1 Categories, Sets and the Nature of Mathematical entities

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2 It is claimed that category theory cannot provide an adequate foundation for mathematics. The main reasons seems to be the following:

1. category theory cannot provide an adequate foundation for mathematics for *epistemological* reasons, i.e. it presupposes other, more simple, concepts for its understanding;

2. Category theory, perhaps useful in certain areas of mathematics, for instance in algebraic topology, homological algebra, algebraic geometry, homotopical algebra, K-theory, theoretical computer science or even mathematical physics, cannot provide a comparable picture of mathematics as set theory does. First, there is an *informal* set theory that provides a framework for mathematics. What this informal set theory amounts to is not entirely clear, but it seems to play an important role. Second, there is a well-known and well-understood *universe*, namely the cumulative hierarchy, and a well-known and well-understood theory written in a well-known and well-understood formal language, namely ZF (of NBG) written in first-order logic. Thus, the objection goes, category theory does not fulfill some obvious philosophical and metamathematical requirements one might expect or ask from a foundational framework.

In this paper, we want to address these issues in the following manner. I want to argue that:

1. category theory, as it already is, is based on a conception of mathematical object which is, from an ontological point of view, radically *different* from the conception underlying set theory; this fact has numerous consequences, one of which is that the epistemological argument against category theory is ill-founded and therefore can be discarded;

2. although many category theorists believe that category theory is fine as it is, even for foundational purposes[1] — a view that I will not examine here,

[1]Probably one of the best illustration of that position can be found in Taylor's fascinating book. See Taylor 1999.

for it would take us away from our main concern — an alternative picture is being developed, mostly by the logician Michael Makkai at McGill and upon which I will rely heavily, a picture that comprises a universe of mathematics based on a *different* conception of sets, radically different from the cumulative hierarchy, although there is a hierarchy of a different nature, and a formal language in which a theory of that universe can be presented and developed. In a nutshell, the universe, technically the universe of weak ω-categories, is highly heterogeneous in the sense that there are various *kinds* of entities and the variety of these kinds is reflected by the variety of criteria of identity for them. The formal language is an extension of first-order logic, namely it is first-order logic with dependent sorts, FOLDS, which in this context takes the form of a diagrammatic language. We should add immediately that this picture extends radically the nature of mathematical objects presented in the first step of the argument. When we get to this stage of the presentation, I submit that, not only do we have answers to the main objections to a categorical framework, but we can see clearly that the views involved are based on radically different conceptions of mathematical objects. At that point, we can evaluate the situation both from a technical point of view, i.e. what are the technical benefits and the drawbacks of each view, from a philosophical point of view, i.e. which view, if any, is philosophically justified, in particular, which view represents best the way mathematicians work and think about mathematical objects.

The nature of mathematical entities Let us start with the nature of mathematical entities in general and with a rough and classical distinction that will simply set the stage for the picture we want to develop. We essentially follow Lowe 1998 for the basic distinctions. We need to distinguish between abstract and concrete entities, on the one hand, and universals and particulars on the other hand. For our purpose, it is not necessary to specify a criterion of demarcation between abstract and concrete entities. We simply assume that such a distinction can be made, e.g. concrete entities can change whereas abstract entities cannot. We assume that a universal is an entity that can be instantiated by entities which themselves are not instantiable, the latter being of course particulars. Given these distinctions, an entity can be a concrete particular, a concrete universal, an abstract particular or an abstract universal.

Our focus here is between the last two possibilities. For we claim that the current conception of sets makes them abstract particulars whereas for objects defined within categories, mathematical entities are abstract universals[2]. This,

[2]Our choice of terminology is radically different from Ellerman 1987 where a similar proposal is made. However, Ellerman argues that category theory is a theory of *concrete* universals whereas set theory is a theory of *abstract* universals. Needless to say, the difference lies in the way the abstract/concrete distinction is articulated. Thus, whereas Ellerman argues that both concepts of category theory and set theory are universals and that the difference lies in the fact that the former is concrete whereas the latter is abstract, we believe that both are abstract and the former are universals and the latter are particulars. See Marquis 2000 for

we claim, is true of category theory as it is.

Sets, as they are generally conceived and as they are represented in ZFC or NBG, are indisputably abstract particulars. We assume that they are abstract. The fact that they are particulars is established by looking at the criterion of identity for sets in these theories, namely the axiom of extensionality. As is well known, a set is completely determined by its elements and two sets are identical if and only if they have the same elements. Thus a specific set cannot be instantiated by another entity and is therefore a particular. It could be claim that sets are particulars *because* there is a unique criterion of identity for them. A theory in which some entities are universal in the previous sense has to have at least two different criteria of identity: one for the universals themselves and one for the particulars that instantiate these universals. In category theory as it is, we find many different criteria of identity. Three are well-known and common to standard category theory. Others arise in more complex situations. In the universe of categories, there is a whole spectrum of criteria of identity: at one end of the spectrum, we have criteria of identity for particulars, and at the other end, we have a hierarchy of criteria of identity for universals, all related to one another in a systematic manner. What is philosophically interesting, is that the traditional distinction between universal and particular is in some ways inadequate.

Criteria of identity in a categorical context

A brief look at the axioms of a category should be enough to convince anyone that there is an implicit criterion of identity at work for morphisms. Indeed, we have that, for instance, f(gh) = (fg)h, the associativity for morphisms, is an *identity*. Morphisms are treated as particulars from the very beginning. Recall that a morphism f: X $\longrightarrow$ Y is an *isomorphism* if there is a morphism g: Y $\longrightarrow$ X such that fg = 1_Y and gf = 1_X. Two objects X and Y are said to be *isomorphic* if there is an isomorphism between them (at least one). Thus a *specific* pair of maps is used to identify objects as being isomorphic. One could of course immediately stop at this point and reflect on the necessity that these morphisms be particulars. It seems reasonable to ask for a certain type of morphisms that should satisfy certain conditions, as is clear already in the case of a homotopy category. We will come back to this point in due course.

A second criterion of identity appears when we consider how objects are *defined* in a category. Consider the simple and well-known case of the definition of a product for two objects X and Y in a category C. A product for X and Y in a category C is an object P of C *together* with morphisms π_X : P $\longrightarrow$ X and π_Y : P $\longrightarrow$ Y such that for any object Q of C together with morphisms f : Q $\longrightarrow$ X and g : Q $\longrightarrow$ Y, there is a *unique* morphism h : Q $\longrightarrow$ P such that π_Xh = f and π_Yh = g. The important point here is that this definition

some critical remarks on Ellerman 1987. We should point out, however, that the terminology is not clarified in Marquis 2000. We should also point out that our terminology is different from the one found in Makkai 1998 and 1999 where Makkai argues that the concept of collection implicit in a categorical framework is abstract. Once more, the terminology is justified if it is made in a certain way. However, our overall point of view owes a great deal to Makkai's technical work.

characterizes P up to a unique isomorphism. This means that for any object Q *isomorphic* to P in C, if P is a product of X and Y, then Q is a product of X and Y and, moreover, if Q and P are both products of X and Y, then they are isomorphic. Thus, this definition does not give us a *particular* object as a product, nor does it characterize P by stipulating what its elements should be. It specifies under what conditions an object is an instance of the universal *product*, when a given object is a *token* of the product *type*. The criterion of identity in this case is given by the unique isomorphism existing between two tokens of the product of two objects of the category. From a global point of view, the criterion of identity is given by the underlying groupoid of the category C.

This is typical of the way objects are defined in category theory. Mathematical entities and their properties *in* a category C are only given up to isomorphism. It is also true of other entities, e.g. adjoint functors. Category theory specifies what are the abstract universals of mathematics and to know the abstract universals of a domain is to know the fundamental features of that domain. For instance, once one has shown that a given category C has finite products, more generally finite limits, then one knows about various constructions and results that hold in C. A more important example is provided by the notion of abelian category, given by the existence of certain limits (and colimits) and what are called "exactness conditions" (the terminology comes from homological algebra). Another example is provided by the categorical definition of the natural numbers: they too are presented as abstract universals[3]. There is no doubt that the use of category theory in certain contexts, e.g. algebraic topology, homological algebra or algebraic geometry, reveals what is *fundamental* in these domains.

Now, collections or sets are a special kind of mathematical entities. Can they be thought of as entities like any other entity in a category? Already in the mid-seventies, Lawvere had suggested a way to think of sets in this context. Unfortunately for us, he called them "abstract sets", using the term "abstract" in the different sense from what we have assumed here[4]. Here is how he describes these "abstract sets":

An abstract set X has elements each of which has no internal structure whatsoever; X has no internal structure except for equality and inequality of pairs of elements, and has no external properties save its cardinality; (Lawvere, 1976, 119)

The first sentence could be reformulated by saying that the elements of an

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[3]Lowe 1998 also argues, but on different grounds, that the natural numbers are abstract universals. For a categorical analysis of the natural numbers, see for instance McLarty 1993.

[4]We are using "abstract" in an ontological sense, whereas Lawvere clearly has an epistemological notion in mind, something which is not uncommon among mathematicians. In the paper we are referring to, Lawvere says that his notion of sets is "less abstract" than the notion of cardinality. Clearly, one entity cannot be more (or less) abstract than another entity in the ontological sense. However, in the epistemological sense, one can have different levels of abstraction, assuredly a common phenomenon within mathematics. This is an issue we will explore elsewhere. See Marquis 2002.

abstract set are "atoms" or faceless points[5]. Nonetheless, there is an internal criterion of identity for the elements of each sets: we can tell, given two elements of a given set whether they are the same or they are different. Thus, a set in this sense comes equipped with a criterion of identity for its elements. There is no *global* criterion of identity for elements: one cannot ask, given two arbitrary objects ("elements"), whether they are the same or not. The criterion of identity is always relative to a given abstract set. Furthermore, there is no global relation of elementhood: one cannot ask, for any object x and any set A, whether x is an element of A or not. Finally, one cannot ask, given two sets X and Y, whether X = Y or not. Sets in the above sense are *isomorphic* or not. This is what the last sentence of the quote means: their only external property is their cardinality, i.e. two sets are "identical" when they are isomorphic. However, as Lawvere remarks, these sets are "more refined (less abstract) than a cardinal number in that it does have elements while a cardinal number does not."(Lawvere, 1976, 119) It is as if such a set would be a *representative* of a cardinal number, or I would like to say a *token* of a cardinal number, but seen as a token of that cardinal number, i.e. with any specific property erased. Notice that this is in general *how* we look at tokens as tokens of a type: we ignore all specific properties of the token and see only the properties that exemplify the type. In Aristotelian terminology, one would say that such a set is a collection *qua* collection.

Should these sets be treated as particulars or as universals? Since we do not have the standard axiom of extensionality — we cannot compare elements that belong to different sets, it seems that we cannot treat them as particulars. One could argue, though, as follows: since these sets can be the domain or the codomain of morphisms, in particular, they are the domain and the codomain of their own identity morphism. Since the latter *are* assumed to be particulars, the sets have to be particulars too. But this argument fails for two reasons. First, it fails because the criterion of identity for sets is given by isomorphisms. Thus, in particular, any automorphism is acceptable, i.e. a set can be identical with itself in more than one way. This might sound odd, but we are perfectly at ease with this idea for *geometrical* objects. Second, it fails because in the universe we will be considering, even morphisms won't be treated as particulars. The identities are replaced by isomorphisms systematically.

I suggest that we call these sets "transcendental sets" since they are purely the *form* of sets[6]. Another possibility would be to call them "perfect sets" or again, following a suggestion also made by Lawvere, "pure sets", but I favor the previous terminology. We have already underlined the fact that the *totality* of these transcendental sets *cannot* constitute a set. (For there is no

[5]It could certainly be argued that Cantor was developing a conception of sets along these lines. For the process of double abstraction underling Cantor's conception yields a " form ", not a particular entity. This is especially clear when one looks at order-*types*. Ordinals are tokens of a type for Cantor, they are not abstract particulars *à la* Von Neumann. Lawvere has himself recently developed this idea along a different line by looking at abstract sets as *Kardinalen*. See Lawvere 1994.

[6]I am using the term "transcendental" based on an analogy with Kant's usage. It has nothing to do with the expression "transcendental numbers".

set-theoretical criterion of identity for them.) As Lawvere has already observed, these transcendental sets can support mappings, the latter notion being taken as a primitive notion. Composition of mappings can be defined and it clearly satisfies the usual axioms of a category. Thus, the *totality* of transcendental sets constitute a category. It is a different *kind* of entity. As Lawvere has argued in his paper, it is reasonable to say that the universe of these transcendental sets form a topos. In fact, in a topos, any object can be considered to be a transcendental set.

The first conclusion we can immediately draw is that a coherent conception of sets can be developed in a categorical context; this conception is different from the conception inherent to traditional set theory; thus, at the very least we can say that there are various conceptions of sets (of course, there is also the *naïve* conception, but to claim that, say ZFC, with the cumulative hierarchy, is the correct formalization of that conception is a problematic claim).

We could stop here and start arguing for the foundational relevance of category theory as it is. We could give various technical results obtained within toposes or about toposes and expose their foundational significance and importance. We could also articulate a view in which mathematics is done in toposes. The main claim, I guess, would be the following: given any piece of mathematics M, it is possible to find a topos E in which the concepts and theorems of M can be defined and proved. This in itself is an interesting position that deserves to be examined carefully. But we will rather move on to a different, emerging, position, a position that has an intrinsic beauty and that can be presented as an alternative to the set theoretical picture.

Systems of categories

Let us come back to categories. Given the importance of isomorphisms *in* categories, one would expect that the notion of isomorphism would provide the criterion of identity *for* categories. However, this is not the case[7]. A criterion of identity for categories is given by the notion of *equivalence* of categories: two categories C and D are *equivalent* if there are functors F: $C \longrightarrow D$ and G: $D \longrightarrow C$ such that the composite FG is isomorphic to 1_D and GF is isomorphic to 1_C. Notice that the identity between the composites FG and GF and the respective identity functors are replaced by isomorphisms. We can immediately conclude two important facts from this situation: a category of categories, no matter what it turns out to be, *cannot* simply be a category. For, as we have seen, in a category, the criterion of identity is given by the isomorphisms and since categories are not individuated by isomorphisms, a category of categories will have to be something else. Second, transcendental sets and categories have *different* criteria of identity; for transcendental sets, it is given by isomorphisms, for categories it is (at least at this level) given by equivalences; hence, categories *cannot* be said to be structured sets, in the same sense, say, that one can say that groups are structured sets. This is now crucial: categories in our universe cannot be said to be (structured) sets.

[7]As far as I can tell, this was first discovered, or at the very least emphasized, by Grothendieck in his Tohoku paper, published in 1957. However, it might have been discovered by Yoneda some times before. This is an open problem.

Before we move on to the universe as a whole, let us briefly consider how the criteria of identity build up in a categorical context.

Often in category theory, the objects of investigation are functors F: $C \longrightarrow D$, G: $A \longrightarrow B$. One has to determine the correct criterion of identity for such functors. It turns out that the right notion can be presented as follows: two functors F: $C \longrightarrow D$ and G: $A \longrightarrow B$ are *equivalent* if there are equivalence of categories E_1: $C \longrightarrow A$ and E_2: $D \longrightarrow B$ and an isomorphism η: $\mathrm{GE}_1 \longrightarrow \mathrm{E}_2\mathrm{F}$. This situation can be represented by the following diagram:

$$\begin{array}{ccc} C & \xrightarrow{F} & D \\ {\scriptstyle E_1}\downarrow & {\scriptstyle\eta}\nearrow & \downarrow{\scriptstyle E_2} \\ A & \xrightarrow[G]{} & B \end{array}$$

$$\begin{tikzcd} C \arrow[r, "F"] \arrow[d, "E_1"'] & D \arrow[d, "E_2"] \\ A \arrow[r, "G"'] \arrow[ur, phantom, "\nearrow\!\!\!\nearrow"] & B \end{tikzcd}$$

We see that this is more involved: we use the notion of equivalence of categories together with the notion of isomorphism of parallel functors.

We could go on like this and introduce the criterion of identity for fibrations, bicategories, etc. In each case, the criterion of identity would be more involved. The thrust should however be clear: the identity of objects in a categorical context is *derived* from that context, i.e. the underlying category in each case. Furthermore, there is a hierarchy of criteria of identity that seems to be endless. This is the first sense in which the categorical universe is heterogeneous: the criterion of identity for objects in a category is not the same as the criterion of identity for categories, which in turn, is not the same as the criterion of identity for functors, which in turn, is not the same as the criterion of identity for fibrations, etc. We cannot refrain at this moment to quote from Lowe:

The idea that one can "introduce" a kind of objects simply by laying down an identity criterion for them really inverts the proper order of explanation. As Locke clearly understood, one must first have a clear conception of what kind of objects one is dealing with in order to extract a criterion of identity for them from that conception. (...) So, rather than "abstract" a kind of objects from a criterion of identity, one must in general "*ex*tract" a criterion of identity from a metaphysically defensible conception of a given kind of objects. (Lowe, 1995, 517)

In categorical practice, the kind of objects one has to deal with are very often clear from the context. One then determines the proper criterion of identity for the objects of that kind[8]. This is strikingly different from the prevalent situation

[8]Indeed, in many textbooks, various notions are defined, examined and developed and the criterion of identity for these objects is not even mentioned, e.g. fibrations. See for instance Borceux 1994, Jacobs 1999.

in set theory. A general theory of identity reflecting the order of presentation we have just given has been proposed by Michael Makkai in the form of FOLDS. It is the very purpose of that formal framework to be able to formulate in a precise and rigorous fashion, for various kinds of mathematical entities, corresponding criteria of identity. We will come back to FOLDS later.

This informal hierarchy of criteria of identity already indicate the heterogeneity of the universe. At the bottom of the universe, we find the transcendental sets with their criterion of identity: isomorphisms. Thus, their totality does not constitute a set. In fact, they form various categories. These categories, in turn, can be collected into totalities: what are these totalities? They certainly cannot be categories. There is more structure involved. They are at least what are called strict 2-categories or 2-categories. Then again, 2-categories form totalities and these totalities, to be described accurately, require more structure: they form weak 3-categories. In order to see this more clearly, let us look at equivalences more carefully.

An equivalence is given by a pair of functors $\mathrm{F}\colon C \longrightarrow D$, $\mathrm{G}\colon D \longrightarrow C$ and natural isomorphisms $\alpha : \mathrm{FG} \longrightarrow 1_D$ and $\beta : \mathrm{GF} \longrightarrow 1_C$. The fact that α and β are isomorphisms means that the identities $\alpha\alpha^{-1} = 1_{1_D}$ and $\alpha^{-1}\alpha = 1_{\mathrm{FG}}$ hold. In other words, we have reintroduced particulars at the last stage. To be entirely consistent with the underlying conception of object we are assuming, these identities should be replaced by isomorphisms (of the right type, i.e. satisfying certain conditions.) A complete description of this situation is given by what are called bicategories, or, more commonly nowadays, weak 2-categories. The next step is provided by tricategories or weak 3-categories[9]. The latter notion takes six pages to be defined and 13 pages are required to present the various conditions that have to be satisfied by the various levels. Fortunately, a general and very compelling picture is emerging. I want to insist on the fact that it is technically and philosophically compelling. The general picture of the resulting universe is given by what are called "higher dimensional categories" or weak ω-categories. (See Leinster 2002 for a review of different definitions.)

Here is an extremely simplistic sketch of the universe of weak n-categories. 0-categories are transcendental sets. One and the same set can be the same as itself in various ways; i.e. it can have various automorphisms. More generally, two sets can be the same in different ways. Each and every isomorphism between them stipulates how they are the same and we can keep track of these various identities. Moreover, 0-categories, i.e. sets, are linked to one another by morphisms and these morphisms compose in the obvious way. Let us call morphisms between 0-categories 1-morphisms. There is a motivation behind the terminology, for 0-categories can be represented as points and 1-morphisms as directed lines between points. It would be tempting to say that 1-morphisms satisfy various identities, e.g. associativity, and that they form a category.

[9]It can be shown that any weak 2-category is equivalent, in a specific sense, to a strict 2-category. Thus, it would appear that weak 2-categories, those we are describing here and that are relevant to this discussion, are dispensable. However, this is no longer true for 3-categories. In the latter case, there are weak 3-categories which cannot be replaced by strict 3-categories.

But as we have seen, that would amount as treating them as abstract *particulars*. Hence, instead of having identities between 1-morphisms, we require that isomorphisms exist between them (with extra conditions). This implies that 2-morphisms between 1-morphisms have to be introduced and that they provide a criterion of identity for 1-morphisms. When this is done, it can be seen that the collection of 1-morphisms form a weak 2-category. How about 2-morphisms? Clearly, once more, we have to go up the ladder and introduce 3-morphisms. These will stipulate how 2-morphisms behave, how they compose and under what conditions they are identical. The general pattern should now be obvious: to connect and identify n-morphisms, (n+1)-morphisms are required. Notice that it is possible to stop at any n and stipulate that at that point, equalities between n-morphisms exist but that for all $j < n$, identities are given by (j+1)-morphisms.

The general picture is therefore this. The collection of 0-categories forms a 1-category. If we were to stop at this stage, it would mean that we take equalities between 1-morphisms and that the latter are treated as abstract particulars. But we can consider the collection of 1-categories and this is a 2-category. Again, if we were to stop at this point, it would mean that we consider equalities between 2-morphisms but that 1-morphisms, that is 1-categories, are now treated as abstract universals. Thus, for each n, there is an (n+1)-category of all n-categories. Of course, one can consider the ω-category of n-categories for all n: this amounts to defining n-categories for all n simultaneously. The ω-category of weak n-categories is the alternative picture to the cumulative hierarchy of sets.

The technical problems involved in the study of higher-dimensional category theory are daunting. We have not even mentioned the simplest obstacle. We refer the reader to the literature. (See Baez 1997, Baez & Dolan 1998a, Baez & Dolan 1998b, Batanin 1998, Makkai 1998, Makkai 1999 and Leinster 2002.) The point I wanted to make is purely conceptual. I am deliberately ignoring the practical motivations underlying actual research into higher dimensional categories, although they are probably more important than the conceptual ones within the research community, for they go from computer science to topological quantum field theory via homotopy theory. What I *do* want to emphasize are the following points:

1. although we have started with a simple opposition between universals and particulars, the final picture forces us to think about this opposition with care. In the original picture, we had abstract universals and abstract particulars. Now, we seem to be forced to think about the realm of abstract universals in a more elaborate way: within abstract universals, there is a complex structure of relationships between kinds of universals. A simple case of a similar hierarchy can be given: start with a *specific* metric space X, given as a particular. Consider its group of automorphisms. As such, the latter group is also a particular. However, as a group, it is a token of a type: a group whose elements are unidentified but with an isomorphic structure. In turn, this group can be seen as a one object category, that is

an object in the universe of categories. At this stage, we are back in the foregoing picture. Now, the elements of the original groups are automorphisms of the one object category and they can be related to one another either by equalities, in which case we treat them as particulars, or they can be related by morphisms of higher order, in which case they are universals. There is a general ontological picture emerging from this analysis that will force us to look more carefully at the nature of universals.

2. The argument against category theory usually rests on the way categories are presented, i.e. as classes or sets with a certain structure. Thus, it is assumed that category theory has to be presented or understood in a set-theoretical framework. As we have seen in the foregoing section, this misses the fundamental aspect of categories; if the nature of categories is revealed by its criterion of identity, then we can say that categories are *not* structured sets.

3. It might very well be that the concept of a collection as an abstract universal rests on our *understanding* of abstract particulars. One cannot but think of representation theory of groups where interplay between an abstract universal, e.g. an abstract group, and abstract particulars, e.g. its representations, is crucial. However, it by no means implies that a coherent conception of such collections cannot be developed and depends for its development upon a specific choice of abstract particulars.

The requirements of a foundation for mathematics might vary, depending upon one's conception of the foundational enterprise[10]. We do believe that category theory is such that it can answer any requirement one might expect from a foundational framework. But one has to look at it properly and see how and in what sense it is universal. John Bell, in his paper on category theory and the foundations of mathematics, claimed that "far from being in opposition to set theory, [category theory] ultimately enables the set concept to achieve a new universality."(Bell, 1981, 358) Bell could not be closer to the point: sets are not particulars in a categorical framework, they *are* universals and they are the first universals in a complex and rich hierarchy that ought to be foundationally appealing.

Bibliography

Baez, J., 1997, An Introduction to n-Categories, in 7^{th} *Conference on Category Theory and Computer Science*, E. Moggi & G. Rosolini, eds., SNCS, vol. 1290, Berlin: Springer-Verlag, 1-33.

Baez, J. & Dolan, J., 1998a, Higher-Dimensional Algebra III. N-Categories and the Algebra of Opetopes, *Advances in Mathematics*, 135, 145-206.

Baez, J. & Dolan, J., 1998b, Categorification, in *Higher Category Theory*, E. Getzer & M. Kapranov, eds., Contemporary Mathematics, vol, 230, Providence: AMS, 1-36.

Batanin, M., 1998, Monoidal Globular Categories as a Natural Environment for the Theory of Weak n-Categories, *Advances in Mathematics*, 136, 39-103.

[10]See, for instance, Mayberry 1994 and Marquis 1995.

Bell, J., 1981, Category Theory and the Foundations of Mathematics, *British Journal for the Philosophy of Science*, 32, 349-358.

Borceux, F., 1994, *Handbook of Categorical Algebra 2: categories and structures*, Cambridge University Press.

Ellerman, D. P., 1988, Category Theory and Concrete Universals, *Erkenntnis*, 28, 409-429.

Jacobs, B., 1999, *Categorical Logic and Type Theory*, New York: Elsevier.

Lawvere, W., 1976, Variable quantities and variable structures in topoi, *Algebra, topology, and category theory,* New York, Academic Press, 101-131.

Lawvere, W., 1994, Cohesive Toposes and Cantor's '*lauter Einsen*', *Philosophia Mathematica*, (3), Vol. 2, 5-15.

Leinster, T., 2002, A Survey of definitions of n-category, *Theory and Applications of Categories*, vol. 10, 1-70.

Lowe,E.J.,1995, The Metaphysics of Abstract Objects, Journal of Philosophy, 92(10), 509-524.

Lowe, E.J., 1998, *The Possibility of Metaphysics*, Oxford: Clarendon Press.

Makkai, M., 1998, Towards a Categorical Foundation of Mathematics, *Logic Colloquium '95(Haifa)*, LNL 11, Berlin: Springer, 153-190.

Makkai, M, 1999, On Structuralism in Mathematics, in *Language, Logic, and Concepts, Essays in Memory of John Macnamara*, R. Jackendoff, P. Bloom, K. Wynn, eds., Cambridge: MIT Press, 43-66.

Marquis, J.-P., 1995, Category Theory and the Foundations of Mathematics : Philosophical Excavations, *Synthese*, 103, 3, 421-447.

Marquis, J.-P., 2000, Three kinds of Universals in Mathematics?, in *Logical Consequence: Rival Approaches and New Studies in Exact Philosophy: Logic, Mathematics and Science*, Vol. II, B. Brown & J . Woods, eds., Oxford: Hermes, 191-212.

Mayberry, J., 1994, What is Required of a Foundation for Mathematics?, *Philosophia Mathematica*, (3), Vol. 2, 16-35.

McLarty, C., 1993, Numbers Can be Just What They Have to, *Noûs*, 27, 487-498.