**Grounding as Minimal Necessitation**

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**Abstract**

Let *NNG* be the claim that necessitation is necessary for grounding, and let *NSG* be the claim that necessitation is sufficient for grounding. The consensus view is that grounding cannot be reduced to necessitation, and this is due to the (approximately) universally-accepted claim that NSG is false. Among deniers of NSG: grounding contingentists think NNG is also false, but they are in the minority compared to grounding necessitarians who uphold NNG. For one who would defend the claim that grounding is reducible to necessitation, the task is formidable: she must defend NSG *and* NNG. I consider two prominent objections against NSG, and two more against NNG before developing a reductive account of grounding as minimal necessitation that avoids not only all four of the previously mentioned objections, but also an additional objection that targets minimal necessitation accounts in particular. If my arguments are compelling, then, insofar as we thereby have a strong prima facie case for thinking that grounding *can* be reduced to (minimal) necessitation, we have a strong prima facie case for thinking the consensus view is mistaken.

**Keywords:** metaphysics, grounding, reduction, minimal necessitation.

1. **Four Arguments Against the Claim that Grounding is Reducible to Necessitation**

Locutions such as ‘depend(s) on’, ‘grounds’, ‘in virtue of’, and ‘is (ontologically) prior to’, are frequently taken to refer to a specifically metaphysical notion of grounding. It has recently been argued that grounding (thus construed) is closely analogous to causation (cf. Schaffer 2015; Sider 2011, 145; Wilson 2018). If so, and, given that comparatively few philosophers defend non-reductive accounts of causation, the desirability of a reductive account of grounding seems clear.[[1]](#footnote-1)

But, insofar as we should not multiply ontological primitives without necessity, a reductive account of grounding (if available) is thereby attractive even to those (cf. Bernstein 2016) who deny that grounding is like causation.[[2]](#footnote-2) Here, then, is an initial pass at such an account. First,necessitation is metaphysical sufficiency: if an entity *x* necessitates another entity *y*, then, necessarily, if *x* exists, then *y* also exists, whichis equivalent to the claim that there is no possible world where *x* exists without *y* (cf. Armstrong 2004, 5-7; Merricks 2007, 5). Second, in its crudest form, the claim that grounding is reducible to necessitation isthe claim that *x* grounds *y* iff *x* necessitates *y*.

The desirability of a reductive account notwithstanding, the consensus view is that grounding cannot be reduced to more familiar modal notions (cf. Dasgupta 2014a, 5, 7; Rosen 2010, 113; Trogdon 2013, 465); if so, then, in particular, grounding cannot be reduced to necessitation.So what is the relationship between grounding and necessitation? The answer varies. Let *NNG* be the claim that necessitation is necessary for grounding, and let *NSG* be the claim that necessitation is sufficient for grounding. On the one hand, there is the quite popular view that NNG is true, but NSG is false: call this *grounding necessitarianism*. On the other, there is the somewhat less popular view that both of NNG and NSG are false: call this *grounding contingentism*. Differences aside, necessitarianism and contingentism converge insofar as each entails the (approximately) universally-held view that NSG is false.[[3]](#footnote-3)

Two main arguments have been given for this dominantview. First, necessary facts (e.g., that1 + 2 = 3) are not grounded in contingent facts (e.g., that my brain is in a particular state), but the latter trivially necessitate the former. For example,there is no possible world where my brain is in a particular state, and yet where it is false that 1 + 2 = 3. Thus, if NSG is true, we get the wrong result: among its other attributes, grounding is frequently taken to be an explanatory relation,[[4]](#footnote-4) but the fact that my brain is in a particular state does not explain the fact that 1 + 2 = 3. Call this the *problem of* *spurious necessitation*(cf. Bliss and Trogdon 2014, §5; Cameron forthcoming;Schaffer 2010, 320).

As for the second argument against NSG, consider contingent entities that necessarily coexist, such as Socrates and his singleton set. Socrates’ singleton set is grounded in Socrates himself, and not vice versa, but if NSG is true, we are unable to distinguish the direction of grounding: Socrates necessitates his singleton set, *and* vice versa. Call this the *problem of symmetric necessitation between asymmetrically grounded entities* (or the *problem of symmetric necessitation*, for short) (cf. Barnes 2018;Bliss and Trogdon 2014, §4; Fine 1995).

Turning to NNG,although there is a variety of objections that are plausibly regarded as being directed at this claim, I will focus on two recent ones.[[5]](#footnote-5) First, it is possible that the existence of an ordinary composite object is grounded in the arrangement of its spatial parts at a given time. But it is also possible that these spatial parts could be rearranged, and then brought back into their original arrangement in such a way that the composite object no longer exists; as necessitarianism entails that necessarily, the object in question exists if its parts do, it follows that NNG is false.

To illustrate, consider the classic example of the Ship of Theseus. Suppose that *Old Ship* consists entirely of planks *a1*, *a2*, …, *an* standing in a particular arrangement, and that this arrangement grounds and so necessitates the fact that Old Ship exists. Suppose further that each of *a1*, *a2*, …, *an* is gradually replaced, one by one, with qualitative duplicate planks *b1*, *b2*, …, *bn*, and that each of *a1*, *a2*, …, *an* is gradually reassembled to make *New Ship*; at the end of this process there are two ships. But suppose we destroy Old Ship. If so, then, contrary to NNG, the arrangement *a1*, *a2*, …, *an­* does not necessitate the fact that Old Ship exists. Call this the *rearrangement problem* (cf. deRosset 2013a, 265-266, n23; Skiles 2015, 722).

The second problem requires some background. Consider the claim that all solid gold spheres are less than one mile in diameter. This generalization, though presumably true, is nevertheless accidentally so, as there is nothing about gold or the laws of nature that prevents such spheres from having diameters of a mile or greater.Let *Γ* be the set of the actual first-order facts.Let [*gold*] be the fact that all solid gold spheres are less than one mile in diameter, and suppose for *reductio* that NNG is true. It is at least initially plausible to suppose thatthe members of Γ ground [*gold*]; if so, it follows that the members of Γ necessitate [*gold*].But it is not the case that the members of Γ necessitate [*gold*], for it is possible that these members coexist with a solid gold sphere whose diameter is at least one mile across.

First, and somewhat informally, *full grounds* suffice for what they ground, while *merely partial grounds* contribute to, but do not suffice for, what they ground. Given the standard assumption that full (as opposed to merely partial) grounds necessitate (Bliss and Trogdon 2014, §5; Correia and Schnieder 2012, 20), it follows that the members of Γ are merely partial grounds for [*gold*]. Thus, as the members of Γ fail to necessitate [*gold*], and, by hypothesis, there are no *other* first-order facts apart from the members of Γ, it follows that there is grounding without necessitation, and so, contrary to our initial assumption, NNG is false.Call this the *problem of accidental generalizations*.

The problem of accidental generalizations admits of a seemingly straightforward response.Let the *totality fact* be the actual second-order fact [*the members of Γ are all of the first-order facts*]. The totality fact fully grounds [*gold*], and so the totality fact also necessitates [*gold*] (cf. Rosen 2010, 120-121;Trogdon 2013a, 478). It seems that there is no insuperable difficulty for NNG.[[6]](#footnote-6)

So much by way of background to the second problem for NNG. Alexander Skiles (2015, 729-736) argues that a variation of the objection from accidental generalizations poses a further difficulty for proponents of NNG.To use one of Skiles’ examples (2015, 730-731), let [*Swiss swans*] be the claim that all Swiss swans are white. Let *w* be some world that is just like ours except that someone has brought some black swans into Switzerland; in particular, all the Swiss swans that are white in the actual world remain so in *w*. The totality fact does not necessitate [*Swiss swans*], because the actual totality fact and the totality fact-in-*w­* are the same fact: again, there is no difference between the actual world and *w* in terms of what exists; rather, the only difference is that there are more Swiss swans in *w* than there are in the actual world, and some of the Swiss swans in *w* are black. As the totality fact does not necessitate [*Swiss swans*], it follows that NNG is false. Call this the *problem of restricted accidental generalizations*.

Taken together, these four objections present a formidable challenge to any attempt to reduce grounding to necessitation. Nevertheless, in the following section, I will articulate such a reductive account of grounding that attempts to meet this challenge.

1. **Grounding as Minimal Necessitation**

Consider some entities *x1*, *x2*, *x3*, …, and another entity *y*. Letting *x1*, *x2*, *x3*, … be the members of a set *S*, we formulate the following sufficient condition for *minimal necessitation*:

(*MN*) *S* minimally necessitates *y* if:

(1) *S* necessitates *y*,

(2) neither *S* nor *y* is a pure set,

(3) *S* does not violate the *proportionality constraint* (*PC*) with respect to *y*,[[7]](#footnote-7) and

(4) there is no set *S\** that (a) satisfies (1) – (3) of MN, and (b) is: either (i)a non-empty proper subset of *S*, or (ii) both disjoint from, and has a lesser cardinality than, *S*.

According to the account I am proposing, a sufficient condition for *full grounds* is:

(*FG*): *x1*, *x2*, *x3*, … fully ground *y* if:[[8]](#footnote-8)

1. no *xi* = *y*, and
2. *x1*, *x2*, *x3*, … are the members of *S*, and *S* and *y* satisfy MN.

As we will see in the next section, cases of grounding involving mathematical entities require distinct sufficient conditions to supplement MN and FG. The additional sufficient condition for minimal necessitation is:

(*MN∅*) *S* minimally necessitates *y* if:

(1) *S* necessitates *y*,

(2) *S* and *y* are both pure sets, and

(3) *S* has no proper subset that also necessitates *y*.

The additional sufficient condition for full grounds is:

(*FG∅*): *S* fully grounds *y* if:

1. *S* ≠ *y*, and
2. *S* and *y* satisfy MN∅.

Thus, MN and MN∅ are individually sufficient and jointly necessary conditions for minimal necessitation; similarly, FG and FG∅ are individually sufficient and jointly necessary conditions for full grounds.

In contrast to minimal necessitation and full grounds, we can provide a comparatively straightforward definition of *merely partial grounds*:

(*PG*): *x* *partially* *grounds* *y* iff:

1. *x* ≠ *y*,
2. *x* is a member (though not the *sole* member) of *S*, and *S* and *y* satisfy MN.

Finally, letting *MNNG* and *MNSG* be the claims that minimal necessitation is necessary and sufficient for grounding, respectively, my account of *grounding-as-minimal-necessitation* (GMN) is the conjunction of MNNG and MNSG.

A few further clarificatory remarks are in order.

First, clause (1) of FG, FG∅, and PG is required to prevent any entity from groundingitself.[[9]](#footnote-9)

Second, according to FG and PG, although *S* itself minimally necessitates *y*, it does not follow that *y* is fully or partially grounded in *S*; rather, it follows that *y* is fully grounded in *the* *members* of *S*, or that *y* is partially grounded in *a member* of *S*.

Third, I assume that: while minimal necessitation is a relation that links a set, on the one hand, with some other entity, on the other, grounding is a relation that links entities from arbitrary categories (e.g., objects, facts, propositions, sets, etc.), and the relata of any particular grounding relation need not belong to the same category (e.g., objects can ground facts, facts can ground propositions, etc.).[[10]](#footnote-10)

Fourth, in various definitions (e.g., FG, FG∅, PG), I use the locution ‘*S* and *y* satisfy MN∅’, and ‘*S* and *y* satisfy MN’ – rather than the more straightforward ‘*S* minimally necessitates *y*’ – so as to distinguish those cases where *S* and *y* are both pure sets (as required by MN∅) from those cases where neither *S* nor *y* is a pure set (as required by MN). That is, although there is only one relation of minimal necessitation, I hold that sometimes the relata are pure sets (as in cases of the grounding of mathematical entities), and sometimes the relata are not pure sets (as in all other cases). This distinction is more clearly expressed by ‘*S* and *y* satisfy MN∅’, or ‘*S* and *y* satisfy MN’, rather than by the ambiguous ‘*S* minimally necessitates *y*’.

Fifth, my initial mention of ‘the proportionality constraint’ in clause (3) of MN requires further elaboration. To begin, there is a rough but intuitive sense in which, if a set *S* is to minimally necessitate some entity *y*, then *S* must have no members that are (so to speak) superfluous with respect to *y*, and *y* must have no non-logical constituents, parts, or members that are superfluous with respect to *S*. That is, and as we will see momentarily, *S* must be proportional to *y*.[[11]](#footnote-11)

Consider the following case. If {[*a*], [*b*]} minimally necessitates [*a* & *b*], then PG entails that each of [*a*] and [*b*] partially grounds [*a* & *b*], just as FG entails that [*a*], [*b*] fully ground [*a* & *b*], as desired. But how can {[*a*], [*b*]} minimally necessitate [*a* & *b*] when there is another set, {[*a* & *b*]} that not only necessitates [*a* & *b*], but whose cardinality is also less than that of {[*a*], [*b*]}? For if {[*a* & *b*]} minimally necessitates [*a* & *b*], FG’s prohibition on self-grounding entails the incorrect result that nothing grounds [*a* & *b*].

In response, first, let a *complex* entity be any of the following: a logically complex fact, a mereologically complex object, a structurally complex proposition, a non-empty set, etc. Second, let an entity be *simple* if it is not complex. Now consider the following definitions. The first applies when *y* is simple:

*S* and *y* are *simply-paired* iff:

*y* is simple, and

the sole member of *S* is identical to *y*.

The next two definitions apply when *y* is complex. Here is the first:

*S* and *y* are *upwardly-paired* iff:

*y* is complex, and

every member of *S* is identical to a unique one of the non-logical constituents, parts, or members of *y*.

The second definition is:

*S* and *y* are *downwardly-paired* iff:

*y* is complex, and

every non-logical constituent, part, or member of *y* is identical to a unique member of *S*.

Given these preliminary definitions, a necessary condition on a given set *S* minimally necessitating an entity *y* is that *S* does not violate the following proportionality constraint with respect to *y*:

(*PC*) *S* is *proportional to* *y* iff:

*S* and *y* are simply-paired, or

*S* and *y* are both (i) upwardly-paired, and (ii) downwardly-paired iff *y* is not disjunctive.

It is easy to show that, with respect to [*a* & *b*], {[*a* & *b*]} violates clause (2) of PC, while {[*a*], [*b*]} does not: {[*a* & *b*]} has only one member (i.e., [*a* & *b*]), which is not identical to either of the non-logical constituents of [*a* & *b*]; in contrast, each of the members of {[*a*], [*b*]} *is* identical with each of the non-logical constituents of [*a* & *b*], and vice versa.

PC helps eliminate further spurious cases of minimal necessitation. Consider necessary facts [*p*], [*q*], [*r*], [*s*], [*p* & *q*], and some contingent fact [*a*]. We want to say that {[*p*], [*q*]} – and none of {[*p*]}, {[*q*]}, {[*r*], [*s*]}, {[*p* & *q*]}, or {[*a*]} – minimally necessitates [*p* & *q*]. But each of {[*p*]}, {[*q*]}, {[*r*], [*s*]}, {[*p* & *q*]}, and {[*a*]} necessitates [*p* & *q*]; furthermore, each of {[*p*]} and {[*q*]} is a proper subset of {[*p*], [*q*]}, while each of {[*r*], [*s*]}, {[*p* & *q*]}, and {[*a*]} is both disjoint from, and has a cardinality that is less than or equal to, {[*p*], [*q*]}.

Once more, clause (2) of PC eliminates each of these cases. First, the sole member of {[*p*]} is identical to only one of the non-logical constituents of [*p* & *q*], but not the other (similar remarks rule out {[*q*]}). Second, none of the respective members of {[*r*], [*s*]}, {[*p* & *q*]}, or {[*a*]} is identical to either of the non-logical constituents of [*p* & *q*]. Only {[*p*], [*q*]} satisfies clause (2) of PC with respect to [*p* & *q*], as desired.

Sixth, and finally, disjunctions are grounded in one or more of their disjuncts. So, for example, [*a* ∨ *b*] is grounded in [*a*] (if [*a*] obtains), or [*b*] (if [*b*] obtains). Thus my account should deliver the result that either {[*a*]} or {[*b*]} minimally necessitates [*a* ∨ *b*]. Clause (2), (i) of PC both prevents {[*a* ∨ *b*]} from minimally necessitating [*a* ∨ *b*], and entails that either {[*a*]} (if [*a*] obtains) or {[*b*]} (if [*b*] obtains) minimally necessitates [*a* ∨ *b*], as desired. Clause (2), (ii) of PC states that *S* and *y* are downwardly-paired iff *y* is not disjunctive. This restriction prevents the implausible result that neither {[*a*]} nor {[*b*]} minimally necessitates [*a* ∨ *b*].[[12]](#footnote-12)

I have just said that [*a* ∨ *b*] is minimally necessitated by {[*a*]} (if [*a*] obtains), or by {[*b*]} if ([*b*]) obtains. But if both of [*a*] and [*b*] obtain, then there is no single set that minimally necessitates [*a* ∨ *b*]. In response, I note that standard accounts of grounding hold that disjunctive facts are fully grounded in each of their obtaining disjuncts, which entails that disjunctive facts are “overdetermined” when more than one of their disjuncts obtains. But this overdetermination is no objection to such accounts of grounding. Similarly, I see no reason to assume that [*a* ∨ *b*] cannot have two minimally necessitating sets.

In the following sections, I will argue that GMN avoids both objections to NSG, both objections to NNG, and an additional objection that targets minimal necessitation accounts in particular.

**The Problem of Spurious Necessitation**

As for the first problem, let [*b*] be the fact that my brain is currently in a particular state, and let [*addition*] be the fact that 1 + 2 = 3. I will now show that [*b*] does not ground [*addition*]. First, no set with [*b*] as a member is a pure set. Second, it is plausible that the natural numbers are identical with pure sets, and so that [*addition*] is to be understood in terms of such sets. But, third, both of MN and MN∅ prohibit such “mixed cases” as those in which an impure set necessitates a pure set, and so no set with [*b*] as a member minimally necessitates [*addition*]; it follows that [*b*] neither fully nor partially grounds [*addition*].[[13]](#footnote-13) On the other hand, as ∅ is a pure set that both necessitates [*addition*], and is also the only set that has no proper subset, MN∅ entails that ∅ minimally necessitates [*addition*]; by FG∅, it follows that ∅ fully grounds [*addition*].

An objector may remind us that the original problem of spurious necessitation says that if NSG is true, then what is implausible about a contingent fact such as [*b*] groundinga necessary fact such as [*addition*] is that, given the explanatory nature of grounding (§1), [*b*] does not explain [*addition*]. But, the objection continues, my proposed solution fares no better, for ∅ also fails to explain [*addition*].

Contrary to the current objection, it is plausible that ∅ explains [*addition*]. Define the natural numbers using the *successor function S*: *S*(*n*) = *n* {*n*}:

0 = ∅

1 = {∅}

2 = {∅, {∅}}

3 = {∅, {∅}, {∅, {∅}}}

etc.

Next, recursively define addition in terms of *S*.[[14]](#footnote-14)The set *S3* ={∅, {∅}, {∅, {∅}}} necessitates [*addition*]; but, as∅ is a proper subset of *S3*, clause (3) of MN∅ entails that*S3* does not *minimally*necessitate [*addition*].

Suppose we try again. For example, define the successor function as *S*(*n*) = {*n*}, and generate the following alternative definitions of the natural numbers:

0 = ∅

1 = {∅}

2 = {{∅}}

3 = {{{∅}}}

etc.

Let *S3\** = {{{∅}}}. Although *S3\** necessitates [*addition*], ∅ is a proper subset of *S3\**, and so, again, clause (3) of MN∅ entails that*S3\** does not *minimally*necessitate [*addition*]. There are other definitions of the natural numbers, but we need not concern ourselves with them: ∅ is a proper subset of every non-empty set that necessitates [*addition*], and so MN∅ entails that ∅ minimally necessitates [*addition*]; but then, as we have already seen, FG∅ entails that ∅ fully grounds [*addition*]. If so, then, insofar as the various set-theoretic constructions of the natural numbers are explanatory, it is plausible that ∅ also *explains* [*addition*].

My argument for claiming that ∅ fully grounds [*addition*] requires holding that the natural numbers are identical to sets (i.e., MN∅entails that [*addition*] itself must be understood in terms of pure sets). But Paul Benacerraf (1965) has given a famous argument that might seem to contradict this. To illustrate, by the transitivity of identity, both of the following claims cannot be true:

*S3* = {∅, {∅}, {∅, {∅}}}

*S3\** = {{{∅}}}

But Benacerraf also showed that it is entirely arbitrary which definition we choose, and so there is no reason to hold that *any* such definition of the natural numbers is correct.

In response, my solution does not contradict Benacerraf, because my solution does not entail the correctness of any particular identification of numbers with sets. So, for example, although my solution entails that the number 3 is a set, it does *not* entail that there is any particular set such that the number 3 is that set.[[15]](#footnote-15) Furthermore,any attempt to model the natural numbers must assume a distinguished starting point, and then define a successor function on this starting point; but every set-theoretic definition of the natural numbers takes ∅ as the starting point. As my solution to the problem of spurious necessitation entails that ∅ minimally necessitates [*addition*], my solution merely entails what is common to every way of identifying numbers with sets.

Nevertheless there is a further worry, which is that I cannot account for the partial grounding of mathematical facts. Let [*math*] be any mathematical fact that is plausibly grounded in some set-theoretic construction(s) or other.As∅ is a proper subset of every non-empty set,clause (3) of MN∅ entails that ∅ is the only set that minimally necessitates [*math*], and so, byFG∅, ∅ fully grounds [*math*]. But surely there are some mathematical facts that are *partially* grounded in some set-theoretic construction or other; if so, then GMN fails to account for them.

In response, there are accounts of grounding according to which various mathematical facts have partial grounds.[[16]](#footnote-16) Though this is not the place to consider a detailed comparison between such accounts and GMN, I grant that it is a legitimate objection to GMN that it fails to accommodate such merelypartial grounding claims.Nevertheless, this objection is both distinct from, and less worrying than, the original problem of spurious necessitation. Clearly, [*addition*] is not grounded (in whole, or in part) in [*b*]; any account that entails otherwise should be rejected. In contrast, with respect to the partial grounding of mathematical facts, if GMN is at a disadvantage compared to other accounts, then this cost must be weighed against other advantages and disadvantages that accompany these competing accounts. As a working hypothesis – which the remainder of this paper is intended to confirm – I tout the overall reductive nature of GMN as its principal advantage over these competitors.

It is worth noting that the problem of spurious necessitation is not confined to the worry that contingent facts spuriously necessitate mathematical facts. Rather, the problem generalises to the objection that contingent facts also spuriously necessitate necessary but non-mathematical facts. For instance, let [*p*] and [*q*] be two such necessary facts. It is plausible that the conjunctive fact [*p* & *q*] is fully grounded in [*p*], [*q*]. If GMN entailed that [*p* & *q*] is (fully or partially) grounded in either of [*b*], or ∅, this would be a sufficient reason to reject my account. It is straightforward, however, to show that GMN entails neither of these implausible results.

First, clause (2) of MN entails that ∅ is ineligible to minimally necessitate [*p* & *q*]. Second, insofar as the sole member of {[*b*]} is not identical to either of the non-logical constituents of [*p* & *q*], {[*b*]}violates clause (2) of PC with respect to [*p* & *q*] (cf. §2). Thus, no set with [*b*] as a member minimally necessitates [*p* & *q*]. In contrast, MN entails the correct result that the set {[*p*], [*q*]} minimally necessitates [*p* & *q*], and so it follows by FG that [*p*], [*q*] fully ground [*p* & *q*].

1. **The Problem of Symmetric Necessitation**

Necessarily, if Socrates exists, then {Socrates} exists.Let the set *A* = {Socrates}.As *A* minimally necessitates {Socrates}, and Socrates is the sole member of *A*,FG entails that Socrates fully grounds {Socrates}.

In contrast, {Socrates} is not a member of any set that minimally necessitates Socrates. First, Socrates is an ordinary composite object composed of various parts standing in an appropriate arrangement, and so it is plausible that Socrates is both fully grounded in some arrangement of his parts, and that he is partially grounded in each of his parts considered individually.[[17]](#footnote-17) Second, let {Socrates} be the sole member of the set *B*; that is, let *B* = {{Socrates}}. If *B* is to minimally necessitate Socrates, then, as Socrates is a mereologically complex object, *B* must satisfy clause (2) of PC with respect to Socrates. But, as {Socrates} is not identical to any of Socrates’ proper parts, this clause is not satisfied. It follows that no set with {Socrates} as a member minimally necessitates Socrates. Third, let *a1*, *a2*, *a3*, … be some arrangement of Socrates’ parts. Fourth, MN entails that{*a1*, *a2*, *a3*, …} minimally necessitates Socrates; FG entails that *a1*, *a2*, *a3*, … fully ground Socrates, and PG entails that each of *a1*, *a2*, *a3*, …, considered individually, partially grounds Socrates. Recall that the problem of symmetric necessitation is that one who regards grounding as necessitation is unable to distinguish the direction of grounding between Socrates and {Socrates} (§1). According to the arguments I have just provided, however, this problem has been solved: Socrates grounds {Socrates}, but not vice versa.

Perhaps my solution to the problem of symmetric necessitation only works because Socrates is a composite object. Let *Simple* be a mereologically simple object. We can resurrect the problem of symmetric necessitation by observing first, that {Simple} is grounded in Simple, and not vice versa, and that, second, it is unclear how GMN can account for this asymmetry.

By way of response, let the set *C* = {Simple}.As *C* minimally necessitates {Simple}, and Simple is the sole member of *C*,FG entails the correct result that Simple fully grounds {Simple}. But {Simple} does *not* fully ground Simple. Let the set *D* = {{Simple}}. As the sole member of *D* is not identical to Simple, *D* violates clause (1) of PC with respect to Simple, and so it follows that no set with {Simple} as a member minimally necessitates Simple.

It bears emphasizing that although *C* minimally necessitates Simple, clause (1) of FG entails that nothing grounds itself; it follows that, in the present case, we have minimal necessitation without grounding. But this is a welcome result: {Simple} is grounded in Simple, but not vice versa.[[18]](#footnote-18)

**The Rearrangement Problem**

I now turn to objections to NNG. Following Skiles (2015, 722), consider the following formalization of the rearrangement problem (where *o* is an ordinary object composed of its various parts, the *a*s):

(P1) The existence of *o* is fully grounded in Φ, some arrangement of the *a*s, at *t*.[[19]](#footnote-19)

(P2) There are “Ship-of-Theseus-style” scenarios of rearrangementwith respect toΦ at the end of which Φ exists, but *o* does not.

Therefore, NNG is false.

I deny (P1).To explain why, I begin with a common illustration. From a distance, there seem to be only one cloud, and it appears to have sharp boundaries; up close, matters are different: the cloud itself is a mass of water droplets, and there is no clear cutoff between the cloud and its surroundings. Rather, at the molecular level, the difference between the cloud and its environment is marked only by a gradual change in the density of the arrangement of various water droplets. Thus we can choose to draw the boundary around the cloud in many different ways. On one way of drawing the boundary, we include a given droplet, while on another we exclude it. Other choices with respect to other droplets result in the drawing of further boundaries. For each choice of boundary, we get a distinct cloud candidate; and, as there is no principled reason for choosing one boundary over another, it seems that every bounded collection of water droplets is a cloud. So, contrary to initial appearances, there is not one cloud, but a multitude. This is *the problem of the many* (Unger 1980). As far as the problem is concerned, there is nothing special about clouds for it arises with all sorts of ordinary composite objects, such as mountains (cf. McGee and McLaughlin 2000), cats (cf. Lewis 1993), and persons’ bodies (cf. Unger 1980, 461-462).

Let *o* be an ordinary composite object, and let *o1*, *o2*, *o3*, … be the many candidate objects that result from each of the many different ways of drawing *o*’s precise boundary at *t*. Let *S1* be the set consisting of *o1*’s suitably arranged molecules; let *S2* be the set consisting of *o2*’s suitably arranged molecules; etc.; thus, ­*S1* minimally necessitates *o1* at *t*, *S2* minimally necessitates *o2* at *t*, and so on. So, for any *Si*, and any *oi*, FG entails that the members of *Si* fully ground *oi* at *t*. Due to the problem of the many, however, it is implausible to hold that *o* is identical to any particular one of the *oi* at *t*. But then, for any *Si*, it is implausible to hold that the members of *Si* fully ground *o* itself at *t.* It follows that (P1) is false.[[20]](#footnote-20)

Perhaps my response merely pushes the rearrangement problem back a step. That is, it might seem that, for any candidate object *oi*, we can offer a revised formulation of the rearrangement problem as follows:

(P1\*) The existence of *oi* is fully grounded in Φ, some arrangement of the *a*s, at *t*.

(P2\*) There are “Ship-of-Theseus-style” scenarios of rearrangement with respect to Φ at the end of which Φ exists, but *oi* does not.

Therefore, NNG is false.

Call this the *revised rearrangement problem*.

Whereas my response to the original rearrangement problem was to deny (P1), my response to the revised variant is to deny (P2\*). Let *a1*, *a2*, …, *an* be *oi*’s molecules. Suppose we replace one of *oi*’s molecules with a qualitative duplicate *a1\**. If so, then we have an object, *oi\**, whose molecules are *a1\**, *a2*, …, *an*. As *a1* ≠ *a1\**, it follows that *oi\** ≠ *oi*. That is, no *oi* can survive the replacement of a single molecule. If *oi* were an ordinary composite object, then I grant that such mereological fragility would be deeply implausible (cf. Skiles 2015, 728). But *oi* is no ordinary composite object. Rather, *oi* is one of many perfectly precise object candidates, and, as the problem of the many has shown us, such candidates are individuated by such arbitrarily small differences as whether they include a single molecule as a part.

Thus, for any *oi*, and contrary to the current objection, there is no Ship-of-Theseus-style scenario of rearrangement in which all of *a1*, *a2*, …, *an* exist, but *oi* itself does not. The revised rearrangement problem fails.

It might seem that my proposal can only handle cases that give rise to “problem-of-the-many”-type scenarios. Let *Old 3-Block* be an object entirely composed of three mereologically simple blocks *a1*, *a2*, and *a3* that are red, yellow, and green, respectively, and suppose that it is essential to Old 3-Block that it be composed of blocks of at least three colours. Suppose further that each of *a1*, *a2*, and *a3* is gradually replaced, one by one, with three respective duplicate blocks *b1*, *b2*, and *b3* through an interval [*t1*, *t3*]. During this interval, *a1*, *a2*, and *a3* are gradually built into a new object, *New 3-Block*, which is an exact duplicate of Old 3-Block. Thus, at *t3*, there are two duplicate objects; at *t4*, Old 3-Block is destroyed. As Old 3-Block cannot be composed of fewer than three blocks, there is, at most, one candidate collection of blocks that can compose Old 3-Block: at *t0*, it is *a1*, *a2*, and *a3*; at *t1*, it is *b1*, *a2*, and *a3*; at *t2* it is *b1*, *b2*, and *a3*; at *t3* it is *b1*, *b2*, and *b3*. It follows that, contrary to NNG, the arrangement *a1*, *a2*, …, *an­* does not necessitate the fact that Old 3-Block exists. Call this the *simpler rearrangement problem*, which we formalise as follows:

(P1\*\*) The existence of Old 3-Block is fully grounded in Φ, some arrangement of the *a*s, at *t*.

(P2\*\*) There are “Ship-of-Theseus-style” scenarios of rearrangementwith respect toΦ at the end of which Φ exists, but Old 3-Block does not.

Therefore, NNG is false.

I deny (P2\*\*). Let *R* be a composition relation. For any collection of three blocks, I make the following two assumptions: first, that the collection instantiate *R* iff the collection compose an object whose parts are essential to it; and, second, that the collection compose an object whose parts are essential to it iff the collection and *R* are members of a particular set that minimally necessitates the composed object.

So *S1\** = {*R*, *a1*, *a2*, *a3*}, and *S1\** minimally necessitates Old 3-Block; *S2\** = {*R*, *b1*, *a2*, *a3*}, and *S2\** minimally necessitates *o2\**; *S3\** = {*R*, *b1*, *b2*, *a3*}, and *S3\** minimally necessitates *o3\**; etc. Furthermore, and contrary to the initial description of the simpler rearrangement problem, both *S1\** and Old 3-Block cease to exist at *t1*, and begin to exist again at *t3*. Insofar as *a1* is replaced by *b1* at *t1*, it follows that *a1*, *a2*, and *a3* no longer instantiate *R* at *t1*, and so that both *S1\** and Old 3-Block cease to exist at *t1*.[[21]](#footnote-21) At *t2*, *a1*, *a2*, and *a3* do not instantiate *R*, and so it remains the case that *S1\** and Old 3-Block do not exist. At *t3*, as *a1*, *a2*, and *a3* have returned to their initial arrangement and so instantiate *R*, *S1\** and Old 3-Block exist again. Finally, at *t4*, the status of *S1\** and Old 3-Block remain unchanged. In sum, and contrary to the simpler rearrangement problem, there is never a time when *S1\** exists, but Old 3-Block does not.

In my earlier discussion of the revised rearrangement problem, I argued that no *object candidate* can survive the loss of a single molecule. I agreed that, with respect to ordinary objects, such mereological fragilitywould be deeply implausible, but I deny that objects like Old 3-Block are mereologically complex enough to be ordinary objects in the relevant sense. That is, while I agree that an ordinary object, such as a person or a table, can survive the loss of a single molecule, I deny that a comparatively simple object, such as Old 3-Block, can survive the loss, and subsequent replacement of, any one of its three composing blocks.

It is not arbitrary to treat these objects differently. An ordinary object can survive the replacement of a single molecule, because an ordinary object has many such molecules, and the replacement of any one of them by a qualitative duplicate seems not to make any relevant difference to its status as an ordinary object. Insofar as Old 3-Block has only three parts, however, it is not an ordinary object. I claim that our intuitions about whether Old 3-Block can survive the replacement of a single part by a qualitative duplicate are not nearly as robust as our intuitions concerning ordinary objects. To illustrate, in the limit case where the object in question is a mereological simple, it seems clear that this object cannot survive being replaced by a qualitative duplicate. It is still plausible, though not as obviously so, that an object composed of only two mereologically simple parts also cannot survive the replacement of one of its parts by a qualitative duplicate. There is no sharp cutoff between, on the one hand, instances in which relatively simple objects cannot survive the replacement of one of their parts, and, on the other, instances in which ordinary objects *can* survive such a replacement. Rather, there are borderline cases. If we suppose *o!* to be an instance of such a borderline case, then there is a minimally necessitating set *S!* whose members are all and only *o!*’s parts. But it is not yet clear whether *o!* is a mereologically fragile *object*, or whether *o!* is instead one among a range of nearby mereologically fragile *object* *candidates*, each of whom has its own minimally-necessitating set.

It is not independently obvious that relatively simple objects like Old 3-Block do *not* have their parts essentially. Nevertheless, that my view commits me to the mereological fragility of such objects is a cost. But it is a cost that must be weighed against the benefits of adopting my view. Obviously, the first benefit is that my view avoids the rearrangement problem. The second, and more general, benefit is that my view, which I am in the process of defending, provides a reductive account of grounding. I think the benefits outweigh the cost.

**The Problem of Restricted Accidental Generalizations**

Let *w@* be our (actual) world, let *Φw@* be the set whose members are all of the white Swiss swans in our world, and let [*Swiss-swans-in-w@*] be the fact that all of the Swiss Swans in *w@* are white. My argument is straightforward: Φw@minimally necessitates [*Swiss-swans-in-w@*], and so it follows that the members of Φw@ fully ground [*Swiss-swans-in-w@*].

The possibility that black swans are brought into Switzerland is irrelevant to my argument: in a possible world *w* where all of the members of Φw@ coexist in Switzerland with some additional black swans, these black and white Swiss swans of *w* are the members of a set, Φ*w*, that is distinct from Φw@, and Φ*w* minimally necessitates a fact, [*Swiss-swans-in-w*], that is distinct from [*Swiss-swans-in-w@*]. That this is so, however, does not alter the grounding relationship that holds in *w* between the members of Φw@ and [*Swiss-swans-in-w@*]. What *does* change is that, in *w*, [*Swiss-swans-in-w@*] plays a different role than it did in *w@*: in *w@*, [*Swiss-swans-in-w@*] is a restricted accidental generalization whose instances are *all* of the Swans in Switzerland; in *w*, this is not so.

I see my proposal as having three primary benefits. First, although I have articulated it in terms of MNNG, my proposal can be straightforwardly recast so as to be consistent with NNG. Second, my proposal allows one to preserve the plausible notion that a restricted accidental generalization is grounded in its instances insofar as, in *w@*, [*Swiss-swans-in-w@*] is grounded in Φw@. The third benefit, which is related to the second, is that my proposal does not require acceptance of any sort of totality fact.[[22]](#footnote-22)

I hold that [*Swiss swans*] has no grounds. Assuming a fact is fundamental iff it has no grounds, the primary cost of adopting my proposal might seem to be that it follows that [*Swiss swans*] is fundamental, which is implausible. In response, I deny the supposed cost, because I deny the relevant entailment. That is, [*Swiss swans*] is not fundamental, but this is because [*Swiss swans*] – unlike [*Swiss-swans-in-w@*] – is not indexed to *w@*; insofar as [*Swiss swans*] is a generalization fact that lacks such an index, it is an incomplete, and so not a well-formed, fact.[[23]](#footnote-23) Facts that are not well-formed lack grounds, but they are not thereby fundamental. I conclude that the problem of accidental generalizations is solved.

**The No Minimality Problem**

Having considered two objections to NNG, I will now discuss another problem that threatens MNNG in particular. Let *e1*, *e2*, *e3*, … be denumerably many distinct electrons, let [*d*] be the fact that there are denumerably many distinct electrons, and suppose [*d*] obtains. If {*e1*, *e2*, *e3*, …} necessitates [*d*], then so does the set consisting of every second electron: {*e2*, *e4*, *e6*, …}, and so does the set consisting of every fourth electron: {*e4*, *e8*, *e12*, …}, and so on. Thus there is an infinite number of proper subsets of {*e1*, *e2*, *e3*, …} that each necessitate [*d*]. If this is so, then clause (4) of MN entails that there is no such *minimally* necessitating set. Given the plausibility of the claim that [*d*] is grounded in denumerably many distinct electrons, it follows that we have grounding without minimal necessitation, and so that MNNG is false (cf. Fine 2012, 57; Restall 1995; Schaffer 2010, 314). Call this the *no minimality problem.*

I reply that each distinct denumerable set of electrons minimally necessitates a corresponding distinct fact. For example, let *S1* = {*e1*, *e2*, *e3*, …}, let *S2* = {*e2*, *e4*, *e6*, …}, etc.; let ‘denumerable*e1, e2, e3*, …’ refer to the members of *S1*, let ‘denumerable*e2, e4, e6,* …’ refer to the members of *S2*, etc.; and let [*d1*] be the fact that there are denumerable*e1, e2, e3*, … distinct electrons, let [*d2*] be the fact that there are denumerable*e2, e4, e6,* … distinct electrons, etc. Thus, *S1* minimally necessitates [*d1*], *S2* minimally necessitates [*d2*], etc.; by FG, it follows that: the members of *S1* fully ground [*d1*], the members of *S2* fully ground [*d2*], etc.

As for [*d*] itself, either it is identical to some [*di*], or it is not.If it is, then, forwhichever [*di*] is identical to [*d*], we have just seen that there is a corresponding *Si* that minimally necessitates [*di*], and so, by FG, it follows that the members of *Si* fully ground [*di*]. On the other hand, if all of [*d1*], [*d2*], …, are equally good candidates such that there is no particular [*di*] to which [*d*] is identical, then it is still true that there is no [*di*] that fails to be fully grounded in the members of a corresponding *Si*. Thus, whether or not [*d*] is identical to some [*di*], and contrary to the no minimality problem, we *can* make sense of the claim that [*d*] is grounded in denumerably many distinct electrons.

1. **Conclusion**

GMN is an account of grounding with substantive commitments. First, it entails that mathematical facts have no partial grounds (§3). Second, it entails that numbers are sets (§3). Third, it entails that relatively mereologically simple objects have their parts essentially (§5). I have already argued that these commitments are less onerous than they appear; nevertheless, I grant that they are costs. But philosophers have not (so far) despaired of providing a reductive account of grounding because of the substantive commitments that inevitably accrue to such an account. Rather, philosophers have given up on such accounts because of their perceived inability to handle a range of counterexamples. With respect to necessitation-based accounts in particular, the consensus view is that they are not viable, and this is because of the (approximately) universally-accepted claim that NSG is false. The two most significant reasons given for this claim are the respective problems of spurious necessitation, and of symmetric necessitation. Grounding contingentists also think NNG is false, and two of the more prominent difficulties that have seemed to tell in favour of their conclusion are the rearrangement problem, and the problem of restricted accidental generalizations. For one who would defend the claim that grounding is reducible to necessitation, the task is formidable for she must defend NSG *and* NNG.

I have argued that GMN avoids not only all four of these objections, but also an additional objection – the no minimality problem – that threatens minimal necessitation accounts in particular. If my arguments are compelling, then, insofar as we thereby have a strong prima facie case for thinking that grounding *can* be reduced to (minimal) necessitation, we have a strong prima facie case for thinking the consensus view is mistaken.[[24]](#footnote-24)

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1. For a good overview of the vast literature on reductive and non-reductive accounts of causation, see Schaffer (2016, §2). [↑](#footnote-ref-1)
2. I take no stand here on the question of whether grounding is analogous to causation. [↑](#footnote-ref-2)
3. Among those who accept necessitarianism are: Audi (2012), Cameron (forthcoming), Dasgupta (2014a), (2014b), deRosset (2010), (2013b), Rosen (2010), Trogdon (2013), and Wilsch (2015). Contingentists include Bricker (2006), Leuenberger (2014), Schaffer (2010), and Skiles (2015). It is somewhat unclear whether Chudnoff (2013) endorses contingentism (see n. 5 for further explanation). [↑](#footnote-ref-3)
4. Note that some, such as Correia (2010) and Fine (2012), do not explicitly accept that grounding is a relation; rather, they accept only the weaker thesis that grounding claims should be regimented using a non-truth-functional sentential connective. [↑](#footnote-ref-4)
5. There are other objections that I will not address. First, Elijah Chudnoff (2013, 186-194) gives plausible examples that seem to show that certain sorts of visual experiences ground certain sorts of justificatory states without necessitating them. As Chudnoff (2013, 194) is careful to emphasise, however, his target is not NNG itself, but the conjunction of NNG and the claim that grounding is an explanatory relation. Though I do not deny that grounding is sometimes explanatory (cf. my response to the problem of spurious necessitation in §3), the account of grounding I will develop is independent of the claim that grounding is always explanatory, and my goal is merely to defend NNG. Second, Stephan Leuenberger (2013) argues that NNG is incompatible with certain plausible physicalist assumptions, on the one hand, and with certain plausible assumptions about dispositions, on the other. Nevertheless, Leuenberger is explicit that one who has antecedent reason to accept NNG thereby has reason to reject his argument for the incompatibility of NNG with physicalism (2013, 158); as we will see, I am defending the claim that a type of necessitation is necessary (and sufficient) for grounding, and so I have such a reason. And although Leuenberger does not say so, presumably one who already accepts NNG also has an analogous reason to reject his argument for the incompatibility of NNG and the plausible claims about dispositions(i.e., the structure of Leuenberger’s two arguments is similar enough to vindicate the plausibility of this interpretation). Third, although each of Philip Bricker (2006), Jonathan Dancy (2004), and Benjamin Schnieder (2006) have also argued against NNG, Kelly Trogdon (2013) has adequately responded to these objections. [↑](#footnote-ref-5)
6. Though see McDaniel (2017) for an opposing view. [↑](#footnote-ref-6)
7. I elaborate below on PC. [↑](#footnote-ref-7)
8. As is standard, I assume that the generic form of a full grounding claim takes a plurality of variables “on the left”, and a single variable “on the right” (i.e., *x1*, *x2*, *x3*, … fully ground *y*), but that this is compatible with cases that take only a single variable on the left (e.g., when *x* fully grounds *y*). [↑](#footnote-ref-8)
9. Though this prohibition on self-grounding is standard, see Jenkins (2011), Thompson (2016), and Wilson (2014) for dissenting views. [↑](#footnote-ref-9)
10. Though Schaffer (2009) rejects the claim that grounding is reducible to necessitation, he also holds a similarly permissive view of the relata of the grounding relation (cf. 2009, 375-376, n33). [↑](#footnote-ref-10)
11. Krämer and Roski (2017, §7) discuss a notion of proportionality with respect to grounding, while Yablo (1992, 277ff) discusses such a notion with respect to causation. Though there are perhaps some similarities between these authors’ respective views of proportionality and my own, I have developed my position independently. [↑](#footnote-ref-11)
12. Clause (2) of PC does not rule out {[*a*], [*b*]} minimally necessitating [*a* ∨ *b*]. Nevertheless, as {[*a*], [*b*]} has a proper subset (i.e., {[*a*]}, if [*a*] obtains, or {[*b*]}, if [*b*] obtains) that minimally necessitates [*a* ∨ *b*], it follows that {[*a*], [*b*]} never minimally necessitates [*a* ∨ *b*]. [↑](#footnote-ref-12)
13. Although I am currently arguing for MNSG, the claim in the text above (i.e., “…no set with [*b*] as a member minimally necessitates [*addition*];it follows that [*b*] neither fully nor partially grounds [*addition*].”) obviously presupposes MNNG.So this argument (and others that make similar presuppositions) must remain a promissory note until I (hopefully) vindicate MNNG in §§5-7. [↑](#footnote-ref-13)
14. For natural numbers *n*, *m*, a standard recursive definition of addition is: *n* + 0 = *n*, and *n* + *S*(*m*) = *S*(*n* + *m*). [↑](#footnote-ref-14)
15. Cf. Lewis’s (1993, 29) remark on supervaluationism: ‘It’s like the old puzzle: I owe you a horse, but there’s no horse such that I owe you that horse’. [↑](#footnote-ref-15)
16. For example, Donaldson (2017), drawing on and extending the work of Rosen (2010), and Schwarzkopf (2011), considers (without ultimately endorsing) an account according to which various mathematical facts have partial grounds. [↑](#footnote-ref-16)
17. In the following section, although I deny the plausible assumption that ordinary composite objects are grounded in some arrangement of their parts, this denial does not affect my argument here. To preview, due to the “problem of the many”, there are many candidate objects in Socrates’ immediate vicinity, and although each of these many objects is (fully and partially) grounded in the arrangements of their respective parts, there is no principled way to determine which of these many is identical to Socrates. [↑](#footnote-ref-17)
18. Note that my solution to the grounding relation that holds between Simple and {Simple} can be generalised to other cases. Let *F* be the property *being mereologically simple*. Let [Simple is *F*] be the *fact* that Simple is *F*, let <Simple, *F*> be the *singular proposition* that Simple is *F*, and let *Simple is F* be the *event* of Simple’s being *F*. It is plausible that Simple grounds [Simple is *F*], but not vice versa; that Simple grounds <Simple, *F*>, but not vice versa; and that Simple grounds *Simple is F*, but not vice versa. My account straightforwardly delivers these results. For example, *C* = {Simple} minimally necessitates [Simple is *F*]; as Simple is the sole member of *C*,FG entails that Simple fully grounds [Simple is *F*]. Let [Simple is *F*] be the sole member of the set *C\**; that is, let *C\** = {[Simple is *F*]}. As [Simple is *F*] is not identical to Simple, *C\** violates clause (1) of PC with respect to Simple. Obviously analogous arguments establish similar results with respect to Simple and < Simple, *F*>, on the one hand, and Simple and *Simple is F*, on the other. [↑](#footnote-ref-18)
19. Skiles (2015, 722) does not explicitly state that the existence of *o* is *fully* grounded in Φ; nevertheless, as full (and not merely partial) grounds necessitate, the ‘fully’ qualifier is necessary for Skiles’ argument to go through. [↑](#footnote-ref-19)
20. One might consider the intersection, *I*, of all of the sets that minimally necessitate each of *o1*, *o2*, *o3*, …, and insist that *I* itself minimally necessitates *o* at *t*. If so, FG entails that the members of *I* fully ground *o* at *t*. But surely there is no such *I*. First, let *I* = {*a1*, *a2*, *a3*, …}. Second, there are many plausible *o* candidates, *on*, *on+1*, *on+2*, …, and, for each *ai*, there is an *oi* that fails to have that *ai* as a part. If so, then, third, it is also plausible that each of *on*, *on+1*, *on+2*, … is minimally necessitated by a corresponding set: e.g., *Sn* = {*a2*, *a3*, *a4*…}, *Sn+1* = {*a1*, *a3*, *a4*, …}, *Sn+2* = {*a1*, *a2*, *a4*, …},…. But then, fourth, it follows that there is no intersection of *Sn*, *Sn+1*, *Sn+2*, …. [↑](#footnote-ref-20)
21. In holding that *S1\** ceases to exist at *t1*, I am assuming that a set’s identity is uniquely determined by its members. [↑](#footnote-ref-21)
22. There may be other reasons to accept a totality fact (e.g., for grounding unrestricted negative existential claims), but discussion of such reasons lies beyond the scope of this paper. [↑](#footnote-ref-22)
23. Thus I offer the following amendment: a well-formed fact is fundamental iff it has no grounds. [↑](#footnote-ref-23)
24. Thanks to all of the following for helpful feedback on various drafts of this paper: Ross Cameron, Louis deRosset, Geoff Goddu, Paul Nedelisky, Donald Smith, and three anonymous referees. [↑](#footnote-ref-24)