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Some notes on the Aristotelian doctrine of opposition and the propositional calculus

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Abstract: We develop some of Williamson's ideas regarding how propositional calculus aids in comprehending Aristotelian logic. Specifically, we enhance the utilisation of truth tables to examine the structure of opposition diagrams. Using 'conditioned truth tables', we establish logical dependency relationships between the truth values of different propositions. This approach proves effective in interpreting various texts of the *Organon* concerning the doctrine of opposition.

Keywords: Aristotle · Ancient logic · Square of opposition · Conditioned truth tables · Paradoxes of material implication · Nonsubstantial particulars.

§ 1. Introduction

Given any two formulae A and C in standard propositional logic, we can condition A to depend on the truth values of C. To do this, we analyze the truth table of C and consider the rows that are true (if any) separately from the rows that are false (if any). When A is independent of C wherever C is true (meaning there are two different truth distributions), we replace the corresponding logical values of A with those of C. We do the same wherever C is false. We leave the remaining rows of A (if any) unchanged. As an excepcion, when A and C are different, and A depends on C wherever C is true and wherever C is false, we take care to replace A with C. This is important, *inter alia*, to prevent a formula from being a logical consequence of its own negation (McCall, 2012; Wansing, Summer 2022). Using this method, we condition the truth values of A on the truth values of C. The resulting formula will be logically equivalent to C or a tautology or a contradiction. We will write A/C to indicate that A is considered under condition C as described. It is worth noting that the relation '/' is reflexive, symmetric, and transitive, ensuring a reliable and consistent approach.

An example of the application of the above informal idea is as follows. Given the propositional variables *s* (0 1 1 0) and *u* (0 0 1 1), the formula (¬s ↛ u) (1 0 0 0) under the condition *s* would be

$$((\neg s \nrightarrow u)\ /\ s)\ (\underline{0}\ 0\ 0\ \underline{0}).$$

The truth values associated with each formula are enclosed in parentheses, where 1 means true and 0 means false. After conditioning, the initial proposition loses its independence from the truth values of the condition in a certain number of rows. In our example, the affected rows (first and fourth) have all become false. We underline the corresponding logical values to make them more visible while preserving their conventional meaning. We generally speak of ‘the conditioned truth table of A given C’ or ‘the truth table of A under the condition C’.

The conditioning of propositions applies to the ‘paradoxes of material implication’. E.g. the proposition ‘If 2 + 2 = 5, then 2 + 2 = 4’ is a logical truth – contrary to what intuition suggests – because the antecedent is false. However, when we represent ‘2 + 2 = 5’ as *s* and ‘2 + 2 = 4’ as *u*, we know that *s* is false while *u* is true. Therefore, we should rewrite the formula (s → u) (1 0 1 1) under the condition (¬s ∧ u) (0 0 0 1) to obtain

$$((s \rightarrow u) \,/\, (\neg s \wedge u))\ (\underline{0}\ \underline{0}\ \underline{0}\ 1).$$

This statement is no longer true when the antecedent is false; in fact, it is only true if both ¬*s* and *u* are simultaneously true. The latter expression is logically equivalent to (u ↛ s) (0 0 0 1), which cannot be misinterpreted. The sentence in natural language is now clear: ‘From 2 + 2 = 4, it does not follow that 2 + 2 = 5’.

This article explores some of Williamson’s ideas about how propositional calculus contributes to a better understanding of Aristotelian logic (Williamson, 1971; 1972; 1988). The use of truth tables to analyse the structure of the traditional square of opposition is improved by the conditioning technique shown above. The interaction between different propositions allows us to construct conditioned truth tables. We use this possibility to propose a new reading of certain passages of the *Organon* concerning the doctrine of opposition.

The formalisation we have adopted affects the interpretation of Aristotelian logic. The quantified expressions and relationships between terms, which are characteristic of syllogistic reasoning, explain why predicate calculus and class logic have dominated approaches to the subject in symbolic logic. Modal logic is used when necessary. However, several Aristotelian texts can be subjected to alternative formalisations. A recurring theme in the *Organon* is the study of the forms of logical opposition that exist between all kinds of propositions. The analyses are not restricted to propositions with quantified expressions. Although the list of scientific publications is extensive (as far as the square of opposition is concerned, there is a series of monographic publications with an almost annual periodicity: Béziau & Payette (eds.), March 2008; Béziau & Payette (eds.), 2011; Béziau & Jacquette (eds.), 2012; Beziau & Read (eds.), 2014; Béziau & Giovagnoli (eds.), June 2016; Béziau & Basti (eds.), 2017; Béziau & Stamatios (eds.), 2017; Béziau & Lemanski (eds.), March 2020; Béziau & Vandoulakis (eds.), 2022; among others), there is still a gap. It is unusual to approach the topic through propositional calculus.

§ 2. Williamson's squares of opposition and the conditioning of propositions

Williamson effectively argued for the connections between syllogistic and propositional logic. One of his significant contributions was demonstrating that there are two plausible variations of the traditional square of opposition in propositional calculus and that the same should apply to the syllogistic. The truth tables show that the number of opposable formulas is greater than previously thought (Williamson, 1972: 498–499). The conditioning of the propositions adds new possibilities to the study of opposition. Accepting a hypothesis influences the truth tables of connected formulas, increasing the identifiable similarity and dissimilarity relations. In this way, we learn more about the internal structure of opposition diagrams.

In formalising passages from the Aristotelian *Organon*, we use two propositional variables (*s* and *u*), along with the logical connectives: negation (¬), material implication (→), material non-implication (↛), biconditional (↔), and exclusive disjunction (↮). Sometimes, it can be useful to have recourse to opposite conjunction (↑) and conjunction (∧). This makes the formalisation of Aristotelian affirmations and negations more intuitive. Technically, the novelty lies in the idea of conditioning. The introduction of assumptions modifies the truth tables of formulas in various ways. The sixteen logical functions for two propositional variables are present through the conditioning symbol (/). Different connectives can provide the same conditioned truth table in each case. The underlining serves as a visual marker of the conditioning of propositions, while the truth values retain their usual meaning. Therefore, conditioned formulas can be combined with normality using logical connectives. With the exception mentioned, propositional calculus and truth tables remain conventional. A more formal definition is not necessary.

Williamson (1972: 497–499) presents four pairs of formulas located in the corners of his squares of opposition. They are as follows, based on the variables *s* (0 1 1 0) and *u* (0 0 1 1) and the connectives already mentioned: (s → u) (1 0 1 1), (s ↛ u) (0 1 0 0), (s → ¬u) (1 1 0 1), (s ↛ ¬u) (0 0 1 0), (¬s → u) (0 1 1 1), (¬s ↛ u) (1 0 0 0), (¬s → ¬u) (1 1 1 0), and (¬s ↛ ¬u) (0 0 0 1). The two squares are shown in Figure 1.

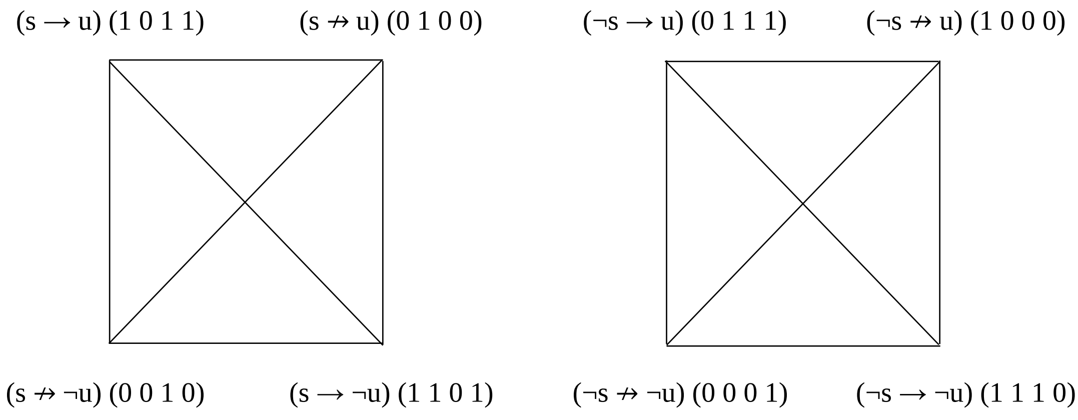


Figure 1: Williamson's squares of opposition adapted to the propositional variables s (0 1 1 0) and u (0 0 1 1) and the connectives →, ↛, and ¬

The inclusion of conditions in the formulae of opposition diagrams is helpful when commenting on selected texts of Aristotle. It allows us to observe the effect of the choice of conditions, interpreting the conditioning of a complex formula as its decomposition into simpler components.

When examining the traditional concept of opposition, we employ six conditions: s (0 1 1 0), $\neg s$ (1 0 0 1), u (0 0 1 1), $\neg u$ (1 1 0 0), ($s \leftrightarrow u$) (1 0 1 0), and ($s \nleftrightarrow u$) (0 1 0 1). These are all the possibilities with two true rows in the truth tables. We impose the restriction that a formula satisfies or fulfils a condition only if its conditioned truth table matches the truth table of the condition itself; e.g. the formula ($s \rightarrow u$) satisfies the condition u:

$$((s \rightarrow u) / u)\ (0\ 0\ \underline{1}\ \underline{1}) \leftrightarrow u\ (0\ 0\ 1\ 1).$$

By being aware of the conditions that the formulas in the squares of opposition fulfil, we can enhance our comprehension of their relationships. Appropriate assumptions make their similarities and dissimilarities evident. Using the biconditional with a sense of identity, we can construct tautologies such as the following:

$$(((s \rightarrow \neg u) / \neg u) \leftrightarrow ((s \nrightarrow u) / \neg u))\ (1\ 1\ 1\ 1).$$

Furthermore, it is possible to form tautologies in which the exclusive disjunction is an indicator of difference:

$$(((s \nrightarrow u) / \neg u) \nleftrightarrow ((s \nrightarrow \neg u) / u))\ (1\ 1\ 1\ 1).$$

§ 3. The presence of underlying conditions in *Prior Analytics* I, 46

As we have seen, by assuming a certain hypothesis, it is possible to check its effect on the truth table of the conditioned formula. A good place to test this idea is in *Prior Analytics* I, 46. Aristotle uses one of the diagrams that may have been instrumental in the development of the traditional square of opposition:

> Let A stand for ‘to be good’, B stand for ‘not to be good’, C stand for ‘to be not good’ (which is below B), and D stand for ‘not to be not good’ (which is below A). Now, either A or B will belong to everything but not <both> to any same thing; and also either C or D will belong to everything, but not < both> to any same thing. (Smith translation, 1989)

The letters A, B, C, and D represent declarative propositions. Smith (1989: 178) has suggested treating affirmations and denials as ‘pairs of *open sentences*: sentences in which the subjects are only variables’. However, because of the way in which truth and falsehood explicitly articulate oppositions in the text, formalisation requires the use of compound propositions. Different conditions provide alternative readings that give more precise information about the truth values. Aristotle supports his arguments with examples such as ‘to be good’, ‘a thing is white’, and ‘a log is white’. All logical formalisation necessarily involves assumptions. Since the priority is to identify the truth tables underlying the oppositions, we must take the licence to add ‘is’ or ‘is not’ to all meaningful expressions, giving them a propositional status (*On Interpretation* 16a17-17a17 and 20b31-21a4; *Categories* 1b25-2a12). We represent by *s* the simple proposition ‘it is a thing’ (or, in its case, ‘it is a log’), and by *u* ‘it is good’ or ‘it is white’. (If the formalisation of the examples raises doubts, they are dispensable: the main theses are restated at the end of the chapter in a general tone independent of any illustration.) Connectives can be used to mark affirmations and denials. The letters A, B, C, and D correspond to the propositions (s → u), (s ↛ u), (s → ¬u), and (s ↛ ¬u), respectively. Formulas involving material implication are interpreted as affirmative, while those connected by the material non-implication are interpreted as negative. B is the negation of A [B ↮ A], and D is the negation of C [D ↮ C]. This leads us to one of Williamson’s squares:

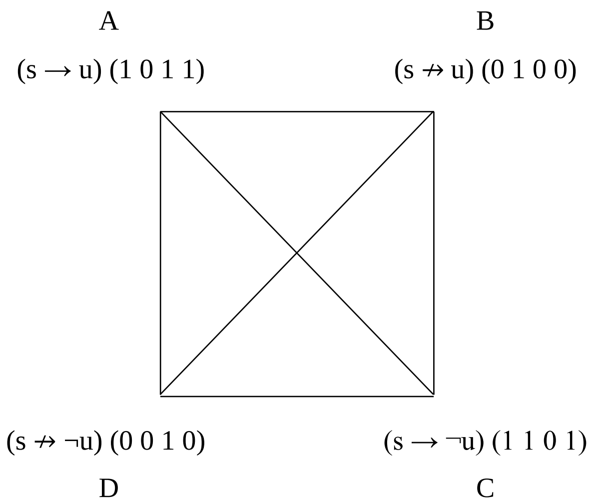


Figure 2: Diagram of Prior Analytics I, 46

It is useful to keep Figure 2 in mind when interpreting Aristotle’s comments on the structure of his diagram. In general, it is sufficient to recognise the logical conditions implicit in the relations between formulae A, B, C, and D. These conditions are, as the case may be, *u*, ¬*u*, *s* and ¬*s*. In Aristotle’s text, formalisations and explanations are inserted in brackets:

> And it is necessary for B to belong to everything to which C belongs: for if it is true to say that something is not-white [((s → ¬u) / ¬u) (1 1 $\underline{0}$ $\underline{0}$)], then it is true to say that it is not white [((s ↛ u) / ¬u) ($\underline{1}$ $\underline{1}$ 0 0), this formula is logically equivalent to the previous one; rows that are not underlined indicate truth values that are not affected by the conditioning] (for it is impossible to be white and to be not-white at the same time [(((s → u) / u) ↮ ((s → ¬u) / ¬u)) (1 1 1 1), the

> exclusive disjunction points to a difference], or to be a not-white log and to be a white log, so that if the affirmation does not belong, then the denial will [if C $(((s \rightarrow \neg u) / \neg u)\ (1\ 1\ \underline{0}\ \underline{0}))$ cannot be fulfilled at the same time as the affirmative proposition A $(((s \rightarrow u) / u)\ (\underline{0}\ \underline{0}\ 1\ 1))$, then the latter should be replaced by its negation B $(((s \nrightarrow u) / \neg u)\ (\underline{1}\ \underline{1}\ 0\ 0))$]. But C does not always belong to B (for what is not a log at all $[((s \rightarrow \neg u) / \neg s)\ (1\ \underline{0}\ \underline{0}\ 1)]$ will not be a not-white log $[((s \nrightarrow u) / s)\ (0\ \underline{1}\ \underline{1}\ 0)]$). Therefore, in reverse order, D belongs to everything to which A belongs: for either C or D belongs to everything, and since it is not possible to be at once not-white and white, D will belong to A (for of that which is white it is true to say that it is not not-white $[(((s \rightarrow u) / u) \leftrightarrow ((s \nrightarrow \neg u) / u))\ (1\ 1\ 1\ 1)]$). But A will not be true of every D (for it is not true to say A – that it is a white log – of what is not a log at all $[(((s \rightarrow u) / \neg s) \nleftrightarrow ((s \nrightarrow \neg u) / s))\ (1\ 1\ 1\ 1)]$; consequently, it is true to say D, but it is not true to say A, i.e. that it is a white log [if there is a disagreement, only one of the opposing formulas can be true]). It is also clear that A and C cannot belong to anything the same $[(((s \rightarrow u) / u) \nleftrightarrow ((s \rightarrow \neg u) / \neg u))\ (1\ 1\ 1\ 1)]$, and that B and D can belong to the same thing $[(((s \nrightarrow u) / s) \leftrightarrow ((s \nrightarrow \neg u) / s))\ (1\ 1\ 1\ 1)]$.

Exclusive disjunction connects formulas that are opposed under certain conditions. If they are logically equivalent, the biconditional signals an identity. The conditions accepted in the formulas of the square of opposition explain the relationships that take shape. The text is convoluted. Aristotle is trying to show how the law of contraposition can be used to reverse the direction of inferences.

Formulas that satisfy the same condition are similar. Their conditioned truth tables are identical. In the following distribution of assumptions, BD serves as a medium between CB and DA (the proportionality symbols can be replaced by the biconditional):

$$((C / \neg u)\ (1\ 1\ \underline{0}\ \underline{0}) : (B / \neg u)\ (\underline{1}\ \underline{1}\ 0\ 0)) :: ((B / s)\ (0\ \underline{1}\ \underline{1}\ 0) : (D / s)\ (0\ \underline{1}\ \underline{1}\ 0)) ::$$
$$((D / u)\ (0\ 0\ \underline{1}\ \underline{1}) : (A / u)\ (\underline{0}\ \underline{0}\ 1\ 1).$$

Aristotle does not complete the proportionality by connecting the extremes with the condition ¬*s*. The reasoning can be reversed by using those contrary conditions that make the formulas dissimilar (the appropriate connective becomes the exclusive disjunction); CA acts as a medium between BC and AD:

$$((B / s)\ (0\ \underline{1}\ \underline{1}\ 0) : (C / \neg s)\ (1\ \underline{0}\ \underline{0}\ 1)) :: ((C / \neg u)\ (1\ 1\ \underline{0}\ \underline{0}) : (A / u)\ (\underline{0}\ \underline{0}\ 1\ 1)) ::$$
$$((A / \neg s)\ (1\ \underline{0}\ \underline{0}\ 1) : (D / s)\ (0\ \underline{1}\ \underline{1}\ 0)).$$

At the end of the passage are the formulas that mediate the chains of reasoning: ‘B and D can belong to the same thing’, but ‘A and C cannot belong to anything the same’. Whether the negative formula follows the affirmative or the reverse is indicated by the expressions: ‘it is necessary for B to belong to everything to which C belongs’, but ‘C does not always belong to B’, and ‘D belongs to everything to which A belongs’, but ‘A will not be true of every D’.

Aristotle collects more complex examples of oppositions that fit the structure of the same diagram. He considers the concepts of equality and inequality:

> Privations also have the same relationship to their predications when put in this arrangement. Let A stand for equal, B stand for not equal, C stand for unequal, [and] D stand for not unequal.

Knowing that an affirmation is implicit in what is *unequal* helps us to understand what Aristotle meant at the beginning of the chapter when he said that in what is *unequal* ‘a subject underlies’, but in negations it does not. (‘Nor are “to be not equal” and “not to be equal” the same: for there is a certain subject for “to be not equal”, that is, the thing which is unequal [((s → ¬u) / ¬s) (1 $\underline{0}$ $\underline{0}$ 1) and ((s → ¬u) / ¬u) (1 1 $\underline{0}$ $\underline{0}$); it is the underlined values that we condition: the formula has truth values 1 by itself], whereas there is not any subject for the other [((s ↛ u) / s) (0 $\underline{1}$ $\underline{1}$ 0) and ((s ↛ u) / ¬u) ($\underline{1}$ $\underline{1}$ 0 0); in this case, it is the truth values 1 that we condition on in the formula]. It is for this reason that not everything is equal or unequal [(((s → u) / ¬s) ↔ ((s → ¬u) / ¬s)) (1 1 1 1); if the formulas were contradictory, they should be connected by the exclusive disjunction instead of the biconditional], though everything is equal or not equal [((s → u) ↮ (s ↛ u)) (1 1 1 1)]’.) The ‘underlying subject’ refers to the formula that has the truth value 1 in some rows of its truth table by itself, without being affected by the condition. This occurs in the affirmations (((s → ¬u) / ¬s) (1 $\underline{0}$ $\underline{0}$ 1) and ((s → ¬u) / ¬u) (1 1 $\underline{0}$ $\underline{0}$), and ((s → u) / ¬s) (1 $\underline{0}$ $\underline{0}$ 1) and ((s → u) / u) ($\underline{0}$ $\underline{0}$ 1 1)), but not in the denials (((s ↛ u) / s) (0 $\underline{1}$ $\underline{1}$ 0) and ((s ↛ u) / ¬u) ($\underline{1}$ $\underline{1}$ 0 0), and ((s ↛ ¬u) / s) (0 $\underline{1}$ $\underline{1}$ 0) and ((s ↛ ¬u) / u) (0 0 $\underline{1}$ $\underline{1}$)).

In the analysis of propositions which it is correct to regard as contradictory, Aristotle presents an even more complex case. He considers some expressions with quantifiers without completing their adaptation to the diagram:

> In the case of a group of things, where the same term belongs to some of them and not to others, the denial might similarly be true either in that they are not all white or in that each of them is no white, though it is false that each is not-white or that all are not-white. In the same way also, the denial of ‘every animal is white’ [*s is taken universally* to mean ‘every animal is’ and *u* means ‘it is white’; in letter A of the square, the values in the second and third rows of the truth table of (s → u) (1 0 1 1) show that ‘every *s* is *u*’] is not ‘every animal is not-white’ [we have to consider the letter C (s → ¬u) (1 1 0 1); according to the truth values of the second and third rows, ‘every *s* is ¬*u*’] (for both are false), but ‘not every animal is white’. [In a group of animals where some are white and some are not, both universal propositions are false. But if they were contradictory opposites, only one should be false. The categorical proposition we are looking for is the particular negative, the letter B of the square: (s ↛ u) (0 1 0 0)].

Formalising quantifier expressions without resorting to predicate logic presents a significant challenge. One approach that deserves further exploration involves considering *On Interpretation* 10, 19b23-34. The diagram from *Prior Analytics* 46 reappears here, as mentioned in the text itself. Aristotle examines the affirmations and negations that the same verb can produce when combined with names and indefinite names. The example he uses to illustrate the various possibilities is ‘a man is just’. It is interesting that he immediately extends its application to the categorical proposition ‘every man is just’. This allows for an unambiguous interpretation of the passage in *Analytics* at hand, using the same square of opposition:

> I mean that ‘is’ will be added either to ‘just’ or to ‘not-just’, and so, too, will the negation. Thus there will be four cases. What is meant should be clear from the following diagram :
>
> | | |
> |---|---|
> | (*a*) ‘a man is just’ | (*b*) ‘a man is not just’
This is the negation of (*a*). |
> | (*d*) ‘a man is not not-just’
This is the negation of (*c*). | (*c*) ‘a man is not-just’ |
>
> ‘Is’ and ‘is not’ are here added to ‘just’ and to ‘not-just’.
>
> This then is how these are arranged (as is said in the *Analytics*). Similarly, too, if the affirmation is about the name taken universally, e.g. :
>
> | | |
> |---|---|
> | (*a*) ‘every man is just’ | (*b*) ‘not every man is just’ |
> | (*c*) ‘not every man is not-just’ | (*d*) ‘every man is not-just’ |
>
> Here, however, it is not in the same way possible for diagonal statements to be true together, though it is possible sometimes. (Ackrill translation, 1963)

Let us examine the formulas that correspond to the previous categorical propositions in Figure 2. In all these formulae, the second and third rows of the truth tables represent the compartments that *s* (0 1 1 0) shares with ¬*u* (1 1 0 0) and *u* (0 0 1 1) respectively. These rows indicate whether something belongs to *s* or not. An interesting aspect is that the name is taken universally in the proposition *s*, which includes a quantifier (‘every man is’, *On Interpretation* 19b11-20). We maintain negation as a monadic operator to express the assertion of an indefinite name (*Prior Analytics* 25b20-24); ¬*u* means ‘it is something other than what is just’ or ‘it is non-just’, while *u* means ‘it is just’. We use the material implication and the material non-implication to formalize affirmative and negative statements respectively. In his diagram, Aristotle assigns A to the universal proposition ‘every *s* is *u*’. The formula has the value 0 in the second row of its truth table and the value 1 in the third row. He associates B with the particular negative ‘not every *s* is *u*’, which can reasonably correspond to the truth values 1 in the second row and 0 in the third row. Similarly, C represents the universal affirmative ‘every *s* is ¬*u*’ and D the particular negative ‘not every *s* is ¬*u*’.

We only have universal affirmatives and particular negatives, with the latter being interpreted as negations of the former. The challenge is to extend this approach to universal negatives and particular affirmatives. Towards the end of Chapter 10 of *On Interpretatione*, Aristotle suggests transforming affirmative propositions into negatives and negatives into affirmatives by obersion. We have employed dyadic connectives to express affirmation or negation. It is essential to obtain logically equivalent formulas but with different connectives. Two strong candidates are the opposite conjunction and the conjunction. The formula (s → ¬u) (1 1 0 1), through which we have formalised the assertion ‘every *s* is ¬*u*’, is equivalent to (s ↑ u) (1 1 0 1). However, in the latter case, by emphasising the truth value 0 of the third row, the connective can signify the denial ‘no *s* is *u*’. In turn, the formula (s ↛ ¬u) (0 0 1 0), used to formalise the proposition ‘not every *s* is ¬*u*', is equivalent to (s ∧ u) (0 0 1 0). Notably, given the position of the value 1 in its truth table, this can denote the affirmation '*u* belongs to some *s*'.

While we will not delve into this issue here, the proposed formalization of categorical propositions would justify the conversion rules of non-modal logic (*Prior Analytics* I, 2). The difficulty lies in determining logically whether the subject and predicate are interchangeable in each categorical proposition. There is no doubt about the conversion of the universal negative and the particular affirmative because the operators used in them (∧ and ↑) have the commutative property. On the other hand, the operators used in the universal affirmative and the particular negative (→ and ↛) are not commutative, which explains why there is no conversion in these cases. If the universal affirmative is true, the particular affirmative is also true. Conversion always preserves the truth of the latter proposition. If the particular negative is true, then either the particular affirmative or the universal negative is true. Conversion always upholds the truth in the latter two cases. (The idea is present in Williamson (1972: 498-499), who avoids the direct transformation of logical connectives of propositional calculus into categorical propositions of Aristotelian logic. Since they are systems subject to different assumptions, he dares only to speak of "analogy" or "direct comparison" between the two. However, he assigns the connectives → and ↑ to particular propositions and ∧ and ↛ to universals. It is not the best choice because the conjunction of the two affirmative propositions ((((s → u) ∧ (s ∧ u)) (0 0 1 0)) must coincide with the particular (s ∧ u) (0 0 1 0), and the disjunction of both ((((s → u) ∨ (s ∧ u)) (1 0 1 1)) with the universal (s → u) (1 0 1 1); can be reasoned similarly with negative propositions).

Now let us return to the passage from the *Analytics*. Aristotle examines what is the contradictory opposite of 'every animal is white' (A). He argues that it cannot be 'every animal is not-white' (C), because both universal propositions would be false if some animals are white and some are not. However, if they were really contradictory opposites, only one of them should be false. So he concludes that the contradictory opposite of 'every animal is white' (A) is 'not every animal is white' (B), which is obviously true in this context. It is easy to see that in any other situation, only one of these two propositions would be true; A and B cannot both belong 'to any same thing': ((s → u) ↮ (s ↛ u)) (1 1 1 1).

What follows in the text of the *Analytics* is an excursus on how to work with affirmative and negative statements in demonstrations. We will not reproduce it. We are interested in how Aristotle formulates new relations in the square of opposition. He includes in the hypotheses the relations that occur on one diagonal and deduces the consequences for the other diagonal. In order to avoid discrepancies in the text, scholars traditionally exchange the position of the letters C and D in the diagram (e.g. Mueller: 2014, 112-114, 132 n. 263; Striker: 2009, 246). But the apparent inconsistencies are not such if we select and combine the assumptions in the formalisation correctly. The conditions are always the touchstone:

> Without qualification, whenever A is so related to B that it is not possible for them to belong to the same thing at the same time but of necessity one or the other of them belongs to everything

[that is, they always relate to each other in a contradictory way and always under opposing assumptions: $(((s \rightarrow u) / u) \nleftrightarrow ((s \nrightarrow u) / \neg u))$ (1 1 1 1) and $(((s \rightarrow u) / s) \nleftrightarrow ((s \nrightarrow u) / \neg s))$ (1 1 1 1)], and C and D, in turn, are likewise related [$(((s \rightarrow \neg u) / \neg u) \nleftrightarrow ((s \nrightarrow \neg u) / u)$ (1 1 1 1) and $(((s \rightarrow \neg u) / \neg s) \nleftrightarrow ((s \nrightarrow \neg u) / s))$ (1 1 1 1)], and A follows C [$(((s \rightarrow u) / \neg s) \leftrightarrow ((s \rightarrow \neg u) / \neg s))$ (1 1 1 1)] and does not convert with it [$(((s \rightarrow \neg u) / \neg u) \nleftrightarrow ((s \rightarrow u) / u))$ (1 1 1 1)], then D will follow B [$(((s \nrightarrow \neg u) / s) \leftrightarrow ((s \nrightarrow u) / s))$ (1 1 1 1)] and will not convert with it [$(((s \nrightarrow u) / \neg u) \nleftrightarrow (((s \nrightarrow \neg u) / u))$ (1 1 1 1)]. Also, it is possible for A and D to belong to the same thing [$(((s \rightarrow u) / u) \leftrightarrow ((s \nrightarrow \neg u) / u))$ (1 1 1 1)] but not possible for B and C [$(((s \nrightarrow u) / s) \nleftrightarrow ((s \rightarrow \neg u) / \neg s))$ (1 1 1 1)].

The above formalisation in square brackets makes Aristotle's words easier to understand. The distribution of similar conditions in different pairs of formulae allows the following proportionalities to be established:

$$((C / \neg s) : (A / \neg s)) :: ((A / u) : (D / u)) :: ((D / s) : (B / s)).$$

If we compare these proportionalities with the previous ones, we find that the order of the items related by the expressions 'follow' and 'belong' can be varied. This is a fact that should not be ignored. Using the law of contraposition, the reasoning based on dissimilar conditions is now as follows:

$$((A / u) : (C / \neg u)) :: ((C / \neg s) : (B / s)) :: ((B / \neg u) : (D / u)).$$

Subsequent reasoning shows that the diagonals of the square of opposition cannot fulfil the same condition. In the diagonal AC, $\neg s$ is satisfied, and in the diagonal BD, s is satisfied:

> First, then, it is evident from the following argument that D follows B. Since one or the other of C and D belongs to everything of necessity, but it is not possible for C to belong to what B does (because C brings along with it A, and it is not possible for A and B to belong to the same thing), it is evident that D will follow B.

The condition that is fulfilled on a diagonal cannot be placed anywhere else in the square (except to generate an opposition with the contrary assumption). This idea can be extended to the sides of the square. The condition $\neg u$ (also $(s \nleftrightarrow u)$) is only satisfied on the BC side, and u (also $(s \leftrightarrow u)$) is only satisfied on the AD. Aristotle focuses on proving the latter case:

> Next, since C does not convert with A [they are contraries given the conditions $\neg u$ and u respectively], but either C or D belongs to everything, it is possible for A and D to belong to the same thing. However, this is not possible for B and C, because A follows along with C (for something impossible results [if A and D are logically equivalent under the condition u, then B

> and C are equivalent under its contrary ¬*u*]). It is evident, then, that B does not convert with D either [under the same assumptions that made C and A contrary to each other, i.e. ¬*u* and *u* respectively, B and D will also be contrary to each other], since it is possible for D and A to belong to something at the same time.

In short, identifying the assumptions under which the formulas are to be interpreted is methodologically helpful for a correct understanding of Aristotelian arguments. The conditioned truth tables are effective.

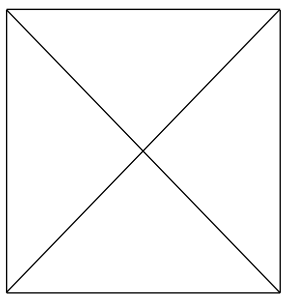
We will not dwell on the last paragraphs of Chapter 46. They show the inconsistencies to which it leads by the wrong distribution of the letters in the diagram. Knowing that A follows C, we err in assigning D the role of affirmation and B the role of denial. We will falsely believe that B follows D, i.e. that the contrary of A follows the contrary of C, in violation of the law of contraposition. When Aristotle later presents the proof, he might also be implying how to introduce a third atomic proposition to *s* and *u* in the oppositions.

In *On Interpretation* 10, 19b36-20a2, Aristotle further states that when contemplating indefinite names, the examination of all conceivable opposites needs to be complemented with an additional diagram:

> There are others [opposites] if something is added to ‘not-man’ as a sort of subject, thus :
>
> (*a*) ‘a not-man is just’ (*b*) ‘a not-man is not just’
>
> (*d*) ‘a not-man is not not-just’ (*c*) ‘a not-man is not-just’
>
> There will not be any more oppositions than these. These last are a group on their own separate from the others, in that they use ‘not-man’ as a name.

The formalisation of this group of opposites in propositional logic coincides with the formulae of Williamson’s square of opposition that were still to be used (Figure 1). Keeping the same propositional variables as before, ¬*s* means ‘it is something that is different from a man’ or ‘it is a not-man’, *u* means ‘it is just’ and ¬*u* ‘it is not-just’:

$(\neg s \rightarrow u)$ (0 1 1 1) $(\neg s \nrightarrow u)$ (1 0 0 0)



$(\neg s \nrightarrow \neg u)$ (0 0 0 1) $(\neg s \rightarrow \neg u)$ (1 1 1 0)

Figure 3: Diagram of On Interpretation 10, *19b36-20a2*

In the new diagram, logical conditioning processes can be studied in a similar way to the previous ones.

§ 4. Oppositions in Chapter 2 of the *Categories*. Nonsubstantial particulars

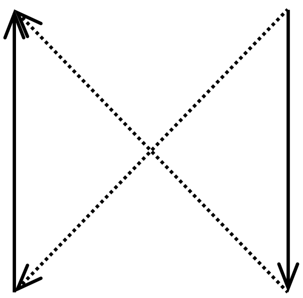
In Chapter 2 of the *Categories*, Aristotle speaks ambiguously of 'things that are said' and 'things there are', but his intention to establish logical relations between opposite expressions is clear. Once again, we give propositional status to Aristotelian terms: s refers to the 'being' of something, u to its 'individual' character, and $\neg u$ to the 'plural'. It is probable the influence of the dialogue *Parmenides*, in which Plato examines the relations between the forms of the One, the Not-One, the Being, and the Not-Being. In the Aristotelian text, the latter form seems to be missing; it will be the conditioning of the formulae that makes the presence of a proposition $\neg s$ visible in the formalisation. Opposite forms are a central element of the Platonic method: it is necessary 'to examine the consequences that follow from the hypothesis, not only if each thing is hypothesised to be, but also if the same thing is hypothesised not to be' (*Parmenides* 135e-136a: Allen translation, 1997). In his analysis of the relationships between forms, Plato systematically varies the contrary conditions he assumes and thoroughly investigates their consequences until all logical possibilities have been exhausted.

It is a pity that Aristotle does not explicitly indicate what the underlying diagram of opposition is in this chapter of the *Categories*. We retain the one used in *Prior Analytics* I, 46, whose structure meets the requirements. We have represented substantial being by s, but it is risky to assign an interpretation to the contrary proposition. In any case, the formalisation of the expression 'be said of' and its negation obliges us to accept $\neg s$ as a possible condition. Less problematic is the relation of the assumptions u (individual) and $\neg u$ (plural) to the expression 'is present in' or its negation. The attribution of meanings to the basic propositions should not disguise the fact that we are still using the same logical resources studied in *Prior Analytics* I, 46. However, the language has changed; there, the expressions 'belong to' and 'follow' prevailed, affirmed, or denied, with indications about the priority of one of the related items.

We will reproduce Aristotle's text, minimally separating the possible situations in the order in which he classifies them. Square brackets show the formalisation of the examples used for illustration. Again, the biconditional and the exclusive disjunction indicate what is the same and what is different between the formulae. The underlying diagram of opposition explains the rearrangement of the four types of 'things' according to logical criteria (*cf*. Studtmann, Spring 2021):

Primary substance   What is not a primary substance

(s → u) (1 0 1 1)   (s ↛ u) (0 1 0 0)



(s ↛ ¬u) (0 0 1 1)   (s → ¬u) (1 1 0 1)

What is not a secondary substance   Secondary substance

Figure 4: Diagram of Chapter 2 of the Categories

As the text confirms, Aristotle establishes two alternative chains of relationships. Both lead from what is not a primary substance to a primary substance, but through a different medium:

(i) A secondary substance (or secondary being) *is said of* a primary substance (or primary being) but *is not present in* it:

> Of things that are said, some involve combination while others are said without combination. Examples of those involving combination are 'man runs', 'man wins'; and of those without combination 'man', 'ox', 'runs', 'wins'.
>
> Of things there are: some are *said of* a subject but are not *in any* subject. For example, man [((s → ¬u) / ¬s) (1 $\underline{0}$ $\underline{0}$ 1)] is said of a subject, the individual man [((s → u) / ¬s) (1 $\underline{0}$ $\underline{0}$ 1)], but is not in any subject [(((s → ¬u) / ¬u) ↮ ((s → u) / u)) (1 1 1 1)]. (Ackrill translation, 1963)

(ii) What is not a secondary substance *is present in* a primary substance but *is not said of* it. In the examples, the 'soul' and 'body' spoken of are 'individuals':

> Some are in a subject but are not said of any subject. (By 'in a subject' I mean what is in something, not as a part, and cannot exist separately from what it is in.) For example, the individual knowledge-of-grammar [((s ↛ ¬u) / u) (0 0 $\underline{1}$ $\underline{1}$)] is in a subject, the soul [((s → u) / u) ($\underline{0}$ $\underline{0}$ 1 1); the rows underlined or conditioned with the value 1 in the truth table of the first formula depend on the rows that are not underlined or conditioned in the second formula: what is not plural *is in* what is individual], but is not said of any subject [((s ↛ ¬u) / s) ↮ ((s → u) / ¬s)) (1 1 1 1)]; and the individual white is in a subject, the body (for all colour is in a body), but is not said of any subject.

(iii) What is not a primary substance *is present in* a secondary substance; the text omits that it *is not said of* it. (In the example, the 'soul' spoken of is now 'universal'.) And (iv) what is not a primary substance *is said of* what is not a secondary substance; it is left out that it *is not present in* it:

> Some are both said of a subject and in a subject. For example, knowledge [((s ↛ u) / ¬u) ($\underline{1}$ $\underline{1}$ 0 0)] is in a subject, the soul [((s → ¬u) / ¬u) (1 1 $\underline{0}$ $\underline{0}$); it is implied that knowledge *is not said of* the 'universal' soul: (((s ↛ u) / s) ↮ ((s → ¬u) / ¬s)) (1 1 1 1)], and is also said of a subject, knowledge-of-grammar [(((s ↛ u) / s) ↔ ((s ↛ ¬u) / s)) (1 1 1 1); Aristotle does not specify that knowledge *is not present in* knowledge-of-grammar: (((s ↛ u) / ¬u) ↮ ((s ↛ ¬u) / u)) (1 1 1 1)].

Finally, Aristotle observes that in some combinations of the diagram, no subject is possible:

> Some [things] are neither in a subject nor said of a subject, for example, the individual man or individual horse ─for nothing of this sort is either in a subject or said of a subject. Things that are individual and numerically one [((s → u) / u) ($\underline{0}$ $\underline{0}$ 1 1); this affirmation is logically equivalent to *u* (0 0 1 1), and the rows with truth value 1 are not underlined or changed by the condition] are, without exception, not said of any subject, but there is nothing to prevent some of them from being in a subject─the individual knowledge-of-grammar [((s ↛ ¬u) / u) (0 0 $\underline{1}$ $\underline{1}$)] is one of the things in a subject [it is in the individual soul ((s → u) / u) ($\underline{0}$ $\underline{0}$ 1 1)].

There is some resistance in the text to identifying a logical criterion that explains what functions as a subject in each case. In principle, this role corresponds to affirmative formulae; their *being* cannot be conditioned and therefore represent the primary or secondary substance. However, there is no logical reason why the primary substance should take precedence over the secondary substance. Moreover, in *Prior Analytics* 51b25-29, Aristotle recognises that there is no subject for the negative expressions he considers there. Now, given the condition *s*, he admits as a linguistic subject what is not a secondary substance, i.e. a negative proposition. There is no clarification in the text. If we accept the priority of the primary substance over the secondary substance, a logical confirmation of this linguistic subject follows from the results of *Prior Analytics* I, 46. It is implicit in a formulation of the law of contraposition: If C (the secondary substance, under condition ¬*s*) is said of subject A (the primary substance, under condition ¬*s*), then the opposite contradictory of A (B: what is not a primary substance, under condition *s*) will be said of the opposite contradictory of C (D: what is not a secondary substance, under condition *s*), which will act as a subject.

We would like to address the controversy surrounding the existence of nonsubstantial particulars (Ackrill, 1963; Owen, 1965; Frede, 1987; Matthews, 1989; Devereux, 1992; Wedin, 1993; and Aranyosi, 2004, among others in a long list). Aristotle acknowledges that some negative expressions lack a subject, whereas in the analogous affirmative expressions, there is a subject. This contrast becomes apparent in truth tables, where the denial corresponds to the value 0, and the affirmation corresponds to 1. When a certain condition is met, we can rework the idea. The underlined parts emphasise where the truth table of the formula is influenced by the conditionning. These are the rows with the truth values 0 in the negatios and 1 in the affirmations. We understand that if the assumption *u* (individual) is satisfied, what is not a secondary substance (((s ↛ ¬u) / u) (0 0 $\underline{1}$ $\underline{1}$)) *is present in* the primary substance (((s → u) / u) ($\underline{0}$ $\underline{0}$ 1 1)). Under this condition, negation and affirmation are logically equivalent, although the subject role shifts towards affirmation, whose truth values 1 are not affected by the conditioning (‘is not plural’ *cannot exist separately* from that which ‘is individual’). However, a dilemma emerges when considering the assumption *s*.

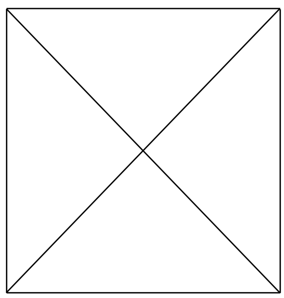
The negative proposition $((s \nrightarrow \neg u)\ /\ s)\ (0\ \underline{1}\ \underline{1}\ 0)$ satisfies the condition *s*. The underlining of the truth values 1 (second and third rows), that have been conditioned, leaves no uncertainlty that what is not a secondary substance lacks any inherent existence. When Aristotle asserts that what is not a primary substance $(((s \nrightarrow u)\ /\ s)\ (0\ \underline{1}\ \underline{1}\ 0))$ *is said of* what is not a secondary substance $(((s \nrightarrow \neg u)\ /\ s)\ (0\ \underline{1}\ \underline{1}\ 0))$, we must focus on identifying what is the same in the corresponding formulas. Since both expressions are negative, the rows with the truth values 1 are only supported by the condition. The subject in language is determined by reasoning based on similarity or proportionality.

§ 5. The diagram of opposition for *Topics* II, 7

A curious variant of the diagram of opposition appears in *Topics* II, 7. The case in question, concerning the obligations that friendship entails, is treated at length by Plato in the dialogue *Lysis* as part of the study of the opposing forms of Likeness and Unlikeness. Aristotle uses only affirmative propositions. However, he had considered denials (‘not to harm our friends’ and ‘not to do good to our enemies’) in a preliminary discussion of the same case in *Topics* I, 10. The morally desirable actions have to be similar to each other and opposed to the objectionable ones that are similar on their side.

Once again, if we reduce the Aristotelian terms to propositions of sentential calculus, truth and falsehood articulate oppositions, ‘to do good’ can be subsumed in *u*, and ‘to be friends’ can be subsumed in *s*. The resulting diagram is as follows:

$(s \rightarrow u)\ (1\ 0\ 1\ 1)$ $(s \rightarrow \neg u)\ (1\ 1\ 0\ 1)$



$(\neg s \rightarrow u)\ (0\ 1\ 1\ 1)$ $(\neg s \rightarrow \neg u)\ (1\ 1\ 1\ 0)$

Figure 5: Diagram of Topics II, 7

It is now easy to obtain a logical formalisation faithful to the Aristotelian text: The assumptions under which we interpret the formulae allow us to identify what is the same and what is different in the opposites. When two opposites of the diagram are equivalent under the same condition, there is no collision. This requires the use of contrary assumptions in the two opposing formulas:

> The first two of the above combinations do not form a contrariety for ‘to do good to friends’ is not the contrary of ‘to do harm to enemies’ [$(((s \to u) / (s \leftrightarrow u)) \leftrightarrow ((\neg s \to \neg u) / (\neg s \leftrightarrow \neg u)))$ (1 1 1 1)]; for both these actions are objects of choice and belong to the same character [the formalisation explains Aristotle’s words: the formulae are logically equivalent under their respective conditions; in turn, the conditions $(s \leftrightarrow u)$ (1 0 1 0) and $(\neg s \leftrightarrow \neg u)$ (1 0 1 0) are equivalent, so they have ‘the same character’]. Nor is ‘to do harm to friends’ the contrary of ‘to do good to enemies’ [$(((s \to \neg u) / (s \leftrightarrow \neg u)) \leftrightarrow ((\neg s \to u) / (\neg s \leftrightarrow u)))$ (1 1 1 1)]; for both these actions are objects of avoidance and belong to the same character [as in the previous case, they are logically equivalent, as are the conditions $(\neg s \leftrightarrow u)$ (0 1 0 1) and $(s \leftrightarrow \neg u)$ (0 1 0 1)], and one object of avoidance is not generally regarded as the contrary of another object of avoidance, unless the one is used to denote excess and the other defect; for excess is generally regarded as an object of avoidance, and so likewise also is defect. But all the other four combinations form a contrariety; for ‘to do good to friends’ is the contrary of ‘to do harm to friends’ [$(((s \to u) / u) \nleftrightarrow ((s \to \neg u) / \neg u))$ (1 1 1 1)], for they proceed from contrary characters, and one is an object of choice and the other of avoidance [the conditions u (0 0 1 1) and $\neg u$ (1 1 0 0) are logically contrary)]. Similarly, also, with the other combinations; for in each pair one is an object of choice, the other of avoidance; one always belongs to a good character, the other to a bad. It is obvious, therefore, from what has been said that the same thing has in fact more than one contrary. For ‘to do good to friends’ has as its contrary both ‘to do good to enemies’ [$(((s \to u) / \neg s) \nleftrightarrow ((\neg s \to u) / s))$ (1 1 1 1)] and ‘to do harm to friends’. In like manner, if we examine them in the same way, it will be apparent that the contraries of each of the others are two in number [$(((\neg s \to \neg u) / s) \nleftrightarrow ((s \to \neg u) / \neg s))$ (1 1 1 1) and $(((\neg s \to \neg u) / \neg u) \nleftrightarrow ((\neg s \to u) / u))$ (1 1 1 1)]. (*Topics* 113a1-19: Forster translation, 1960)

If we decompose the formulae with material implications into the atomic propositions they fulfil, we can take each of the latter as a condition for creating opposites. This explains why ‘the same thing had more than one contrary’. The text is particularly interesting because it contains the six possible conditions considered for the squares in Figure 1: $(s \leftrightarrow u)$, $(s \nleftrightarrow u)$, $\neg u$, u, $\neg s$, and s ($(s \nleftrightarrow u)$ exists under the equivalent formulae $(\neg s \leftrightarrow u)$ and $(s \leftrightarrow \neg u)$).

§ 6. Conclusions

We can condition the propositions of the Aristotelian diagrams to depend on the truth values of different assumptions and then analyse how they behave towards each other. The implementation of this idea allows a homogeneous reading of some texts in relation to the Aristotelian doctrine of opposition. This reading reveals the existence of distinctions to be faced outside predicate logic. We have made progress in Williamson’s line of work: modern truth tables and propositional calculus can contribute to a better understanding of Aristotelian logical analysis.

The possibilities offered by the relations between propositions under the principle of conditioning deserve careful study. Many passages in the *Organon* are based on barely expressed diagrams of opposition. It remains a challenge to adapt the idea to categorical propositions or even to Aristotelian modal logic. (Given two conditioned formulae, the fact that one of them is or is not necessary, possible, admissible, or impossible affects the modal qualification of the other).

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REFERENCES

Ackrill, J. L. (1963). *Aristotle, Categories and De Interpretatione: Translated with Notes by J. L. Ackrill*. Oxford: Clarendon Press.

Allen, R. E. (1997). *Plato's Parmenides. Revised edition*. New Haven and London: Yale University Press.

Aranyosi, I. A. (2004). 'Aristotelian Nonsubstantial Particulars'. *Philosophical Writings* 26: 3–12.

Béziau, J. & Basti, G. (eds.) (2017). *The Square of Opposition: A Cornerstone of Thought*. Birkhäuser Cham. <https://doi.org/10.1007/978-3-319-45062-9>

Béziau, J. & Giovagnoli, R. (eds.) (June 2016). *Logica universalis* 10, 2–3, Special Issue: The Vatican Square.

Béziau, J. & Jacquette, D. (eds.) (2012). *Around and Beyond the Square of Opposition*. Birkhäuser Basel. < https://doi.org/10.1007/978-3-0348-0379-3>

Béziau, J. & Lemanski, J. (eds.) (March 2020). *Logica universalis* 14, 1, Special Issue: Cretan Square

Béziau, J. & Payette, G. (eds.) (March 2008). *Logica universalis* 2, 1.

Béziau, J. & Payette, G. (eds.) (2011). *The Square of Opposition: A General Framework for Cognition*. Peter Lang.

Beziau, J. & Read, S. (eds.) (2014). *History and Philosophy of Logic* 35, 4 <https://doi.org/10.1080/01445340.2014.917836>

Béziau, J. & Stamatios, G. (eds.) (2017). *New Dimensions of the Square of Opposition*. Philosophia. https://doi.org/10.2307/j.ctv2nrzhd7>

Béziau, J. & Vandoulakis, I. (eds.) (2022). *The Exoteric Square of Opposition. The Sixth World Congress on the Square of Opposition*. Birkhäuser Cham. <https://doi.org/10.1007/978-3-030-90823-2>

Cook, H. P. & Tredennick, H. (1962). *Aristotle: The Categories. On Interpretation. Prior Analytics. Topica*. With facing Greek and English pages. Cambridge: Harvard University Press.

Devereux, D. (1992). 'Inherence and Primary Substance in Aristotle's *Categories*'. *Ancient Philosophy* 12: 113–131. <https://doi.org/10.5840/ancientphil199212147>

Frede, M. (1987). *Essays in Ancient Philosophy*, Minneapolis: University of Minnesota Press.

Matthews, G. B. (1989). 'The Enigma of *Categories* 1a20ff and Why It Matters'. *Apeiron* 22 (4): 91–104. <https://doi.org/10.1515/APEIRON.1989.22.4.91>

McCall, S. (2012). 'A History of Connexivity', in D. M. Gabbay, F. J. Pelletier, J. Woods (ed.). *Handbook of the History of Logic* 11: 415-449. North-Holland, <https://doi.org/10.1016/B978-0-444-52937-4.50008-3>

Mueller, I. (2014). *Alexander of Aphrodisias: On Aristotle Prior Analytics 1.32–46*. Bloomsbury.

Owen, G. E. L. (1965). 'Inherence'. *Phronesis* 10 (1): 97–105. <https://doi.org/10.1163/156852865X00095>

Smith, R. (1989). *Aristotle's Prior Analytics*. Translation with introduction, notes, and commentary. Indianapolis/Cambridge: Hackett Publishing Company.

Striker, G. (2009). *Aristotle's Prior Analytics: Book I*. Translation and commentary. Oxford: Clarendon Press.

Studtmann, P. (Spring 2021). 'Aristotle's *Categories*'. *The Stanford Encyclopedia of Philosophy*, Edward N. Zalta (ed.). <https://plato.stanford.edu/archives/spr2021/entries/aristotle-categories/>

Tredennick, H. & Forster, E. S. (1960). *Aristotle: Posterior Analytics. Topica*. Facing Greek and English pages. Cambridge: Harvard University Press.

Wansing, H. (Summer 2022). 'Connexive Logic'. *The Stanford Encyclopedia of Philosophy*, Edward N. Zalta (ed.). <https://plato.stanford.edu/archives/sum2022/entries/logic-connexive/>

Wedin, M. V. (1993) "Nonsubstantial Individuals". *Phronesis* 38 (2): 137–165. <https://doi.org/10.1163/156852893321052361>

Williamson, C. (1971). 'Traditional logic as a logic of distribution-values'. *Logique et Analyse* 14 (56): 729–746.

Williamson, C. (1972). 'Squares of opposition: Comparisons between syllogistic and propositional logic'. *Notre Dame Journal of Formal Logic* 13 (4): 497–500. <https://doi.org/10.1305/ndjfl/1093890711>

Williamson, C. (1988). 'How many syllogisms are there?'. *History and Philosophy of Logic* 9 (1): 77–85. <https://doi.org/10.1080/01445348808837127>