Kant on Negative Quantities, Real Opposition and Inertia

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Kant's obscure essay entitled *An Attempt to Introduce the Concept of Negative Quantities into Philosophy* has received virtually no attention in the Kant literature. The essay has been in English translation for over twenty years, though not widely available. In his original 1983 translation, Gordon Treash argues that the *Negative Quantities* essay should be understood as part of an ongoing response to the philosophy of Christian Wolff. Like Hoffmann and Crusius before him, the Kant of 1763 is at odds with the Leibnizian-Wolffian tradition of deductive metaphysics. He joins his predecessors in rejecting the assumption that the law of contradiction alone can provide proof of the principle of sufficient reason. The recognition that Kant's *Negative Quantities* essay is part of a response to the tradition of deductive metaphysics is, without doubt, an important contribution to the Kant literature. However, there is still more to be said about this neglected essay. The full significance of the paper becomes known through its ties to a second, empiricist line of succession. Clues to this second line of succession can be found in Kant's prefatory remarks concerning Euler's 1748 *Reflections on Space and Time* and Crusius' 1749 *Guidance in the Orderly and Careful Consideration of Natural Events*. As I will show, Kant's prefatory remarks suggest a reading of his *Negative Quantities* paper that reaches beyond German deductive metaphysics to engage a debate regarding the application of mathematics in philosophy that traces back to George Berkeley.

Mathematics and Philosophy

Evidence to support the claim that there are two lines of succession for Kant's *Negative Quantities* essay can be found in the opening sentence of the paper. There, Kant introduces his topic as the two uses of mathematics in philosophy. One use, he says, concerns the imitation of mathematical method. The other concerns the application of mathematical propositions to the objects of philosophy. (2:167) This distinction between imitation and application can be linked to two different threads in the disputes between mathematicians and metaphysicians that followed the publication of Newton's *Principia*. The first originates with Leibniz and his deductive metaphysics; the second originates with George Berkeley and his critique of the calculus. Let's consider each thread in turn, with the object of explaining how Kant's *Negative Quantities* essay engages these two uses of mathematics in philosophy.

The first thread concerns the imitation of mathematical method in philosophy. This use of mathematics can be seen in the argumentative strategy deployed by Leibniz against Newtonian physics. According to the imitative method, philosophical principles are granted the same certain status as mathematical propositions, and they provide a similar guarantee of certainty in metaphysical proofs. Hence, Leibniz appealed to the laws of contradiction and sufficient reason to reduce central claims of Newtonian physics, such as the appeal to an unobservable absolute space, to absurdity. If absolute space were real, Leibniz reasoned, then no point in space would differ from any other, and God would not have had a sufficient reason for the placement of bodies in any particular order of co-existence. Since this supposition is absurd and contradictory, absolute space cannot be real. Leibniz offered similar reductio-style proofs against other features of Newtonian physics, including Newton's material atoms.

The empiricist response to Leibniz's attacks, as seen in the Leibniz-Clarke correspondence, was one of quick rejection; the rationalist response, by way of contrast, was gradual and hesitant. Leibniz's successor, Christian Wolff, adopted much of Leibniz's metaphysics, but went on to introduce elements into his cosmology that were clearly incompatible with the Leibnizian system. Wolff's modifications raised epistemological questions for his early critics. How could two incompatible views of substance be derived from the same first principles? Kant's immediate predecessors, Hoffmann and Crusius, argued that such difficulties stemmed from the misguided assumption that all philosophical truths, including the truths of physics, could be deduced from the law of contradiction. They proposed that different criteria of truth govern the ideal world of metaphysics and the real world of physics. By 1763, Kant is evidently sympathetic to their critique. In the *Negative Quantities* essay, he charges that metaphysics has `often armed itself against mathematics' in an effort to render the mathematician's concepts `as nothing but subtle illusions which have little truth outside their field'. (2:167) However, the imitative method, it turns out, has been of little use in settling metaphysical disputes. For, `despite the great advantage initially promised from it', says Kant, ` [the] philosophical propositions decorated out of jealousy for geometry have gradually fallen away'. (2:167) Hence, we find that by 1763, Kant expresses dissatisfaction with the imitation of mathematical method in philosophy.

The second thread in the disputes between mathematicians and metaphysicians concerns the application of mathematical concepts and propositions to the objects of philosophy. This thread originates with George Berkeley. In his early work, Berkeley had argued that concepts such as absolute space were little more than useful ways of representing features of the physical world. They could not be traced back to sensation, and, as such, they were meaningless abstractions --- false constructions that were not literally true of things in nature. In his 1734 publication, *The Analyst,* Berkeley turns this empiricist critique against the mathematical calculus itself --- the analysis of motion seen by many as the linchpin of theory-data fit in Newton's `system of the world'. One of Berkeley's criticisms was that its concept of the infinitely small was not given in sensation. As Berkeley sarcastically noted, it `requires a marvellous sharpness of Discernment, to be able to distinguish between evanescent Increments and infinitesimal Differences'. (Berkeley, 1734, § XVII) Berkeley also wondered what sort of quantity an infinitesimal might be. An infinitesimal greater than zero would not define anything instantaneous; an infinitesimal less than zero would not define anything like speed. Hence, the Newtonian explanation of motion depends on an appeal to calculus, but the calculus invokes fictions that cannot easily be reconciled with the demands of a coherent philosophy of nature. To make matters worse, Berkeley charged that there were serious methodological problems underlying the calculus. His main complaint in this regard was that the calculus violates the canons of both logic and mathematics when it arbitrarily shifts between positing and ignoring infinitesimal quantities. To posit a real increment as something and then later suppose that it is nothing is an absurd and contradictory method. (Berkeley, 1734, § XIV - XV) Berkeley then, had raised several substantial questions concerning the application of the mathematical calculus to the objects of philosophy.

What we find in the *Negative Quantities* essay, curiously enough, is support for both Berkeley's line of criticism and the introduction of the mathematical concepts into philosophy. To wit, Kant thinks it quite proper to base observations of the nature of space on the data of geometry, and we find Kant applauding the `invaluable assistance' that the `infallibly demonstrated data' of geometry lend to philosophy. What we learn from geometry, Kant says, is that `space does not at all consist of simple *parts*'. (2:167) Here Kant appears to side with the mathematicians' analysis of space. However, in introducing the mathematician's concepts, he says, we must also heed the empiricist requirement that mathematical concepts conform to objects of sensation. As Kant stipulates, the concepts of objects introduced into philosophy must not be `conceived in an entirely abstract fashion' or `arbitrarily invented'. (2:168) Otherwise one loses the advantages associated with certain and exact starting points. It seems then, that the Kant of 1763 holds a complex position with respect to the application of mathematical concepts to the objects of philosophy. On the one hand, he is sympathetic to the empiricist's evidential requirement; on the other hand, he views the mathematician's concepts as valuable assets to metaphysics.

What then, can we say by way summary about the manner in which Kant's *Negative Quantities* essay engages the two uses of mathematics in philosophy? First, we can say that Kant expresses antagonism toward the imitative method. Second, we can say that Kant expresses support for the empiricist worry about importing abstract and arbitrary concepts into philosophy. Finally, we can also say that Kant seems to favour a Newtonian physics. In order to see how all of these elements come together in the *Negative Quantities* essay, we will need to consider some additional clues supplied by the text. These clues relate to Kant's prefatory remarks concerning Euler and Crusius. We will take them up in order.

Euler and Kant

Euler's 1748 *Reflections on Space and Time* is, arguably, the most important clue for interpreting Kant's *Negative Quantities* paper. Kant praises the `the famous Euler' for his defense of Newtonian concepts of space and time. Metaphysics, he tells us, would do well to accept assistance from the direction of mathematics; for the concepts and propositions of mathematicians `exceed all others combined in certainty and clarity'. (2: 168) More specifically, what Euler's piece shows, Kant says, is that `the mathematical observation of motion joined with the knowledge of space give data to keep the metaphysical observation of time on the track of truth'. (2:168) Undoubtedly, Kant saw a stroke of genius in Euler's 1748 paper. However, given Kant's reservations regarding the application of mathematical concepts in philosophy, the specific basis for Kant's approval of Euler remains unclear. Indeed, we have already seen that Kant denounces the introduction of `arbitrarily invented' and `abstractly conceived' concepts into philosophy. As it happens, Euler's 1748 defence of Newton overlooks these very objections. However, in order to assess properly Kant's response to Euler, we will need to identify both the flaws and merits of Euler's 1748 analysis, as Kant would have seen them. Given that Euler's piece is not widely read, and that it is an important clue to the *Negative Quantities* paper, we will review the main arguments of Euler's essay before we reflect any further upon Kant's response to it.

In his *Reflections on Space and Time*, Euler directs his arguments against Newton's detractors --- especially Berkeley and Leibniz. Berkeley, like Leibniz, was partial to a relativistic conception of motion and space. He held that relative motion is the apparent motion of a body from one place to another relative to surrounding bodies, while relative space is the space in which such motion occurs. In addition, Berkeley rejected Newton's claim to have provided support for absolute space with his so-called `bucket experiment'. The experiment itself purported to show that an explanation of all of the effects of forces requires a distinction between relative and absolute motion, and hence, the introduction of absolute space. Berkeley, however, argued that it was not necessary to invoke the framework of absolute space to explain the effects in question; rather, all that was needed was the framework of the fixed stars. Euler rejects this relativistic position in his *Reflections on Space and Time,* and adjudicates the controversy by appeal to the two principles of inertia, which he takes to be `indisputable truths':

> It is thus an indisputable truth that a body once at rest will remain perpetually at rest, unless it is disturbed from this state by some external force. It is similarly certain that a body once put in motion will continue to move with the same velocity and in the same direction, provided that it does not encounter any impediment to the conservation of this state. (Euler, 1748, § I)

Proceeding on the assumption that Newton's detractors and supporters alike are committed to the above propositions, Euler goes on to show, contra the metaphysicians, that we cannot explain the observed phenomena in a way that is consistent with the two propositions concerning inertia *unless* we introduce Newtonian concepts of absolute space and time.

Euler begins his argument with the claim that whatever is said on the nature of body must conform to his first proposition concerning inertia. He argues that the Newtonian concept of body is required to explain the tendency of a body to maintain its position unless acted upon by an outside force. On the relativistic conception, we are required to say, by way of contrast, that a body in a flow of water maintains its position by actually *moving* downstream with the flow. This is absurd. (Euler, 1748, § VI-XI) Moreover, Euler claims, specifically in reply to one of Berkeley's criticisms, it is not possible to give an adequate account of position in relation to the fixed stars. In each case, we must appeal to the framework of absolute space to properly describe the body's position. (Euler, 1748, § XII)

It is worth pointing out that Euler's arguments regarding position would not budge a relativist such as Berkeley or Leibniz. For, the relativist entirely rejects the idea of position apart from the concept of the place that a body occupies. Moreover, both Leibniz and Berkeley would reject Euler's question-begging appeal to the Newtonian concept of the inertia of bodies. Whatever it was that Kant found laudable in Euler's 1748 piece, it is not likely to be this circularity in his argument. Moreover, Kant would have held that the appeal to inertia must not be based on a concept conceived in an abstract or arbitrary fashion. Euler misses these points of philosophical subtlety.

What Euler says concerning the second proposition may get him closer to his mark. In this case, Euler's strategy is a clever one, since both Newton and his detractors agree to the principle of inertial motion. All are interested in the analysis of observed circular motions, and all agree that circular motions are composed of a linear inertial component and a motion of descent towards a centre. What Euler here argues is that the representation of a body travelling in the same direction in a uniform and rectilinear motion requires that space and position be explained in terms of something more than the co-existence of bodies. Specifically, the description requires the designation of a direction of motion, and this, in turn, requires an appeal to the framework of absolute space. (Euler, 1748, § XVII) To this argument, Euler adds a further consideration. To argue, with the relativists, that time is nothing more than a succession of ideas in the subject is absurd and contrary to the principle of inertial motion, which requires that we postulate absolute time. For, absolute time `flows and serves as a measure for the duration of things' --- meaning that it enables us to describe inertial motion as the linear motion of a body through equal spaces in equal times. (Euler, 1748, § XVIII) In thus grounding the Newtonian concepts of space and time in an appeal to inertial motion, Euler is attempting to ground them in an aspect of the mathematicians' theory that is well entrenched, but nonetheless acceptable to the metaphysicians. Moreover, he is quite right to claim that the principle of inertial motion requires the fixed framework of absolute space and time --- a point that Newton himself raised against the Cartesians. Unfortunately, however, this defence of absolute space and time is, once again, circular. This circularity is sure to have been noticed by Kant.

Despite Kant's expressed admiration for Euler, there is reason to think that he saw limitations in Euler's answer to the metaphysicians. The difficulty is not just that the appeals to inertia are circular, but also that Euler fails to consider the sorts of evidential objections raised by the metaphysicians ---

evidential objections that Kant took seriously. Indeed, Euler's only escape from circularity would appear to involve a controversial appeal to the calculus. For, Euler declares at the outset of his essay that the first principles of mechanics --- the principles concern inertia --- are established `beyond all doubt' by the `remarkable agreement between the results derived by means of the mathematical calculus and the motions of [terrestrial and celestial] bodies'. (Euler, 1748, § I) While such a declaration may sound fine to us, Kant was obviously preoccupied with the application problem. He alludes to the controversy surrounding infinitesimals in the 1763 paper, saying that `not enough has yet been understood about [infinitesimals] in order to bring down a judgment'. (2:168) The salient point is that while Kant does not take Berkeley's side, he is sure to have noticed the shortcomings in Euler's reply. One of the principal tasks of Kant's *Negative Quantities* paper, I argue, is to address empiricist concerns about the application of mathematical concepts to the objects of philosophy. On my reading, Euler's appeal to inertia raises concerns about the grounding of Newtonian concepts involved in such an appeal. The *Negative Quantities* paper offers a pre-critical account of how such representations are possible --- an account that invokes negative quantities and real opposition.

Crusius and Kant

What Kant says at the outset of his 1763 essay is that he hopes to show that the concept of negative quantities can be profitably introduced into philosophy. He announces that, `I have for now the intention of studying the relation to philosophy of a concept which is well enough known in mathematics but still quite foreign to philosophy'. (2:169) He elaborates further on his intentions, explaining that the presentation made of the concept so far has been `strange and contradictory'. (2:170) In fact, negative quantities were not well understood in Kant's day, and were often mistaken for quantities derived through the subtraction of a quantity from zero. Insofar as zero was taken to be a point below which there was in fact nothing, it was thought that a quantity below zero could represent neither a thing nor a number. Hence, negative quantities were sometimes called `false quantities'. Many mathematicians were convinced that it would be better to eliminate them altogether. Berkley, for example, wondered `whether the bringing Nothing under the notion of Quantity may not have betrayed Men into false Reasoning?' (Berkeley, 1734, Query 40) The main source of difficulty was that no satisfactory account had been given of the grounds upon which conclusions could be drawn from them. Were such concepts to be supported by induction and analogy or by mathematical demonstration?   Neither, it turns out. For, as Reid (1748) and later Kant realized, mathematical concepts must simply be defined. They are granted --- rather than given in experience or proven by demonstration. Any further justification for their use must come through establishing their usefulness. It is presumably with this in mind that Kant devotes the first section of his *Negative Quantities* essay to the task of defining the concept of negative quantities and then proceeds, in the second and third sections of the essay, to demonstrate their usefulness.

With this in view, Kant begins his 1763 paper with a definition of negative quantities. The first conceptual issue that he raises relates to the confusion between logical and real opposition. Berkeley, for example, sometimes appears to confuse the notions of logical and real opposition in his appraisal of the calculus. The treatment of a quantity first as something and then as nothing, Berkeley claims, quite apart from any difficulties associated with the lack of sensible basis for the quantities, is sophistical and contradictory. (Berkeley, 1734, § XIV-XV) However, what Berkeley does not appear to realize is that there can in fact be a conceptually sound basis for treating a quantity as something and then nothing in mathematics. The process of `crediting' and `debiting' real opposing quantities, for example, does not produce a logical contradiction. (2:172) Indeed, logical opposition, Kant explains, is something quite distinct from real opposition. In cases of logical opposition, there is no corresponding representation. A logical opposition, Kant says, yields absolutely nothing, or *nihil negativum irrepraesentabile*. (2:171) However, a real opposition yields something that can be represented --- a cogitable privation, or *nihil privatum repraesentabile*. (2:172) A negative quantity represents a real opposition, hence, it represents a cogitable privation rather than absolutely nothing.

Kant next goes on to address several other misconceptions about negative quantities. One of these underlies Crusius' criticisms of Newton's appeal to negative quantities. (2:169) As Kant's discussion makes apparent, a basic confusion regarding the meaning and use of the negative sign underlies Crusius' rejection of Newton's equations as `false to the point of astonishment'. Crusius is apparently laboring under the common confusion that negative quantities are negations of quantities. But, as Kant corrects, `negative quantities are not negations of quantities as the similarity of expression allowed him to conjecture. They are rather something that is truly positive in itself and which is opposed to the other'. (2:169)

Kant offers still more by way of clarification. He places emphasis, for example, on the distinction between the reciprocal relationship between opposing quantities and the arithmetic operations of addition and subtraction. In arithmetic, placing the sign of an operator before a quantity tells us to add or subtract one quantity relative to another. However, the significance of the positive and negative signs differs in the case of real opposing quantities. Where there is a real opposition, the result may be zero (if positive and negative quantities cancel one another), a positive quantity (if the positive exceeds the negative quantity), or a quantity designated by the negative sign as its symbol (if the negative exceeds the positive quantity). `From this the mathematical concept of negative quantities develops'. (2:174)

In sum, on the understanding that neither inductive reasoning nor deductive proof can establish what must be taken for granted, Kant's *Negative Quantities* essay begins with the definition of a term that has heretofore been applied only through the use of `special rules'. Negative quantities, Kant explains, result when two positive quantities cancel each other such that the negative quantity exceeds the positive quantity. What remains to be shown in justifying the application of this concept to the objects of philosophy is the extent of its usefulness to philosophy.

<u>Negative Quantities, Real Opposition and Inertia</u>

With his definition of negative quantities in hand, Kant turns in the second and third parts of his essay, to the usefulness of negative quantities to philosophy. His survey of the various objects that might be represented by appeal to real opposition and negative quantities is extensive and varied. As it turns out, the concept of negative quantities may be profitably applied to the objects of natural philosophy, moral philosophy, metaphysics, aesthetics, and so on. While it is not possible to consider all of Kant's examples here, we can follow the main focus of his discussion, which concerns the objects of natural philosophy.

Kant mentions a number of cases of real opposition in natural philosophy. He claims, for example, that the concept of rest is properly explained as a state that results from the complete cancellation of opposing forces. (2:178-9) Similarly, the concept of impenetrability, he says, `presupposes a genuine power in the parts of the body by means of which these parts together occupy a space, as that power does with which another [body] attempts to move into this space'. (2:179) He further claims that the concept of a body is the concept of something that occupies a space `through the conflict of two forces opposing each other'. (2:180) Later in his paper, Kant reflects at some length on the concept of an alteration, another concept, he says, that presupposes the sort of cancellation of one thing by another that is characteristic of real opposition. (2:191-2) It is here that Kant distinguishes between two kinds of real opposition; namely, `actual opposition' and `potential opposition'. In the former case, there is an existing opposition; in the latter, the objects in question `only stand in potential opposition'. They `are such as to belong to different things [so that] one does not immediately cancel the result of the other'. (2: 193) Hence, for two bodies moving toward each other with equal force in a straight line communicate forces upon impact, one force can be said to negate the other. For two bodies moving away from each other with equal force in the same straight line, no force is communicated, and the forces merely stand in potential opposition to one another. (2:193) The point of introducing this distinction, it soon becomes apparent, is to prepare the way for a new foundation for inertial motion. Indeed, Kant next claims that the rules of mechanics --- regardless of their derivation in physics --- actually depend upon a metaphysical foundation build upon an appeal to potential opposition and negative quantities. (2:195) The fundamental principle at issue, he claims, is a principle that governs how we must think alterations,

> In all natural alterations of the world, the sum of positive things is neither increased nor diminished insofar as these are computed by adding positions which are in agreement (not those opposed to each other) and subtracting from one another those that are really opposed. (2:194)

According to Kant, this principle governing representations of alteration is not intended as a metaphysical claim. Rather, it is a rule for representing the actual or potential opposition of things. (2:198) Representations of alterations are foundational to mechanics, and the above rule supplies the true ground for representations of quantity of motion and of force. Hence, we should judge that, `The quantity of motion is not altered by reciprocal activity ... if the forces of bodies that direct this motion towards the same side are taken together, and those which are directed towards the opposed side are subtracted'. (2:195) `Even if this rule', he insists, `is not deduced in pure mechanics from the metaphysical foundation from which we have derived the universal proposition, *in fact*, its veracity rests upon this basis.' `For the law of inertia which in the usual proof constitutes the fundamental proposition [of mechanics], merely borrows its truth from the basis of the proof cited here as I could easily show were I allowed to be prolix.' (2:195)

Although Kant does not fully explain the sense in which real oppositions and negative quantities ground the principle of inertia in his 1763 essay, it is clear that he took the provision of a foundation for this principle to be an important task. Indeed, Kant's new foundation for inertia not only addresses the application problem; it also provides a non-circular basis for an appeal to inertia in defense of absolute space and time. In light of this, we can make sense of Kant's claim that the concept of negative quantities --- a concept that is grounded in either the real or potential opposition of positive quantities --- holds great promise for the objects of philosophy.

Conclusion

What was the purpose of Kant's 1763 *Negative Quantities* essay? In one sense, Kant's aims are continuous with those of Hoffmann and Crusius. Like his predecessors, Kant rejects the view that the true concepts of nature follow from logical laws such as the law of contradiction or identity. It is one thing, Kant says, to grasp an analytic connection between a ground and consequence, but quite another to explain the criteria of real possibility. `But how something follows from something else, yet not according to the rule of identity' Kant writes, `--- that is something which I would be glad to be able to make plain'. (2:202) Hence, as Treash correctly notes, the *Negative Quantities* essay is part of a critique of the deductive style of metaphysics that takes ideal grounds as the sole arbiters of reality. No less important to our understanding of the 1763 essay is its second, empiricist line of succession. For, Kant evidently intends the paper to address the issues raised by Berkeley's criticisms. Contra Berkeley, Kant has shown that there are indeed mathematical concepts consistent with the demands of a meaningful representation *and* the principles of Newtonian mechanics. What we find then, is that the *Negative Quantities* paper responds to Newton's detractors by justifying the application of mathematical propositions to the objects of philosophy, and hence, that the paper seems to involve not one, but two distinct lines of succession. With this dual perspective in view, we gain a much clearer understanding of the issues that Kant hoped to address in the 1763 paper. What we also find, is that the Kant of 1763 offers a pre-critical version of the same double-edged criticism of German dogmatism and skeptical empiricism that will follow in his *Critique of Pure Reason*. From a strategic point of view then, Kant owes a significant debt of gratitude to Euler and his 1748 *Reflections on Space and Time*.