ON JUSTIFIED CREDENCE

Abstract. Brief, unfinished draft of a proposal for a novel and demanding, yet arguably well motivated, necessary condition for justified credence.

1. Pettigrew on Justified Credence

Accuracy is an epistemic virtue. If an agent's credence in X is x, and X turns out to be true, the agent's accuracy is given by $\log_2(x)$. Or, if one prefers, the agent's inaccuracy is given by $-\log_2(x)$.[1] Note however that one's accuracy can, sometimes, be decidedly unmerited. If the ideal credence[2] for me to have in X is $y < x$ and X turns out to be true then although my accuracy is $\log_2(x)$, this accuracy is not, in some sense, merited. Since I "should have had" credence y in X, must $\log_2(y)$ be taken as an upper bound on what we might call my forecast's "meritorious accuracy"?

My initial interest in meritorious accuracy was picqued by a desire, which arose whilst studying some Sarah Moss stuff, to quantify "degrees of knowledge" for degreed beliefs, i.e. credences. This ought to involve some degreed notion of "justified belief". One might think that a degreed belief, i.e. a credence, is justified just when it is numerically equal to the ideal credence, given one's evidential situation. Indeed, Pettigrew (2021), seems to think just this. Here's one of his (jargon-laden) formulations:

Reliabilism for Strongest-Grounds Justified Credence (epistemic value version)
A credence of x in proposition X by agent S is strongest-grounds justified iff
($\mathrm{ERC}_{1_{ag}}$) g is the most inclusive ground that S has;
($\mathrm{ERC}_{2_{ag}}$) the objective probability of X given that the agent has ground g approximates or equals x – that is, $P(X|S \text{ has } g) \approx x$.

There are several reasons why this definition is inadequate, however. First is a worrisome ambiguity in the formulation. Suppose X is known to S under the description "S has the most inclusive ground that T has". Unknown to S, the most inclusive ground that T has is g. Then in one sense the objective probability of X given that the agent has ground g is 1, but that needn't be S's ideal credence to have in X.

That problem is perhaps not so serious. It probably goes away entirely if one dispenses with the fancy jargon and just says what I said before....that a credence is justified just when it is numerically equal to the ideal credence, given one's evidential situation. (I don't think Pettigrew's more formal-sounding definition has any clear advantages over this, at any rate...nor do I think it is intended to be substantially different.) This

[1] There other measures of (in)accuracy, but as I have demonstrated elsewhere, these other measures have fatal flaws.

[2] Assume the universe is infinite. Then there is an infinite sequence of agents in evidential situations just like mine (which is unique when ordered by, say, distance from me, for some suitable notion of distance). The ideal credence in X, given my evidence, is then just the asymptotic frequency of X agents (agents for whom their version of the indexical "X" is true) in this sequence.

too seems inadequate, however. Suppose that X is the exclusive disjunction "A or B, but not both", where A and B look to be obviously independent, with B objectively much likelier than A. (A is that a strangely shaped die lands smallest face down; B is that the price of rice in China is above some threshold.) Let's say I have credence .8 in "A, but not B", and credence .01 in "B, but not A". As it turns out I have these precisely backwards. The objective probability of "A, but not B" is .01, and the objective probability of "B, but not A" is .8. So the objective probability of X is .81...just my credence in X. According to the proposal we are considering, my .81 credence in X should come out justified. That seems wrong, though.

Purely probabilistic fixes to this "disjunction problem" face obstacles. Suppose that I am considering three mutually exclusive scenarios for the price of some derivative. A: its price is at least 100 on Jan. 1; B: its price is strictly between 80 and 100 on Jan. 1; C: its price is at most 80 on Jan. 1. I look in the newspaper and, from the current price of certain stock options, am able to (correctly, say) discern that the objective probability of A is .37. Without prices for some other exotic financial instruments, however, I'm just guessing as to relative likelihoods of B and C. That doesn't mean my credence of .63 in the disjunction X of B and C isn't justified. It is justified, even if my credences in the disjuncts aren't.

Another example doesn't overtly involve disjunction. Elementary math students often confuse the formulas for area and perimeter. Suppose a student assigns near 1 credence to the proposition that the area of a square of side length 4 is 16, a value they arrive at not by squaring the side length, but by multiplying the side length by 4. (I.e. by computing the perimeter of the square, which in this case is also 16.) My reasoning here is something like "the square has side length 4, and I think the area of a square is 4 times the side length". But just the first part of this, that the square has side length 4, entails that the area of the square is 16, and this implies that a near 1 credence is justified, according to the proposal we are considering. That seems wrong, because the processes by which the student arrived at their credence isn't robust. In nearby cases, where the side length is 5 or 3, the student has near 1 credence in a false proposition. On the other hand, if the student is thinking "I'm pretty sure the area is $4s$, but if it isn't it's s^2, and that gives the same answer", then their near 1 credence is arguably justified.

Here is a final example that is perhaps most humbling of all. Imagine two agents, A and B, who have identical evidence. Each has credence .4 in X, which is in fact ideal given their epistemic position. However, neither has complete confidence that their .4 credence is ideal. A holds that the ideal credence is somewhat normally distributed around .4, with standard deviation .1. B holds that the ideal credence is somewhat normally distributed around .4, with standard deviation .01. So B is much more certain that, say, the ideal credence falls in $[.35, .45]$ than is A. Are we to nevertheless say that A's credence is "justified"? Whatever "justified" means, it seems that it should be an ideal notion. Suppose that B consults with C, who is completely certain that .4 is the ideal value, asking her "is the ideal credence greater than .41?" C answers "no". Isn't B now in a better epistemic position than before? So it seems...and yet, B no longer has credence .4 in X. That is, B no longer has the ideal credence. We assumed from the outset that having the ideal credence is necessary for having a justified credence. If so, B would not now have a justified credence in X. But B is better off epistemically than before...and surely B was at least as well off as A before. So how can A have a justified credence in X?

These considerations appear to push one to the following proposal, according to which justified credences are harder to come by than one might have thought:

Justified Credence

Suppose that agent S has a justified credence of x in proposition X. Then, necessarily:

(1) The objective chance of X, given S's total evidence, is x.

(2) S is certain that x is the ideal credence to have in X, given her evidence. That is, S assigns full probability to the proposition "The objective chance of X, given my total evidence, is x".

Joint satisfaction of (1) and (2) is surely not sufficient for justification. Cf. the area 16 square or, more trivally, suppose S is irrationally disposed to always assign X credence .4, regardless of her evidence, and to always be certain that this is the ideal choice. Suppose next that S's ideal credence in X happens, by lucky accident, to be .4 given her current evidence. Then S's credence is surely unjustified, being the result of a dogmatic attitude that is only occasionally, like a stopped clock, correct.

An interesting question is how to measure, purely probabilistically and in the relevant sense, degree of departure from the state where (1) and (2) are satisfied. (To be used as a lower bound on one's departure from justification, say.) Here's a promising, albeit first blush counterintuitive, proposal: for $i = 1, 2$, suppose that S_i's probability density function for the ideal credence to have in X, given her evidential condition, is $f_i(x)$. Suppose, moreover, that S_1 and S_2 are in the same evidential situation, and that each of the f_i is continuous at α, where α is ideal credence in X given the relevant evidential situation. Then S_1's credential attitude toward X exhibits greater "formal justification" than S_2's does precisely when $f_1(\alpha) > f_2(\alpha)$. This basis of comparision has the interesting property that if S_1 and S_2 were to observe an independent sequence of counterfactual resolutions to X (say in a sequence of randomly chosen counterparts inhabiting other portions of an actual infinite universe), S_1's distribution for α would, in the limit, be more tightly concentrated around the true value.[3]

References

Pettigrew, Richard. 2021. What is Justified Credence? *Episteme* 18:16-30.

[3] Apparently I'm done, so that's the end. To elaborate on the last paragraph, though, the idea is the same as that behind logarithmic scoring. The agent who fares best in logarithmic accuracy is she who assigns the actual state of big enough portions of the world greatest prior probability. Same here...the agent for whom $f(\alpha)$ is greatest assigns greatest probability to the sequence of truths and falsities of those versions of "X" contemplated by any large enough set of her actual evidential counterparts. Of course, what's fascinating is how far we've strayed from the intuitions expressed in the first paragraph of this note, once "justified" gets interpreted loosely. (One's confidence that the ideal credence be near α becomes a more important determinant of loose justification than that one's credence be near α!) Were those intuitions after all bad? If not, are there two distinct notions of quantified justification? Fine questions if there were any community to intelligently address them.