Witnesses

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Abstract

The meaning of definite descriptions is a central topic in philosophy and linguistics. Indefinites have been relatively neglected by philosophers, under the Russellian assumption that they are simply existential quantifiers. However, a well-known set of patterns suggest that this assumption is wrong. In this paper I develop a new approach to (in)definites which aims to capture these patterns. On my theory, truth-conditions are classical. But in addition to truth-conditions, meanings comprise a second dimension, which I call *bounds*. It is at the level of bounds, not truth-conditions, that the characteristic coordination between indefinites and definites takes place. My system has a classical logic, thus avoiding serious problems which face the most plausible extant account of these patterns, namely, dynamic semantics. More generally, my approach yields a new perspective on the relation between truth-conditions and dynamic effects in natural language.

1 Introduction

The meaning of definite descriptions like 'the King of France' or 'the dog' has been a central topic in philosophy and linguistics for the past century. Indefinites ('Someone called', 'A child sat down', etc.) have been relatively neglected by philosophers, who have mostly adopted the Russellian assumption that indefinites are simply existential quantifiers. However, a robust set of patterns suggest that the Russellian model is untenable: indefinites cannot be modeled simply as existential quantifiers.[1] To see the basic issue, compare (1-a) and (1-b):

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1 See Egli 1979 for some history, which goes back to Stoic logic.

(1) a. Everyone who has a child loves them.
b. Everyone who is a parent loves them

(1-a) can be interpreted as saying that every parent loves their child; while (1-b) can only be naturally interpreted as saying that some salient person is loved by every parent. But if indefinites are just existential quantifiers, then 'has a child' and 'is a parent' should mean exactly the same thing. But then it is hard to see how we could account for differences in how they embed in pairs like (1). Parallel problems arise with 'the child' in place for 'them'. Minimal pairs like this one form the heart of the case against the identification of indefinites with existential quantifiers.

There are two broad responses to this puzzle. *E-type* approaches argue that indefinites are existential quantifiers after all; contrasts like those in (1) are explained by the syntactic differences between the two indefinite expressions. In particular, (1-a) makes salient a predicate ('child') missing from (1-b), which can then—the thought is—be recruited to license anaphora with subsequent definites.[2] *Dynamic* approaches instead argue that the behavior of definites and indefinites shows that meanings are more fine-grained than truth-conditions: sentential meanings are not sets of indices, but instead functions from contexts to contexts. Contexts, in turn, are not just sets of worlds, but sets of world-assignment pairs. Finally, while 'x has a child' and 'x is a parent' have, in some sense, the same 'worldly' content, they have different meanings, because they update contexts differently: only the first yields a context which supports subsequent anaphora to a child.[3]

Both of these approaches require revisionary approaches to content. In the dynamic system, contents are, again, functions from contexts to contexts; in the e-type approach, they are sets of situations or events. And both approaches must adopt non-classical treatments of the connectives.[4] The proper treatment of patterns like those in (1) thus

2 See e.g. Geach 1962, Evans 1977, Parsons 1978, Cooper 1979, Neale 1990, Heim 1990, Ludlow 1994, Büring 2004, Elbourne 2005; see Lewis 2012, 2019, Mandelkern and Rothschild 2020, Lewerentz 2020 for more recent developments and criticism.

3 E.g. Karttunen 1976, Kamp 1981, Heim 1982, Groenendijk and Stokhof 1991, Dekker 1993, 1994, van den Berg 1996, Muskens 1996, Aloni 2001, Beaver 2001, Nouwen 2003, Brasoveanu 2007, Charlow 2014.

4 I will explain this presently vis-à-vis dynamic semantics; see Mandelkern and Rothschild 2020 for this point in the context of e-type theories.

raises central questions, not just about the meaning of (in)definites, but also, more generally, about logic and the nature of content.

In this paper I develop a new approach, which answers these questions differently from either dynamic or e-type theories. On my theory, the *bounded theory*, contents are sets of indices, connectives are classical, and indefinites have the truth-conditions of existential quantifiers; in this respect, my theory is very classical. But, on the bounded theory, indefinites also have a secondary dimension of meaning: they have what I call a *witness bound* which requires that, if the indefinite is true, then a witness to its truth is assigned to the indefinite's variable. Definites have as their truth-conditional content simply the conjunction of their scope and restrictor. But they, too, have a secondary meaning: a bound which requires that the content of their scope be *familiar*—that is, true throughout the definite's local context. This approach builds on the insights of existing theories, especially dynamic semantics. But by locating the characteristic coordination of indefinites and definites in a dimension of meaning separate from truth-conditions, I avoid well-known problems that arise from the revisionary logical framework of dynamic semantics. More generally, the resulting system yields a new perspective on the source and nature of dynamic effects in natural language.

2 Problems for the classical picture

I will begin by summarizing the puzzle for the classical approach to (in)definites; the dynamic response to that puzzle; and some serious problems it faces. Then I will develop my own system.

On the classical treatment (due, roughly, to Frege, Russell, Quine, and Strawson) an indefinite sentence like ⌜Some F is G⌝ is equivalent to $\exists x(Fx \& Gx)$, where $\exists$ is the classical existential quantifier.[5] Definite descriptions have the same meaning plus a uniqueness inference: so ⌜The F is G⌝ says that something is both F and G, and also says (or presupposes) that there is exactly one (relevant) F-thing. Pronouns are treated as variables.

Given these assumptions, and given the intuitive meaning of 'parent' and 'child', 'Sue is a parent' and 'Sue has a child' will mean exactly the same thing. But they seem

5 (Corner) quotes are omitted for expressions of formal language.

to pattern differently in terms of their interaction with subsequent definites (descriptions and pronouns). We have already seen this in the context of quantifiers in (1) (so-called *donkey sentences*). We can see this in an even simpler way by looking at minimal pairs like (2):

(2) a. Sue has a child. She is at boarding school.
 b. Sue is a parent. She is at boarding school.

The prominent interpretation of (2-a) says that Sue has a child at boarding school, while (2-b) is much more naturally interpreted as saying that Sue herself is at boarding school. (This is not to say that (2-b) is impossible to interpret in the same way as (2-a); the observation is rather that there is a striking *contrast* in the availability of these interpretations which needs to be accounted for.) This pair is modeled on Partee's famous marble sentence from Heim 1982, so I'll call pairs like this *Partee pairs*. For another example, compare:

(3) a. Sue has a twin. She lives in Dubuque.
 b. Sue is a twin. She lives in Dubuque.

Again, having a twin and being a twin are, under classical assumptions, plausibly the same property. Yet these indefinite expressions set up very different anaphoric potential: (3-a) is naturally heard as saying that Sue has a twin who lives in Dubuque, while (3-b) is naturally heard as saying that Sue is a twin who lives in Dubuque.

Note that Partee pairs cannot be accounted for simply by saying that the indefinite in these cases takes wide-scope over the two sentences and binds 'she'. Of course, the idea that indefinites can take scope over whole discourses—sequences of sentences—would already require a huge revision to the classical approach to logic. But more importantly, this would be a local solution to a very global problem, which would do nothing to help explain corresponding contrasts in other embedded environments, like the donkey sentences in (4), or the corresponding examples with 'is a twin'/'has a twin' in (5):

(4) a. Everyone who has a child loves $\left\{\begin{array}{c}\text{them}\\ \text{the child}\end{array}\right\}$.
 b. Everyone who is a parent loves $\left\{\begin{array}{c}\text{them}\\ \text{the child}\end{array}\right\}$.

(5) a. Everyone who has a twin loves $\left\{ \begin{array}{c} \text{them} \\ \text{the twin} \end{array} \right\}$.

b. Everyone who is a twin loves $\left\{ \begin{array}{c} \text{them} \\ \text{the twin} \end{array} \right\}$.

To see the problem, focus on the variant of (4-a) with 'them'. The classical assumptions above would yield (6) as the gloss for (4-a) (with $\forall$ the classical universal quantifier and $\rightarrow$ the material conditional):

(6) $\forall x((\exists y(\mathit{child\text{-}of}(y,x))) \rightarrow \mathit{loves}(x,y))$

The problem with (6) is that the variable y in the consequent is unbound, so we don't get the intended covariation between 'a child' and 'them'. A natural thought is to give the existential quantifier wide scope over the material conditional, but that doesn't help: while y would end up bound, we would get absurdly weak truth-conditions for (4-a) (which would end up being true provided that no one is everyone's parent). So it is not at all obvious how to derive the intended meaning of (4-a) given the classical assumptions above.

Donkey sentences with definite descriptions rather than pronouns raise slightly different, but equally serious, issues, as Heim (1982) showed. Consider the variant on (4-a) with a definite description:

(7) Everyone who has a child loves the child.

Given the classical picture above, (7) will say that every parent loves their child, but will also assert (or presuppose, depending which version of the classical picture you adopt) that every parent has exactly one child. Clearly, though, (7) does not lead to a uniqueness inference like that. This is brought out clearly by a variant on Heim's sage-plant sentence, in (8):

(8) Everyone who bought a sage plant bought seven others along with the sage plant.

(8) clearly does not license the incoherent inference that everyone who bought a sage plant bought exactly one sage plant.

3 Dynamic semantics

Dynamic semantics responds to these problems by proposing that predicates like 'has a child' and 'is a parent' in fact have different meanings, and adopting revisionary treatments of definites, connectives, and contents more generally. To explain the basic ideas behind this response—which, in turn, form the background for my own account—I will informally sketch a simplified version of Heim 1982's dynamic system. There are many dynamic systems in the literature; the one I will present gives a representative sense of the key ideas.[6]

On this approach, sentence meanings are functions from contexts to contexts. A context, in turn, is a set of pairs of (possibly partial) variable assignments and worlds. The worlds represent the possibilities treated as live in the conversation (the conversation's *context set*, in roughly the sense of Stalnaker 1974). The variable assignments track anaphoric relations between indefinites and definites.

The role of indefinites is to extend the contextual variable assignments so they are defined on a new variable; the role of definites is to pick up on a variable that has been introduced this way. So, as long as x is novel in c (that is, nowhere defined in c), 'There is a_x cat(x)' denotes the function that takes c and extends every variable assignment in c to an assignment which assigns x to a cat. More carefully, the resulting context will be the set of pairs $\langle g, w\rangle$ such that $g(x)$ is a cat in w, and for some pair $\langle g', w'\rangle \in c$, $w = w'$, and g' and g agree everywhere except on x.

This captures the idea that indefinites "open a file card", in Heim's metaphor, or establish a new "discourse referent", in Karttunen's: an indefinite indexed to x extends a context c so that every variable assignment is defined on x, and, in particular assigns x to a witness of the indefinite.

Definites, in turn, are used to talk about variables that have already been assigned to a witness throughout the context. In the Heimian metaphor: they are used to add information to file cards that have already been opened (that are already "familiar").

6 I will not discuss e-type approaches here. E-type approaches struggle to deal with Partee pairs in general. The issue is that in a pair like (3), the relationship of being a twin is equally salient, linguistically and cognitively, in both constructions, but only 'has a twin', not 'is a twin', licenses subsequent anaphora to the twin. Dynamic treatments of these contrasts have been developed much more explicitly, and so make a more effective foil.

So, for instance, ‘The$_x$ cat(x) is sleeping(x)’ presupposes that x is defined throughout c and moreover assigned to a cat through c. Where this presupposition is satisfied, ‘The$_x$ cat(x) is sleeping(x)’ updates c so that the resulting context includes exactly the pairs $\langle g,w\rangle \in c$ where $g(x)$ is a sleeping cat in w. Pronouns are treated analogously, but with only the requirement that x is defined throughout the input context: so, for instance, ‘He$_x$ is sleeping(x)’ presupposes that x is defined throughout c, and, where that presupposition is satisfied, takes c to the set of pairs $\langle g,w\rangle \in c$ such that $g(x)$ is sleeping in w.

Updating with an indefinite thus sets the stage for subsequent updates with definites: indefinites open file cards that definites can then add to. Suppose we have updated the context with ‘There is a$_x$ cat(x)’. This guarantees that, at every point $\langle g,w\rangle$ in the resulting context c, $g(x)$ is a cat in w. So the familiarity presupposition of the definite ‘The$_x$ cat(x)’ is satisfied in c. Hence updating c with ‘The$_x$ cat(x) is sleeping(x)’ will take us to the context comprising just those pairs $\langle g,w\rangle$ from c where $g(x)$ is a sleeping cat. By contrast, a definite out of the blue will not have its familiarity presupposition satisfied, since (we assume) default starting contexts include only empty assignments, and hence include, for any variable x, assignments where x is undefined.

This puts us in a position to see how dynamic semantics accounts for Partee pairs. Focus, for concreteness, on (2). Updating a context c where x is novel with either ‘Sue has a$_x$ child(x)’ or ‘Sue is a$_x$ parent(x)’ will have the same *worldly* effect, in the sense that a world in any pair in c will be in some pair in the updated context just in case Sue is a parent in that world. But these updates have very different effects on variable assignments. Updating c with ‘Sue has a$_x$ child(x)’ will result in a context comprising exactly the pairs $\langle g,w\rangle$ such that $g(x)$ is a child of Sue’s in w and for some $\langle g',w'\rangle \in c$, $w = w'$ and g' agrees with g except on x. By contrast, updating with ‘Sue is a$_x$ parent(x)’ will instead result in a context which comprises exactly the pairs $\langle g,w\rangle$ such that $g(x)$ is Sue, Sue is a parent in w, and for some $\langle g',w'\rangle \in c$, $w = w'$ and g' agrees with g except on x. So the first update opens a file on Sue’s child, while the second instead opens a file on Sue.

That means that only the first update licenses subsequent anaphora to Sue’s child; the second, by contrast, licenses anaphora only to Sue. A context that has been updated with 'Sue has a_x child(x)' can then be updated with 'She$_x$ is at boarding school(x)'; the resulting context will comprise exactly the pairs from the intermediate context which take x to a child of Sue's who is at boarding school. By contrast, when a context that has been updated with 'Sue is a_x parent(x)' is updated with 'She$_x$ is at boarding school(x)', 'she$_x$' will refer to Sue, not her child: the subsequent context will comprise exactly the pairs from the intermediate context where Sue is at boarding school.

This account of Partee pairs extends to conjunctive versions of Partee pairs, like those in (9):

(9) a. Sue has a child and she is at boarding school.
    b. Sue is a parent and she is at boarding school.

Conjunction is treated by dynamic semantics as successive context update, first with the meaning of the left conjunct, then the right. That is: where $[p]$ is the dynamic meaning of any sentence p, and $c[p]$ is the application of $[p]$ to context c, the dynamic conjunction says $c[p\&q] = (c[p])[q]$ (wherever defined). So, 'Sue has a_x child(x) and she$_x$ is at boarding school(x)' first extends c by assigning x everywhere to a child of Sue's, then further updates this context by keeping just those pairs $\langle g, w\rangle$ where $g(x)$—that is, Sue's child—is at boarding school. By contrast, 'Sue is a_x parent(x) and she$_x$ is at boarding school(x)' first extends c by assigning x everywhere to Sue and keeping just those points where Sue is a parent; then further updates this context by keeping just those points $\langle g, w\rangle$ where $g(x)$—that is, Sue—is at boarding school.

A general way of characterizing the dynamic account of Partee pairs is to say that, in dynamic semantics, indefinites have *open scope* to their right (Egli 1979): they can "bind" co-indexed definites, whether or not the definite is in their syntactic scope. Schematically:

- *Open scope of indefinites*: The following are equivalent:

(10) a. Some F is G and H.
     b. Some F is G, and $\left\{\begin{smallmatrix}\text{it}\\ \text{the } F\end{smallmatrix}\right\}$ is H.
     c. Some F is G. $\left\{\begin{smallmatrix}\text{It}\\ \text{The } F\end{smallmatrix}\right\}$ is H

So, for instance, ‘Sue has a child, and she is at boarding school’ will be equivalent to ‘Sue has a child who is at boarding school’; while ‘Sue is a parent, and she is at boarding school’ will be equivalent instead to ‘Sue is a parent who is at boarding school’.

This explanation of Partee pairs extends naturally to the corresponding contrasts in donkey sentences. For reasons of space I won’t spell this out in detail, but the intuition is the same: predicates like ‘has a child’ and ‘is a parent’ update contexts’ assignments differently, making available different possibilities for subsequent anaphora.

4 Logical problems

Dynamic semantics thus has a compelling account of Partee pairs. It also has well-known problems involving negation and disjunction, which I will explain in this section.

The problems, abstractly, stem from the fact that the logic of dynamic semantics is non-classical. In classical logic, $\neg\neg p$ and p are equivalent. Likewise, $\neg p \vee q$ is equivalent to $\neg p \vee (p \& q)$. But these equivalences fail in dynamic semantics, and those failures do not match intuition.

To work up to the problem, start by considering how to extend the dynamic system to negation. A natural first thought is that $c[\neg p] = c \setminus c[p]$ where defined. But this doesn’t work. Consider a negated sentence like (11):

(11) It’s not the case that Sue has a_x child(x).

Negated indefinites have strong truth-conditions: (11) intuitively communicates that Sue is childless. Thus what we want, when we update c with (11), is to keep just those pairs $\langle g, w \rangle$ in c such that Sue has no children in w. But the current proposal gives us something much weaker: in fact, since ‘Sue has a_x child(x)’ is (wherever defined) a pointwise extension of pairs in c, no pair in c will survive unchanged after update with [Sue has a_x child(x)], meaning that c itself would remain unchanged after update with (11).

A more general way of putting the point is this. If indefinites are existential quantifiers, as in the classical approach, the universal meaning of negated indefinites follows from their interaction with Boolean negation. But in dynamic semantics, indefinites are not themselves quantificational; hence, if we are to capture the strong meaning of negated indefinites, we need negation itself to quantify over assignments.

Hence the standard approach to negation in dynamic semantics says $c[\neg p]$ is the set of pairs from c which *can't be extended* in any way to be a part of $c[p]$. That is, provided $c[p]$ is defined, $c[\neg p]$ is $\{\langle g,w\rangle \in c : \neg\exists g' \geq g : \langle g',w\rangle \in c[p]\}$ (where $g' \geq g$ iff g' and g agree everywhere that g is defined). Given this treatment of negation, when we update c with (11), we keep just those pairs $\langle g,w\rangle$ in c such that no extension of g assigns a child of Sue's in w to x. Provided that x is novel (which is required for the whole update to be defined), that means that we keep exactly those pairs $\langle g,w\rangle$ from c such that Sue has no children in w, as desired.

This approach captures the intuitive truth-conditions of (11). But it has a problematic upshot: double negation elimination is not valid in this system.[7] Because negation quantifies over assignments, 'Not (Not (Sue has a_x child(x)))' doesn't have any effect on assignments in the resulting context. A quick way to see this is that negation is eliminative—$c[\neg p]$ is always a subset of c, where defined—and so, if x is novel in c, it will also be novel in $c[\neg p]$ and in $c[\neg\neg p]$.

In particular, updating c with 'Not (Not (Sue has a_x child(x)))' will yield a context containing exactly the pairs $\langle g,w\rangle$ from c where Sue has a child in w. This update puts no constraints on variable assignments, and hence does not set up subsequent anaphora dependencies—unlike updating with 'Sue has a_x child(x)'.

In short, dynamic semantics predicts that doubly-negated indefinites will not license anaphora with subsequent definites. But, as Karttunen (1976) already observed, doubly-negated indefinites do license subsequent definites, contrary to the present prediction. While stacked negations are not terribly natural, there is still a striking contrast in pairs like (12):

(12) a. It's not the case that Sue doesn't have <u>a child</u>. <u>She's</u> at boarding school.
     b. It's not the case that Sue isn't <u>a parent</u>. <u>She's</u> at boarding school.

7 There are different ways to define validity in dynamic semantics, but double negation elimination is not valid on any of them. A common approach says that the inference from p to q is valid just in case for any context c, $c[p] = (c[p])[q]$ whenever both sides are defined. But $c[\neg\neg p]$ will not always equal $(c[\neg\neg p])[p]$. For instance, when p is 'Sue has a_x child(x)' and x is novel in c, $c[\neg\neg p]$ will be the set of points $\langle g,w\rangle \in c$ where Sue has a child in w, while $(c[\neg\neg p])[p]$ will be the set of points $\langle g,w\rangle$ such that $g(x)$ is a child of Sue's in w and for some $\langle g',w'\rangle \in c : w = w'$ and g agrees with g' except on x.

The doubly-negated indefinite in (12-a), like the non-negated indefinite 'Sue has a child', seems to license subsequent anaphora to Sue's child, while the doubly-negated indefinite in (12-b) only seems to naturally license subsequent anaphora to Sue. This is brought out more naturally in exchanges like those in (13):

(13) a. - Sue doesn't have a child. - That's not true! She's at boarding school.
b. - Sue isn't a parent. - That's not true! She's at boarding school.

So the prediction of dynamic semantics that doubly-negated indefinites do not license anaphora does not match observations.

Furthermore, this problem with negation infects other—somewhat more natural—environments, in particular disjunction. As Evans (1977) and Roberts (1987: crediting Partee) observed, negated indefinites in left disjuncts license definites in right disjuncts. Compare:

(14) a. Either Sue doesn't have a child, or she's at boarding school.
b. Either Sue isn't a parent, or she's at boarding school.

Only in (14-a) can 'she' be naturally interpreted as referring to Sue's child. But this is not captured by dynamic semantics. The standard dynamic disjunction says $c[p \vee q]$ is (where defined) the set of pairs in c which have extensions in $c[p] \cup c[\neg p][q]$ (Groenendijk et al. 1996). Thus $c[\neg p \vee q]$ is the set of pairs in c which have extensions in $c[\neg p] \cup c[\neg\neg p][q]$. What we want is for this to come out equivalent to $c[\neg p] \cup c[p][q]$. Then indefinites in p would be accessible to definites in q. But, because double negation elimination is not valid, this equivalence does not hold, and so we won't be able to predict that the pronoun in the right disjunct of (14-a) is licensed by the negated indefinite in the left disjunct.

Schematically, in other words, to account for the contrast in (14), we want $\neg p \vee q$ to be equivalent to $\neg p \vee (p \& q)$. Then (14-a) would be equivalent to 'Either Sue doesn't have a child, or she has a child and she is at boarding school'. But, while this equivalence holds in classical logic, it doesn't hold in dynamic systems. In particular, the negated indefinite in the left disjunct of (14-a) is not predicted to license the definite in the right disjunct. Generally speaking, then, dynamic semantics will not be able to explain contrasts like those in (14).

5 The bounded theory

These problems with negation and disjunction are worrying enough to motivate taking a skeptical look at the foundations of dynamic semantics. If we want to follow dynamic semantics in holding that indefinites have open scope to their right—as I think we should—then we need *something* non-classical in our system. But the present problems suggest that dynamic semantics overdoes the non-classicality.

In the rest of the paper I will develop a new theory, the *bounded theory*, which, like dynamic semantics, predicts the open scope of indefinites—and thus accounts for the key contrasts motivating dynamic semantics—but which is logically more conservative than dynamic semantics.[8] I start with the classical treatment of indefinites as existential quantifiers. Then I propose that indefinites with the form $\ulcorner \text{Some}_x\ F \text{ is } G \urcorner$ have a *witness bound*, operating in a separate dimension from truth-conditions. The witness bound requires, at a point $\langle g, w \rangle$, that, if *anything* is F and G in w, then $g(x)$ is. In other words, a witness must be assigned to the indefinite's variable if the indefinite is true. The witness bound ensures that indefinites license subsequent definites, which require that their scope be familiar, in roughly the sense of dynamic semantics. Anaphora is thus coordinated in the dimension of bounds. At the level of truth-conditions, however, everything is classical, which means that this approach avoids the problems just sketched for dynamic semantics.

Before I dive in, let me briefly situate it in relation to the existing literature. My system builds on a variety of proposals in the literature. Krahmer and Muskens (1995), van den Berg (1996), Rothschild (2017), Chatain (2017), Heim (2018), Elliott (2020)

8 My system crosscuts the standard boundaries between dynamic vs. static systems. At the level of truth-conditions, everything is purely static. But, as we will see, the projection of bounds is calculated by way of recursively specified local contexts, which, while not formulated in the traditional functional or relational architecture of dynamic semantics, is still a tool characteristic of dynamic semantics. There are various precise criteria of dynamicness (see van Benthem 1996, Rothschild and Yalcin 2015, 2016), as well as various broader heuristics for what counts as dynamic (cf. Chierchia 1995). An interesting question, which I leave for further work, is where my system falls vis-à-vis those criteria, and more generally whether we should think of it as a dynamic theory or not.

all propose solutions to the double negation problem which likewise exploit partiality or multidimensionality (see also related discussion in Onea 2013). Chatain (2017) contains, to my knowledge, the origin of the idea of disaggregating the dimension of truth-conditional content from the dimension of anaphoric tracking, and my approach builds most directly on his, along with Rothschild 2017 and Krahmer and Muskens 1995. The idea that witnesses play a central role in the semantics of indefinites also has an important precedent in the system of Dekker 1994 and following,[9] as well as in the class of approaches which use Hilbert's epsilon calculus to analyze indefinites (e.g. Egli and von Heusinger 1995;[10] compare also the arbitrary object approach of Fine 1986, 2020). Gotham (2019) develops an approach to the double negation problem which, while superficially very different from mine, has an abstract similarity in that it aims to close the gap between indefinite extensions and anti-extensions via a presupposition. Finally, Schlenker (2011) also develops a system which exploits statically-conceived local contexts to deal with anaphora. My approach is deeply indebted to all these, but also differs from all of them. I will not go through those differences in detail, nor will I argue that my approach is superior to these. My goal, rather, will be to concisely lay out and explore a new system which I think deserves serious consideration.

9 See Dekker 2000, 2002, 2004a,b, 2008, 2012 for developments; see Jäger 2005 for a type-theoretic implementation. The most salient difference between Dekker's system and mine is that Dekker's system is developed with a conjunction which is capable of extending the input sequence, and (thus) with a negation which quantifies over possible extensions of the input sequence; in the resulting system, double negation elimination is logically invalid, as in dynamic systems. Dekker (2001, 2015), however, develops the system so that anaphora resolution is sensitive to information structure, in a manner that could be used to deal with double negation and disjunction.

10 See Avigad and Zach 2020 for a general overview of the epsilon calculus. Superficially, in our system, indefinites and definites are sentence operators, not terms as in the epsilon calculus. But deep differences remain even if we translate our approach into something syntactically more like the epsilon calculus. For instance, we don't validate the characteristic epsilon axiom $\exists x Px \equiv P(\varepsilon x Px)$. Of course, $\exists x Px \equiv P(\text{ɜ} x Px)$ is not well-formed for us anyway, but we could naturally extend our system by making $\text{ɜ} x Px$ a term by letting $\text{ɜ} x Px = g(x)$ iff $g(x) \in \mathfrak{I}(P,w)$, $\perp$ (an object of which everything is false) if $\mathfrak{I}(P,w) = \varnothing$, and # otherwise. By contrast, in the epsilon calculus, $\varepsilon x Px$ is an *assignment-insensitive* thing in $\mathfrak{I}(P,w)$, if $\mathfrak{I}(P,w) \neq \varnothing$, and otherwise some arbitrary individual. So, for instance, $P(\varepsilon x Px)$ will be true whenever $\mathfrak{I}(P,w) \neq \varnothing$, while $P(\text{ɜ} x Px)$, on the proposed interpretation, can be undefined, invalidating the characteristic axiom of the epsilon calculus.

5.1 Truth and falsity

On the bounded theory, again, meanings have two dimensions: a dimension of truth-conditions and a dimension of bounds. But bounds never affect truth-conditions, and so we can start, in this subsection, by introducing truth-conditions, ignoring bounds for the moment.

I work with a standard first-order language with a two-place definite operator ιx and two-place indefinite operator εx, for any variable x (I reserve $\exists$ for the classical existential quantifier). $\iota x(p,q)$ and $\varepsilon x(p,q)$ are well-formed just in case p and q are formulae free in x. $\varepsilon x(p,q)$ stands for ⌜Some p is q⌝. $\iota x(p,q)$ stands for ⌜The p is q⌝. Pronouns are treated as definites with tautological restrictors, so e.g. ⌜He is p⌝ is rendered $\iota x(\top x, p)$. The sentences are, as usual, the formulae whose variables are all bound (in the classical syntactic sense).

The truth-conditions of our language are essentially those of classical logic: in particular, indefinites have the truth-conditions of existential quantifiers. Definites have the truth-conditions of the conjunction of their restrictor and scope. That is, for any (possibly partial) assignment g, world w, and atomic valuation $\mathfrak{I}$, where $g_{[x\to a]}$ is the assignment which takes x to a and otherwise is just like g, we have:

- *Atoms*: $[\![A(x_1,x_2,\ldots x_n)]\!]^{g,w} = 1$ iff $\langle g(x_1), g(x_2), \ldots g(x_n)\rangle \in \mathfrak{I}(A,w)$
- *Conjunction*: $[\![p\&q]\!]^{g,w} = 1$ iff $[\![p]\!]^{g,w} = [\![q]\!]^{g,w} = 1$
- *Disjunction*: $[\![p \vee q]\!]^{g,w} = 1$ iff $[\![p]\!]^{g,w} = 1$ or $[\![q]\!]^{g,w} = 1$
- *Negation*: $[\![\neg p]\!]^{g,w} = 1$ iff $[\![p]\!]^{g,w} = 0$
- *Indefinites*: $[\![\varepsilon x(p,q)]\!]^{g,w} = 1$ iff $\exists a : [\![p\&q]\!]^{g_{[x\to a]},w} = 1$
- *Definites*: $[\![\iota x(p,q)]\!]^{g,w} = [\![p\&q]\!]^{g,w}$

I assume bivalence: if a formula is not true at $\langle g,w\rangle$, it is false there.[11]

11 So $A(x_1,x_2,\ldots x_n)$ is false at $\langle g,w\rangle$ if g is undefined at any $x_i : i \leq n$.

5.2 The witness bound

The second dimension of meaning is the dimension of bounds. It is in this dimension that I capture the characteristic interaction of indefinites and definites. Crucially, bounds, again, never affect truth-conditions. This is a crucial fact about the system, which lets us preserve classical logic while still capturing the dynamic interaction of indefinites and definites.

Treating bounds as a separate dimension of content from truth-conditions builds on multi-dimensional approaches developed in the literature on presupposition and anaphora.[12] Bounds are indeed akin to presuppositions, but, as we will see, the pragmatics of bounds are different from the standard pragmatics of presupposition: by contrast to the standard treatment of presupposition, we do not say that bounds must be satisfied throughout the antecedent context. We might still identify them as presuppositions (and revise the standard pragmatics of presupposition, as I have argued in Mandelkern 2016 we need to do anyway), or as something else (for instance, Brasoveanu (2013)'s post-suppositions), or as a sui generis kind of content; while I am hesitant to introduce a new term, I do so because I want to remain neutral for the present on this question. While other implementations of the basic idea are possible, the two-dimensional approach provides a simple and illuminating version which brings out clearly the logical points which are central to my interests here.[13]

Starting with indefinites: I propose that indefinites have a *witness bound* which says that if their scope is true relative to *any* assignment, then their scope is true (and has its bounds satisfied) relative to the starting assignment. For brevity, I will use 'satt' for 'has its bounds satisfied'; so, where c is a context, g an assignment, and w a world, the witness bound says:[14]

12 See Herzberger 1973, who credits Buridan, and Karttunen and Peters 1979 for classical multi-dimensional approaches to presupposition; see Dekker 2008 for a multi-dimensional approach to anaphora.

13 An alternative would be to develop this system in a one-dimensional but trivalent setting, by treating unsatt sentences as lacking truth-values. I discuss this route in Mandelkern ta. Spector (2021), however, points out a serious problem for that implementation, and develops a one-dimensional system along different lines, using plural assignments. There is much more here to explore, and hopefully future work will bring out the pros and cons of various implementations.

14 Many thanks to Keny Chatain for suggesting this formulation of the witness bound, based on a much more tortuous earlier version.

- *Witness bound*:
  $\text{з}x(p,q)$ is satt at $\langle c,g,w\rangle$ only if, if $\exists a : [\![p\&q]\!]^{g[x\to a],w} = 1$, then $p\&q$ is true and satt at $\langle c,g,w\rangle$.

We can equivalently reformulate the witness bound in terms of the existential quantifier:

- *Witness bound*:
  $\text{з}x(p,q)$ is satt at $\langle c,g,w\rangle$ only if, if $[\![\exists x(p\&q)]\!]^{g,w} = 1$, then $p\&q$ is true and satt at $\langle c,g,w\rangle$.

So, for instance, an indefinite with the form $\text{з}x(cat(x), sleeping(x))$ is *true* at $\langle g,w\rangle$ just in case there is some sleeping cat in w; and it is satt at $\langle c,g,w\rangle$ only if, if there is some sleeping cat in w, then $g(x)$ is a sleeping cat in w. So the witness bound coordinates the variable assignment with a witness for the indefinite whenever there is one. This is our way of ensuring that an indefinite indexed to x "opens a file" on x.

5.3 The familiarity bound

Definites have a complementary bound requiring that their scope is *familiar*, just as in Heim's system. In other words, in Heim's metaphor, while indefinites *open* files, definites presuppose that a file has *already* been opened, and that whatever information is in their scope is already contained in that file.

To implement this idea, we exploit a context parameter. Contexts will be modeled like Heimian contexts, as sets of pairs of (possibly partial) variable assignments and worlds. As in the Heimian model, worlds represent the context set, and assignments are used to track discourse referents. In the Heimian model, the notion of a null context was a context which pairs each world with the empty assignment; assignments are then gradually extended as conversation proceeds. By contrast, in our system, the null context is the context which contains every pair of a world and a (possibly partial) assignment; pairs are eliminated, rather than extended, as conversation proceeds.

The role of the context parameter is to provide the domain throughout which we require that the definite's scope is true (and satt). That is, the definite's familiarity bound runs as follows:

- *Familiarity*:
  $\iota x(p,q)$ is satt at $\langle c,g,w\rangle$ only if $\forall \langle g',w'\rangle \in c : p$ is true and satt at $\langle c,g',w'\rangle$.

So a definite description like 'The$_x$ sleeping cat(x) is tabby(x)' is only satt in a context c if, at every point $\langle g,w\rangle \in c$, $g(x)$ is a sleeping cat in w. Pronouns, again, can be treated as definites with tautological restrictors, so a sentence like 'She$_x$ is tabby', parsed $\iota x(\top x, tabby(x))$, is satt in c only if, at every point $\langle g,w\rangle \in c$, $g(x)$ is defined. In short: pronouns require that their variable be familiar, in the sense of being defined throughout their context; while definite descriptions require that the variable be familiar in that sense, and also that their restrictor be true and satt at every point in the context.[15]

5.4 Updating

Although there is more to do in laying out the bounded theory, we have enough in place for me to pause to explain how things fit together over the course of updates.

First, let me explain the update rule I will assume. In Heimian dynamic semantics, the basic update rule is just functional application: c updated with p is $c[p]$. In my system, updating is instead much closer to the standard eliminative update rule of Stalnaker 1978. I assume that updating a context c with a sentence p results in a subsequent context which comprises exactly the points from c where p is true and has its bounds satisfied relative to c: that is, the points $\{\langle g,w\rangle \in c : p$ is true and satt at $\langle c,g,w\rangle\}$.[16] Again, this rule treats bounds differently from the way that Stalnaker (1974) treats presuppositions. On that account, to update c with p, c must entail p's presuppositions; if p's presuppositions fail at any point in c, then the update is undefined. If we generalized this Stalnakerian assumption to bounds, then updating with indefinites would lead to constant crashes. Consider a null context, and suppose we update it with $\mathfrak{z}x(Fx,Gx)$. The result I want is the set of points $\langle g,w\rangle$ such that $g(x)$ is in $F_w \cap G_w$ (for any one-place predicate A, I use A_w for the extension of A at w). The Stalnakerian

15 Plausibly, there is also a uniqueness-presupposing definite. Schwarz (2009) has influentially argued that these are two different lexical items. A uniqueness-presupposing definite can of course straightforwardly be incorporated into our system. I will continue to focus on the familiarity-definite throughout, since it is that definite whose interaction with indefinites is of particular interest.

16 Of course, the context can be updated by other means. I assume, following Stalnaker, that the context is in some sense derivative on joint attitudes of some kind, so that e.g. commonly believing p suffices for the context to entail p.

assumption would lead to a crash, however, since the null context contains every point, including ones where $F_w \cap G_w$ is non-empty but $g(x) \notin F_w \cap G_w$. This is why I assume that when we update c with p, we simply keep all the points from c where p is true and satt, relative to c.

This raises the important question of how to integrate theories of presupposition into my framework. I will not take up this question here, partly because I don't know the answer, and partly because there is no need to take a stand on this as far as my goals here are concerned. It is of course consistent with the present approach that presuppositions are treated exactly as usual.[17]

With this in hand, let's turn to an example. Suppose we update a null context c with the indefinite $\mathrm{ɜ}x(\mathit{cat}(x), \mathit{exists}(x))$. Consider an arbitrary point $\langle g, w\rangle \in c$. Recall that ɜ has the truth-conditions of the existential quantifier, so $\mathrm{ɜ}x(\mathit{cat}(x), \mathit{exists}(x))$ is *true* at $\langle g, w\rangle$ iff there is a cat in w. Suppose first that w has no cats; then our sentence is false at $\langle g, w\rangle$ and so we eliminate this point. Suppose next that there is a cat in w. Then the sentence is true at $\langle g, w\rangle$. But this isn't enough for this point to survive; we must also check whether the sentence's witness bound is satisfied. That bound says that, if w has a cat, then $g(x)$ is a cat in w. Since w does have a cat, the witness bound is thus satisfied iff $g(x)$ is a cat in w. So, $\langle g, w\rangle$ survives update with $\mathrm{ɜ}x(\mathit{cat}(x), \mathit{exists}(x))$ iff $g(x)$ is a cat in w.

This, in turn, is the key to the subsequent licensing of anaphora. Let c' be the context that results from updating c with $\mathrm{ɜ}x(\mathit{cat}(x), \mathit{exists}(x))$. Note that, in c', not only does every world contain cats, but also every variable assignment assigns x to a cat in its paired world. Suppose that someone now says 'The cat is tabby', $\iota x(\mathit{cat}(x), \mathit{tabby}(x))$. Consider an arbitrary point $\langle g, w\rangle$ in c'. $\iota x(\mathit{cat}(x), \mathit{tabby}(x))$ has its familiarity bound satisfied at $\langle c', g, w\rangle$ iff, for every point $\langle g', w'\rangle \in c'$, $g'(x)$ is a cat in w'. But this is guaranteed to hold because of our prior update with the corresponding indefinite. $\iota x(\mathit{cat}(x), \mathit{tabby}(x))$ is true at $\langle g, w\rangle$ iff the corresponding open formula $\mathit{cat}(x) \& \mathit{tabby}(x)$ is true, iff $g(x)$ is a tabby cat in w. So, putting these two

17 A reviewer for this journal suggests an interesting idea: perhaps the familiarity requirement is a presupposition, not a bound, and presuppositions can be identified as just those bounds that must be satisfied throughout a context, rather than at a given point in a context.

updates together, a point $\langle g,w\rangle$ in c survives update with $\exists x(cat(x), exists(x))$ and then $\iota x(cat(x), tabby(x))$ just in case $g(x)$ is a tabby cat in w.

Things work in essentially the same way for pronouns. In fact, updating c with 'There is a$_x$ cat. It$_x$ is tabby' has exactly the same effect as updating with 'There is a$_x$ cat. The$_x$ cat is tabby'. The familiarity bound of $\iota x(\top x, tabby(x))$ requires that, for all points $\langle g,w\rangle$ in c', $g(x)$ is defined. But this will hold thanks to the preceding update with the indefinite. The truth-conditions of 'It$_x$ is tabby' require simply that $g(x)$ is tabby. So updating c with either sequence takes us to a context which comprises exactly those points $\langle g,w\rangle$ from c where $g(x)$ is a tabby cat in w.

Note that, since the familiarity bound quantifies universally over points in the context, it will hold at either all or none of them. This is what accounts for the infelicity of asserting a definite without a corresponding indefinite.[18] This is very different from the witness bounds of indefinites, which, crucially, can be satisfied at some points in a context and not at others.

There is a deep similarity between this system and Heim's file card system. In our system, asserting $\exists x(p,q)$ 'opens a file' on x by making x defined at every point in the updated context, and adds to this file the information that x is p and q. Asserting a definite $\iota x(p,r)$ or $\iota x(\top x, r)$ is then licensed: its familiarity bound is satisfied throughout the context. The definite adds to the file the further information that x is r. So, abstractly, the system has much in common with Heim's; but the mechanics are very different.

5.5 Projection

Let's turn from cross-sentential anaphora to intra-sentential anaphora, where I will argue that, similarities aside, my system is a clear improvement over dynamic semantics. First, we need to say how bounds project out of complex sentences. To do this, we extend the local context approach to the corresponding question about presuppositions, developed in Karttunen 1973, 1974, Stalnaker 1974, Schlenker 2009, 2010. In particular, we say that a negation is satt at $\langle c,g,w\rangle$ just in case the negatum is satt at $\langle c,g,w\rangle$. A conjunction $p\wedge q$ is satt at $\langle c,g,w\rangle$ just in case the left conjunct is satt at $\langle c,g,w\rangle$ and the right conjunct is satt at $\langle c^p,g,w\rangle$, where c^p is the set of points $\langle g',w'\rangle \in c$ such that

18 Of course, definites can also be accommodated, as Heim 1982 and others discuss.

p is true and satt at $\langle c, g', w' \rangle$. A disjunction $p \vee q$ is satt at $\langle c, g, w \rangle$ just in case the left disjunct is satt at $\langle c, g, w \rangle$ and the right disjunct is satt at $\langle c^{\neg p}, g, w \rangle$. As a base case, we say that an atomic sentence $A(x_1, x_2, \ldots x_n)$ is satt at $\langle c, g, w \rangle$ iff g is defined on all $x_i : i \in [1, n]$.

This is all straightforward. There are two trickier questions about projection that I want to flag here. The first concerns order. Traditional dynamic systems are asymmetric: the local context for a left junct is the global context, while the local context for the right junct is the global context updated in a suitable way with the left junct. This is motivated by contrasts like those in (15):

(15) a. A man came in and he sat down.
b. He sat down and a man came in.

There is a striking contrast here: it is very difficult to interpret the 'he' as anaphoric on 'a man' in (15-b).

So order seems to matter for anaphora. But the data are very subtle. While there is an indisputable contrast in (15), in the case of bathroom disjunctions, both orders are clearly interpretable anaphorically:

(16) a. Either there isn't a bathroom, or $\left\{ \begin{smallmatrix} \text{it} \\ \text{the bathroom} \end{smallmatrix} \right\}$ is upstairs
b. Either $\left\{ \begin{smallmatrix} \text{it} \\ \text{the bathroom} \end{smallmatrix} \right\}$ is upstairs, or there isn't a bathroom.

To see that the indefinite is doing work here, and the anaphoric interpretation of (16-b) is not simply a matter of accommodation, we can look at our running minimal pair:

(17) a. Either $\left\{ \begin{smallmatrix} \text{she} \\ \text{the child} \end{smallmatrix} \right\}$ is at boarding school, or Susie doesn't have a child.
b. Either $\left\{ \begin{smallmatrix} \text{she} \\ \text{the child} \end{smallmatrix} \right\}$ is at boarding school, or Susie isn't a parent.

The negated indefinite in the right disjunct clearly facilitates an anaphoric reading in (17-a) which is much less available in (17-b).

Moreover, as many have noted, left-to-right anaphoric dependencies are possible across conjunction, even if somewhat less accessible, as in (18):

(18) a. It took him a long time, but a student of mine wrote a really good paper.

b. He complained all the way to the fair, and then one of my kids just disappeared.

So it's unclear what to make of order asymmetries.

In fact, the question of order is modular in my system, in the sense that (unlike in dynamic semantics) it is straightforward to implement both an asymmetric version and a symmetric version, depending how we define local contexts. So, instead of saying that $p\&q$ is satt at $\langle c, g, w\rangle$ iff p is satt at $\langle c, g, w\rangle$ and q is satt at $\langle c^p, g, w\rangle$, as we have it above, we could say, symmetrically, that $p\&q$ is satt at $\langle c, g, w\rangle$ iff p is satt at $\langle c^q, g, w\rangle$ and q is satt at $\langle c^p, g, w\rangle$; likewise for disjunction. We could also, following Schlenker, posit that both symmetric and asymmetric systems are present, with a preference for the asymmetric one. I do not know what to make of the question of order. For reasons having to do with modality, I am sympathetic to a symmetric approach to local contexts in general (see Mandelkern 2019). Since most existing systems in the literature are asymmetric, however, I will spell out an asymmetric version of my system for ease of comparison, emphasizing again that a symmetric implementation of the basic ideas can just as easily be developed.

The second question concerns how bounds should project out of (in)definites. Consider first indefinites. Here is my proposal about how indefinites project the bounds of their complements:

- *Indefinites*: $\text{з}x(p, q)$ is satt at $\langle c, g, w\rangle$ only if $\exists g' : p\&q$ is satt at $\langle c, g', w\rangle$.

We don't want to require that $p\&q$ itself is satt at $\langle c, g, w\rangle$, since then updating with, say, $\neg\text{з}x(Fx, Gx)$ would make x familiar, since $Fx\&Gx$ is satt at g only if $g(x)$ is defined. But negated indefinites do not generally license subsequent anaphora (*modulo* modal subordination, which I address in the conclusion):

(19) We don't have a cat. # She is a tabby.

On the other hand, we don't want з to have trivial projection conditions, since that would mean that negated indefinites could contain novel definites in their scope. A middle ground is to require that the indefinite's restrictor and scope are satt relative to *some* assignment, holding fixed the local context. That means that a sentence like $\neg\text{з}x(Fx, Gx)$

will be satt even if $g(x) = \#$; but a negated indefinite taking scope over a novel definite will not be satt, since varying the starting assignment will not change whether a definite is familiar or not (which depends only on the context). In short, embedded witness bounds will fail to project through negated indefinites, while embedded familiarity bounds will project.

With this in hand, consider the update effect of a negated indefinite, like 'There is not a cat', $\neg \exists x(cat(x), exists(x))$. Consider any point $\langle g, w \rangle$. $\neg \exists x(cat(x), exists(x))$ is true at $\langle g, w \rangle$ iff its negatum is false; since the negatum has the truth-conditions of the existential quantifier, that holds iff there is no cat in w. Suppose that there is no cat at w. $\neg \exists x(cat(x), exists(x))$ is satt iff (i) its scope is satt at some assignment, which it is (namely, any assignment defined on x); and (ii) its witness bound (which projects through negation) is satt, which holds just in case, if there is a cat in w, then $g(x)$ is a cat in w. But, by assumption, there's no cat in w; so the witness bound is (trivially) satisfied. So $\langle g, w \rangle$ survives update with $\neg \exists x(cat(x), exists(x))$ just in case w has no cats. In general, whenever an indefinite sentence is false, its witness bound is trivially satisfied, since the witness bound is a material conditional whose antecedent asserts the truth of the indefinite. So *negated indefinites* are always satt (*modulo* any familiarity bounds in their scope), meaning that negated indefinites have exactly the same update effect as the corresponding negated existential quantifiers. We thus capture the strong, universal meaning of negated indefinites, as well as their inability to license subsequent definites.

We come, finally, to the question of how definites pass on the bounds of their scope (the bounds of the restrictor are already required, by the familiarity bound, to be satisfied throughout the context). I propose that they do so in a conditionalized fashion:

- *Definites*: $\iota x(p, q)$ is satt at $\langle c, g, w \rangle$ only if, if p is true at $\langle c, g, w \rangle$, then q is satt at $\langle c^p, g, w \rangle$.

We don't want $\iota x(p, q)$ to require that q be satt unconditionally. Consider a bathroom sentence with the form, say, $\neg \exists x(Fx, Gx) \vee \iota x(\top x, Hx)$ (more on these in a moment). We don't want a sentence like this to make x familiar, since updating with 'Either there isn't a bathroom, or it is upstairs' doesn't license subsequent anaphora to the bathroom (#'It has horrible wall paper'). On the other hand, we can't simply ignore the bounds of q, since witness bounds of indefinites in q should be satisfied once we update with $\iota x(p,q)$ ('The bathroom has a sink' licenses subsequent anaphora to the sink). Hence I propose that q must be satt whenever p is true—intuitively, q has to be satt provided we're in a case where we are keeping a point because of how it values p.[19]

5.6 Open scope of indefinites

With this in hand, we can look at some of the characteristic features of intra-sentential anaphora in my system.

Recall that one way to formulate the generalization about indefinites which motivates dynamic semantics is to say that indefinites have open scope to their right: ⌜Some F is (G and H)⌝ is in some sense equivalent to ⌜Some F is G, and $\left\{\begin{smallmatrix}\text{it}\\ \text{the } F\end{smallmatrix}\right\}$ is H⌝. Thus, for instance, 'Sue has a child, and she is at boarding school' is intuitively equivalent to 'Sue has a child at boarding school'; while 'Sue is a parent, and she is at boarding school' is intuitively equivalent to 'Sue is a parent at boarding school'.

In our language, the claim that indefinites have open scope to their right can be formulated as the claim that the three variants in (20) are equivalent:[20]

(20) a. $\exists x(Fx, Gx \& Hx)$ *Some F is G and H*
b. $\exists x(Fx, Gx) \& \iota x(Fx, Hx)$ *Some F is G, and the F is H*
c. $\exists x(Fx, Gx) \& \iota x(\top x, Hx)$ *Some F is G, and it is H*

(20-a)–(20-c) are not *logically* equivalent in my system.[21] For instance, consider a point $\langle g, w\rangle$ where $F_w \cap G_w \cap H_w$ is non-empty, and $g(x) \notin H_w$. Then (20-a) will be true at $\langle g, w\rangle$, since (20-a) is truth-conditionally equivalent to the corresponding true existentially quantified sentence. By contrast, (20-b) and (20-c) will both be false at

19 On this approach, a disjunct can contain locally novel definites without that making the sentence unsatt, as in $\neg\exists x(Fx, Gx) \vee \iota x(\top x, \iota y(\top y, R(x,y)))$. This will be satt at $\langle c, g, w\rangle$ when $g(x) = \#$ and nothing is F and G in w, even if y is novel in c, since the restrictor of ιx (that is, $\top x$) is not true here. Still, if y is novel, then this sentence will be equivalent to $\neg\exists x(Fx, Gx)$ whenever satt, which will suffice to account for its infelicity via standard theories of redundancy.

20 I focus on conjunction here, but the same points apply to sequences, and to any substitution instances of Fx, Gx, and Hx by formulae free in x.

21 p and q are logically equivalent whenever, for any index i in any intended model, p is true at i iff q is true at i.

$\langle g, w \rangle$, since Hx is false there; and $\iota x(Fx, Hx)$ and $\iota x(\top x, Hx)$ are both true only if Hx is true.

But you will have noticed that, while (20-a) is is true at $\langle g, w \rangle$, its witness bound is not satisfied—there is something in $F_w \cap G_w \cap H_w$, but $g(x)$ is not in $F_w \cap G_w \cap H_w$. This points the way towards the sense in which (20-a)–(20-c) are equivalent: they have the same truth-value *provided their bounds are satisfied*. To characterize this kind of equivalence, we can generalize von Fintel (1999)'s notion of Strawson entailment (which he applies to presupposition) to bounds as follows:[22] *p bound-entails q* iff, for any index (context/assignment/world-triple) i in any intended model, if p and q are both satt at i and p is true at i, then q is true at i (I'll write $p \models_L q$ for logical entailment, and $p \models_B q$ for bound-entailment). p and q are bound-equivalent iff each bound-entails the other. (20-a)–(20-c) are not pairwise logically equivalent, but they are pairwise bound-equivalent.

The reasoning behind this is simple. For any point $i = \langle c, g, w \rangle$, suppose (20-a) is satt and is true at i. As we have seen, this holds just in case $g(i) \in F_w \cap G_w \cap H_w$. Whenever (20-b) or (20-c) are satt, they are also true just in case $g(i) \in F_w \cap G_w \cap H_w$. Focusing on (20-b): assuming its witness bound is satt, the left conjunct of (20-b) is true at $\langle g, w \rangle$ iff $g(x) \in F_w \cap G_w$. The right conjunct is true iff $Fx \& Hx$ is true, that is, iff $g(x)$ is also in $F_w \cap H_w$; together, this is equivalent to the requirement that $g(x) \in F_w \cap G_w \cap H_w$. The reasoning for (20-c) is similar. All three sentences are both satt and true iff $g(x) \in F_w \cap G_w \cap H_w$.[23]

Since it is the combination of bounds plus truth-conditions that matters for pragmatics, this account of the open scope of indefinites carries enough weight to account for the intuition that indefinites have open scope.

5.7 Classicality

It is crucial, however, that we do *not* predict the pairs in (20) to be *logically* equivalent. By making these bound-equivalent but not logically equivalent, we can validate the

22 Thanks to a reviewer for suggesting this helpful terminology.

23 In addition to being bound-equivalent, there is another, stronger sense in which (20-a)–(20-c) are equivalent: if any one of (20-a)–(20-c) is satt and true, then all three are satt and true.

open scope of indefinites (in a pragmatically relevant sense) while still retaining a classical logic.

Recall the two closely related problems for dynamic semantics discussed in §4: in dynamic semantics, $\neg\neg p$ and p are not always equivalent; nor are $\neg p \vee q$ and $\neg p \vee (p \& q)$. By contrast, since our connectives are classical, the logic of our system is the logic of classical predicate logic (under the obvious translation schema between the two slightly different languages). And so it follows from this more general fact that the two classical rules $\neg\neg p \mathrel{{}_{L}{\dashv}{\vDash}_{L}} p$ and $\neg p \vee q \mathrel{{}_{L}{\dashv}{\vDash}_{L}} \neg p \vee (p \& q)$ are valid in my system.

Moreover, the bounded logic of any system is always a superset of the system's logic: that is, if $p \vDash_{L} q$, then $p \vDash_{B} q$. This is for the obvious reason that, if $p \vDash_{L} q$, then q is true at any point in any model where p is, and thus a fortiori q is true at any point where p and q are satt and p is true. So we also have $\neg\neg p \mathrel{{}_{B}{\dashv}{\vDash}_{B}} p$, and likewise $\neg p \vee q \mathrel{{}_{B}{\dashv}{\vDash}_{B}} \neg p \vee (p \& q)$.

The basic reasoning behind this is very simple: our connectives are, at the level of truth and falsity, just the classical connectives. And bounds, again, do not affect truth-conditions, which means that they can never disrupt classically valid reasoning.

More concretely, that means that the bounded system avoids the empirical problems that these logical failures lead to in dynamic semantics. First, doubly-negated indefinites will license subsequent definites, as desired. Consider 'It's not the case that Susie doesn't have a child', $\neg\neg($ɜ$x(\mathit{child}(x), \mathit{Susie's}(x)))$. This will be semantically equivalent to ɜ$x(\mathit{child}(x), \mathit{Susie's}(x))$, since our truth-conditions for negation are classical and since bounds project out of negation; and thus this will license subsequent definites like $\iota x(\mathit{child}(x), \mathit{at\text{-}boarding\text{-}school}(x))$.

Similar reasoning applies to disjunctions like 'Either Susie doesn't have a child, or $\left\{\begin{smallmatrix}\text{she}\\ \text{the child}\end{smallmatrix}\right\}$ is at boarding school' , $\neg$ɜ$x(\mathit{child}(x), \mathit{Susie's}(x)) \vee \iota x(\mathit{child}(x), \mathit{at\text{-}boarding\text{-}school}(x))$. The local context for the right disjunct will be the set of points from the global context where the negation of the left disjunct, $\neg\neg$ɜ$x(\mathit{child}(x), \mathit{Susie's}(x))$, is true and satt. But, thanks to the validity of double negation elimination, that holds at a point iff ɜ$x(\mathit{child}(x), \mathit{Susie's}(x))$ is true and satt there. Thus the local context for the right disjunct will only contain pairs $\langle g, w\rangle$ where $g(x)$ is a child of Susie's in w. That means that the familiarity bound of the definite description (or a corresponding pronoun) will be satisfied. The whole sentence will thus be true and satt at $\langle c, g, w\rangle$ iff either Susie is childless in w; or (i) $g(x)$ is Susie's child in w (this follows from the indefinite's witness bound, which projects to the whole sentence by our projection rules above) and (ii) $g(x)$ is at boarding school in w.

The bounded theory thus avoids the problems that negation and disjunction pose for dynamic systems. Importantly, this is not due to a local fix, but rather follows from the much more general fact that the underlying architecture of the bounded system is fully classical. Because indefinites are, at the level of truth-conditions, existential quantifiers, we do not need to make negation a quantifier over assignments in order to get the intuitively universal readings of negated indefinites. A truth-conditionally classical treatment suffices, letting us avoid dynamic semantics' logical problems.

5.8 Quantifiers

This is the basic system. The final element that I will discuss is how to add generalized quantifiers like 'every' and 'most'. Extending the system to account for the basic phenomenon of donkey sentences is fairly straightforward. There are many further subtleties involving quantifiers, but I will keep my discussion of this complicated area brief, since these issues largely go beyond my central concerns here.

Recall the core data we are trying to capture in the interaction between quantification and anaphora: the availability of covariation between indefinites and definites in donkey sentences like (21-a), and the contrasting unavailability of covariation in variants like (21-b) without the indefinite:

(21) a. Everyone who has a child loves $\left\{\begin{array}{c}\text{them}\\\text{the child}\end{array}\right\}$.

b. Every parent loves $\left\{\begin{array}{c}\text{them}\\\text{the child}\end{array}\right\}$.

As usual, quantifiers are indexed to a domain variable. But, instead of corresponding to a set of individuals, the domain variable's value in our system is a non-empty set of pairs of individuals and variable assignments. Then we proceed in the natural way: for instance, 'every' and 'most' get the following truth-conditions:

- $[\![\text{EVERY}x_\delta(p,q)]\!]^{g,w} = 1$ iff $\forall \langle a,g'\rangle \in g(\delta) : [\![p]\!]^{g'_{[x\to a]},w} = 1 \to [\![p\&q]\!]^{g'_{[x\to a]},w} = 1$
- $[\![\text{MOST}x_\delta(p,q)]\!]^{g,w} = 1$ iff for most $\langle a,g'\rangle \in g(\delta)$ s.t. $[\![p]\!]^{g'_{[x\to a]},w} = 1$, $[\![p\&q]\!]^{g'_{[x\to a]},w} = 1$

Quantifiers also have bounds, which require that the value of the domain parameter (i) contain exactly one pair per individual in the domain of individuals (we thus avoid the "proportion problem" which arises for some versions of dynamic semantics); (ii) contain only pairs that make the restrictor and the conjunction of restrictor and scope satt, which ensures that definites in $p \wedge q$ are satt, and indefinites have their witness bounds satisfied relative to the relevant x-variants on assignments in $g(\delta)$; and (iii) contain only assignments which agree with the starting assignment on any variables that are not novel in the starting context (we don't get to vary the values of already familiar variables).

For sentences without (in)definites, the variable assignments in $g(\delta)$ don't do any interesting work. So, e.g., $\text{EVERY}x_\delta(\mathit{farmer}(x), \mathit{tall}(x))$ is true and satt at $\langle g, w \rangle$ just in case every individual in the domain who is a farmer in w is tall in w.

The extra apparatus becomes relevant in cases of donkey sentences like (21-a), repeated here, with the parse in (22-b):

(22) a. Everyone who has <u>a child</u> loves <u>them</u>.

b. $\text{EVERY}x_\delta(\underbrace{\mathrm{з}y(\mathit{child}(y), \mathit{of}(y,x))}_{p}, \underbrace{\iota y(\top y, \mathit{loves}(x,y))}_{q})$

First, note that (22-b) can be satt even when y is novel, because the local context for q will entail $\mathrm{з}y(\mathit{child}(y), \mathit{of}(y,x))$, ensuring that y is defined throughout its local context, and hence that the definite's familiarity bound is satt. (y is in fact guaranteed to be a child throughout its local context, so the variant with 'the child' in place of 'them' works identically.) Now suppose that (22-b) is satt in $\langle c, g, w \rangle$. (22-b) is true in $\langle c, g, w \rangle$ iff for every pair $\langle a, g' \rangle \in g(\delta)$, if p is true at $\left\langle c, g'_{[x \to a]}, w \right\rangle$, then so is $p\&q$. Consider an arbitrary pair $\langle a, g' \rangle \in g(\delta)$. Given that p is satt at $\left\langle c, g'_{[x \to a]}, w \right\rangle$, it is true at $\left\langle c, g'_{[x \to a]}, w \right\rangle$ iff $g'(y)$ is a child of a, false iff a is childless. If false, then $\langle a, g' \rangle$ doesn't count against the truth of (22-b). If true, then $p\&q$ must also be true at $\left\langle c, g'_{[x \to a]}, w \right\rangle$ in order for (22-b) to be. Given that $p\&q$ is satt at $\left\langle c, g'_{[x \to a]}, w \right\rangle$, it is true there iff a loves their child $g'(y)$. So, (22-b) is true and satt at $\langle c, g, w \rangle$ iff, for every a in the domain, if a has a child in w, then a loves the paired child in w. We thus derive a covarying interpretation of donkey sentences.

By contrast, a co-varying reading will not be available for a sentence like (21-b):

(21-b) Every$_x$ parent$_x$ loves them$_y$.

Without an indefinite corresponding to the definite, the only way (21-b) can be satt is if y is not novel. In that case (thanks to our third bound), (21-b) will be interpreted as saying that some particular person is loved by every parent. But a covarying reading will never be available, since if y is novel, (21-b) will not be satt, since the familiarity bound of 'them' will fail.

For connoisseurs: the covarying interpretation we obtain for donkey sentences cross-cuts the two "readings" which are standardly distinguished. The "weak" reading says that everyone who has a child loves some child of theirs; the "strong" one says that everyone who has a child loves every child they have. There is controversy about whether these are really two readings or whether the two interpretations instead arise from something pragmatic.[24] This is a complicated issue, which I won't explore in detail here. But it is interesting to note that our interpretation cross-cuts these two readings. As long as the weak reading is true at a world w, then the sentence will be true at w relative to *some* assignment—namely, one which matches up each parent with a child they love. By contrast, if the strong reading is true at w, then the sentence will be true at w paired with *any assignment* where it is satt, since any way of matching will do. This might help explain the pull of each reading: if the strong reading is true, then we don't have to find an appropriate pairing; while if only the weak reading is true, the donkey sentence can be true, but only with an appropriate pairing schema. More thorough discussion of this tricky issue, and other phenomena involving quantification (like quantificational subordination and plural anaphora) will have to wait.

6 Conclusion

There is a difference between indefinites like 'has a child' and 'is a parent'. This poses a challenge for the classical analysis of indefinites as existential quantifiers. Both dynamic semantics and e-type theories captures this difference by rejecting, in different ways, classical notions of meaning and treatments of connectives.

24 See e.g. Heim 1982, Root 1986, Rooth 1987, Schubert and Pelletier 1989, Chierchia 1992, Kanazawa 1994, Chierchia 1995, Champollion et al. 2019.

The bounded system captures the coordination between indefinites and corresponding definites with very different tools. My theory separates the characteristic contributions of indefinites into two dimensions of contents: their existential import is captured by their quantificational truth-conditions, while their ability to license subsequent anaphora is captured by their witness bound. This system avoids dynamic systems's problems negation and disjunction. More generally, it shows that we can pull apart the insights of dynamic semantics from its revisionary approach to content and connectives. Dynamic semantics departs in deep ways from classical logic, invalidating not just double negation elimination and the equivalence of $\neg p \vee q$ with $\neg p \vee (p \wedge q)$, but also laws like non-contradiction and excluded middle (van Benthem 1996, Mandelkern 2020). By contrast, my system is logically conservative, because all the dynamic action happens at the level of bounds, and never at the level of truth-conditions.

There is obviously much more work to do in exploring the bounded system. We should look, for instance, at extensions of the system to other domains, like modality, attitude reports, quantificational subordination, adverbial quantifiers, conditionals, and plural anaphora. I take up these topics in Mandelkern 2021b. A particularly important extension, given the main focus of this paper, is to modal subordination, which interacts with negation in complex ways,[25] as in (23), due to an anonymous referee:

(23) Sue doesn't have a child. You would know them by now.

Cases like this pose a *prima facie* challenge to the standard generalization that singly negated indefinites do not license subsequent definites (though not, obviously, to the more limited generalization that singly negated indefinites do not license subsequent matrix definites).

While I don't have space for a full discussion, I think a natural extension of our approach can deal with cases like this, by combining our system with ideas recently developed (in a standard dynamic framework) by Hofmann (2019). The idea is to have the value of a variable, relative to an assignment, be a partial individual concept rather than an individual (following Stone 1999, Aloni 2000). Then we can generalize the witness bound across worlds: $\exists x(p,q)$ is satt at $\langle g,w\rangle$ only if, *for any world* w', if

25 See e.g. Karttunen 1976, Roberts 1987, Kibble 1994, Frank 1996, Brasoveanu 2007, 2010, Hofmann 2019.

$\exists x(p\&q)$ is true at $\langle g, w' \rangle$, then $p\&q$ is true at $\langle g, w' \rangle$. That means that the witness bound is not inert in the case of negated indefinites: $\neg\exists x(p,q)$ can license subsequent anaphora inside the scope of an operator that quantifies only over worlds where $\exists x(p\&q)$ is true. That means that singly negated indefinites will still not license subsequent matrix definites, but they will license definites in a sequence like (23), given any reasonable theory of worldly modal subordination (that is, one on which the local context for the indefinite under 'would' entails that Sue has a child). This is no more than a sketch, which, again, I develop in Mandelkern 2021b; but I think this suggests a promising route for dealing with the interaction of anaphora with intensional operators.[26]

Another area for further exploration is variations on the bounded theory. For one thing, we should explore questions of order. As we saw above, the empirical situation is complicated, and local contexts can just as easily be specified in a symmetric or asymmetric fashion. This means that we have more flexibility than standard dynamic systems in accounting for order symmetries, but how we should use that flexibility remains unclear to me. We should also compare the bounded theory to other implementations of similar ideas, as in trivalent systems (on which see again Mandelkern ta, Spector 2021). We should explore how the bounded system fits broadly into theories of anaphora, presupposition, and modality, where it contributes to a developing research program which aims to capture the insights of dynamic semantics with more conservative foundations (e.g. see Schlenker 2008, 2009 on presupposition and Dorr and Hawthorne 2013, Mandelkern 2019, 2021a on modality). In Mandelkern 2021b I try to bring these strands together, arguing that there are common arguments across these domains for locating the dynamic effects of modals, conditionals, and anaphora in a dimension of bounds rather than truth-conditions.

26 A harder case comes from what Kibble (1994), citing Paul Dekker, calls *negative subordination*—cases like (24):

(24) John doesn't have <u>a car</u> so he doesn't wash $\left\{ \begin{array}{c} \underline{\text{it}} \\ \underline{\text{the car}} \end{array} \right\}$.

I am not sure what to make of cases like this, which are not straightforwardly accounted for in our account, even with the cross-world witness bound. One possibility is to appeal to something like Heim (1982, 1983)'s notion of local accommodation, or, relatedly, to analyze this as a kind of meta-linguistic negation. Obviously this needs further exploration; while I don't think my view shines any special light on this issue, I think it is amenable to whatever general solution one prefers. See Lewis (2020) for interesting further problems involving negation.

Let me close with a high-level comment on the structure of bounded theory. In an illuminating discussion, Cumming (2015) identifies what he calls *the dilemma of indefinites*. On the one hand, they seem to have existential import: whether an indefinite sentence is true or false apparently depends just on the truth or falsity of the corresponding existential quantifier. Intuitively 'Sue has a child' is true just in case Sue is a parent, false otherwise, whether or not the speaker has a particular child in mind. On the other hand, indefinites license subsequent anaphora in ways not predicted by a purely existential account, as we have seen. Crudely speaking, the two main approaches to indefinites, dynamic and e-type, generalize to one of these two aspects. The bounded theory suggests a synthesis: both faces of indefinites are present, but in different dimensions of content. At the level of truth-conditions, indefinites are existential quantifiers, accounting for the existential import of indefinites and explaining the validity of classical inference patterns. But at the level of bounds, indefinites do more: they require the presence of a witness to their truth, which allows us to track anaphoric relations through the twists and turns of conversation.

A Appendix: Semantics

For ease of reference, I summarize the semantics given in the text. We specify the truth-conditions for our language, relative to pairs of a world and a variable assignment: where g is a variable assignment, w a world, and sentence p, $[\![p]\!]^{g,w}$ is the bivalent truth-value of p at $\langle g,w\rangle$. We simultaneously specify the bounds of sentences relative to a context (set of assignment-world pairs), an assignment, and a world. Bounds never influence truth-conditions, so contexts play no role in determining the latter. For any context c and sentence p, c^p is the set of world-assignment pairs in c where p is true and satt relative to c. $\mathfrak{I}$ is an atomic valuation taking n-ary atoms and worlds to sets of n-tuples.

- *Atoms*:
  $[\![A(x_1,x_2,\dots x_n)]\!]^{g,w} = 1$ iff $g(x_1),\dots g(x_n)$ are all defined and $\langle g(x_1),\dots g(x_n)\rangle \in \mathfrak{I}(A,w)$, 0 otherwise
  $A(x_1,x_2,\dots x_n)$ is satt at $\langle c,g,w\rangle$ iff $g(x_1),\dots g(x_n)$ are all defined

- *Conjunction*:
  $[\![p\&q]\!]^{g,w} = 1$ iff $[\![p]\!]^{g,w} = [\![q]\!]^{g,w} = 1$
  $p\&q$ is satt at $\langle c,g,w\rangle$ iff p is satt at $\langle c,g,w\rangle$ and q is satt at $\langle c^p,g,w\rangle$

- *Disjunction*:
  $[\![p \vee q]\!]^{g,w} = 1$ iff $[\![p]\!]^{g,w} = 1$ or $[\![q]\!]^{g,w} = 1$
  $p \vee q$ is satt at $\langle c,g,w \rangle$ iff p is satt at $\langle c,g,w \rangle$ and q is satt at $\langle c^{\neg p},g,w \rangle$

- *Negation:*
  $[\![\neg p]\!]^{g,w} = 1$ iff $[\![p]\!]^{g,w} = 0$
  $\neg p$ is satt at $\langle c,g,w \rangle$ iff p is satt at $\langle c,g,w \rangle$

- *Indefinites*:
  $[\![\text{ɜ}x(p,q)]\!]^{g,w} = 1$ iff $[\![\exists x(p\&q)]\!]^{g,w} = 1$ iff $\exists a : [\![p\&q]\!]^{g[x\to a],w} = 1$
  $\text{ɜ}x(p,q)$ is satt at $\langle c,g,w \rangle$ iff $\exists g' : p\&q$ is satt at $\langle c,g',w \rangle$ and $[\![\exists x(p\&q)]\!]^{g,w} = 1 \to p\&q$ is true and satt at $\langle c,g,w \rangle$

- *Definites*:
  $[\![\iota x(p,q)]\!]^{g,w} = [\![p\&q]\!]^{g,w}$
  $\iota x(p,q)$ is satt at $\langle c,g,w \rangle$ iff $\forall \langle g',w' \rangle \in c : p$ is true and satt at $\langle c,g',w' \rangle$ and, if p is true at $\langle c,g,w \rangle$, then q is satt at $\langle c^{p},g,w \rangle$

- *Quantifiers*:
  $[\![\text{EVERY}x_\delta(p,q)]\!]^{g,w} = 1$ iff $\forall \langle a,g' \rangle \in g(\delta) : [\![p]\!]^{g'_{[x\to a]},w} = 1 \to [\![p\&q]\!]^{g'_{[x\to a]},w} = 1$
  $\text{EVERY}x_\delta(p,q)$ is satt at $\langle c,g,w \rangle$ iff

  $\forall a : \exists! \langle a',g' \rangle \in g(\delta) : a' = a;$

  $\forall \langle a,g' \rangle \in g(\delta) : p$ and $p\&q$ are satt at $\left\langle c, g'_{[x\to a]}, w \right\rangle$; and

  $\forall \langle a,g' \rangle \in g(\delta) : g' \sim_c g$, where $g' \sim_c g$ iff g' agrees with g on the values of all variables which are not novel in c.

  $[\![\text{MOST}x_\delta(p,q)]\!]^{g,w} = 1$ iff $\frac{|\{\langle a,g' \rangle \in g(\delta) : [\![p\&q]\!]^{g'_{[x\to a]},w} = 1\}|}{|\{\langle a,g' \rangle \in g(\delta) : [\![p]\!]^{g'_{[x\to a]},w} = 1\}|} > .5$
  $\text{MOST}x_\delta(p,q)$ is satt at $\langle c,g,w \rangle$ iff

  $\forall a : \exists! \langle a',g' \rangle \in g(\delta) : a' = a;$

  $\forall \langle a,g' \rangle \in g(\delta) : p$ and $p\&q$ are satt at $\left\langle c, g'_{[x\to a]}, w \right\rangle$; and

  $\forall \langle a,g' \rangle \in g(\delta) : g' \sim_c g.$

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