Chapter 9
Paradigms and Self-reference: What Is the Point of Asserting Paradoxical Sentences?

Jakub Mácha

9.1 Paradoxes

Let me begin with the notion of a paradox. A paradox is usually defined as a self-contradictory (or at least unacceptable) conclusion that is derived from true premises by sound reasoning. For example, Russell's paradox is derived from the axioms of naïve set theory, and this derivation is sound. A self-contradictory conclusion indicates that there is something wrong with the original premises (or with the rules of reasoning). The original Greek meaning of 'paradox' is, however, different. $\pi\alpha\rho\acute{\alpha}\delta o\xi o\varsigma$ (parádoxos) is something contrary to common belief (doxa). A paradox is, thus, something unexpected, strange, something surprising.[1]

Wittgenstein's understanding of paradox goes back to this original Greek meaning. We can find passages in his writings where the word 'paradox' is used as an alternative for 'surprise': '[such a] representation of it makes a situation surprising, or astonishing, even paradoxical' (RFM I, appdx. II, §1). Here is Wittgenstein's most prominent characterization of a paradox: 'Something surprising, a paradox, is a paradox only in a particular, as it were defective, surrounding. One needs to complete this surrounding in such a way that what looked like a paradox no longer seems one' (RFM VII, §43). Or, in a similar manner: 'A sentence [Satz] is a paradox only if we abstract from [absehen] its use' (RPP I, §65). Hence, paradoxes arise when we take sentences out of their context of use. A sentence appears paradoxical, i.e. surprising, only if we abstract away from its context of use—only if its context of use is missing. One way of dealing with paradoxical sentences is just to imagine the missing context of use, i.e. the missing language-game.

There is a long history of paradoxes of self-reference, going back to the Epimenides paradox. I will begin, however, with Frege's concept 'horse' paradox. For

[1] Cf. Kutschera (1989) for an overview.

J. Mácha (✉)
Masaryk University, Brno, Czech Republic
e-mail: macha@mail.muni.cz

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Frege (1892), concepts are functions, unsaturated entities. Functions are saturated by objects, which are saturated entities. This distinction between concepts and objects is derived from a grammatical analysis of the use of definite and indefinite articles. A noun with the definite article refers to an object [‘auf einen Gegenstand hinweist’], whereas a noun with the indefinite article refers to a concept. Frege was aware of several exceptions to this rule (due to the logical imperfections of ordinary language). This intuitive distinction leads, as Bruno Kerry observed, to a paradox if we consider the expression ‘the concept “horse”’. In this expression, ‘concept()’ is a concept and ‘horse’ is an object. This concept is then saturated in the expression ‘the concept “horse”’ and becomes an object. Thus, the concept ‘horse’ is an object. Or, in other words, the concept ‘horse’ is not a concept. This is an absurd consequence, which Frege was not able to deal with. It was, for him, ‘a certain inappropriateness of linguistic expression’, that is, a distortion of natural language that could be overcome in a logically perfect language.

For Frege, an object cannot be a concept and *vice versa*. There is an unbridgeable gap between these two classes of entities. Kerry’s suggestion, however, is that there is an entity that is both a concept and an object at the same time. ‘Horse’ is an object if it is an argument of the function ‘concept()’ and it is a concept, at the same time, if the application of the same function is supposed to yield the truth.

This paradox of the concept ‘horse’ is circumvented in the *Tractatus*. For Wittgenstein, the problem lies in the idea of assimilating philosophical (as well as logical and linguistic) vocabulary into language itself.[2] This philosophical vocabulary includes concepts such as ‘concept’, ‘object’ and ‘name’. ‘Concept’ is a formal concept (or pseudo-concept) which—unlike proper concepts—is not supposed to be represented by a function or by a class, but by a propositional variable (4.126). ‘The concept “horse”’ is, thus, *not* an application of the function concept() to the name ‘horse’, and therefore ‘the concept “horse”’ is not a name, but a variable. It makes no sense to consider the application of a formal concept to something, because formal concepts are, quite simply, not something that can be applied. To be a concept is a formal characteristic (or feature) of the name ‘horse’. This feature cannot be expressed in a proposition, but can only be shown. The introduction of formal concepts thus effectively avoids the ‘horse’ paradox. This circumvention is incorporated into the conceptual framework of the *Tractatus*. Names refer to objects. If there were a name X of a name Y, then Y would be an object. Objects, however, are simple; they cannot mean anything. Hence, there is no name of Y.

The gap between concepts and objects is not closed in the *Tractatus*. It is instead transformed into the gap between what can be said and what can be shown: ‘What can be shown, *cannot* be said’ (4.1212). Let us now move on to Wittgenstein’s later philosophy, where, I will argue, this gap is bridged.

[2]Cf. Diamond (1995, p. 184): ‘The predicate “concept” cannot be predicated of any concept: we think we want it in order to say things about concepts, but to think that it will enable us to do so is confusion, confusion about what a concept is. Wittgenstein thought that the *whole* philosophical vocabulary reflected confusion.’

9.2 Paradigms, Rules and Private Language

Let a *paradigm* of X be a *material object* together with the *praxis* of applying this object in a given situation. I call the object of a paradigm a *paradigmatic sample*. The term 'object' is understood in the broadest way possible. Paradigmatic samples are real material things, ranging from clearly defined objects such as the metre stick or a colour plate to intricate structures such as formalizations of mathematical proofs or works of art. A paradigmatic sample is a model of a situation (what Wittgenstein called a 'model' in his early *Notebooks*, and an 'image' [Bild] in the *Tractatus*). A paradigm is, thus, a model along with a method for its projection or, rather, a method of comparison.[3] A paradigmatic sample is an object of comparison (PI, §130).[4]

Wittgenstein considers many instances of giving a definition of a certain expression by means of a paradigm. The most familiar ones are the definition of 'one metre' and 'sepia' by reference to the standard metre rod (or another yardstick [Maßstab]) and the standard sepia sample. Is this technique in any way general? Are we qualified to say that every expression must be defined by means of a paradigm? The later Wittgenstein's method involves comparing language to (or looking at language as) a rule-governed activity.[5] There are two points to bear in mind. Firstly, Wittgenstein often compares using language and measuring with a yardstick [Maßstab].[6] This is a general analogy. Note that Wittgenstein conceives of paradigms and yardsticks at two distinct levels: at the general level of this analogy and at the local level of defining a particular expression by means of a paradigm. Second, Wittgenstein at times uses the expressions 'rule' and 'paradigm' (or 'yardstick', 'model', 'object of

[3]Cf. Wittgenstein's remark that a paradigmatic sample (yardstick) alone is a dead object: 'Lay the yardstick alongside a body; it doesn't say that the body is such-and-such a length. Rather, I want to say, in and of itself it is dead, and it achieves nothing of what thought achieves' (BT, pp. 70e–71e). On the other hand, the materiality of a paradigmatic sample must not be taken as its imperfection: 'In the case of the measuring stick you can't say: "Yes, the measuring stick measures length in spite of its corporality; but a measuring stick that had only length would be the ideal, would be the *pure* measuring stick". No, if a body has length, then there can't be any length without a body—and even if I do understand that in a certain sense only the length of the measuring stick does the measuring, yet what I put into my pocket is still the measuring stick—the body and not the length.' (BT, p. 352e).

[4]Every rule and concept has a pictorial (iconic, natural) component and a conventional (habitual, customary) component (the praxis).

[5]Though there are other objects of comparison that do not involve rules; cf. Kuusela (2019, §§5.5 and 6.1) for details.

[6]Cf. the following very early remark from 1914: 'Proposition and situation are related to one another like the yardstick and the length to be measured' (NB, p. 32). This idea is developed in the *Tractatus* (2.1512) and further in Wittgenstein's later thought. For instance, 'der Satz wäre wie ein Maßstab an die Wirklichkeit angelegt' (Ms-109, 272), or in the *Big Typescript*: 'When I compared a proposition to a yardstick, then strictly speaking all I did was to take a proposition that relies on a yardstick to state a length and use it as an example of a proposition./If one conceives of propositions as instructions for making models, their picture-like quality becomes even clearer' (BT, p. 67e). In Ms-109, 176, Wittgenstein says: '"In der gleichen Sprache ausdrücken" heißt mit dem gleichen Maßstab messen.' Or simply: 'Vergleiche die Grammatik von "Satz", "Maßstab" und "Gedanke".' (TS-219, 3).

comparison') interchangeably or for textual variation. So, for instance, in the *Big Typescript* we read that 'the steps in the chain of equations have been taken in accordance with *these* particular rules (or paradigms)' (BT, p. 460e).[7] It thus seems that comparing language to a rule-governed activity is close to comparing it to measuring with a yardstick or paradigm. These two objects of comparison are very close to each other. In fact, I would like to claim that, beyond these exegetical considerations, they amount to the same thing.

I shall now argue that paradigms involve rules and rules involve paradigms. The former claim is fairly obvious. Defining an expression using a paradigm certainly involves some rules; namely the rules of how the comparison with (or the measurement by) the paradigmatic sample is to be carried out. In PI, §53, Wittgenstein restates paradigms in the form of a chart [Tabelle] and further explains that 'we call such a chart the expression of a rule of the language-game' (cf. also PI, §§73, 74, 162, 265). The latter claim merits closer consideration. Can any rule be restated in terms of a paradigm? In other words, does every rule involve some comparison with a paradigmatic sample? Let us consider public and private rules separately. Can there be a public rule that does not involve any paradigm? The negative answer to this question is guaranteed by a certain flexibility of the concepts of paradigm and rule. The notion of a paradigmatic sample is defined very broadly. A paradigm can be thought of as any rule-based technique that, at some points, takes or depends on external objects as its inputs. In that case, the concepts of a public rule and of a paradigm are effectively equal.

What about private rules? The considerations discussed in the previous paragraph would entail that a rule of this sort that does not make use of any paradigmatic sample would be a private rule. The argument could be made by pointing to the result of the private language argument, namely that private rules are impossible. I think an even stronger conclusion is available if the private language argument is examined more closely. We have to realize that the aim of an ostensive definition of a general term (as opposed to a proper name) is to introduce a sign by pointing at a paradigmatic sample and stipulating in which respect entities of that kind are similar to the sample (i.e. stipulating the method of comparison or measurement).[8]

There have been many interpretations of the private language argument. I can focus only on those that are relevant to the overall topic of this essay, and shall leave others aside without commenting on them. Several commentators (McGinn 1997; Stroud 2001; Ahmed 2017) have restated the problem with private rules as a failure

[7]Here is another formulation from the early 1930 s period: talking about the equation a + (b + c) = (a + b) + c, Wittgenstein says: 'For if the proposition is a *rule*, a *paradigm*, which every calculation has to follow, then it makes no more sense to talk of working out the equation, than to talk of working out a definition' (PG, p. 395, my italics).

[8]This precise idea is expressed in the *Big Typescript*: 'in an ostensive definition I do not state anything about the paradigm (sample), but only use it to make statements; that it belongs to the symbolism and is not one of the objects to which I apply the symbolism' (BT, p. 408e). Or we can refer to §1 of the *Investigations*: the shopkeeper possesses paradigmatic samples for 'apple', 'red' and numbers, and uses them as described in the passage. 'It is in this and similar ways that one operates with words.'

of a *private* ostensive definition. In PI, §258, Wittgenstein considers establishing the meaning of the sign 'S' by concentrating one's attention on a sensation. This is effectively a kind of ostensive definition. This private ostensive definition is, however, futile; it fails to determine the subsequent use of 'S'. But why is this so? McGinn and Stroud argue that the problem with a private ostensive definition is that the private diarist fails to specify the respect in which a future episode will resemble the original sensation. The private diarist fails to specify that 'S' is 'a concept of a certain kind, i.e. [has] a particular use or grammar' (McGinn 1997, p. 131). Ahmed provides another reason for the failure of a private ostensive definition. A private ostensive definition is underdetermined between the following cases: '"S" refers to the *original* felt quality of the initial sample', and 'at any later time *t*, "S" refers to his *memory at time t* of the initial sample' (Ahmed 2017, p. 47). That is to say, 'S' refers to the initial sensation or to the diarist's memory of it. On all these interpretations, the problem with a private language or private rules thus boils down to defining a sign by means of a private paradigm (McGinn and Stroud effectively say that what is problematic is the method of comparison, whereas Ahmed says that the problem lies in the indeterminacy of the paradigmatic sample).

Before moving on, let us pause for a moment and consider the very idea of a private sample. In the early 1930s, Wittgenstein contrasts a yardstick [Maßstab] or a measuring instrument [Meßinstrument] with an *Augenmaß*, which, in this context, can be translated as 'private sample' or 'private yardstick'.[9,10] He maintains that such a private yardstick is not an inaccurate yardstick, but something completely different from a physical yardstick (Ms-109, 244[2]). A private table [Tabelle] is of no use: 'Looking up a table in the imagination is no more looking up a table than the image of the result of an imagined experiment is the result of an experiment' (PI, §265). Here, the idea of a private paradigm is not wholly inconsistent. It is clear, however, that we can use private samples only if there is a way of using a public paradigm in the same situation (or we must at least be able to imagine an actual measurement of this sort).

Let me conclude this section by saying that, for Wittgenstein, the grammar of the words 'paradigm', 'yardstick' and 'measure' is closely related to the grammar of the word 'rule' (cf. PI, §150).[11]

[9]Cf. BT, p. 43; Ms-107, 264[3]; Ms-109, 244[2]; Ms-110, 280[2].

[10]There is an analogous idiom in English: to *eyeball* something means to judge it by eye. Here, however, the intimate connection to a measurement is lost.

[11]See Diamond (2001, pp. 125–132) for another elaboration of this parallel between the understanding of measurement and of rules.

9.3 Paradigm Paradox

We can find a restatement of the concept 'horse' paradox in terms of paradigmatic samples in Wittgenstein writings: 'one sentence can never describe the paradigm in another, unless it ceases to be a paradigm' (PG, p. 346).[12] Let me call this the *paradigm paradox*.

This way of putting things is striking for its generality, which is unusual for Wittgenstein even in the 1930s. There are several instances scattered throughout Wittgenstein's writings: 'one cannot say of a group of strokes serving as a paradigm of 3, that it consists of 3 strokes' (PG, p. 346)—or of the standard metre rod that it is or is not one metre long, or of the standard sepia sample that it is this colour or it is not (PI, §50). Why can we not say this? To be one metre long means being the same length as the standard metre when compared—and there must be a standard method of comparison.[13] To say that the standard metre is one metre long requires us to compare the standard metre with itself. For this peculiar kind of comparison to make sense, we have to specify how it works. We can say that the length of the standard metre is always identical with itself. That is, the standard metre is always one metre long, whatever happens. But this is an *ad hoc* stipulation that, moreover, does not explain its negation, i.e. saying that the standard metre is not one metre long.

What we can do, however, is describe a paradigm by using another paradigm. Then, of course, the original paradigm ceases to be a paradigm in this usage. We can say, for example, of the standard metre that it is 1.0936133 yards long. Then, however, the paradigm of length is the standard yard, not the standard metre.

Remember that, according to Wittgenstein, paradoxes arise when we take sentences out of their context of use. When someone says this rod is one metre long, there is always an implicit commitment to the act of comparing the rod with the standard metre. This praxis is part of the meaning of 'one metre long'. In everyday situations, we do not do this, of course (no real shopkeeper uses paradigmatic samples as the shopkeeper from §1 of the *Investigations* does). But this praxis must at least be possible. A proposition may be a paradox, if we abstract from this praxis, from this context of use, and take 'one metre long' to be a function ranging over all material objects. Then the situation may occur that we apply this function to an object that appears in its definition, that is, to the standard metre rod.

[12] 'Und ein Satz kann das Paradigma im andern nie beschreiben, sonst ist es nicht das Paradigma.' From the German wording it is clear that the pronoun in the second clause 'es' refers to the paradigm from the first clause.

[13] This is, of course, a crude simplification of the actual praxis, which, however, does not impact on the overall argument.

9.4 Russell's Paradox

The (intuitive) belief that we can express in language any property whatsoever has been captured by various *principles of unrestricted abstraction* or comprehension (e.g. Frege's Basic Law V). The principle states that for any property nameable in language, there is a concept expressing this very property. This principle implies that we can express in language properties such as 'being one metre long', 'being sepia-coloured' or 'having three elements'. We can, of course, do this intuitively. Our intuition says (or tempts us to accept) that these concepts (whose existence is postulated by the principle) can be truthfully applied to *any* objects having this or that property. The principle of abstraction is *unrestricted*; this means that nothing can be excluded from the range of the universal quantifier. For instance, the concept 'being one metre long' can be truthfully applied to *any* object with a length of one metre. The standard metre cannot be excluded. This captures our everyday intuition that the standard metre is one metre long.

This intuition is wrong, however. It is well known that the principle of unrestricted comprehension leads to a contradiction. *Russell's paradox* follows from a set-theoretical version of the principle:

> For any formula $\phi(x)$ containing x as a free variable, there will exist the set $\{x\colon \phi(x)\}$ whose members are exactly those objects that satisfy $\phi(x)$.

And again, this principle is very intuitive. If $\phi(x)$ stands for 'x is one metre long', then $\{x\colon \phi(x)\}$ will be the set containing all objects that are one metre long, including the standard metre. Russell's paradox follows if we let $\phi(x)$ stand for $\neg(x \in x)$. Russell's initial solution or, rather, response to his paradox is to adopt the *vicious circle principle*. This principle states that 'whatever involves all of a collection must not be one of the collection'; that is, a collection must not contain members that are presupposed by the whole collection.[14]

Let us return to paradigms. We can regard paradigms as characteristic functions of collections. A paradigm generates a collection of objects that are in accord with this paradigm. Accordingly, a paradigmatic sample is precisely an object that is presupposed by the collection of objects that are defined by this paradigm. The question of whether we can describe a paradigmatic sample using the very same paradigm is structurally equivalent to the question of whether a set contains itself as a member. From this structural equivalence it follows that the vicious circle principle effectively counteracts the paradigm paradox as well. Or, in other words, Wittgenstein's claim that 'one proposition can never describe the paradigm in another' is a version of the vicious circle principle.

Russell's vicious circle principle is a prohibition on possible members of a collection. The principle states something we *must not* do.[15] Wittgenstein's own principle, however, continues: 'unless it ceases to be a paradigm'. This means that if we attempt to apply a paradigm to its own paradigmatic sample, it ceases to be a paradigm in this

[14] This account follows Irvine and Deutsch's (2016) entry on Russell's paradox.

[15] Russell's theory of types is a more sophisticated version of this prohibition.

usage. This is the point that goes beyond Russell's vicious circle principle. There is no such absolute prohibition in Wittgenstein's version of the principle. Propositions describing the paradigm are not prohibited. They are, however, paradoxical. But remember, again, that a sentence will only appear paradoxical if we are not aware of its context of use. Our task is to identify a language-game that involves such sentences, and the paradox will then vanish.[16] What I shall now attempt to do is work out what a use of paradoxical sentences of this kind might be.

9.5 The Self-reference of a Paradigm, the Self-membership of a Set

We shall look for sentences that assert the self-reference of a paradigm or the self-membership of a set. Let us consider Wittgenstein's own examples first. §50 of the *Philosophical Investigations* is well known. In this passage, we read: 'There is *one* thing of which one can say neither that it is one metre long, nor that it is not one metre long, and that is the standard metre in Paris.' But this prohibition is restricted to 'the language-game of measuring with a metre-rule' (ibid.) I take this to mean that we cannot ascribe the property of being one metre long to the standard metre by using the sentence 'The standard metre is one metre long'. This does not, however, exclude the possibility that there is another use of the sentences 'The standard metre is one metre long' and 'The standard metre is not one metre long'. Let us move on to a noteworthy passage from the final part of the *Remarks on the Foundations of Mathematics*, which opens as follows:

> What *sort* of proposition is: 'The class of lions is not a lion, but the class of classes is a class'? How is it verified? How could it be *used*?—So far as I can see, only as a grammatical proposition. To draw someone's attention to the fact that the word 'lion' is used in a fundamentally different way from the name of a lion; whereas the class word 'class' is used like the designation of one of the classes, say the class *lion*. (RFM VII, §36)

This remark says that paradoxical sentences like 'The class of lions is not a lion, but the class of classes is a class' can be used as a grammatical reminder.[17] Or—as Wittgenstein also suggests—we can employ the Tractarian method of analysis and say that 'lion' is used in two different ways or with two different meanings here (e.g. 'Lion' as the name of a particular lion or "Cat" as the name of a cat). Another suggestion is that we can 'ascribe a sense out of politeness' to these sentences. I take this sense to be whatever sense is ascribed by an accidental stipulation—analogous to the stipulation that the standard metre is (always) one metre long.

These are, however, ways to analyse the paradox away by getting rid of the self-referential structure. Wittgenstein wants to take the sentences in the right way [richtig auffaßen]—not to analyse them away but to find language-games for them.

[16] This could be seen as a general characterization of the later Wittgenstein's method.

[17] Cf. PI, §127: 'The work of the philosopher consists in assembling reminders for a particular purpose.'

Animal fables [Tierfabeln] are the first context of language-games that Wittgenstein considers:

> The fable says: 'The lion went for a walk with the fox', not a lion with a fox; nor yet the lion so-and-so with the fox so-and-so. And here it actually is as if the species lion came to be seen as a lion.

Wittgenstein undertakes a grammatical analysis based on the use of the definite and indefinite articles. The very same analysis is the basis of Frege's distinction between concepts and objects, as we saw above. The definite article refers to an object (for Frege) or to an instance of a noun that is identifiable from the context. 'An instance of a noun' is, in this context, analogous to 'an element of a set'. There are, however, many exceptions to this rule.[18] We can analyse the definite article as referring to every member of a set (e.g. in 'The fox is a predatory animal'), i.e. as the universal quantifier. Yet this is clearly not the analysis that fits the context of the fable. Wittgenstein considers and opposes another analysis that was suggested by Lessing:

> (It isn't as Lessing says, as if a particular lion were put in the place of some lion or other. 'Reynard the Fox' ['Grimmbart der Dachs'] does not mean: a fox of the name 'Reynard'.) (RFM VII, §36)

Lessing says, in his *Essays on Fable*, that the fox represents a distinctive feature of its species (e.g. cunningness), and continues: 'It would be a childish abuse of words to say that the particular is *similar* to the general, the individual to the species, and the species to the genus' (Lessing 1825, p. 72). Lessing's view is, thus, that in a fable a particular animal *exemplifies* a certain feature of its species. Accordingly, Reynard the Fox represents the characteristics of being thoughtful and calm, for instance. However, exempla are not mere instances; by exemplifying a certain feature, they involve every member of its species (of a set, of a collection), and in this respect they are analogous to paradigms. Lessing's view is not as simple as putting a particular animal (lion, fox, etc.) in the place of some animal in a fable—as Wittgenstein seems to read him. Despite this inaccurate interpretation of Lessing's views, Wittgenstein's point is that the definite article must not be analysed away by interpreting it in terms of the indefinite article and that it is not childish to say that the particular is similar to the general—as we shall see in the next section.

9.6 Asserting Self-referential Sentences

Finally, let us move on to Wittgenstein's positive examples of this kind of self-reference. The passage quoted above continues:

[18] This rule is rather complex. There are differences between particular languages, such as English and German. Moreover, there are languages that do not use articles, such as Latin and Slavic languages.

> Imagine a language in which the class of lions is called 'the lion of all lions', the class of trees 'the tree of all trees', etc.—Because people imagine all lions as forming *one* big lion. (We say: 'God created man [Gott hat den Menschen geschaffen]'.) (RFM VII, §36; Ms-124, 125–126)

In this passage, which curiously enough has escaped the attention of commentators, Wittgenstein discusses language-games that deal with self-membered classes. In 'God created man' (in German with the definite article 'den Menschen', accusative singular), 'man' refers to all men and at the same time to a particular man Adam. The quotation from Genesis 1:27 continues with 'in his own image, in the image of God created he him'. The first man and man, the class of men, is created as an image of God. The first man is himself an image of man; he is, so to speak, an archetype of man—a paradigmatic sample. In the same way, the lion of all lions can refer to the paradigm of a lion, or the song of songs to a paradigmatic song.[19]

I take from these considerations that there is a structural analogy between a noun being employed as a self-membered set and a paradigmatic sample being included in or excluded from the set it generates. If there is a use for employing self-membered sets, there might be a use for describing paradigmatic samples using the very same paradigm. Wittgenstein's vicious circle principle says that if we attempt to use such a self-referential description, the paradigm ceases to be a paradigm. What, then, would be the use of the sentence 'the standard metre is *not* one metre long'? To point out two different uses of 'metre'? This would be too confusing, and besides, there is a better way of putting this point. It cannot mean that the standard metre has changed its length either, for this must be put in terms of another paradigm, e.g. by saying that the standard metre is not 3.28 feet long any more. The only employment of this sentence I can imagine is to indicate a change in the praxis with the standard metre or, more precisely, a change in the method of comparison (e.g. with respect to the accuracy of the comparison, the material the standard metre is made of, the storing of the standard metre in a certain environment, or its replacement by a beam of light). Let us imagine that we can use the standard metre to measure another object using the old method, and subsequently we can use this object to measure the standard metre using the new method. Then we might say that the standard metre is *not* one metre long.

Is the sentence 'The standard metre is not one metre long' paradoxical? It may be if 'paradox' means *surprise* and if we are not envisioning its context of use, i.e. an attempt to compare a stick with itself. However, even if we are aware of the context, we can nevertheless be surprised, because then this sentence indicates that the method of comparison (or the praxis in general) has changed. Moreover, there is another surprising element: the way in which the praxis has changed is completely arbitrary,

[19]The construction 'X of Xs' ('song of songs', 'holy of holies', 'king of kings', etc.) is common in the Old Testament. It expresses a *superlative*. Although it is in the singular, it does not necessarily express a single element (cf. Keel 1994, pp. 38–39). Today, there is a similar idiomatic construction: 'the mother of all somethings', e.g. Douglas Engelbart's 'The Mother of All Demos'. This refers to something's being superlative not in the sense of having the greatest degree of a certain characteristic, but rather in having the greatest number of characteristics of X. There is a single thing that has the greatest number of characteristics of X, the most X, the paradigmatic sample of X.

if we have no further context than this sentence. The old paradigm has ceased to exist and no new paradigm has been established by asserting the sentence.[20]

It is often said that a paradox can cause confusion if it goes undetected. If 'paradox' means *surprise*, there are no undetected paradoxes. We cannot be surprised without knowing it. However, one can cause confusion by describing paradigmatic samples using the very same paradigm *without realizing it*. This situation arises if we derive a measuring instrument (e.g. a ruler) from the standard metre and subsequently use this instrument to measure the standard metre without realizing that the measured object is the standard metre. That leads to the conclusion that one can confuse a paradigmatic sample with an object generated by this paradigm. This is analogous to confusing a set with its member (which, as we saw above, is also possible) and, of course, to confusing a concept with an object.

This series of (possible) confusions leads to the following question: can one confuse the concept of X with an X itself? Given the discussion above, the answer must be affirmative. But how is this possible? How can one possibly confuse the word 'lion' with a lion? Neither the graphic (written) form of 'lion', nor the acoustic (spoken) form of 'lion' resembles a real lion. Such a resemblance occurs only in rare onomatopoeic expressions such as 'bark', 'cry', 'whisper' or 'beep', or shape words (phenomimes) such as 'U-turn'. Or we can imagine a fully pictorial language where every sign is a picture of the thing it refers to. However, this is not the confusion in question here. The quotation marks refer to the graphic form of an expression, not to a concept; they refer to a sign, not to a symbol (in the terminology[21] of the *Tractatus*). The question, however, is: how can one confuse the symbol of X, i.e. 'X' plus the sense of X, with an X? 'X' refers to an X in a given context. This amounts to saying that the reference of 'X' is an X. If 'X' is actually referring, then all we get is an X, and then the confusion ensues. In order to avoid the confusion, what we need is to capture (or, better, freeze) the sense of X when it is not actually referring.[22] The sense of X can be taken as a paradigmatic sample of X together with the *praxis* of applying this paradigmatic sample. To take the sense of X when it is not referring is to disregard the praxis. What remains then is the paradigmatic sample alone. To confuse

[20] Cf. Antonio Gramsci's famous remark: 'the old is dying and the new cannot be born; in this interregnum a great variety of morbid symptoms appear.' (Gramsci 1971, pp. 275–276).

[21] 'The sign is the part of the symbol perceptible by the senses.' (TLP 3.32).

[22] This idea is captured by Agamben (1993, p. 72): 'Actually, since every term refers by definition to every and any member of its extension, and can, furthermore, refer to itself, one can say that all (or almost all) words can be presented as classes that, according to the formulation of the paradox, both are and are not members themselves.

It is not worth objecting against this that one never mistakes the term "shoe" for a shoe. Here an insufficient conception of self-reference blocks us from grasping the crux of the problem: What is in question is not the word "shoe" in its acoustic or graphic form (the *suppositio materialis* of medieval logicians), but the word "shoe" precisely in its signifying the shoe (or, *a parte objecti,* the shoe in its being signified by the term "shoe"). Even if we can completely distinguish a shoe from the term "shoe," it is still much more difficult to distinguish a shoe from its being-called-(shoe), from its *being-in-language.* Being-called or being-in-language is the non-predicative property par excellence that belongs to each member of a class and at the same time makes its belonging an aporia.'

a paradigmatic sample of X with an X itself is something we can now imagine very clearly.

It seems, then, that self-membership is much more common than one would expect. Every predicate is prone to cause the confusion between its sense and its reference, and thus generate a paradox. We can, however, intentionally force people to run up against this paradox in order to indicate a change in the praxis with this predicate, i.e. to generate a new sense.[23]

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