(CON_Z^*(Л)) \Rightarrow Л$$

where Z denotes conversation participants who belong to an index of the participants. The above expression can be expanded to the form

$$AP(CON_Z^*(AP(CON_Z) \vee AP(D_Z))) \Rightarrow CON_Z \vee D_Z$$

Activation of either a concept or description leads to the activation of some procedure or description-building processes, such that a concept or description is produced.

An example of a short conversational interaction between two participants A and B (where $A, B \in Z$) is given as follows: This is done to illustrate a snapshot of a basic conversational interaction that may be represented through algebra. To begin with, let:

$$AP(CON_A^+(AP(CON_A))) \Rightarrow D_A$$

Where A applies a description building CON^+ to their CON to produce a description of their topic, and let

$$AP(CON_B^-(AP(D_A))) \Rightarrow CON_B(D_A)$$

Indicate B, having received a given topic, attempting to guess what A means by D_A such that B yields $CON_B(D_A)$. In doing so, B compiles an interpretation $INTER$ based manipulation and compiles a series of $PROG \in \pi$ in order to infer what A means by D. Once the interpretation yields what B thinks is A's topic of discussion, let B respond via the following conversational move:

$$AP(CON_B^+(AP(CON_B(D_A)))) \Rightarrow D_B$$

Where:

$$AP(CON_A^-(AP(D_B))) \Rightarrow CON_A(D_B)$$

Thus, A receives D_B and attempts to infer B's concept of its initial concept. Conversations ultimately rest on such discursive moves, in which both participants attempt to infer what the other person means. Given that $CON = \langle CON^0, CON^1 \rangle$ and $D = \langle D^0, D^1 \rangle$ such that concepts and descriptions are demarcated along L^0 and L^1 lines of discourse (see: Figure 1), such conversational moves entails participants being able to engage in the giving and asking of reasons. Conversation theory permits giving and asking for reasons through a command and question language, which allows participants to ask questions and give explanations and commands to each other during a conversation.

Commands, Questions, Explanations

The object language L permits the giving of commands, questions, and explanations; because of this "it must be a command and question language and it must admit ostension and predication" (Pask, 1975b, p. 22). It follows that commands, questions, and explanations then occur at the two levels of discourse, L^0 and L^1 (p. 72). At L^0 a command might be "Solve this problem" or

"Do this task," while a question might be "How do we do this task" or "What needs to be done". In L^1, commands might be "Show me your reasoning," and questions might be "Why did you do this task" or "What the significance of doing this?". Explanations at L^0 meanwhile, might be "I tried using such-and-such" and explanations at L^1 might be "I thought it would help us deal with so-and-so".

Let the command and question language L contain procedures corresponding to commands $COMM$, questions $EQUEST$ and explanation $EXPL$. Such utterance procedures operate under L^0 and L^1 assumptions (p. 190). It is assumed that, at the very minimum, commands and questions correspond to the following utterance procedure script:

$$\langle Z!\ X \mid Y \rangle$$

This reads along the lines of "Addressee! Do X, Given Y", or "Addressee! Bring about X, in relation to Y", where Y is a precondition $PRECON$ (containing conditional topic relation; Pask, 1975a, p. 215).[1] Commands for Pask (1975b) are issued *on authority* to Z, and activated if a given precondition is satisfied (p. 470). Therefore, they are deontic imperatives or obligations that the listener *should* acknowledge and correctly respond to the

[1] This form on expression, is ultimately borrowed by Pask from *The Logic Of Commands* (Reshcer, 2020)

contents of the intention. Therefore, $Z!$ should be read as "*The addressee should*...', rather than merely signaling the addressee's attention.

Commands can be treated as imperatives to perform such-and-such, that is, imperative to execute $EXEC(\ldots)$ a behavior (where the execution operator is treated in this text as corresponding to $\&\langle AP(\ldots)\rangle$ in the above sections). In this sense, a command $COMM$ is a double imperative, containing a prescription for the addressee in the form of a second-order prescription "The addressee should…," and the first-order prescription "Do such-and-such" Do such-and-such' that is requested. Questions are treated as commands to explain, such that they may also be considered imperatives to performing the action of giving an explanation.

Finally, explanations are treated as descriptions of Model M (p. 471). It is supposed that a model is the result of a procedure acting upon a specific universe of modeling $\mathbb{U}$ by which participants interact with a modeling facility (such as that seen in Figure 1; Pask, 1975b, p. 148). Let a^0, b^0 be partial models conceived using such a facility. Following Pask (1975b, p. 148), $PROC_Z^0 i(\mathbb{U}) = \langle a^0, b^0, \ldots, M_Z i\rangle$ if and only if such models can be represented as topics R_i from here if:

$$PROC_Z^0 i(\mathbb{U}) \Rightarrow M_Z i$$

Then:

$$M_Z i = PROC_Z^1 i(M_Z i)$$

It is said to hold that a model can be represented as a topic R_i as expressed in $D(M_Z i) = D(R_i)$; thus, a description of a model is merely a description of a topic. From here on, it is possible to represent the simplified forms of commands, questions, and explanations as follows:

$$\begin{aligned} COMM\ i &= \langle Z!\ EXEC\ i \mid PRECON \rangle \\ EQUEST\ i &= \langle Z!\ EXPL\ i \mid PRECON \rangle \\ EXPL\ i &= PROC(R_i) \end{aligned}$$

where $COMM\ i$ is expected to induce $PROC_Z^0 i(\mathbb{U})$ in an addressee, $EQUEST\ i$ is expected to induce $PROC(R_i)$, and $EXPL\ i$ is expected to induce $M_Z i$ or R_i in the addressee. The execution of such commands, questions, and explanation procedures in an L-processor gives rise to utterance D which is intended to cause a participant to respond in some way (either through implicit or explicit acknowledgment of T, or by asking for clarity as to the nature of what has been said).

While commands, questions, and explanations can be conceived through a variety of other forms, what I believe is important here is that this conception of utterances is evidenced in preverbal linguistic development (Bruner, 1983). Bruner's (1983) division of utterances in proto-conversational pseudo-conversational interactions between a caregiver and a preverbal infant distinguishes between attentional vocatives, queries, labels, and feedback (p. 79). Aside from feedback (which is postulated here to be reducible to the other categories), attentional vocatives can be considered commands, queries as questions, and label explanations within algebra. Considering the importance of such proto-conversational interactions in establishing conversation proper later on, there is good reason to suspect that conversation can be carried out with a set of commands, questions, and explanations.

No Doppelgangers

Finally, the text examines Pask's last theorem or the Laing-Pask theorem, which asserts that no one concept conceived is the same. The basis for the theorem originates in Laing *et al.* 's(1964) work on interpersonal perception, whereby we can have our perceptions of others and our perceptions of others' perceptions of ourselves. Pask applied the work of Laing *et al.* (1964) to the conversation and interaction of actors' theories. Let there be some general notion of participant Z that acts on topic T) such that T_Z can be said to be their topic. If the participants are A and B, then the following holds.

$$A(T) = T_A = \langle CON_A(T), D_A(T) \rangle$$
$$B(T) = T_B = \langle CON_B(T), D_B(T) \rangle$$

Let T be either A or B in the expressions below to designate A and B's topics. From this, it follows that:

$$A(A) = A_A = \langle CON_A(A), D_A(A) \rangle$$
$$A(B) = B_A = \langle CON_A(B), D_A(B) \rangle$$
$$B(B) = B_B = \langle CON_B(B), D_B(B) \rangle$$
$$B(A) = A_B = \langle CON_B(A), D_B(A) \rangle$$

From which it can be inferred:

$$A(A) = A_A \neq B(B) = B_B$$
$$A(A) = A_A \neq B(A) = A_B$$
$$B(B) = B_B \neq A(B) = B_A$$

For any concept of a topic, it holds that A's concept of A's concept is not equivalent or a substitute for B's concept of B's concept; neither is A's concept of A's concept equivalent or a substitute for B's concept of A's concept, or vice versa. Thus, there are no doppelgangers in the production and reproduction of concepts, as all concepts are unique to the discursive being utilizing them (Pask and de Zeeuw, 1992, pp. 42-43). This also holds for temporal iterations of concepts, for if $A = A$ and $B = A'$, then A's current concept cannot be substituted with the current concept. Thus, “The Now is pointed out. ‘Now’; it has already ceased to be in the act of pointing to it” (Hegel, 1977, p. 63).

Conclusion

The text compiles a variety of texts from Pask and associates’ collaborations in order to further elaborate on the basis for discursive algebra. The benefits of such discursive algebra would primarily be of use to those working on conversational approaches to multi-agent systems learning. The text covers how an object language may be established to permit concept formation and concept sharing between two or more participants. It has also been conceived here that concepts are inferentially self-generating and cyclical structures that use inferences in the form of heuristics to infer topics, to which a selection of such topics may be used to form the predicate of the initial subject topic. It has been argued, following Vygotsky’s (1934/1962) work on educational psychology, that concepts proper must be conceived of as utilizing concepts and descriptions through the form of the л set. The form of cyclic conversational interactions has also been documented through the use of algebra, as well as the nature of command, question, and explanation giving (which has similarities to some of Bruner’s (1983) research on proto-conversational interactions). Finally, algebra can be made to establish the non-equivalence of participants' concepts, such as my concept is not the concept of my own. While aspects of algebra have not been developed here (specifically the establishment of material analogies and common understandings), I believe I have established the core considerations of Pask's algebra such that these topics may be elaborated upon through subsequent readings of his work by the reader.

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