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Kory Matteoli
University of Massachusetts Amherst
bmatteoli@umass.edu

November 2, 2023

Conflict of Interest Statement

No funding was received to assist with the preparation of this paper.

I am thankful to Rohan French, Cody Gilmore, Hanti Lin, and the anonymous referees at *Synthese* for their helpful comments and discussions while developing this paper.

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Kory Matteoli*

November 15, 2023

Abstract

Palle Yourgrau (1987, 2000, 2019) defends what he calls the principle of Prior Possibility: nothing comes to exist unless it was previously possible that it exists. While this seems like a plausible principle, it's not strong enough; it allows the impossible to come to exist. I argue for a stronger principle: nothing exists unless its existence has always been possible. Further, I argue that we then have reason to accept a surprising result: nothing exists unless its existence is always possible. Or, more generally, that nothing is the case unless it's always possible that it's the case.

1 Introduction

Here's an undeniable modal principle: nothing exists unless it's also possible that it exists. That's all well and good for the modal status of things that exist right now, but what about the modal status of things at other times, or the modal status of things over time? Here's what might seem like a plausible principle: nothing comes to exist unless its existence was previously possible. That is, things that were once impossible can't come to exist. Palle Yourgrau (1987, 2000, 2019) calls this the principle of Prior Possibility:[1]

Prior Possibility If a thing x comes to exist, then it was the case that it's possible that x exists.

If it wasn't previously possible, in the broadest sense of 'possible', that something exists, then it seems as if it couldn't ever have come to exist, otherwise it couldn't have been impossible. Likewise, if something did come to exist, then it seems as if its existence couldn't have been impossible, otherwise it wouldn't have come to exist. Or so we might think.

*Special thanks to Rohan French, Cody Gilmore, and Hanti Lin for their helpful discussion and advice. Thanks also to the anonymous referees at *Synthese*.

[1]Yourgrau talks in terms of existence (interchangeably with actuality), but I mainly talk in terms of cases. This is because Yourgrau takes Prior Possibility to apply only to individuals, but I think that both it and the principles I argue for apply to more than just individuals. More on this in section 6. Further, I take it that cases of existence are a subset of cases in general. For more, see Lewis 1990; Menzel 2021; Routley 2018 and Williamson 1987, 2002.

In this paper I argue for two main things. One, that Prior Possibility is not strong enough to be the right principle for preventing the impossible from coming to exist. Instead, we should accept the principle of Having Possibility:

Having Possibility If x is the case, then it has always been the case that it's possible that x is the case.

Two, I argue that we should accept two additional principles:

T If x is the case, then it's possible that x is the case.

Going Possibility If x is the case, then it will always be the case that it's possible that x is the case.

all three of which together entail:

Always Possible If x is the case, then it's always possible that x is the case.

This result might surprise you! But if we accept the three principles I argue for, we also have to accept what they together entail.

We should care about Prior Possibility, and we should care about the principles I argue for too. Here's why we should care. For Yourgrau, it helps us understand what it means for a thing to come to exist. Against views descending from those of Aquinas, specifically views from Geach (1969a,b), Marcus (1985), and Prior (1960), Yourgrau (2019) argues that Prior Possibility supports that a thing coming into existence is in fact a genuine change in the thing itself—a change from the state of non-existence to the state of existence. Further, Yourgrau takes his principle to support the controversial distinction between being and existence. Existence is just one property among others, and being is a bridge between existence and nonexistence. Caesar's having come to exist implies there being such a person as Caesar before Caesar existed (2019, p. 77). Importantly for Yourgrau, the dead are just one of the many things that have being but lack existence. We should also care about Prior Possibility, and about the principles I argue for, because it gives us insight into connections between time, tense, and modality. A topic which has been discussed in, for example, the work of Byrd (1980), Dorr and Goodman (2020), Fine (2005), French (2008), Guéron and Lecarme (2008), King (2003), Prior (1957), Rescher and Urquhart (1971), Rini and Cresswell (2012), Woolhouse (1973), and Yourgrau (2019). On my part, I think that we should accept my principles, rather than accept Prior Possibility alone. It's not just the *past* that's relevant to modality, it's *all* the times. Accepting these principles has interesting consequences worth exploring.

I just introduced the topic we're discussing in section 1. In section 2 I argue that, for the purposes of what I'm doing early on in the paper, we should accept a particular principle that will lead us to also accept Having Possibility. In section 3 I explain Having Possibility in more detail, and argue why we should accept it. In section 4 I introduce and defend the alternative principle of Prior Possibility. In section 5 I argue that we should accept Having Possibility instead of Prior Possibility. In section 6 I connect the results so far back to Yourgrau's project. In section 7 I show how further interesting results are entailed by the principles that I argue for. In section 8 I address a concern about the relations between the principles I argue for and views like Origin Essentialism. I conclude in section 9. The appendices contain various formal things that relate to what I argue for, and I direct the reader to there when applicable. Do note that the informal arguments stand on their own, and the formal regimentations are meant to aid the reader in their understanding.

2 Not a Bridge too Far

In this section I argue for the following principle connecting necessary falsehoods and future times:[2]

Necessarily Not Future If it's necessarily not the case that x, then it will not be the case that x.

Necessarily Not Future* If x will be the case, then it's possible that x is the case.

This is meant to capture part of the thought that there's some kind of close connection between the temporal and the modal.[3] Namely, the thought that things that are relevant for temporal evaluations about the future are, or are connected to, things that are relevant for modal evaluations. If x is necessarily not the case, then x can't also be the case in the future. For example, $2+2=5$ is necessarily not the case. Since $2+2=5$ is necessarily not the case, then there couldn't be any future time when it's the case that $2+2=5$. If it were ever the case that $2+2=5$, then it just wouldn't be necessarily not the case that $2+2=5$, since at that point $2+2$ does equal 5. Another way to put the point is that since we know that it's necessarily false that $2+2=5$, we can infer, say, that we won't wake up tomorrow to find that $2+2$ now equals 5. Necessarily false things are just not the kinds of things that can ever be true; this seems to just be part of the notion of possibility itself.

As further support, suppose that Necessarily Not Future doesn't hold for some proposition p. If Necessarily Not Future doesn't hold for p, then it's not the case that: if it will be the case that p, then p is possible. But this is equivalent to it being the case that both: it will be the case that p, and it's not the case that it's possible that p. So, if Necessarily Not Future doesn't hold for some proposition p, it follows that it both will be the case that p, and that p is impossible. This is to say that, without Necessarily Not Future, the impossible will be the case. But the impossible can't ever be the case—that's just what it means to be impossible. Since the impossible can't ever be the case, the impossible can't be the case in the future.

[2]There's an analogous principle for past times: Necessarily Not Past. More in section 7.

[3]This thought has been around since at least Diodorus Cronus. Diodorean accounts reduce modality to tense; what is necessary is that which is and always will be the case. See Bobzien and Duncombe 2023, §4.2; Rice 2023, §2; Rescher and Urquhart 1971, Chapter 12. Yourgrau (2019) accounts for this connection in arguing for Prior Possibility.

Necessarily Not Future makes it easy for something to be possible. If something will be the case, then it's possible. For example, if there is going to be a sea battle tomorrow, then it's possible that there is a sea battle tomorrow (and it's possible that there are even sea battles at all). We can't have impossible sea-battles happening, otherwise they wouldn't be impossible. In other words, if we know that something will be the case, then we should be certain that it's possible. If I'm uncertain whether sea battles are possible, but then I learn that there will be a sea battle tomorrow, I should stop being uncertain and now think that sea battles are possible.

Necessarily Not Future makes it hard for something to be necessary. In order for something to be necessary, its negation must never be the case in the future. That is, things that are necessary don't later stop being the case. For example, if it's impossible for me to shake hands with the number 2, then it better not be the case that I do so tomorrow (or on any later day). We can't have impossible handshakes happening, otherwise they wouldn't be impossible. In other words, if we know that something is impossible, we should be certain that it won't come to happen. If I know that handshakes with numbers are impossible, I should be certain that I won't shake hands with a number tomorrow.

Now let's turn to two alternative principles to Necessarily Not Future that we might find appealing, and see why we should prefer Necessarily Not Future instead.

The first alternative is Necessarily Always:

Necessary Always If it's necessarily the case that x, then it's always the case that x.

This seems like a plausible principle. For example, it's a necessary truth that $2 + 2 = 4$, and so we can reasonably infer from this that it's at all times the case that $2 + 2 = 4$. If there were a time when it's the case that $2 + 2 \neq 4$, then $2 + 2 = 4$ wouldn't be a necessary truth. However, Necessarily Always does more than we need right now, in that it concerns too many times (namely, all of them), so we'll set it aside.

The second alternative is Sometime Possible:

Sometime Possible If x is the case at some time, then it's possible that x is the case.

This also seems like a plausible principle.[4] If there's some time when it's the case that my cat is asleep, then surely I can infer that it's possible that my cat is asleep. But, like Necessarily Always, Sometime Possible covers too many times (all of them). Further, it's easy to see that Sometime Possible entails Necessarily Not Future,[5] but not vice versa. If we were to accept Sometime Possible, we'd have Necessarily Not Future anyway. So we can set Sometime Possible aside too, and just stick with Necessarily Not Future, for now.

However, I argue later that we have good reason to accept three principles that together entail Sometime Possible:

[4] Necessarily Always and Sometime Possible are actually logically equivalent.

[5] For a formal explanation, see appendix F.2.

Necessarily Not Past If it's necessarily not the case that x, then it was not the case that x.

T If x is the case, then it's possible that x is the case.

Necessarily Not Future If it's necessarily not the case that x, then it will not be the case that x.

I hold off on this because the only one of the three principles that's needed to address Yourgrau and Prior Possibility is Necessarily Not Future.

Note that all I really need to get to Always Possible and Necessary Always are Sometime Possible and the modal B-axiom.[6] For now, what we're working with is strictly weaker, since B is not entailed by our principles (at all), nor is Sometime Possible (until we accept those other principles I just mentioned). If you think my principles now are implausible, then you think that they're implausible either because they're too weak, or because they're too strong. If you think my principles now are too weak, then just accept the stronger principles that together entail my principles now; you'll still end up where we're going in the end. If you think my principles now are too strong, then since they are entailed by Sometime Possible and B, you must have a problem with either Sometime Possible or B. Part of the project of this paper is to argue that we should accept Sometime Possible, while issues with B are outside the scope of this paper.

3 Having Possibility

In this section I argue for the principle of Having Possibility:[7]

Having Possibility If x is the case, then it has always been the case that it's possible that x is the case.

This principle says that nothing is the case without it *having always* been possible that it's the case. That is, if x is the case, then it can't have been impossible that x is the case. Having Possibility follows directly from the principles I've argued for above. Further, as I argue in section 5, my principle has advantages over Prior Possibility.

Having Possibility follows from the principles I've argued for so far.[8] If x exists, then it has always been the case that it will be the case that x exists. That is, there is no past time when it will always be the case that x does not exist. According to Necessarily Not Future, if it will be the case that x exists, then it's possible that x exists. It then follows that it has always been the case that: if it will be the case that x exists, then it's possible that x exists. And so if it has always been the case that it will be the case that x exists, then it has

[6] B: $\varphi \rightarrow \Box\Diamond\varphi$. I say a bit more on this in footnote 18.

[7] It would be more accurate to call this the principle of Having Always Been Possible, but that just doesn't have the same ring to it. Likewise for similar principles later.

[8] For a formal explanation, see appendix F.3.

always been the case that it's possible that x exists. And so, since if x exists then it has always been the case that it will be the case that x exists, it follows that if x exists then it has always been the case that it's possible that x exists. That is, Having Possibility follows.

Here are two other, brief, arguments for Having Possibility. Even though Having Possibility follows given the principles I've argued for so far, what these two arguments show is that Having Possibility is also plausible on its own.

Way one. We can reason directly. Consider something that exists, like my cat. Could it have failed to be the case that it has always been the case that it's possible that my cat exists? No. Suppose that it had; that is, that it's not the case that it was not at some time the case that it's not possible that my cat exits. If so, then this means that it was at some time the case that it's not possible that my cat exists. Further, this also means that it was at some time the case that, necessarily, my cat does not exist. This should seem odd, given that my cat exists and is sitting here next to me. If it was the case that my cat, necessarily, doesn't exist, then it must not be the case that he could ever have come to exist. Because, if so, then it couldn't be necessary that he doesn't exist. So, we should conclude here that it must have always been the case that it's possible that my cat exists, as Having Possibility says.

Way two. We can reason from the contrapositive. Consider something such that it has not always been the case that it's possible that it exists, like a square circle. Could it also be the case that this square circle exists? No. By the same reasoning as before, we have that it was the case that, necessarily, this square circle doesn't exist. If it were the case now that this square circle exists, then it wouldn't be the case that it, necessarily, doesn't exist. Fortunately, this square circle doesn't exist at all, in line with what the contrapositive of Having Possibility says.

I give further defense of Having Possibility in light of the competing principle of Prior Possibility in section 5, but my hope in this section is that these arguments show you that not only does Having Possibility follow from principles I've already argued for, but also that Having Possibility is plausible on its own. We have good reason to accept Having Possibility.

4 Prior Possibility

In this section I clarify Prior Possibility, and then defend Prior Possibility. Yourgrau (2019) does a lot to defend Prior Possibility, and this section presents his arguments in a clear and concise way, while making it clear to the reader exactly what Prior Possibility says. This section also contains some of my own arguments for Prior Possibility.

Yourgrau's statement of Prior Possibility is that "[n]othing becomes actual unless previously possible" (2019, p. 79). This gets the general idea across, which we might state as:

Prior Possibility If a thing x comes to exist, then it was the case that it's possible that x exists.

That is, if x comes to exist at time t, then at some time t' earlier than t, it must also be possible that x exists. This is a good starting point.

The tricky part here is 'a thing x comes to exist'. Yourgrau (2019, p. 74) argues against Geach (1969a,b), Peirce (1931), and Prior (1960) that coming to exist is a genuine change in the thing that comes to exist. While I find Yourgrau's arguments appealing, we can get by with a much thinner notion of coming to exist, i.e. of becoming.

How should we read 'comes to exist'? A flat-footed way is that it didn't exist, but now it does. That it, that it was at some time the case that x does not exist, and it's the case that x exists. However, this is too weak.[9] If x exists in 2022, and if x has existed since 1922, then on this reading 'x comes to exist' is true in 2022. That is, this reading really means 'either x comes to exist now, or x came to exist and currently exists', which is probably not what Yourgrau has in mind. We're concerned with the very *first* time that x comes to exist, so this reading won't work.

The better way to understand 'comes to exist' is that x hasn't existed, but now it does. That is, it's both: not the case that it was at some time the case that x exists, and x exists. But this is just to say that both: it has always been the case that x does not exist, and it's the case that x exists. This excludes cases like the one we just considered. So, this seems like the reading we want.

We now have a more precise understanding of Prior Possibility:

Prior Possibility* If it both: was not at some time the case that x exists and it's the case that x exists, then it was at some time the case that it's possible that x exists.

Keep in mind that this is the way to understand Prior Possibility when we talk about Prior Possibility.

Now that we're clear on what Prior Possibility says, we can defend it. In defense of Prior Possibility, Yourgrau says (2019, p. 82, original emphasis):

> This compelling image [regarding an alternative principle], however, must yield to the logic behind the Principle of Prior Possibility, which rests on the very concepts of possibility and actuality. Some questions of modality can be disputed, or admit alternatives. Some cannot. If someone denies that what is necessary is possible, he or she simply fails to grasp the concepts of necessity and possibility. The same is true, I believe, when someone denies that what is actual must have previously been possible. Can we really accept what Prior and Peirce are maintaining, that Caesar, though "nonpossible", somehow managed to become actual—i.e. that the *impossible can become actual?* (I take it that Prior's 'nonpossible' can be rendered as 'impossible'.)[10]

[9]Thanks to Cody Gilmore.

[10]I'm inclined to agree. However, Prior might think that there's a truth-value gap between the nonpossible and the impossible. See Ishiguro and Skorupski 1980 for a related discussion.

Just as it's undeniable that whatever is necessary must also be possible, it's undeniable that anything that's not possible can't come to exist. In order for something to come to exist, it must first be possible that it exists. A square circle could never come to exist, because it can't first be possible that a square circle exists. If it were first possible that a square circle exists, this would mean that square circles aren't actually impossible. This is a compelling defense, but I'll elaborate on some of the details.

Imagine that it's the case that something that was previously impossible came to exist. That is, imagine that it has always been the case that it's impossible that a exists, and it's the case that a came to exist. Since it has always been the case that it's impossible that a exists, then there is no past time when it's possible that a exists, as it seems the objectors to Prior Possibility are saying. So, the best restatement of 'the impossible can come to exist' is 'it has always been the case that it's impossible that a exists, and it's the case that a came to exist'.

Notice that *it has always been the case that it's not that case that it's possible that a exists* is equivalent to *it has always been the case that, necessarily, a doesn't exist*. But if so, then it couldn't be the case that a ever came to exist, since then there would be a world where it's possible that a exists. So, if it has always been the case that, necessarily, a does not exist, then it's not the case that both: it was the case that a does not exist, and a exists, since it's false that a exists. Necessarily, square circles don't exist. So, it couldn't also be the case that some square circle came to exist. If it did, then there would be a world where it's possible that a square circle exists (the very world where it does exist), which can't be right.

To clarify how the argument in the previous paragraphs works, notice that if both: it has always been the case that it's not possible that a exists and both: it was not the case that a exists and a exists, then it follows that it has always been the case that falsum.[11] While *it has always been the case that falsum* isn't plain-old falsum, it's just as bad. It would be terrible if it turned out that it has always been the case that falsum! But if what has always been impossible comes to exist, then it has always been the case that falsum. Clearly the problem lies in our original conjunction then, as the informal argument shows. The real problem is caused by having both *it has always been the case that it's not possible that a exists* and *a exists*; *it was not the case that a exists* is not why there is a problem.

So far so good for Prior Possibility.

5 Replacing Prior Possibility

It's not so far so good for Prior Possibility. In this section, I argue that we have good reason to prefer Having Possibility over Prior Possibility. Not only is Having Possibility able to capture the core motivation behind Prior Possibility—that it being possible that something exist is a necessary condition for it coming to

[11] For a formal explanation, see appendix F.4.

exist—but Having Possibility is better than Prior Possibility at doing the exact things that Yourgrau wants his principle to do. In particular, Having Possibility properly handles the modal status of things at first times, the modal status of eternal existents, and doesn't allow the once impossible to come to be. Prior Possibility fails all of these tasks. Note that some of these examples are not meant to be counterexamples, strictly speaking, to Prior Possibility. Rather, they are cases where Prior Possibility seems to give us the wrong result, but Having Possibility seems to give us the right result. I take it that it's a disadvantage of Prior Possibility that it gives the wrong results, and an advantage of Having Possibility that it gives the right results. However, do note that the last case in this section is one where Prior Possibility holds but something that was once impossible comes to exist.

Prior Possibility is false at first times, but Having Possibility isn't. Note that if it has always been the case that it's possible that a exists, this doesn't imply that it was at some time the case that it's possible that a exists. Imagine a rock at the beginning of time. The rock exists then, and it's not the case that the rock existed at some earlier time because there aren't any such times. However, it was not the case that it's possible that this rock exists, for the same reason. So, Prior Possibility is violated in what seems like a simple, conceivable scenario. However, Having Possibility is not violated. It has always been the case that it's possible that this rock exists, since there are no previous times when it was not possible that this rock exists. At the first time, all statements of the form *it was the case that...* are false.

Prior Possibility also struggles with things that have always existed, while Having Possibility doesn't. Imagine something that always exists, like some chunk of matter, or God. Either there is a first time, or there isn't. Suppose there is a first time. As we just saw, Prior Possibility will yield the wrong result about the modal status of the eternal existent, but Having Possibility won't. Suppose there isn't a first time. For an eternal existent, it won't ever have been the case that this thing didn't exist, and so Prior Possibility will be trivially true. But this is unhelpful. We'd like our principle to be able to properly distinguish what's going on with the modal and temporal statuses of things that exist at all times, like special lumps of matter or God, and things that don't, like cats and coffee cups. Having Possibility is sensitive to this, but Prior Possibility isn't; so we have reason to prefer Having Possibility here.

Here's another way to draw out the argument. Suppose that there is no first time. Suppose also that a always exists. Since there is no first time, and since a always exists, it's always true that it was the case that a exists. Since it's always true that it was the case that a exists, Prior Possibility will always be satisfied, but in an uninformative way: because a never actually *came* to exist, x just always existed. Having Possibility doesn't have this problem since it only uses *existence* rather than *coming to exist*, though Prior Possibility could easily be changed to accommodate this. More importantly, however, the consequent of Prior Possibility is also true in the same uninformative way. The consequent of Prior Possibility needs only one such time in the past in order for it to be true, and so doesn't distinguish between things that have always existed and things that haven't. Having Possibility is sensitive to this, as it requires that all—not just one—past times are such that it was possible that this thing exists.

Let's look at another problem. Yourgrau wants Prior Possibility to show that the once impossible can't come to exist (2019, p. 82). That is, if it was the case that it's impossible that a exists, then at no time is it the case that a comes to exist—things must endure possibility before they can ever achieve existence. It's not the case that a's creation makes its existence possible, but rather the reverse is true. It's a's existence having always been possible (i.e. a's existence not having previously been impossible) that is a necessary condition for it to ever come to exist. Having Possibility captures this idea.

Consider a square circle. Surely it's impossible that a square circle exists. But this is not merely because it wasn't previously possible that a square circle exists. It's because it has always been impossible that a square circle exists. By contrast, consider the tree outside my window. Surely it's possible that this tree exists, since it exists right now. It's also reasonable to think that there was a time in the past when it was possible that this tree exists (say, yesterday). But we can say a lot more than just this. It must also not have been impossible that this tree exists, else it would go the way of the square circle. Having Possibility captures both of these cases, but Prior Possibility doesn't.

Here's another way to put the argument. If it has always been possible that a exists, then according to Having Possibility it must also not be the case that there was a time when it was not possible that a exists. If a exists, not only must a have endured possibility, a must have also never endured impossibility. Having Possibility, like Prior Possibility, gives us that a's existence having been possible is a necessary condition on its coming to exist, but it also gives us—unlike Prior Possibility—that a's existence *never* having been impossible is also a necessary condition on its existence. This is a better way of understanding that the once impossible can't come to exist, so we should prefer Having Possibility.

For another reason to think that Having Possibility does the job here better than Prior Possibility, let's try to consider a case where where the once impossible comes to exist.[12] I will show that cases of the impossible coming to exist can satisfy Prior Possibility, but will violate Having Possibility. Since cases of the impossible coming to exist can satisfy Prior Possibility but will violate Having Possibility, Prior Possibility is not the correct principle for preventing the impossible from coming to exist. Having Possibility is the correct principle for preventing the impossible from coming to exist.

Imagine two worlds: one where I exist, and one where I don't.[13] Call the world where I exist w, and the world where I don't exist w'. Imagine three times in w': t_1, t_2, and t_3. Imagine also two additional times in w: t_4 and t_5. At all the times in w' it's not possible that I exist, since I do not exist at any (earlier) time in w'. At all the times in w it is possible that I exist, since I do exist at all (earlier) times in w. At t_5 it was the case that it's possible that I exist, since it's possible that I exist at t_4 and t_4 is earlier than t_5. Further, at t_5

[12] Thanks to an anonymous referee for pressing this point.

[13] For a presentation of this argument using a model, see appendix E. The use of two worlds is to avoid a contradiction that would otherwise result; I explain this along with the model.

it's not the case that it has always been the case that it's possible that I exist, since it's not possible that I exist at t_3. Notice now that Having Possibility does not hold at t_5, but that Prior Possibility does hold at t_5. Prior Possibility is satisfied at t_5, since at t_5 it was the case that it's possible that I exist. Having Possibility is not satisfied at t_5, since at t_5 I exist, but it has not always been the case that it's possible that I exist. That is, at t_5 the impossible has come to exist. Since it can be the case that the impossible has come to exist while Prior Possibility is satisfied, Prior Possibility can't be the right principle for preventing the impossible from coming to exist. Since in making it the case that the impossible has come to exist Having Possibility is violated, this gives us reason to think that Having Possibility is the right principle for preventing the impossible from coming to exist. So if we, with Yourgrau, want to rule out the once impossible coming to exist, then we want Having Possibility, not Prior Possibility.

Note here that a statement of the form *it has always been the case that...* only implies that *it was the case that...* with the additional (plausible) assumption that if something has always been the case, then it was the case. But we don't need this assumption for the discussion in this paper. Instead, what we have is that if it has always been the case that x, then it was not at some time the case that not x. There is an analogous case with necessity ($\Box$), possibility ($\Diamond$), and the modal T-axiom ($\Box\varphi \to \Diamond\varphi$). So, without the additional assumption that if something has always been the case then it was the case, Having Possibility does not entail Prior Possibility. Having Possibility only entails Prior Possibility if we add further assumptions.

6 Being and Existence

Yourgrau uses Prior Possibility to argue for the controversial distinction between being and existence.[14] Yourgrau (2019, pp. 26–27) suggests that both being and existence are primitive notions. Being is represented by the existential quantifier and by the words 'there is'. Existence is represented by the existence predicate and by the words 'there exists'. Being is not a kind of existence, but instead is understood as a bridge between existence and nonexistence. For example, before Socrates existed, Socrates was an individual that did not exist. That is, there was such a person as Socrates, who is an identifiable (by us, now) individual. Socrates had being, but did not exist.

Here's how Yourgrau (2019) argues that Prior Possibility supports the distinction between being and existence. Prior Possibility concerns individuals. That is, Prior Possibility says that no *individual* can come to exist without it having been previously possible that that *individual* exists. For example, since Socrates did in fact come to exist, it must have been previously possible for Socrates to exist. If it were not previously possible for Socrates to exist, then Socrates would not have been able to come to exist, since it would not have been possible for Socrates to exist. That is, according to Yourgrau (2019,

[14]Thanks to an anonymous referee for pressing this point.

pp. 27–28), Socrates couldn't have come to exist if it wasn't already possible that Socrates exists, and it couldn't have been possible that Socrates exists if there wasn't already such a person as Socrates. In order for it to be the case—before Socrates' coming to exist, before Socrates even has a history as a material thing—that it's possible that Socrates exists, there must be some such individual as Socrates. That is, Socrates must have being before he can have existence. If Socrates did not have being before he had existence, then there would not be any such individual for there to come to exist at all. But clearly an identifiable individual—Socrates—did come to exist. So, Yourgrau argues, in order to make sense of individuals coming to exist, we have to accept Prior Possibility. Given the close connection between possibility and being, if we accept Prior Possibility, then we should also accept the distinction between being and existence.

What I argue for in this paper gives further support to the distinction between being and existence. Having Possibility, and the other related principles I argue for, concern individuals, just like Prior Possibility does. However, Having Possibility is not restricted strictly to individuals. Rather, Having Possibility is concerned with anything that could exist, such as properties, temporal parts, person-stages, four-dimensional space-time worms, and so on. Some of the very things that Prior Possibility does not address. As I've argued, Having Possibility is a better way to make sense of things, individuals and more, coming to exist than Prior Possibility is. If we accept Having Possibility, then we should accept the distinction between being and existence for the same reasons as we saw with Prior Possibility. The arguments for the distinction between being and existence using Prior Possibility work just as well using Having Possibility, and the additional support I've given for preferring Having Possibility over Prior Possibility adds to this. That is, if accepting Prior Possibility means we should accept the distinction between being and existence, and we should accept the stronger principle Having Possibility instead of Prior Possibility, then we should still accept the distinction between being and existence. There's nothing substantially different between Prior Possibility and Having Possibility with respect to understanding coming to exist. All this is to say that it's not merely that Socrates couldn't have come to exist if it *wasn't already* possible that Socrates exists, but rather that Socrates couldn't have come to exist if it *hadn't always been* possible that Socrates exists. In other words, it's not the Socrates must have *previously* had being in order to come to exist, but rather that Socrates must have *always* had being in order to come to exist. Since we have more reason to accept Having Possibility over accepting only Prior Possibility, and since Having Possibility is, with good reason, about more kinds of things than Prior Possibility, this gives us more reason to accept the distinction between being and existence (or at least that the distinction between being and existence is applicable to more kinds of things than just individuals). This is also the case for the further principles I argue for, and their entailments, in the following section.

7 Diamonds Really are Forever

In this section I show that Having Possibility, along with analogous principles for the present and for the future, entail a surprising result: nothing can come to exist without its existence always being possible. That is, nothing is the case unless it's always possible that it's the case.

First the future (by way of the past). There is a principle analogous to Necessarily Not Future, but for past times. This is Necessarily Not Past:

Necessarily Not Past If it's necessarily not the case that x, then it was not the case that x.

Necessarily Not Past* If x was the case, then it's possible that x is the case.

Things that happened are possible. My arguments for Necessarily Not Future apply just as well to Necessarily Not Past; just change from the future to the past as needed. In the interest of time we won't go through this.

Like how Having Possibility follows from Necessarily Not Future, an analogous principle follows from Necessarily Not Past.[15] This is Going Possibility:

Going Possibility If x is the case, then it will always be the case that it's possible that x is the case.

Nothing can come to exist without its existence remaining possible. As with Necessarily Not Future and Necessarily Not Past, my arguments for Having Possibility apply just as well to Going Possibility; just change from the past to the future as needed. In the interest of time we won't go through this.

We have principles for past and future times, so now we need one for present times. This is just the modal T axiom:

T If x is the case, then it's possible that x is the case.

This is uncontroversial; the modal T axiom is part of any tenable modal logic.

Putting Having Possibility, T, and Going Possibility together, a final principle follows.[16] This is Always Possible:

Always Possible If x is the case, then it's always possible that x is the case.

The reasoning is straightforward. So, suppose that x is the case. Since x is the case and Having Possibility, T, and Going Possibility are all true, it follows that: it has always been possible that x is the case, it's possible that x is the case, and it will always be possible that x is the case. But if x has always been the case, is the case, and will always be the case, then x must always be the case. So, it's always the case that it's possible that x is the case. Therefore, if x is the case, then it's always possible that x is the case. Nothing is the case unless it's always possible that it's the case.

[15] For the derivation, see appendix F.5.
[16] For the derivation, see appendix F.6.

Our overall result is that if something is the case, then it's always possible that it's the case. This is exactly what we need in order to completely stop the once impossible from occurring. To completely stop the impossible from occurring, we need *it's the case that* x and *it's at some time the case that it's impossible that* x to be inconsistent. So, we need it to be a principle that it's not the case that both: it's the case that x and it's at some time the case that it's not possible that x. But *it's not the case that both: it's the case that* x *and it's at some time the case that it's not possible that* x is equivalent to *if it's the case that* x*, then it's the case that it's always possible that* x. And *If it's the case that* x*, then it's the case that it's always possible that* x is just Always Possible.[17] So, if we really do want to stop the once impossible from occurring, then we should accept AP.

8 Always Possible and Origin Essentialism

At this point we might be concerned about whether the principles I've argued for play nicely with Origin Essentialism.[18] That is, for example, you might wonder whether it can be both: the case that it's always possible that my cat exists and essential to my cat that he have the particular origin that he in fact has. However, we might actually be concerned more broadly with whether the principles I've argued for play nicely with a very general, standard idea about essences, of which Origin Essentialism is an instance:[19]

Necessary Essences Necessarily, if being F is essential to being x, then at all times when x exists, x is F.

Necessary Essences is just a very flat-footed analysis of what it means for F to be essential to x. If being human is essential to being Socrates, then any time that Socrates exists, Socrates must also be human. If having four sides is essential to being a square, then any time a square exists, that square must have four sides. If all that sounds right to you, then you probably accept some version of Necessary Essences.

Origin Essentialism is a version of Necessary Essences that's just about the origins of things. A direct way to state Origin Essentialism is this:[20]

Origin Essentialism Necessarily, if x is an individual with origin o, then having o is essential to x.

We might think that my having been born to my particular parents is essential to me. Any time that I exist, I must have had the particular parents that I

[17] For a proof of this equivalence, see appendix F.7.

[18] Thanks to an anonymous referee for pressing this point. I've separately been working on a similar line of argument from just Sometime Possible and the modal B axiom (as Sometime Possible and B are enough to get Always Possible) in a current work in progress, which I began around April 2023. This paper was submitted for review the prior December.

[19] For more the topic, see Robertson Ishii and Atkins 2023.

[20] For more on the topic, see Kripke 1972; Robertson Ishii and Atkins 2020.

in fact have. We might also think that having been made in a certain place is essential to my desk. Any time that my desk exists, it must have been made in the certain place that it was in fact made. If you find those examples plausible, then you'd probably accept some form of Origin Essentialism. Many instances of Necessary Essences play nicely with the principles I've argued for. Origin Essentialism is not one of them.

Here's why Origin Essentialism doesn't play nicely with the principles I've argued for. Recall that what I've argued for ultimately entails Always Possible: if x is the case, then it's always (i.e. at all times) possible that x is the case. If we like Origin Essentialism, then we'd be inclined to think that it's essential to my cat, Kal, that he be born in 2020, since that's when he was in fact born. Any cat born at any other time must not be Kal. If being born in 2020 is essential to Kal, then according to Necessary Essences any time that Kal exists, Kal is such that he is born in 2020. However, according to Always Possible, since Kal exists, it's always possible that Kal exists. This means that it's possible that Kal exists at times before he was born. For example, if Kal were to exist only from 1920 to 1940, then he could not have also been born in 2020; he must have been born in 1920.[21] But if Kal could have been born in 1920, then it's possible that Kal is such that he was born in 1920. But if it's possible that Kal is such that he was born in 1920, then being such that he was born in 2020 is not essential to Kal. So, either Origin Essentialism or Always Possible must be false in this case. As I've argued in this paper, Always Possible is true in all cases. So, it must be Origin Essentialism that's false in this case.

I can say for now that, broadly, the versions of Necessary Essences that don't play well with the principles I've argued for are just those versions that allow for things with temporal components to be essences, as we just saw. If essences are restricted to things without temporal components, like haecceities (e.g. the property of being Kal) or certain species kinds (e.g. being a feline),[22] then the argument above doesn't go through.

There's nothing wrong with it being the case that it's both: essential to Kal that he has the property of being Kal and possible that Kal exists at times other than those when he in fact does. Likewise, there's nothing wrong with it being the case that it's both: essential to Kal that he is a feline and possible that Kal exists at times other than those when he in fact does. But there is something wrong with it being the case that it's both: essential to Kal that he was born in 2020 and possible that Kal exists at times other than those when he in fact does.

Temporal things just aren't good candidates for essences, given Always Possible. A things origins are, in a sense, temporal things. So, a things origins

[21] You may think that for it to be the case that Kal was born in 1920, Kal's parents must have been born in 1915, say. But Kal's parents were in fact born in 2015. (I don't *actually* know when Kal's parents were born, but that's not really important here; 2015 is probably close enough.) According to Always Possible, since Kal's parents were in fact born in 2015, it's always possible that Kal's parents are born in 2015. This means that it's possible that Kal is born in 1920, but also possible that Kal's parents are born in 2015 (*possible*, not *actual*). So, it doesn't need to also be the case that Kal's parents must have been born in 1915.

[22] There's some weirdness with caterpillars and butterflies, for example.

aren't good candidates for it's essences.

9 Conclusion

To recap, in this paper I've argued that we should prefer my principle of Having Possibility to Yourgrau's principle of Prior Possibility. Further, I've also argued that we then will have to accept the principle Always Possible.

Though Yourgrau's principle of Prior Possibility sounds very plausible, it isn't, in the end, going to do what he wants. Instead, what Yourgrau needs is Always Possible. Further, insofar as Prior Possibility supports the distinction between being and existence, so do Having Possibility, Going Possibility, and Always Possible. This means, then, that if something has being, it must also have being at all times. Things can neither begin nor cease to be.

One path for further development is already in progress, as mentioned in footnote 18. Sometime Possible and B are both very plausible principles, but they jointly entail the same results we've seen in the paper. Note, as I mentioned at the end of section 2, that the principles I've argued for in this paper are strictly weaker than Sometime Possible and B. If you found my principles strange, then maybe you'd be more comfortable with just Sometime Possible and B. But if you accept Sometime Possible and B, then the results I've argued in this paper still stand. If you don't accept even the weaker principles that I've argued for, then you have to give up either Sometime Possible or B. I don't think that we can reasonably give up either Sometime Possible or B.

Another path for further development is to compare some of my results to similar things others have argued for. For example, Dorr and Goodman (2020) argue for Perpetuity:

Perpetuity If a proposition is necessarily true, then it's always true.

This is similar to Always Possible, which I've argued for in this paper, and is, fundamentally, just a version of Necessarily Always. At minimum, their arguments add additional weight to the plausibility of my project here.

If you can't possibly accept the results I've found, you have three options: (i) reject the modal T axiom; (ii) reject temporal logic; or, (iii) reject my principles.[23] The first two are hard roads to walk. I take the modal T axiom to be untouchable, and temporal logic (or, more precisely, the basic temporal inferences that make it up) at least nearly so. That said, rejecting Necessarily Not Future, Necessarily Not Past, or whichever ones you want to get rid of isn't much easier! We are better off, I think, accepting Always Possible: that if something is the case then it has always, is, and will always be possible that it's the case. Or, in words Yourgrau might prefer, that nothing becomes actual without always being possible.

[23] Or, as I mentioned in footnote 18, (iv) reject either Sometime Possible or B.

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A The Formal Language

In this appendix I set out the basic formal system that I use in support of various arguments. A minimal, alethic modal-logic KT:[24]

$$\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi) \qquad (\mathrm{K}_\Box)$$

$$\Box\varphi \to \varphi \qquad (\mathrm{T}_\Box)$$

$$\varphi \to \Diamond\varphi \qquad (\mathrm{T}_\Diamond)$$

$$\text{If } \vdash \varphi, \text{ then } \vdash \Box\varphi \qquad (\mathrm{NEC}_\Box)$$

Plus Arthur Prior's (1957) basic temporal logic TL.[25] The sentence operators:

[24]See Garson 2021.

[25]See Goranko and Rumberg 2021.

P: It was the case that...

F: It will be the case that...

H: It has always been the case that...

G: It will always be the case that...

A: It is at all times the case that...

S: It is at some time the case that...

$A\varphi$ is equivalent to $H\varphi \wedge \varphi \wedge G\varphi$, and $S\varphi$ to $P\varphi \vee \varphi \vee F\varphi$. A and S are duals; P and H are duals; and, F and G are duals. Also from TL, the following axiom schemata and inference rules:

$$H(\varphi \to \psi) \to (H\varphi \to H\psi) \tag{K_H}$$
$$G(\varphi \to \psi) \to (G\varphi \to G\psi) \tag{K_G}$$
$$\varphi \to HF\varphi \tag{HF-ax}$$
$$\varphi \to GP\varphi \tag{GP-ax}$$
$$\text{If } \vdash \varphi, \text{ then } \vdash H\varphi \tag{NEC_H}$$
$$\text{If } \vdash \varphi, \text{ then } \vdash G\varphi \tag{NEC_G}$$

Plus the standard rules for the connectives. I occasionally use an existence predicate, $E!$.

A model M is a tuple $\langle W, T, R^W, R^T, \prec, V\rangle$ with a set of worlds W, a set of times T,[26] a modal accessibility-relation R^W, a temporal accessibility-relation R^T, a two-place earlier-than relation on world-time pairs $\prec$, and a valuation function V that maps formulas and world-time pairs to the truth-values $\{T, F\}$. Formulas are evaluated at world-time pairs $\langle w, t\rangle$. A formula is true in a model iff it's true at all world-time pairs in the model. A formula is valid iff it's true in all models. Modal accessibility is illustrated with a dashed arrow, temporal accessibility with a dotted arrow, and joint modal and temporal accessibility with a solid arrow.

I assume that modal accessibility is reflexive. However, our the temporal and modal accessibility relations don't need to obey all the same rules.[27] In order to validate HF-ax and GP-ax, R^T needs to be connected to $\prec$ in some way. This is a result of my differentiating between temporal accessibility and the temporal ordering. I propose this way:

$$\text{If } \langle w,t\rangle \prec \langle w',t'\rangle \vee \langle w',t'\rangle \prec \langle w,t\rangle, \quad \text{then } R^T\langle w,t\rangle t' \to R^T\langle w',t'\rangle t \tag{Weak Temporal Symmetry}$$

[26]For simplicity, lonely times $\langle \varnothing, t\rangle$ and lonely worlds $\langle w, \varnothing\rangle$ are prohibited.

[27]Assuming that temporal accessibility is reflexive entails Prior Possibility. See appendix F.1.

That is, that if $\langle w,t\rangle$ stands in the temporal ordering in some way to $\langle w',t'\rangle$, then if t' is temporally accessible to $\langle w,t\rangle$, then t is temporally accessible to $\langle w',t'\rangle$.

I use a slightly unconventional semantics:[28]

- $\langle w,t\rangle \models p$ iff $V(p) = T$.
- $\langle w,t\rangle \models \neg p$ iff $\langle w,t\rangle \not\models p$.
- $\langle w,t\rangle \models (\varphi \vee \psi)$ iff either: $\langle w,t\rangle \models \varphi$, or $\langle w,t\rangle \models \psi$.
- $\langle w,t\rangle \models (\varphi \wedge \psi)$ iff both: $\langle w,t\rangle \models \varphi$, and $\langle w,t\rangle \models \psi$.
- $\langle w,t\rangle \models (\varphi \rightarrow \psi)$ iff either: $\langle w,t\rangle \models \neg\varphi$, or $\langle w,t\rangle \models \psi$.
- $\langle w,t\rangle \models \Diamond\varphi$ iff $\exists w' \exists t'$ such that both:
  (i) $R^W\langle w,t\rangle w'$, and
  (ii) $\langle w',t'\rangle \models \varphi$.
- $\langle w,t\rangle \models P\varphi$ iff $\exists w' \exists t'$ such that:
  (i) $\langle w',t'\rangle \prec \langle w,t\rangle$,
  (ii) $R^T\langle w,t\rangle t'$, and
  (iii) $\langle w',t'\rangle \models \varphi$.
- $\langle w,t\rangle \models F\varphi$ iff $\exists w' \exists t'$ such that:
  (i) $\langle w,t\rangle \prec \langle w',t'\rangle$,
  (ii) $R^T\langle w,t\rangle t'$, and
  (iii) $\langle w',t'\rangle \models \varphi$.

Notice in particular that in the clause for $\Diamond$, t is allowed to vary; likewise for w in P and F.

I realize that some of my choices here might seem odd, but there are advantages that my choices have over some more familiar options. The rest of this appendix is devoted to showing this.

What exactly the two accessibility relations are needs further explanation first. They are two-place relations that relate some world-time pair to either a world (in the case of R^W) or a time (in the case of R^T). R^W picks out worlds that are relevant for modal evaluation, while R^T picks out times that are relevant for temporal evaluation. To say that a world is modally accessible to a world-time pair is just to say that how things are at that world are relevant to the truth or falsity of modal statements at that world-time pair. Likewise, to say that a time is temporally accessible to a world-time pair is just to say that how

[28] It's at least sound; see appendix C. I haven't proven either completeness or incompleteness. Canonical model completeness proofs don't quite work here, but there seems to be a route to completeness by way of proving correspondence with some more familiar $T \times W$ models.

things are at that time are relevant to the truth or falsity of tensed statements at that world-time pair. If these accessibility relations were relations between world-time pairs, we'd be building into these relations that times or worlds are relevant for modal or temporal evaluation, respectively. While they are in some way relevant, this relevance is only incidental, not essential.

Here's a mereological analogy to illustrate this, and to emphasize why they are relations between world-time pairs and either a world or a time, rather than between world-time pairs. Think of a world-time pair as a mereological fusion of a world and a time (suppose also that worlds and times are distinct).

What R^W says is that some *fusion* stands in a relevant-for-modal-evaluation relation to the *world-part* of some fusion. On its own, it doesn't say that the fusion stands in a relation to the time-part of the fusion, nor that the fusion stands in relation to the fusion. It also doesn't say anything about relevance-for-temporal-evaluation; that's what R^T is for.

What R^T says is that some *fusion* stands in a relevant-for-temporal-evaluation relation to the *time-part* of some fusion. On its own, it doesn't say that the fusion stands in a relation to the world-part of the fusion, nor that the fusion stands in relation to the fusion. It also doesn't say anything about relevance-for-modal-evaluation; that what R^W is for.

Note that R^T and $\prec$ are distinct. R^T is about what's relevant for temporal evaluation, but $\prec$ is about temporal ordering. Temporal ordering is not all that matters for temporal evaluation. For example, imagine a world just like ours except everything there occurs one day earlier than it does here. Barring additional information, the times in that other world are not relevant for temporal evaluation in our world (they may turn out to be relevant for modal evaluation in our world, but that's a separate matter). If they were, we'd get odd results when making temporal evaluations. For example, imagine that later today you finish reading this paper for the first time. If $\prec$ was all that mattered to temporal evaluation, then since the other world is one day "ahead" of us, it would be true now—in our world—that it was the case that you finish reading the paper for the first time. But that can't be right, not if you haven't actually finished reading this paper for the first time yet! So, since temporal ordering can't be the only thing that's relevant to temporal evaluation, we have reason to want R^T rather than just $\prec$.

Another difference is that $\prec$ is transitive, but R^T doesn't need to be. World War I is earlier than World War II, and World War II is earlier than today, so World War I must be earlier than today. If $\prec$ was all that mattered to temporal evaluation, then it would be true now that it was the case that World War I happened. Using an unmodified R^T however, what is true now is that it was the case that it was the case that World War I happened (assuming backwards accessibility). Now, I think it's likely that R^T should be transitive. However, I don't need to make this assumption for my purposes in this paper, so I won't.

A final difference is that $\prec$ assumes forward accessibility, but R^T doesn't need to. If $\prec$ was all that mattered to temporal evaluation, then for all the things that will happen in the future, it's true now that it will be the case that those things happen. However, using R^T allows you to not assume that our future is accessible to us now, if you want. As with transitivity, I think it's likely that R^T should go forward. However, I also don't need to make this assumption for my purposes in this paper, so I won't.

Even if the time in the accessed world-time pair changes, this may not affect whether it's relevant for modal evaluation. R^W is about what's relevant for modal evaluation: worlds. Even if the time changes, this may have no impact; R^W is not, itself, directly concerned with times. Modal relations should be *modal*; if times figure into it at all, they should only do so incidentally, not essentially. Likewise for a change in the world and relevance to temporal evaluation. R^T is about what's relevant for temporal evaluation: times. Even if the world changes, this may have no impact; R^T is not, itself, directly concerned with worlds. Temporal relations should be *temporal*; if worlds figure into it at all, they should only do so incidentally, not essentially. My choice to allow the other parameter in the clauses for the modal and temporal operators to vary, though it may seem odd at first, captures this.

It's easy to see that, given my semantics, the various modal axioms will be valid. However, it's not as obvious that this is the case for the temporal axioms. Lest you worry, they are, in fact, valid. The full proof of soundness for my proof system relative to my semantics is in appendix C.

Next, it's worth noticing a further, interesting result. Given my semantics, we have that if $\langle w,t\rangle \models \varphi$, then, for any t', $\langle w,t'\rangle \models \Diamond\varphi$.[29]

Proof. Suppose, for reductio, that:

$$\langle w,t\rangle \models \varphi \tag{1}$$

and:

$$\langle w,t'\rangle \not\models \Diamond\varphi \tag{2}$$

From (2), it follows that for all w' and all t'':

$$R^W\langle w,t'\rangle w' \rightarrow \langle w',t''\rangle \not\models \varphi \tag{3}$$

But, since R^W is reflexive, and letting $w' = w$ and $t'' = t$, it follows that:

$$\langle w,t\rangle \not\models \varphi \tag{4}$$

which contradicts (1). Therefore, by reductio, it follows that:

$$\langle w,t\rangle \models \varphi \rightarrow \langle w,t'\rangle \models \Diamond\varphi \tag{5}$$

as desired. □

This is relevant to section 5, but note that it's not as strange a result as it may seem. Remember, as I argued earlier, barring any kind of explicit bridge between modality and time, what matters to modal evaluation is just worlds.

[29] Note also that since R^T is not reflexive, we *don't* have an analogous result that if $\langle w,t\rangle \models \varphi$, then, for any w', $\langle w',t\rangle \models S\varphi$.

So, if p is true at a world, then $\Diamond p$ is going to be true there too. But, according to R^W alone, this is all that matters to modal evaluation. Times don't matter to R^W. So, regardless of what time we're considering, the modal results will be the same. As I argue in section 2, we have reason to accept certain bridge principles that tell us how modality and time relate, but, absent those bridges, what matters to modal evaluation and what matters to temporal evaluation are separate.

B Formalizations of Various Principles

Necessarily Not Past:

$$\Box\neg\varphi \to \neg P\varphi \quad \text{(NNP)}$$
$$P\varphi \to \Diamond\varphi \quad \text{(NNP*)}$$

Necessarily Not Future:

$$\Box\neg\varphi \to \neg F\varphi \quad \text{(NNF)}$$
$$F\varphi \to \Diamond\varphi \quad \text{(NNF*)}$$

Sometime Possible:

$$S\varphi \to \Diamond\varphi \quad \text{(SP)}$$

Necessarily Always:

$$\Box\varphi \to A\varphi \quad \text{(NA)}$$

Having Possibility:

$$\varphi \to H\Diamond\varphi \quad \text{(HP)}$$

The modal T axiom:

$$\varphi \to \Diamond\varphi \quad \text{(T)}$$

Going Possibility:

$$\varphi \to G\Diamond\varphi \quad \text{(GP)}$$

Always Possible:

$$\varphi \to A\Diamond\varphi \quad \text{(AP)}$$

C Soundness

Theorem 1 (Soundness)**.** *If* $\Gamma \vdash \varphi$*, then* $\Gamma \models \varphi$*.*

Proof. By strong induction on the structure of derivations, if $\Gamma \vdash \varphi$ then $\Gamma \models \varphi$.

1. If the derivation consists only of the assumption φ, then we have $\varphi \vdash \varphi$, and want to show that $\varphi \models \varphi$. So, suppose that $\langle w, t\rangle \models \varphi$. Then, trivially, $\langle w, t\rangle \models \varphi$, as desired.

2. Suppose that the derivation ends in $\wedge$I, concluding $\varphi \wedge \psi$. This means that this derivation has the premises φ and ψ, with undischarged assumptions Γ and Δ, respectively. So we have that $\Gamma \vdash \varphi$ and $\Delta \vdash \psi$. By the inductive hypothesis, we have $\Gamma \models \varphi$ and $\Delta \models \psi$. It remains to show that $\Gamma \cup \Delta \models \varphi \wedge \psi$.

   So, suppose that $\langle w, t\rangle \models \Gamma \cup \Delta$. Since $\langle w, t\rangle \models \Gamma$ and $\Gamma \models \varphi$, it follows that $\langle w, t\rangle \models \varphi$. Since $\langle w, t\rangle \models \Delta$ and $\Delta \models \psi$, it follows that $\langle w, t\rangle \models \psi$. Since both $\langle w, t\rangle \models \varphi$ and $\langle w, t\rangle \models \psi$, it follows that $\langle w, t\rangle \models \varphi \wedge \psi$, as desired.

3. Suppose that the derivation ends in $\wedge$E, concluding φ. This means that this derivation has the premise $\varphi \wedge \psi$, with undischarged assumptions Γ. So, we have that $\Gamma \vdash \varphi \wedge \psi$. By the inductive hypothesis, we have $\Gamma \models \varphi \wedge \psi$. It remains to show that $\Gamma \models \varphi$.

   So, suppose that $\langle w, t\rangle \models \Gamma$. Since $\langle w, t\rangle \models \Gamma$ and $\Gamma \models \varphi \wedge \psi$, it follows that $\langle w, t\rangle \models \varphi \wedge \psi$. Since $\langle w, t\rangle \models \varphi \wedge \psi$, it follows that $\langle w, t\rangle \models \varphi$, as desired.

   Likewise for concluding ψ, by parity of reasoning.

4. Suppose that the derivation ends in $\vee$I, concluding $\varphi \vee \psi$. This means that this derivation either has the premise φ or the premise ψ, with undischarged assumptions Γ. So, we have either that $\Gamma \vdash \varphi$ or $\Gamma \vdash \psi$. By the inductive hypothesis, we have either that $\Gamma \models \varphi$ or $\Gamma \models \psi$. It remains to show that $\Gamma \models \varphi \vee \psi$.

   If the premise is φ first. Suppose that $\langle w, t\rangle \models \Gamma$. Since $\langle w, t\rangle \models \Gamma$ and $\Gamma \models \varphi$, $\langle w, t\rangle \models \varphi$. Since $\langle w, t\rangle \models \varphi$, it follows that $\langle w, t\rangle \models \varphi \vee \psi$, as desired.

   Likewise if the premise is ψ, by parity of reasoning.

5. Suppose that the derivation ends in $\vee$E, concluding χ. This means that the derivation has the premise $\varphi \vee \psi$ with undischarged assumptions Γ, a derivation ending in χ from φ and undischarged assumptions Δ_1, and a derivation ending in χ from ψ and undischarged assumptions Δ_2. So, we have that $\Gamma \vdash \varphi \vee \psi$, $\Delta_1 \cup \varphi \vdash \chi$, and $\Delta_2 \cup \psi \vdash \chi$. By the inductive hypothesis, we have that $\Gamma \models \varphi \vee \psi$, $\Delta_1 \cup \varphi \models \chi$, and $\Delta_2 \cup \varphi \models \chi$. It remains to show that $\Gamma \cup \Delta_1 \cup \Delta_2 \models \chi$.

   So, suppose that $\langle w, t\rangle \models \Gamma \cup \Delta_1 \cup \Delta_2$. Since $\langle w, t\rangle \models \Gamma$ and $\Gamma \models \varphi \vee \psi$, it follows that $\langle w, t\rangle \models \varphi \vee \psi$. So, it follows that either $\langle w, t\rangle \models \varphi$ or $\langle w, t\rangle \models \psi$. Consider each case:

   (a) If $\langle w, t\rangle \models \varphi$, then since $\langle w, t\rangle \models \Delta_1$ it follows that $\langle w, t\rangle \models \Delta_1 \cup \varphi$. Since $\langle w, t\rangle \models \Delta_1 \cup \varphi$ and $\Delta_1 \cup \varphi \models \chi$, it follows that $\langle w, t\rangle \models \chi$.

   (b) If $\langle w, t\rangle \models \psi$, then since $\langle w, t\rangle \models \Delta_2$ it follows that $\langle w, t\rangle \models \Delta_2 \cup \psi$. Since $\langle w, t\rangle \models \Delta_2 \cup \psi$ and $\Delta_2 \cup \psi \models \chi$, it follows that $\langle w, t\rangle \models \chi$.

So, in either case, $\langle w, t\rangle \models \chi$, as desired.

6. Suppose that the derivation ends in $\rightarrow$I, concluding $\varphi \rightarrow \psi$. This means that we have a derivation from φ and undischarged assumptions Γ to ψ. So, we have that $\Gamma \cup \varphi \vdash \psi$. By the inductive hypothesis, we have $\Gamma \cup \varphi \models \psi$. It remains to show that $\Gamma \models \varphi \rightarrow \psi$.

   So, suppose that $\langle w, t\rangle \models \Gamma$. We then need to show that on the further assumption that $\langle w, t\rangle \models \varphi$, it follows that $\langle w, t\rangle \models \psi$. So, suppose that $\langle w, t\rangle \models \varphi$. Since $\langle w, t\rangle \models \Gamma$ and $\langle w, t\rangle \models \varphi$, it follows that $\langle w, t\rangle \models \Gamma \cup \varphi$. Since $\langle w, t\rangle \models \Gamma \cup \varphi$ and $\Gamma \cup \varphi \models \psi$, it follows that $\langle w, t\rangle \models \psi$. So, since on the assumption that $\langle w, t\rangle \models \varphi$, it follows that $\langle w, t\rangle \models \psi$, it follows that $\langle w, t\rangle \models \varphi \rightarrow \psi$, as desired.

7. Suppose that the derivation ends in $\rightarrow$E, concluding ψ. This means that the derivation has the premises $\varphi \rightarrow \psi$ and φ, with undischarged assumptions Γ and Δ, respectively. So, we have that $\Gamma \vdash \varphi \rightarrow \psi$ and $\Delta \vdash \varphi$. By the inductive hypothesis, we have that $\Gamma \models \varphi \rightarrow \psi$ and $\Delta \models \varphi$. It remains to show that $\Gamma \cup \Delta \models \psi$.

   So, suppose that $\langle w, t\rangle \models \Gamma \cup \Delta$. Since $\langle w, t\rangle \models \Gamma$ and $\Gamma \models \varphi \rightarrow \psi$, it follows that $\langle w, t\rangle \models \varphi \rightarrow \psi$. Similarly, since $\langle w, t\rangle \models \Delta$ and $\Delta \models \varphi$, it follows that $\langle w, t\rangle \models \varphi$. Since $\langle w, t\rangle \models \varphi \rightarrow \psi$, we have that either $\langle w, t\rangle \models \neg\varphi$ or $\langle w, t\rangle \models \psi$. Consider each case:

   (a) If $\langle w, t\rangle \models \neg\varphi$, this contradicts that $\langle w, t\rangle \models \varphi$, and so $\langle w, t\rangle \models \psi$.
   (b) If $\langle w, t\rangle \models \psi$, then, trivially, $\langle w, t\rangle \models \psi$.

   So, in either case, $\langle w, t\rangle \models \psi$, as desired.

8. Suppose that the derivation ends in $\neg$I, concluding $\neg\varphi$. This means that we have a derivation from φ and undischarged assumptions Γ to $\bot$. So, we have that $\Gamma \cup \varphi \vdash \bot$. By the inductive hypothesis, we have that $\Gamma \cup \varphi \models \bot$. It remains to show that $\Gamma \models \neg\varphi$.

   So, suppose that $\langle w, t\rangle \models \Gamma$. Suppose, for reductio, that $\langle w, t\rangle \models \varphi$. Since $\langle w, t\rangle \models \Gamma$ and $\langle w, t\rangle \models \varphi$, it follows that $\langle w, t\rangle \models \Gamma \cup \varphi$. Since $\langle w, t\rangle \models \Gamma \cup \varphi$ and $\Gamma \cup \varphi \models \bot$, it follows that $\langle w, t\rangle \models \bot$. But, since $\bot$ is a contradiction, it follows that $\langle w, t\rangle \models \neg\varphi$, as desired.

9. Suppose that the derivation ends in $\neg$E, concluding $\bot$. This means that the derivation has premises φ and $\neg\varphi$ with undischarged assumptions Γ and Δ, respectively. So, we have that $\Gamma \vdash \varphi$ and $\Delta \vdash \neg\varphi$. By the inductive hypothesis, we have that $\Gamma \models \varphi$ and $\Delta \models \neg\varphi$. It remains to show that $\Gamma \cup \Delta \models \bot$.

   So, suppose, for reductio, that $\langle w, t\rangle \models \Gamma \cup \Delta$ but $\langle w, t\rangle \not\models \bot$. Since $\langle w, t\rangle \models \Gamma$ and $\Gamma \models \varphi$, it follows that $\langle w, t\rangle \models \varphi$. Likewise, since $\langle w, t\rangle \models \Delta$ and $\Delta \models \neg\varphi$, it follows that $\langle w, t\rangle \models \neg\varphi$. Since $\langle w, t\rangle \models \varphi$ and $\langle w, t\rangle \models \neg\varphi$, it follows that $\langle w, t\rangle \models \bot$, which contradicts our assumption that $\langle w, t\rangle \not\models \bot$. So, it follows that $\langle w, t\rangle \models \bot$, as desired.

10. Suppose that the derivation ends in $K_\Box$, concluding $\Box\varphi \to \Box\psi$. This means that this derivation has the premise $\Box(\varphi \to \psi)$ with undischarged assumptions Γ. So, we have that $\Gamma \vdash \Box(\varphi \to \psi)$. By the inductive hypothesis, we have that $\Gamma \models \Box(\varphi \to \psi)$. It remains to show that $\Gamma \models \Box\varphi \to \Box\psi$.

    So, suppose that $\langle w, t\rangle \models \Gamma$ and that $\langle w, t\rangle \models \Box\varphi$, with the aim of showing that $\langle w, t\rangle \models \Box\psi$. Since $\langle w, t\rangle \models \Gamma$ and $\Gamma \models \Box(\varphi \to \psi)$, it follows that $\langle w, t\rangle \models \Box(\varphi \to \psi)$. This means that for all w' and all t', if $R^W\langle w, t\rangle w'$, then $\langle w', t'\rangle \models \varphi \to \psi$. Since $\langle w, t\rangle \models \Box\varphi$, this means that for all w' and all t', if $R^W\langle w, t\rangle w'$, then $\langle w', t'\rangle \models \varphi$.

    From these results, it follows that for all w' and all t', if $R^W\langle w, t\rangle w'$, then $\langle w', t'\rangle \models \psi$. But this just means that $\langle w, t\rangle \models \Box\varphi$, which is what we wanted.

11. Suppose that the derivation ends in $T_\Box$, concluding φ. This means that the derivation has the premise $\Box\varphi$ and undischarged assumptions Γ. So, we have that $\Gamma \vdash \Box\varphi$. By the inductive hypothesis we have that $\Gamma \models \Box\varphi$. It remains to show that $\Gamma \models \varphi$.

    So, suppose that $\langle w, t\rangle \models \Gamma$. Since $\langle w, t\rangle \models \Gamma$ and $\Gamma \models \Box\varphi$, it follows that $\langle w, t\rangle \models \Box\varphi$. Since $\langle w, t\rangle \models \Box\varphi$, for all w' and all t', if $R^W\langle w, t\rangle w'$, then $\langle w', t'\rangle \models \varphi$. But since R^W is reflexive, it follows that $\langle w, t\rangle \models \varphi$, as desired.

12. Suppose that the derivation ends in $T_\Diamond$, concluding $\Diamond\varphi$. This means that the derivation has the premise φ with undischarged assumptions Γ. So, we have that $\Gamma \vdash \varphi$. By the inductive hypothesis we have that $\Gamma \models \varphi$. It remains to show that $\Gamma \models \Diamond\varphi$.

    So, suppose that $\langle w, t\rangle \models \Gamma$. Since $\langle w, t\rangle \models \Gamma$ and $\Gamma \models \varphi$, it follows that $\langle w, t\rangle \models \varphi$. Since R^W is reflexive and $\langle w, t\rangle \models \varphi$, it follows that $\langle w, t\rangle \models \Diamond\varphi$, as desired.

13. Suppose that the derivation ends in K_H, concluding $H\varphi \to H\psi$. This means that this derivation has the premise $H(\varphi \to \psi)$ with undischarged assumptions Γ. So, we have that $\Gamma \vdash H(\varphi \to \psi)$. By the inductive hypothesis, we have that $\Gamma \models H(\varphi \to \psi)$. It remains to show that $\Gamma \models (H\varphi \to H\psi)$.

    So, suppose that $\langle w, t\rangle \models \Gamma$ and that $\langle w, t\rangle \models H\varphi$, with the aim of showing that $\langle w, t\rangle \models H\psi$. Since $\langle w, t\rangle \models \Gamma$ and $\Gamma \models H(\varphi \to \psi)$, it follows that $\langle w, t\rangle \models H(\varphi \to \psi)$. Since $\langle w, t\rangle \models H(\varphi \to \psi)$, it follows that for all w' and all t', if $\langle w', t'\rangle \prec \langle w, t\rangle$ and $R^T\langle w, t\rangle t'$, then $\langle w', t'\rangle \models \varphi \to \psi$. Since $\langle w, t\rangle \models H\varphi$, it follows that for all w' and all t', if $\langle w', t'\rangle \prec \langle w, t\rangle$ and $R^T\langle w, t\rangle t'$, then $\langle w', t'\rangle \models \varphi$.

    From these results it follows that for all w' and all t', if $\langle w', t'\rangle \prec \langle w, t\rangle$ and $R^T\langle w, t\rangle t'$, then $\langle w', t'\rangle \models \psi$. So it follows that $\langle w, t\rangle \models H\psi$, as desired.

14. Suppose that the derivation ends in K_G, concluding $G\varphi \to G\psi$. This means that this derivation has the premise $G(\varphi \to \psi)$ with undischarged assumptions Γ. So, we have that $\Gamma \vdash G(\varphi \to \psi)$. By the inductive hypothesis, we have that $\Gamma \models G(\varphi \to \psi)$. It remains to show that $\Gamma \models (G\varphi \to G\psi)$.

    So, suppose that $\langle w, t\rangle \models \Gamma$ and that $\langle w, t\rangle \models G\varphi$, with the aim of showing that $\langle w, t\rangle \models G\psi$. Since $\langle w, t\rangle \models \Gamma$ and $\Gamma \models G(\varphi \to \psi)$, it follows that $\langle w, t\rangle \models G(\varphi \to \psi)$.

    Since $\langle w, t\rangle \models G(\varphi \to \psi)$, it follows that for all w' and all t', if $\langle w, t\rangle \prec \langle w', t'\rangle$ and $R^T\langle w, t\rangle t'$, then $\langle w', t'\rangle \models \varphi \to \psi$. Since $\langle w, t\rangle \models G\varphi$, it follows that for all w' and all t', if $\langle w, t\rangle \prec \langle w', t'\rangle$ and $R^T\langle w, t\rangle t'$, then $\langle w', t'\rangle \models \varphi$. From these results it follows that for all w' and all t', if $\langle w, t\rangle \prec \langle w', t'\rangle$ and $R^T\langle w, t\rangle t'$, then $\langle w', t'\rangle \models \psi$. So it follows that $\langle w, t\rangle \models G\psi$, as desired.

15. Suppose that the derivation ends in HF-ax, concluding $HF\varphi$. This means that this derivation has the premise φ with undischarged assumptions Γ. So, we have that $\Gamma \vdash \varphi$. By the inductive hypothesis, we have that $\Gamma \models \varphi$. It remains to show that $\Gamma \models HF\varphi$.

    So, suppose, for reductio, that $\langle w, t\rangle \models \Gamma$ and $\langle w, t\rangle \not\models HF\varphi$. Since $\langle w, t\rangle \models \Gamma$ and $\Gamma \models \varphi$, it follows that $\langle w, t\rangle \models \varphi$. Since $\langle w, t\rangle \not\models HF\varphi$, it follows that $\langle w, t\rangle \models PG\neg\varphi$. Since $\langle w, t\rangle \models PG\neg\varphi$, it follows that there is some w' and some t' such that: $\langle w', t'\rangle \prec \langle w, t\rangle$, $R^T\langle w, t\rangle t'$, and $\langle w', t'\rangle \models G\neg\varphi$. Since $\langle w', t'\rangle \models G\neg\varphi$, it follows that for all w'' and all t'', if $\langle w', t'\rangle \prec \langle w'', t''\rangle$ and $R^T\langle w', t'\rangle t''$, then $\langle w'', t''\rangle \not\models \varphi$.

    But this result together with $\langle w', t'\rangle \prec \langle w, t\rangle$, $R^T\langle w, t\rangle t'$, Weak Temporal Symmetry, and letting $w'' = w$ and $t'' = t$, it follows that $\langle w, t\rangle \not\models \varphi$. This contradicts our initial assumption that $\langle w, t\rangle \models \varphi$, and so it follows that $\langle w, t\rangle \models HF\varphi$, as desired.

16. Suppose that the derivation ends in GP-ax, concluding $GP\varphi$. This means that this derivation has the premise φ with undischarged assumptions Γ. So, we have that $\Gamma \vdash \varphi$. By the inductive hypothesis, we have that $\Gamma \models \varphi$. It remains to show that $\Gamma \models GP\varphi$.

    So, suppose, for reductio, that $\langle w, t\rangle \models \Gamma$ and $\langle w, t\rangle \not\models GP\varphi$. Since $\langle w, t\rangle \models \Gamma$ and $\Gamma \models \varphi$, it follows that $\langle w, t\rangle \models \varphi$. Since $\langle w, t\rangle \not\models GP\varphi$, it follows that $\langle w, t\rangle \models FH\neg\varphi$. Since $\langle w, t\rangle \models FH\neg\varphi$, it follows that there is some w' and some t' such that: $\langle w, t\rangle \prec \langle w', t'\rangle$, $R^T\langle w, t\rangle t'$, and $\langle w', t'\rangle \models H\neg\varphi$. Since $\langle w', t'\rangle \models H\neg\varphi$, it follows that for all w'' and all t'', if $\langle w'', t''\rangle \prec \langle w', t'\rangle$ and $R^T\langle w', t'\rangle t''$, then $\langle w'', t''\rangle \not\models \varphi$.

    But this result together with $\langle w, t\rangle \prec \langle w', t'\rangle$, $R^T\langle w, t\rangle t'$, Weak Temporal Symmetry, and letting $w'' = w$ and $t'' = t$, it follows that $\langle w, t\rangle \not\models \varphi$. This contradicts our initial assumption that $\langle w, t\rangle \models \varphi$, and so it follows that $\langle w, t\rangle \models GP\varphi$, as desired.

So, by strong induction on the structure of derivations, if $\Gamma \vdash \varphi$, then $\Gamma \models \varphi$. □

D Formulas and Their Model Constraints

D.1 NNF

Constraint: $\forall w \forall t \forall w' \forall t' [(\langle w,t\rangle \prec \langle w',t'\rangle \wedge R^T \langle w,t\rangle t') \rightarrow R^W \langle w,t\rangle w']$.

Proof. Suppose that:

$$\langle w,t\rangle \models F\varphi \tag{6}$$

with the aim of showing that $\langle w,t\rangle \models \Diamond\varphi$. From (6), it follows that there is some w' and some t' such that:

$$\langle w,t\rangle \prec \langle w',t'\rangle \wedge R^T \langle w,t\rangle t' \wedge \langle w',t'\rangle \models \varphi \tag{7}$$

So, from (7) and the constraint, it follows that:

$$R^W \langle w,t\rangle w' \tag{8}$$

Therefore, from (7) and (8) it follows that:

$$\langle w,t\rangle \models \Diamond\varphi \tag{9}$$

as desired. □

D.2 NNP

Constraint: $\forall w \forall t \forall w' \forall t' [(\langle w',t'\rangle \prec \langle w,t\rangle \wedge R^T \langle w,t\rangle t') \rightarrow R^W \langle w,t\rangle w']$.

Proof. Suppose that:

$$\langle w,t\rangle \models P\varphi \tag{10}$$

with the aim of showing that $\langle w,t\rangle \models \Diamond\varphi$. From (10), it follows that there is some w' and some t' such that:

$$\langle w',t'\rangle \prec \langle w,t\rangle \wedge R^T \langle w,t\rangle t' \wedge \langle w',t'\rangle \models \varphi \tag{11}$$

So, from (11) and the constraint, it follows that:

$$R^W \langle w,t\rangle w' \tag{12}$$

Therefore, from (11) and (12) it follows that:

$$\langle w,t\rangle \models \Diamond\varphi \tag{13}$$

as desired. □

D.3 AP

Constraint: $\forall w \forall t \forall w' \forall t' (R^T \langle w, t \rangle t' \rightarrow R^W \langle w', t' \rangle w)$.

Proof. Suppose, for reductio, that:

$$\langle w, t \rangle \models \varphi \tag{14}$$

and:

$$\langle w, t \rangle \not\models A\Diamond\varphi \tag{15}$$

From (15) it follows that:

$$\langle w, t \rangle \models S\neg\Diamond\varphi \tag{16}$$

From (16) it follows that there is some w' and some t' such that:

$$R^T \langle w, t \rangle t' \tag{17}$$

and:

$$\langle w', t' \rangle \models \neg\Diamond\varphi \tag{18}$$

But by (17) and the constraint, it follows that:

$$R^W \langle w', t' \rangle w \tag{19}$$

and from (14) and (19), it follows that:

$$\langle w', t' \rangle \models \Diamond\varphi \tag{20}$$

which contradicts (18). So, by reductio, it follows that:

$$\langle w, t \rangle \models A\Diamond\varphi \tag{21}$$

as desired. □

E A Model Against Prior Possibility

Consider two worlds, w and w', and five times $t_1, \ldots, t_5$. Let $\langle w', t_1 \rangle$ and $\langle w', t_2 \rangle$ be earlier than $\langle w', t_3 \rangle$, and let $\langle w', t_3 \rangle$ and $\langle w, t_4 \rangle$ be earlier than $\langle w, t_5 \rangle$. Let $R^T \langle w', t_3 \rangle t_1$, $R^T \langle w', t_3 \rangle t_2$, $R^T \langle w, t_5 \rangle t_3$, and $R^T \langle w, t_5 \rangle t_4$; modal accessibility is reflexive. Let $E!a$ be false at $\langle w', t_1 \rangle$, $\langle w', t_2 \rangle$, and $\langle w', t_3 \rangle$, and true otherwise. This model is illustrated in figure 1.

This assignment makes it the case that $\langle w', t_3 \rangle \not\models \Diamond E!a$, since all worlds that are modally accessible to $\langle w', t_3 \rangle$ are $\neg E!a$ worlds. It also makes it the case that $\langle w', t_1 \rangle \not\models \Diamond E!a$ and $\langle w', t_2 \rangle \not\models \Diamond E!a$, since if $\langle w, t \rangle \models \varphi$, then for any time t', $\langle w, t' \rangle \models \Diamond\varphi$. By the same reasoning, this assignment makes it the case that $\langle w, t_5 \rangle \models \Diamond E!a$ and that $\langle w, t_4 \rangle \models \Diamond E!a$. Further, this assignment makes it the case that $\langle w, t_5 \rangle \models P\Diamond E!a$, since $\langle w, t_4 \rangle \models \Diamond E!a$ and $R^T \langle w, t_5 \rangle t_4$. Similarly, this assignment makes it the case that $\langle w, t_5 \rangle \not\models H\Diamond E!a$, since $\langle w', t_3 \rangle \not\models \Diamond E!a$ and $R^T \langle w, t_5 \rangle t_3$.

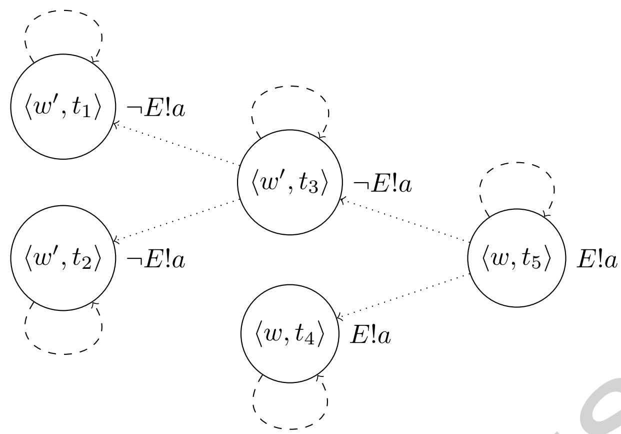

Figure 1: The impossible comes to exist.

The use of two worlds here is to avoid a contradiction that would otherwise result. Consider the same model, but where w' is w. It would still be the case that $\langle w, t_3 \rangle \not\models \Diamond E!a$, as before. However, since it's the case that: if $\langle w, t \rangle \models \varphi$ then for any t', $\langle w, t' \rangle \models \Diamond\varphi$, it would also be the case that $\langle w, t_3 \rangle \models \Diamond E!a$, since $\langle w, t_5 \rangle \models \Diamond E!a$. This would be a contradiction.

Notice that Prior Possibility is satisfied at $\langle w, t_5 \rangle$, since $\langle w, t_5 \rangle \models P\Diamond E!a$. However, also notice that Having Possibility is not satisfied at $\langle w, t_5 \rangle$, since $\langle w, t_5 \rangle \models E!a$ but $\langle w, t_5 \rangle \not\models H\Diamond E!a$. So, at $\langle w, t_5 \rangle$, the impossible has come to exist, Prior Possibility is satisfied, but Having Possibility is not.

F Various Proofs

F.1 Reflexivity Entails Prior Possibility

$$\dfrac{\dfrac{\dfrac{\dfrac{}{P\neg\varphi \wedge \varphi}\,{}^{1}}{\Diamond\varphi}\,\text{Logic}}{P\Diamond\varphi}\,\text{Reflexivity}}{(P\neg\varphi \wedge \varphi) \rightarrow P\Diamond\varphi}\,\rightarrow\text{I, } 1$$

F.2 SP Entails NNF

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overline{\Box\neg\varphi}\;{}^{1}}{\neg\Diamond\varphi}\;\text{Dual} \qquad \dfrac{\overline{S\varphi \to \Diamond\varphi}\;{}^{\text{SP}}}{\neg\Diamond\varphi \to \neg S\varphi}\;\text{Contraposition}}{\neg S\varphi}\;\to\text{E}}{\neg(P\varphi \vee \varphi \vee F\varphi)}\;\text{df. S}}{\neg F\varphi}\;\text{Logic}}{\Box\neg\varphi \to \neg F\varphi}\;\to\text{I}$$

F.3 HP Is Derivable

$$\dfrac{\overline{\varphi \to HF\varphi}\;{}^{\text{HF-ax}} \qquad \dfrac{\dfrac{\dfrac{\overline{\Box\neg\varphi \to \neg\varphi}\;{}^{\text{NNF}}}{F\varphi \to \Diamond\varphi}\;\text{Logic}}{H(F\varphi \to \Diamond\varphi)}\;\text{NEC}_{\text{H}}}{HF\varphi \to H\Diamond\varphi}\;\text{K}_{\text{H}}}{\varphi \to H\Diamond\varphi}\;\text{Logic}$$

F.4 Having Always Been Impossible but Coming to Be Has Always Been Bad

Note that H distributes over conjunction:

$$\dfrac{\dfrac{\dfrac{\overline{(\varphi \wedge \psi) \to \varphi}\;{}^{\text{Tautology}}}{H((\varphi \wedge \psi) \to \varphi)}\;\text{NEC}_{\text{H}}}{H(\varphi \wedge \psi) \to H\varphi}\;\text{K}_{\text{H}} \qquad \overline{H(\varphi \wedge \psi)}\;{}^{1}}{H\varphi}\;\to\text{E} \tag{HD1}$$

$$\dfrac{\dfrac{\dfrac{\overline{(\varphi \wedge \psi) \to \psi}\;{}^{\text{Tautology}}}{H((\varphi \wedge \psi) \to \psi)}\;\text{NEC}_{\text{H}}}{H(\varphi \wedge \psi) \to H\psi}\;\text{K}_{\text{H}} \qquad \overline{H(\varphi \wedge \psi)}\;{}^{1}}{H\psi}\;\to\text{E} \tag{HD2}$$

$$\dfrac{\dfrac{\overline{H\varphi}\;{}^{(\text{HD1})} \qquad \overline{H\psi}\;{}^{(\text{HD2})}}{H\varphi \wedge H\psi}\;\wedge\text{I}}{H(\varphi \wedge \psi) \to (H\varphi \wedge H\psi)}\;\to\text{I, 1} \tag{HD3}$$

and so:

$$\dfrac{\dfrac{\dfrac{\dfrac{H\neg\Diamond\varphi\wedge(\neg P\varphi\wedge\varphi)}{H\neg\Diamond\varphi}\,{\scriptstyle \wedge\text{E}}}{H\Box\neg\varphi}\,{\scriptstyle \text{Duals}}}{H\neg F\varphi}\,{\scriptstyle \text{NNF}} \qquad \dfrac{\dfrac{\dfrac{H\neg\Diamond\varphi\wedge(\neg P\varphi\wedge\varphi)}{\neg P\varphi\wedge\varphi}\,{\scriptstyle \wedge\text{E}}}{\varphi}\,{\scriptstyle \wedge\text{E}}}{HF\varphi}\,{\scriptstyle \text{HF-ax}}}{H\neg F\varphi\wedge HF\varphi}\,{\scriptstyle \wedge\text{I}} \qquad \text{(HB1)}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{(\neg F\varphi\wedge F\varphi)\to\bot}\,{\scriptstyle \text{Tautology}}}{H((\neg F\varphi\wedge F\varphi)\to\bot)}\,{\scriptstyle \text{NEC}_\text{H}}}{H(\neg F\varphi\wedge F\varphi)\to H\bot}\,{\scriptstyle \text{K}_\text{H}}}{(H\neg F\varphi\wedge HF\varphi)\to H\bot}\,{\scriptstyle \text{Logic}} \qquad \text{(HB2)}$$

$$\dfrac{\dfrac{}{H\neg F\varphi\wedge HF\varphi}\,{\scriptstyle \text{(HB1)}} \qquad \dfrac{}{(H\neg F\varphi\wedge HF\varphi)\to H\bot}\,{\scriptstyle \text{(HB2)}}}{H\bot}\,{\scriptstyle \to\text{E}} \qquad \text{(HB3)}$$

F.5 GP is Derivable

$$\dfrac{\dfrac{}{\varphi\to GP\varphi}\,{\scriptstyle \text{GP-ax}} \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\Box\neg\varphi\to\neg P\varphi}\,{\scriptstyle \text{NNP}}}{P\varphi\to\Diamond\varphi}\,{\scriptstyle \text{Logic}}}{G(P\varphi\to\Diamond\varphi)}\,{\scriptstyle \text{NEC}_\text{G}}}{GP\varphi\to G\Diamond\varphi}\,{\scriptstyle \text{K}_\text{G}}}{\varphi\to G\Diamond\varphi}\,{\scriptstyle \text{Logic}}$$

F.6 AP is Derivable

$$\dfrac{\dfrac{}{\varphi\to H\Diamond\varphi}\,{\scriptstyle \text{HP}} \qquad \overline{\varphi}^{\,1}}{H\Diamond\varphi}\,{\scriptstyle \to\text{E}} \qquad \text{(AP1)}$$

$$\dfrac{\dfrac{}{\varphi\to\Diamond\varphi}\,{\scriptstyle \text{T}} \qquad \overline{\varphi}^{\,1}}{\Diamond\varphi}\,{\scriptstyle \to\text{E}} \qquad \text{(AP2)}$$

$$\dfrac{\dfrac{}{\varphi\to G\Diamond\varphi}\,{\scriptstyle \text{GP-ax}} \qquad \overline{\varphi}^{\,1}}{G\Diamond\varphi}\,{\scriptstyle \to\text{E}} \qquad \text{(AP3)}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{H\Diamond\varphi}\,{\scriptstyle \text{(AP1)}} \qquad \dfrac{}{\Diamond\varphi}\,{\scriptstyle \text{(AP2)}}}{H\Diamond\varphi\wedge\Diamond\varphi}\,{\scriptstyle \wedge\text{I}} \qquad \dfrac{}{G\Diamond\varphi}\,{\scriptstyle \text{(AP3)}}}{H\Diamond\varphi\wedge\Diamond\varphi\wedge G\Diamond\varphi}\,{\scriptstyle \wedge\text{I}}}{A\Diamond\varphi}\,{\scriptstyle \text{df. A}}}{\varphi\to A\Diamond\varphi}\,{\scriptstyle \to\text{I, 1}} \qquad \text{(AP4)}$$

F.7 Not Is and Once Impossible Is Equivalent to AP

From $\neg(\varphi \wedge S\neg\Diamond\varphi)$ to $\varphi \rightarrow A\Diamond\varphi$:

$$
\begin{array}{cl}
\neg(\varphi \wedge S\neg\Diamond\varphi) & \\ \hline
\neg\varphi \vee \neg S\neg\Diamond\varphi & \text{Equivalent} \\ \hline
\neg\varphi \vee A\Diamond\varphi & \text{Duals} \\ \hline
\varphi \rightarrow A\Diamond\varphi & \text{Equivalent}
\end{array}
$$

From $\varphi \rightarrow A\Diamond\varphi$ to $\neg(\varphi \wedge S\neg\Diamond\varphi)$:

$$
\begin{array}{cl}
\varphi \rightarrow A\Diamond\varphi & \\ \hline
\neg\varphi \vee A\Diamond\varphi & \text{Equivalent} \\ \hline
\neg\varphi \vee \neg S\neg\Diamond\varphi & \text{Duals} \\ \hline
\neg(\varphi \wedge S\neg\Diamond\varphi) & \text{Equivalent}
\end{array}
$$