A methodological note on proving agreement between the Elementary Process Theory and modern interaction theories

Marcoen J.T.F. Cabbolet*

Center for Logic and Philosophy of Science, Vrije Universiteit Brussel

Abstract — The Elementary Process Theory (EPT) is a collection of seven elementary physical principles that give an exact yet abstract description of what happens in elementary processes at supersmall scale. One of the two main problems of the EPT is that there is no proof that the four fundamental interactions (gravitational, electromagnetic, strong, and weak) as we know them can take place in the processes described by these elementary principles. This paper sets forth the method by which it can be proven that the EPT agrees with the knowledge that derives from the successful predictions of a modern interaction theory T. This determines a fundamentally new research program in theoretical physics.

Keywords: scientific method, categorical model theory, relativity, interaction

1 Introduction

The Elementary Process Theory (EPT) is a first-order scheme together with a (speculative) physical interpretation given in the form of interpretation rules, which yields the view that the axioms of the EPT are non-classical, non-quantum, elementary principles in a (hypothetical) universe with the feature that massive antiparticles are *repulsed* by the gravitational field of bodies of ordinary matter [1, 2, 3]. In a sentence, the seven elementary principles of the EPT give an abstract yet exact description of what happens in individual processes at supersmall scale in a universe with repulsive gravity—it follows that these processes are in essence all the same, regardless of the type of interaction that takes place. The question is then: is this relevant for physics? There are then two main issues with the EPT, both mentioned in [1], which are causes for a genuine concern that the answer to that question is 'no':

(i) the EPT has in essence been developed from a Gedankenexperiment with an outcome (matter-antimatter repulsive gravity) that cannot possibly be true from the perspective of modern physics;

(ii) thus far there is no proof that the four fundamental interactions—at least as far as we know them—can take place in the individual processes described by the EPT.

Concerning the first issue, the crux is that the theoretical arguments against a matter-antimatter repulsive gravity—see [4, 5] for an overview—*all* lean on the assumption that theories of modern physics are valid beyond their established area of application. But as Feynman already remarked, "experiment is *the sole judge* of scientific truth" [6]. The issue whether or not repulsive gravity exists will thus have to be decided by experimental research; the current state of affairs is then that there are at least four sizeable experimental projects going on to establish the coupling of massive antimatter particles with the gravitational field of the earth: three projects at CERN using anti-hydrogen, AEgIS [7], GBAR [8], and ALPHA [9], and one at the PSI using muonium (an exotic atom made up of an antimuon and an electron) [10].

*e-mail: Marcoen.Cabbolet@vub.ac.be

Concerning the second issue, the crux is that the EPT is an example of an *abstract physical theory*, a notion that can be formally defined as follows:

Definition 1.1. Let S be a foundational theory for mathematics, such as ZF, with formal language $\mathcal{L}$; then an **abstract physical theory *T* formalized within *S***, or shortly an **abstract physical theory** T, consists of

(i) the **language** $\mathcal{L}(T)$ for T, which is a sublanguage of $\mathcal{L}$ determined by

- a nonempty set U_T, whose elements are called **individual constants** of T;
- a nonempty set R_T, each element of which is an n-ary **relation** $R \subset (U_T)^n$ of T;

(ii) a collection of **formal axioms** of T:

- for every abstract constant $\phi \in U_T$, an axiom $\exists x(x = \phi)$;
- for every n-ary relation $R \in R_T$, an axiom $\exists x(x = R \wedge R \subset (U_T)^n)$;

(iii) a collection of well-formed formulas in $\mathcal{L}(T)$, which are called the **physical axioms** of T;

(iv) a collection of statements in ordinary language, called the **interpretation rules** of T, which give a physical meaning to the individual constants and relations of T.

Let Σ_T be the total collection of formal and physical axioms of T; a **theorem of** T is then any formula Ψ that can be inferred from Σ_T within the framework of S as in

$$\Sigma_T \vdash_S \Psi \tag{1}$$

and the condition then has to be satisfied that all theorems that can inferred from the physical axioms of T by eliminating all quantifiers, are expressed in terms of abstract constants of U_T. □

The name 'Elementary Process Theory' refers to seven physical axioms: as such the EPT could be viewed as a 'theory' from the perspective of the syntactic view on theories. But it is emphasized that the other attributes mentioned in Def. 1.1 are all essential: there is thus more to the EPT than 'just' the collection of its physical axioms. In particular, as Halvorson also noted in [11], without interpretation rules the physical axioms would have no physical meaning whatsoever.[i] That means that in the context of the EPT, we have to distinguish between the *material object*, i.e. the (postulated) thing in the physical universe that is referred to, and the *formal object*, i.e. the thing in the mathematical universe that refers to the material object.[ii] Furthermore, an essential feature of the EPT is that the individual constants that refer to ultimate constituents of the physical world are *abstract sets*, i.e. sets whose elements are not specified: these stand in contrast to *concrete sets*, i.e. sets whose elements *are* specified (such as the empty set $\emptyset$ or the set $\omega = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \ldots\}$). Now the ontological status of abstract constants can probably be debated forever, but here the following position is taken. A formal axiom $\exists x(x = \phi)$ of the EPT guarantees that there is an object in the mathematical universe whose name is ϕ, but without elaborating on which object that precisely is. The important point here is that we do not have assumed *new objects*: therefore the language of the EPT is merely a *sublanguage* of the language of mathematics—*in casu* the language of set matrix theory, a generalization of ZF [13]—in which we have assumed *new symbols* for existing objects.

Newtonian mechanics, classical electrodynamics, special relativity (SR), general relativity (GR), and standard quantum mechanics (QM) are examples of theories that *do not* qualify as an abstract physical theory. The crux is that the condition of Def. 1.1 is not satisfied: the individual constants of these theories that refer to components of the physical world are *concrete* mathematical objects, not *abstract* ones as required. As a consequence, the EPT has a higher degree of abstractness than the theories just mentioned.

That said, at the degree of abstractness of the EPT, a formal object thus *designates* a material object *without* representing its state, that is, *without* containing information of (expectation values of) quantitative properties of the material object—with an abstract physical theory we want to have the largest possible degree of freedom of expression, we want to express physical principles that hold *regardless of* the properties (like position, momentum, etc.) that the involved components of the physical universe have in the reference frame of an observer. As a consequence, the successful predictions of modern interaction theories cannot ever be reproduced quantitatively by deriving theorems directly from the EPT: this impossibility gives rise to the issue at hand—i.e. issue (ii) mentioned in the beginning of this section. The purpose of this paper is to develop a method by which this issue can be solved, that is, by which it can be proved that the fundamental interactions as we know them can take place in the processes described by the EPT. The next section describes this method, the final section describes the research program thereby determined.

2 Method of proof

To get verifiable predictions on the basis of the EPT, a standard tool is to develop a concrete set-theoretic model: below we give a general definition of a concrete set-theoretical model of an abstract physical theory, comparable to the standard definition of a model of a first-order theory as given e.g. in [14].

Definition 2.1. Let T be an abstract physical theory; then a **concrete set-theoretic model** M of T is an interpretation of the individual constants and relations of T in a concrete set-theoretical domain $\mathcal{D}$ such that the interpretation of the axioms of T in the language of M are true in M. The **interpretation function** is a function $I : \mathcal{L}(T) \to \mathcal{L}(M)$ such that

(i) every abstract object $\phi \in U_T$ that designates a material object is interpreted as a <u>concrete</u> object $I(\phi) \in I[U_T] \subset \mathcal{D}$ representing the state of that object in the reference frame of an observer;

(ii) every n-ary relation $R \subset (U_T)^n$ is interpreted as a relation $I(R) \subset I[U_T]^n$ for which

$$\langle \phi_1, \ldots, \phi_n \rangle \in R \Leftrightarrow \langle I(\phi_1), \ldots, I(\phi_n) \rangle \in I(R) \tag{2}$$

(iii) for any axiom Ψ of T, its interpretation $I(\Psi)$ in the language $\mathcal{L}(M)$ of M is true in M:

$$M \models I(\Psi) \tag{3}$$

□

The thing is, however, that specifying a single set-theoretical model M of the EPT will only yield verifiable predictions in the coordinate system of **one observer**.

Suppose, for example, that in a set-theoretic model M of the EPT the initial state of an individual process is modeled as a point-particle with position X_0 in the coordinate system of an observer $\mathcal{O}$ and with momentum $\vec{p}_0$, and suppose that M predicts that the final state produced by that process is a point-particle with position X_1 in the coordinate system of $\mathcal{O}$ and with momentum $\vec{p}_1$: this is a verifiable prediction. However, for another observer $\mathcal{O}'$ the initial state of that same system will have to be modeled as a point-particle with some position X'_0 in the coordinate system of $\mathcal{O}'$ and with momentum $\vec{p}'_0$, and the predicted final state of the process will be a point-particle with a position X'_1 in the coordinate system of $\mathcal{O}'$ and with momentum $\vec{p}'_1$. The one model M, however, does not contain the initial state of the process in the coordinate system of $\mathcal{O}'$: it <u>only</u> contains the initial state of the process in the coordinate system of $\mathcal{O}$—for the observer $\mathcal{O}'$ <u>*another*</u> set-theoretic model m' of the EPT is required. Moreover, the model M

is incapable of predicting what the values of the aforementioned position X_1' and momentum $\vec{p}_1'$ will be: a single set-theoretic model of an abstract physical theory is thus **insufficient** because it can never predict relativity of spatiotemporal characteristics of motion.

That insufficiency provides the motivation for introducing the notion of a *categorical model* of an abstract physical theory: this does contain a model of a physical system for every observer.

Definition 2.2. Let T be an abstract physical theory; then a **categorical model** of T is a (small) category $\mathscr{C}$ for which

(i) the collection of objects of $\mathscr{C}$ is a family $\{M_i\}_{i\in F_1}$ of concrete set-theoretic models of T, such that each M_p in $\{M_i\}_{i\in F_1}$ is specified in a common background language $\mathcal{L}(\mathscr{C})$;

(ii) the collection of arrows of $\mathscr{C}$ is a family $\{A_j\}_{j\in F_2}$ of structure isomorphisms, so that for any arrow A_k in $\{A_j\}_{j\in F_2}$ there is a domain $M_p \in \{M_i\}_{i\in F_1}$ with interpretation function I_p and a codomain $M_q \in \{M_i\}_{i\in F_1}$ with interpretation function I_q such that

- A_k bijectively maps the universe $I_p[U_T]$ to the universe $I_q[U_T]$;
- for any n-ary relation $R \subset (U_T)^n$ of T we have

$$(A_k(\alpha_1), \dots, A_k(\alpha_n)) \in I_q(R) \Leftrightarrow (\alpha_1, \dots, \alpha_n) \in I_p(R) \tag{4}$$

□

The notion of a categorical model of a first-order theory has already been discussed in the literature, see e.g. [11] and the references therein. Applied to the EPT, this gives the following definition of a categorical model of the EPT:

Definition 2.3. A **categorical model of the EPT** is a (small) category $\mathscr{C}$ such that

(i) the collection of objects of $\mathscr{C}$ is a family $\{M_i\}_{i\in F_1}$ of set-theoretical models of the EPT, so that any model M_p in $\{M_i\}_{i\in F_1}$ is a structure $M_p = \langle |M_p|, I_p(M_E), I_p(R)\rangle$ for the EPT specified in a common background language $\mathcal{L}(\mathscr{C})$, with

- I_p being the interpretation function from the language of the EPT to the language of M_p, which is a sublanguage of $\mathcal{L}(\mathscr{C})$;
- $|M_p|$ being the universe of individuals of M_p, which for any constant ϕ of the EPT contains an interpretation $I_p(\phi) \in |M_p|$ that is *mathematically concrete*;
- $I_p(M_E) \subset |M_p|$ being the interpretation of the unary existence relation M_E of the EPT ;
- $I_p(R) \subset |M_p| \times |M_p| \times |M_p|$ being the interpretation of the ternary relation R of the EPT.

(ii) the collection of arrows of $\mathscr{C}$ is a family $\{T_j\}_{j\in F_2}$ of structure isomorphisms, so that for any arrow T_k in $\{T_j\}_{j\in F_2}$ there is a domain $M_p \in \{M_i\}_{i\in F_1}$ and a codomain $M_q \in \{M_i\}_{i\in F_1}$ such that

- T_k bijectively maps the universe of individuals $|M_p|$ to the universe of individuals $|M_q|$;
- $T_k(\alpha) \in I_q(M_E) \Leftrightarrow \alpha \in I_p(M_E)$;
- $(T_k(\alpha_1), T_k(\alpha_2), T_k(\alpha_3)) \in I_q(R) \Leftrightarrow (\alpha_1, \alpha_2, \alpha_3) \in I_p(R)$.

□

There are, however, a plethora of categorical models that are *not interesting* for physics. For the EPT, for example, we can define a categorical model whose objects are structures interpreting the constants of the EPT referring to material objects as *real numbers*.

Example 2.4. To get a structure M_p as in Def. 2.3, we can interpret the constants of the EPT for positive integers n and k for example as follows:

- $I_p({}^{EP}\Phi_k^n) = 1 + n\pi + k\pi^2$;
- $I_p({}^{NW}\Phi_k^n) = 2 + n\pi + k\pi^2$;
- $I_p({}^{NP}\Phi_k^n) = 3 + n\pi + k\pi^2$;
- $I_p({}^{LW}\Phi_k^n) = 4 + n\pi + k\pi^2$;
- $I_p({}^{S}\Phi_k^n) = 5 + n\pi + k\pi^2$

There is then a structure M_p for each choice of integers N, Ω, so that $|M_p|$ is just the set of these numbers $a + n\pi + k\pi^2$ for which $1 \leq a \leq 5$, $n \leq N$, $k \leq \Omega$; the structure can then trivially be completed to yield a set-theoretic model in which the axioms of the EPT are true. However, *physically* this makes no sense since particles are not numbers. □

We are, thus, only interested in categorical models that are interesting from the point of view of physics. To single out such interesting categorical models, the notion of *empirical reduction*, introduced by Rosaler in [15], can be applied: it allows to compare a categorical model $\mathscr{C}$ of the EPT to an existing interaction theory T.

Definition 2.5. Let $\mathscr{C}$ be a categorical model of the EPT; then $\mathscr{C}$ **reduces empirically** to an existing interaction theory T if and only if for every experiment that has confirmed a prediction of T, the experimentally successful predictions of T can be reproduced by $\mathscr{C}$. □

Note that T does not have to be an axiomatized theory: Def. 2.5 holds for a scientific theory in the sense of a generally accepted body of explanatory principles that has been tested by the scientific method—in that sense, e.g. quantum electrodynamics (QED) is a scientific theory although it is not axiomatized. That brings us to the following formulation of the method by which issue (ii) as stated in the Introduction can be solved:

> *to* ***prove*** *that a fundamental interaction as we know it from the scientific theory T can take place in the processes as described by the EPT, the* ***method*** *is to specify a categorical model $\mathscr{C}$ of the EPT such that $\mathscr{C}$ reduces empirically to T.*

The idea is thus that the knowledge that derives from the successful predictions of T has to be incorporated in a (categorical) model $\mathscr{C}$ of the EPT: a negative result is then that no such categorical model exists. Note that *only* the empirically successful predictions of T have to be reproduced by $\mathscr{C}$: it is, thus, not the case that a categorical model $\mathscr{C}$ of the EPT has to be developed such that $\mathscr{C}$ *reduces formally* to T, that is, such that T emerges from the mathematical formulation of $\mathscr{C}$ by applying some limiting procedure.

3 A new research program in theoretical physics

The method set forth in the preceding section determines in a natural way a fundamentally new research program in theoretical physics—a 'research program' as meant by Lakatos [16]. In this section we specify its hard core, positive and negative heuristics, and aims.

Hard core The hard core of the research program consists of the EPT and its mathematical foundations—these give the language $\mathcal{L}$ in Def. 1.1 in case the EPT is the abstract physical theory T. In this research program, the EPT is then considered to be fundamental—that is, the physical axioms of the EPT are the fundamental laws in this research program. This is supplemented by the examples of how the EPT applies, at an abstract level, to real world problems: the application to the Planck era of the universe and the application to the mind-body problem. This hard core already corresponds to what Kuhn called a paradigm (disciplinary matrix) [17].

Heuristics First of all, an 'initial' categorical model $\mathscr{C}_0$ of the EPT proving agreement with SR has to be fully specified; such a categorical model is already in the works [18]. The natural *positive heuristic* is then to develop successors $\mathscr{C}_1$, $\mathscr{C}_2$, ... of $\mathscr{C}_0$ that are theoretically and empirically progressive. Lakatos has defined notions of theoretical progression and empirical progression for theories [16], but these notions can be defined similarly for categorical models of the EPT:

Definition 3.1. A categorical model $\mathscr{C}_{n+1}$ of the EPT is **theoretically progressive** compared to a categorical model $\mathscr{C}_n$ of the EPT when not only <u>all</u> observations, which could be expressed as predictions in the language of $\mathscr{C}_n$, can also be expressed as predictions in the language of $\mathscr{C}_{n+1}$ but also <u>some</u> observations, which could not be expressed as predictions in the language of $\mathscr{C}_n$, can be expressed as predictions in the language of $\mathscr{C}_{n+1}$. Likewise, a categorical model $\mathscr{C}_{n+1}$ of the EPT is **empirically progressive** compared to a categorical model $\mathscr{C}_n$ of the EPT when in the framework of $\mathscr{C}_{n+1}$ predictions can be formulated that are impossible in the framework of $\mathscr{C}_n$ <u>and</u> some of these predictions have been verified. □

The natural *negative heuristic* is to refrain from developments that are inconsistent with the physical axioms of the EPT in the hard core. For example, the EPT is inconsistent with standard QM: it is therefore not interesting to (attempt to) develop a categorical model of the EPT that *unifies* the EPT and standard QM. Note that this is something else than developing a categorical model of the EPT that *reduces empirically* to standard QM! Likewise, the EPT is inconsistent with the classical concept of continuous motion: it is therefore not interesting to (attempt to) develop a categorical model of the EPT that *unifies* the EPT and a theory that applies this concept of continuous motion (e.g. GR)—again, this is something else than developing a categorical model of the EPT that *reduces empirically* to GR.

Aims The short-term aim is to develop a categorical model of the EPT that reduces empirically to GR: that would be both theoretically and empirically progressive compared to $\mathscr{C}_0$. This comes down to developing a (relativistic) model of an elementary process in which a gravitational interaction takes place, such that it not only predicts a matter-antimatter gravitational repulsion, but also quantitatively reproduces the empirically successful predictions of GR.

If that aim can be achieved, and that's a big 'if', the medium-term aim becomes to develop a categorical model of the EPT that reduces empirically to GR *and* to QED: although a unification of QED and GR—in the sense of a single theoretical framework in which QED and GR are both universally valid—is impossible, the EPT could then be called a *unifying scheme* with the unifying principles (the physical axioms of the EPT) at a more abstract level.

If that medium-term aim can be achieved, and that's an even bigger 'if', the long-term aim becomes to develop a categorical model of the EPT that *in addition* reduces empirically to quantum chromodynamics (QCD) and electroweak theory (EW). The EPT could then be called a *Grand Unifying Scheme*. This notion can thus be defined as follows:

Definition 3.2. The EPT is a **Grand Unifying Scheme** if and only if it has a categorical model $\mathscr{C}$ that reduces empirically to GR, QED, QCD, and EW—that is, if and only if it has a categorical model $\mathscr{C}$ such that <u>all</u> observations on physical systems governed by the fundamental interactions can be formulated as predictions in the language of that category $\mathscr{C}$. □

This notion of a Grand Unifying Scheme is thus related to Van Fraassen's idea of *empirical adequacy*, introduced in [19]: the EPT is a Grand Unifying Scheme if and only if it has a categorical model that is empirically adequate when applied in the area of physical systems of elementary particles whose behavior is governed by the fundamental interactions. This notion of a Grand Unifying Scheme should, thus, **absolutely not** be confused with the idea of a Grand Unified Theory: a Grand Unified Theory is a merger of the three gauge interactions of the Standard Model (electromagnetic, weak, strong) in a single interaction model. So, a Grand Unified Theory is thus confined to the framework of the Standard Model, while the above definition of a Grand Unifying Scheme does not assume that objects of the category (which are models of the EPT) have to be formalized in the framework of quantum field theory.

Concluding, a method has been presented by which it can be proven that the physically abstract EPT agrees with the knowledge of the physical world that derives from the successful predictions of modern interaction theories. This method gives in a natural way rise to a fundamentally new research program in theoretical physics aimed at establishing whether the EPT is a Grand Unifying Scheme—which is, ultimately, its intended relevance for physics.

Notes

[i]Def. 1.1 should not be mistaken for an attempt to come up with an all-encompassing definition for the notion of 'theory'. There are several views on what a 'theory' is, and here no position is taken in the debate on how the notion can be defined such that it fits to all ideas we have about 'theories'.

[ii]Tegmark's view that mathematics is an external reality [12] is thus rejected: there is no physical reality to the mathematical universe—mathematics provides the language for physics and that's it.

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