ON A NO DEFEAT EVIDENCE PRINCIPLE OF TAL AND COMESAÑA

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ABSTRACT. We offer a critical evaluation of a recent proposal of E. Tal and J. Comesaña on the topic of when evidence of evidence constitutes evidence. After establishing that attempts of L. Moretti and W. Roche to discredit the proposal miss their mark, we fashion another, which does not.

E. Tal and J. Comesaña (2017) consider variants of "EEE" (evidence of evidence is evidence) principles. First, they discuss ambiguities and problems with an EEE principle introduced by R. Feldman (2014), together with further problems with a purported counterexample to it due to B. Fitelson (2012). In part to get a handle on the ambiguities, they stress a distinction (cf. W. Roche 2014) between *de re* (evidence for a specific proposition which is evidence for a proposition p is evidence for p) and *de dicto* (evidence for the existential proposition that there is evidence for p is evidence for p) formulations, concluding that the following is "the relevant version of EEE"[1]:

(Existential EEE1 *de dicto*): $\forall(e)\forall(p)\forall(\alpha{>}0)\forall(\beta{>}0)$
$\big(F\big(e, \exists(e')\big(T(e') \wedge F(e', p, \alpha)\big), \beta\big) \rightarrow \exists(y > 0)\big(F(e, p, y)\big)\big)$

In words, Existential EEE1 *de dicto* says that whenever e is evidence for the existential proposition "there exists a true proposition that is evidence for p", e is evidence for p. T is a truth predicate; $F(e, p, \alpha)$ indicates that "e is evidence for p to degree α." Our interpretation of this will be that $\text{Prob}(p){<}1$ and $\text{Prob}(p|e){\geq}(1-\alpha)\text{Prob}(p) + \alpha$.

Tal and Comesaña (henceforth T&C) attempt two counterexamples to Existential EEE1 *de dicto*. The measure space in which the first is couched is not defined to our satisfaction. We shall, therefore, concentrate on the second. T&C define:

$E1 =$ *c is Black*,

$H =$ *c is the Ace of Spades*,

$E5 =$ *c is the Jack of Spades*.

They now write: "$E5$ entails (and so is evidence for) $E1$ (that c is black), and $E1$ is evidence for H (that c is the Ace of Spades). Yet, far from being evidence for H, $E5$ is conclusive evidence against it."

However, it's not relevant that $E5$ is evidence for $E1$; what is required is an $\alpha > 0$ such that $E5$ is evidence for $\exists(e')\big(T(e') \wedge F(e', H, \alpha)\big)$. Clearly though

[1]Although we take issue with their handling of the Fitelson counterexample to Feldman's principle as well, we are confining our attention to Tal and Comesaña's treatment of just those versions of EEE that they deem most important. Note: we've taken liberty to remove an extraneous quantifier from the original formulation of Existential EEE1 *de dicto*, and have repaired unmatched parentheses in Existential EEE1 *de dicto* no defeat (see below).

there is no such α; if c is the Jack of Spades then $\exists(e')\big(T(e') \wedge F(e', H, \alpha)\big)$ will be true if and only if $\alpha \leq \frac{1}{2}$. But that's the case if c is any card other than the Ace of Spades, and if c is the Ace of Spades it is true for every $\alpha \leq 1$. So in fact, for every α one has

$$\text{Prob}\big(\exists(e')\big(T(e') \wedge F(e', H, \alpha)\big)|E5\big) \leq \text{Prob}\big(\exists(e')\big(T(e') \wedge F(e', H, \alpha)\big)\big).$$

We propose the following fix: assume that the probability of *c is the Jack of Spades* is one-half that of the other cards. That is:

Prob (*c is the Jack of Spades*) $= \frac{1}{103}$;

Prob (*c is x*) $= \frac{2}{103}$, x = any single card other than the Jack of Spades.

Now $E5$ is indeed evidence (to degree $\beta = 1$) for $\exists(e')\big(T(e') \wedge F(e', H, \alpha = \frac{200}{303})\big)$. Indeed, $E5$ entails that there exists e' (namely *c is the Jack of Spades or c is the Ace of Spades*) that is true and has the property that

$$\text{Prob}(H|e') = \frac{2}{3} = (1-\alpha)\text{Prob}(H) + \alpha;$$

prior to learning $E5$ the probability of there being a true e' with $\text{Prob}(H|e') \geq \frac{2}{3}$ was $\frac{3}{103}$. By Existential EEE1 *de dicto*, then, $E5$ should be evidence for H. But, it is not.

T&C's make an attempt to repair Existential EEE1 *de dicto* that runs as follows:

(Existential EEE1 *de dicto* no defeat): $\forall(e)\forall(p)\forall(\alpha{>}0)\forall(\beta{>}0)\forall(y{>}0)$ $\big(F\big(e, \exists(e')\big(T(e') \wedge F(e', p, \alpha)\big), \beta\big) \wedge F\big(e \wedge \exists(e')\big(T(e') \wedge F(e', p, \alpha)\big), p, y\big)\big) \rightarrow$ $\exists(\delta > 0)\big(F(e, p, \delta)\big)$

T&C paraphrase Existential EEE1 *de dicto* no defeat as "Evidence that there is *de dicto* evidence for p is itself evidence for p when it is not at the same time a defeater for the support that the proposition that there is evidence for p provides to p." They then add, "Doesn't quite roll off the tongue, but it has not yet been shown false."

Indeed, at least two published attempts to discredit the principle fail. (Whether because natural language intuitions fail to match the formalism or vice versa, we shall not speculate.) W. Roche (2018) gives an example on an algebra of propositions generated by e, p and H (E, H_2 and H_1 in the original) with the distribution given in Table 1.

TABLE 1

e	H	p	Prob
T	T	T	3/5
T	T	F	1/15
T	F	T	0
T	F	F	2/27
F	T	T	0
F	T	F	0
F	F	T	1/4
F	F	F	1/108

Roche's subsequent claim that e is evidence for (on our interpretation of evidence degree) $H^* = \exists(e')\big(T(e') \wedge F(e', p, \alpha = \frac{1}{3})\big)$ is false; H^* is the tautology. (Setting α higher, say $\frac{1}{2}$, would not do; in this case H^* would consist in the atoms having measures 3/5, 1/15, 1/4, 1/108 and 0, yielding $\text{Prob}(H^*|e) = \frac{9}{10} < \text{Prob}(H^*) = \frac{25}{27}$.)

Though we cannot be sure, Roche's contention that $H^* = H$ may be based on the fact that H is defined to be the proposition that "John's total evidence is evidence-HP" for p. (*Evidence-HP* is "evidence in the sense of high probability".) But of course if p holds, John's assigning it a lowish probability is no indication that there isn't evidence for p. There is such evidence–e.g. p itself. In light of this, the attempted counterexample collapses.

L. Moretti (2016), meanwhile, proposes to simply let e and p be propositions "from two disparate domains". (He suggests $e =$ *Aristotle used to snore* and $p =$ *there is a mouse in my house.*) Suppose however that e and p are statistically independent with $\text{Prob}(e) = \frac{2}{3}$ and $\text{Prob}(p) = \frac{1}{2}$. Working in the algebra of propositions generated by e and p, one has $\exists(e')\big(T(e') \wedge F(e', p, \alpha)\big) = p \vee \neg e$ for $\frac{1}{5} < \alpha \leq \frac{1}{2}$. To see this, note that $\text{Prob}(p|p \vee \neg e) = \frac{3}{4}$ (evidence to degree $\frac{1}{2}$) but $\text{Prob}(p|p \vee e) = \frac{3}{5}$ (evidence to degree $\frac{1}{5}$). But of course e is not evidence for $p \vee \neg e$. Nor is e evidence for the tautology; note that $\exists(e')\big(T(e') \wedge F(e', p, \alpha)\big)$ is tautologous when $0 < \alpha \leq \frac{1}{5}$. Finally, e is not evidence for p; note that $\exists(e')\big(T(e') \wedge F(e', p, \alpha)\big)$ is p when $\alpha > \frac{1}{2}$. It follows that for this e and p the first conjunct of Existential EEE1 *de dicto* no defeat's antecedent is true for no pair (α, β), so this e and p cannot ground any counterexample.

Notwithstanding these failures, however, Existential EEE1 *de dicto* no defeat *is* false. For consider a lottery machine with five balls marked v, w, x, y, and z which will be drawn with probabilities (owing to their differing masses, say) $\frac{1}{8}$, $\frac{1}{8}$, $\frac{1}{8}$, $\frac{2}{8}$ and $\frac{3}{8}$ respectively. Let now:

$p =$ *ball v or ball w is drawn.*

$e =$ *ball w or ball x or ball y is drawn.*

Conditionalization on e raises the probability of

$$q = \exists(e')\big(T(e') \wedge F(e', p, \tfrac{13}{21})\big) = \exists(e')\big(T(e') \wedge \text{Prob}(p|e') \geq \tfrac{2}{3}\big) = \{v, w, x\}$$

from $\frac{3}{8}$ to $\frac{1}{2}$, so e is evidence for q (to degree $\frac{1}{5}$). Conditionalization on $(e \wedge q)$, meanwhile, raises the probability of p from $\frac{1}{4}$ to $\frac{1}{2}$, so $(e \wedge q)$ is evidence for p (to degree $\frac{1}{3}$). According to Existential EEE1 *de dicto* no defeat, then, e must be evidence for p. But e and p are independent.

That the measure space is atomic is of course the source of the trouble; $\exists(e')\big(T(e') \wedge F(e', p, \frac{13}{21})\big)$ is false when y or z is drawn (these are weighty atoms). Restricting to non-atomic measures, on the other hand, provides no respite. For in this case, $\exists(e')\big(T(e') \wedge F(e', p, \alpha)\big)$ is true with probability 1 for any $\alpha \in (0, 1)$. In particular, no e can be evidence for this proposition to degree $\beta > 0$, and the principle becomes vacuous.[2]

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