# The Meta-Reversibility Objection

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**Abstract**

One popular approach to statistical mechanics understands statistical mechanical probabilities as measures of rational indifference. Naive formulations of this "indifference approach" face reversibility worries – while they yield the right prescriptions regarding future events, they yield the wrong prescriptions regarding past events. This paper begins by showing how the indifference approach can overcome the standard reversibility worries by appealing to the Past Hypothesis. But, the paper argues, positing a Past Hypothesis doesn't free the indifference approach from all reversibility worries. For while appealing to the Past Hypothesis allows it to escape one kind of reversibility worry, it makes it susceptible to another – the Meta-Reversibility Objection. And there is no easy way for the indifference approach to escape the Meta-Reversibility Objection. As a result, reversibility worries pose a steep challenge to the viability of the indifference approach.

## 1 Introduction

Statistical mechanics is an inherently probabilistic theory. It explains thermodynamic phenomena – the diffusion of milk poured into a cup of hot cocoa, the cooling of the cup of hot cocoa when it's carried out into the snow – by assigning high probabilities to such events. And it explains the absence of anti-thermodynamic behavior – the separation and ejection of all of the milk from a cup of hot cocoa, the spontaneous warming of a cup of hot cocoa left outside – by assigning small probabilities to such events.

But how to understand these probabilities is a contentious issue. The recent proposals in the literature can be roughly divided into two camps.[1] One view of statistical

[1]Those who defend a "typicality" approach to statistical mechanics might not seem to fit into either of these camps (see Goldstein (2001) and Maudlin (2007) for a description of such views). As I understand it, however, this approach is not a competitor to the ones described here. Rather, the typicality approach crosscuts the divide between these two camps, and could be adopted (or rejected) by proponents of both nomic and indifference approaches.

mechanical probabilities, recently defended by Albert (2000) and Loewer (2001), understands them as *chances*, i.e., physical probabilities given by the laws of nature.[2] I'll call this the *nomic approach* to statistical mechanics (and accounts of statistical mechanics in this vein *nomic accounts*), since it maintains that statistical mechanical probabilities are lawful or nomic features of the world.

The other view of statistical mechanical probabilities understands them as measures of rational indifference. On this view, the statistical mechanical probabilities represent the credences that an ideally rational agent would adopt about a system, given only certain information about what that system is like. These probabilities don't come from the laws, they come from *a priori* constraints on rational belief (often called "Indifference Principles"). I'll call this the *indifference approach* to statistical mechanics (and accounts of statistical mechanics in this vein *indifference accounts*), since it maintains that statistical mechanical probabilities are the credences prescribed by Indifference Principles.

Initial formulations of nomic and indifference accounts face "reversibility worries" – while these accounts yield the right results regarding future events, they yield the wrong results regarding past events (see section 3). But in both cases it seems one can overcome these reversibility worries by modifying these accounts to include something like the Past Hypothesis – a law requiring a particular low-entropy initial condition to obtain – as discussed by Albert (2000).

In this paper, I argue that this parity between nomic and indifference accounts is illusory. For while appealing to a Past Hypothesis suffices to free nomic accounts from reversibility worries, it doesn't suffice to free indifference accounts from these worries.

More precisely, I show that both nomic and indifference accounts can escape the standard reversibility worries – what I'll call the Reversibility Objection – by appealing to the Past Hypothesis. But by positing a Past Hypothesis, indifference accounts become susceptible to another kind of reversibility worry – what I'll call the Meta-Reversibility Objection. And I argue that there is no easy way for indifference accounts to escape the Meta-Reversibility Objection. Thus reversibility considerations give us a strong reason to favor nomic accounts over indifference accounts.

The rest of the paper will proceed as follows. In section 2 I provide some background regarding Bayesianism and statistical mechanics, present some assumptions, and describe the accounts of statistical mechanics I'll focus on in more detail.

In section 3 I discuss the Reversibility Objection to the initial formulations of the nomic and indifference accounts. I begin the section with a rough description of the worries motivating the Reversibility Objection. In section 3.1 I provide a more rigorous characterization of the Reversibility Objection. This allows us to see the possible replies to the Reversibility Objection, and provides us with some of the tools we need to set up the Meta-Reversibility Objection. In section 3.2 I assess these replies, and

[2] A number of other people have followed Albert and Loewer in advocating something like the nomic approach to statistical mechanics, including Winsberg (2008), Callender and Cohen (2009), and Frigg and Hoefer (2010).

show why there's pressure to follow Albert in adopting a lawful constraint on initial conditions, the Past Hypothesis.

In section 4 I turn to the Meta-Reversibility Objection. I begin with a rough description of the worries motivating the Meta-Reversibility Objection, and explain why similar worries don't arise for nomic accounts. In section 4.1 I provide a more rigorous characterization of the Meta-Reversibility Objection. In section 4.2 I assess the possible replies to the Meta-Reversibility Objection. I argue that none of the available options provide the proponent of the indifference approach with a satisfying reply. I conclude with some brief remarks in section 5.

For those already familiar with these issues, here is a more detailed sketch of the dialectic that leads to the Meta-Reversibility Objection. The initial formulations of the nomic and indifference accounts, which impose no constraints on initial conditions, are unsatisfactory because they assign the wrong values to past events. This is the Reversibility Objection. We can escape this objection by adopting something like the Past Hypothesis (*PH*), a lawful constraint that requires a particular low-entropy initial condition to obtain. With this addition, the nomic and indifference accounts will assign the right values to past events.

But note that there are other lawful constraints on initial conditions one could adopt, such as the "Past Hypothesis$^*$" ($PH^*$), which requires that some *high*-entropy initial condition obtain. Given something like $PH^*$, the nomic and indifference accounts will again assign the wrong values to past events. I'll argue in section 4 that there is pressure on the indifference approach (though not the nomic approach) to take $PH^*$ to be much more likely than *PH*. And if the indifference approach takes $PH^*$ to be much more likely than *PH*, then the wrong values it assigns to past events given $PH^*$ will swamp the right values it assigns to past events given *PH*. So, when all is said and done, it'll again assign the wrong values to past events. This, in a nutshell, is the Meta-Reversibility Objection.

# 2 Background

## 2.1 Rational Agents

For the purposes of this paper, I'll be restricting my attention to rational agents. I assume that the belief states of such agents can be represented with a *credence function*, *cr*, which takes propositions as arguments, and spits out real numbers between 0 and 1, which represent the agent's degree of confidence that the proposition is true.

I assume that rational agents are *Bayesian*, in that their credences satisfy the following pair of constraints:

**Probabilism:** An agent's credences should satisfy the probability axioms.[3]

[3]That is, an agent's credence function *cr*, defined over an algebra $\mathcal{A}$ over $\Omega$, should be such that: (1) $cr(\Omega) = 1$, (2) $\forall A \in \mathcal{A},\ cr(A) \geq 0$, (3) $\forall A, B \in \mathcal{A}$, if $A \cap B = \emptyset$, then $cr(A) + cr(B) = cr(A \cap B)$.

**Conditionalization:** If an agent with credences $cr$ receives $E$ as evidence, then her new credences $cr^+$ should be:

$$cr^+(A) = cr(A|E), \text{ if defined.}^4 \tag{1}$$

Note that Conditionalization doesn't care about how you break up evidence; conditionalizing on $E$ and then conditionalizing on $F$ yields the same result as conditionalizing on $E \wedge F$. Thus we can reformulate Conditionalization in terms of an agent's initial credence function $ic$ and her total evidence $E$ as follows:[5]

**Conditionalization:** If an agent with initial credences $ic$ has total evidence $E$, then her credences should be:

$$cr_E(A) = ic(A|E), \text{ if defined.} \tag{2}$$

I further assume that rational agents satisfy something like Lewis's (1986) Principal Principle, a principle that links their beliefs about propositions to their beliefs about the chances of those propositions. In particular, let $T$ be a complete chance theory, let $K$ be some background information that $T$ requires in order to produce a chance distribution, and let $ch_{TK}(A)$ be the chance that $T$ assigns to $A$ given $K$. Then I assume that rational agents satisfy the following constraint:

**Chance-Credence Principle:** An agent's initial credences $ic$ should be such that:

$$ic(A|T \wedge K) = ch_{TK}(A), \text{ if defined.}^6 \tag{3}$$

Proponents of the indifference approach to statistical mechanics assume that rational agents also satisfy a further constraint, an *Indifference Principle* of some sort. There are many Indifference Principles, but all of the Indifference Principles compatible with Bayesianism take a similar form. We can formulate such principles in terms of an agent's initial credences $ic$ and a measure of rational indifference $\mu$, as follows (where different choices of $\mu$ yield different Indifference Principles):

**Indifference Principle:** An agent's initial credences $ic$ should be such that:

$$ic(A) = \mu(A).^7 \tag{4}$$

[4] Where $cr(A|B) := \frac{cr(A \wedge B)}{cr(B)}$.

[5] Though, of course, this extension only works for agents who *have* initial credence functions.

[6] This principle is essentially that proposed by Lewis (1986), with one slight amendment – I've replaced Lewis's "complete history up to a time" $H$ with a more general background argument $K$, in order to allow for statistical mechanical chances. (For some considerations in favor of this kind of formulation of the chance-credence principle, see Arntzenius (1995), Meacham (2005), Nelson (2009) and Meacham (2010).) As Lewis and others have noted, this principle is problematic if one adopts a Humean account of chance (see Lewis (1994), Hall (1994) and Thau (1994)); for the purposes of this paper, I'll put these issues aside.

[7] This assumes that we can treat $\mu$ as a probability function. Strictly speaking, this isn't correct (see Meacham (2005)), and accommodating this fact would require formulating the Indifference Principle in terms of (say) an equation relating a pair of primitive conditional probabilities. I'll ignore these complications here.

An Indifference Principle and Bayesianism together will determine the unique rational credence function that an agent with total evidence $E$ should have (i.e., $cr_E(A) = \mu(A|E)$).

Not all Indifference Principles are compatible with Bayesianism. A number of authors, such as Friedman and Shimony (1971) and Dias and Shimony (1981), have argued that some popular Indifference Principles conflict with Bayesianism.[8] But I take conflicting with Bayesianism to be a *reductio* of a proposed Indifference Principle. So I restrict my attention here to Indifference Principles that are compatible with Bayesianism.

## 2.2 Statistical Mechanics

Although the issues discussed in this paper are relevant to both classical and quantum statistical mechanics, I restrict my attention here to classical statistical mechanics.[9]

Consider a classical world with $n$ particles and three spatial dimensions. The *phase space* of this world is a $6n$ dimensional space representing all of the possible positions and velocities of these particles.[10] Each dimension of the phase space corresponds to the location or velocity of one of the particles in one of the three spatial directions. (Thus the locations and velocities of each particle correspond to 6 dimensions.) And each point in the phase space corresponds to a possible arrangement of particle positions and velocities, with the point's position along each dimension encoding the value of the corresponding location or velocity.

Let a *microstate* be a complete specification of the locations and velocities of each particle – this corresponds to a particular point in phase space. Let a *macrostate* be an incomplete specification of the locations and velocities of each particle – this corresponds to the region of phase space containing all and only those microstates compatible with that specification.[11]

To fix on a particular phase space, we need to fix the values of certain parameters, such as the number of particles, the number of spatial dimensions and the spatial extension of each dimension. But these details have little bearing on the points at issue. So to simplify my presentation, I'll assume that the phase space we're working with

[8] Whether these arguments are correct depends on how these principles are understood (see Uffink (1996)).

[9] For more on classical statistical mechanics, see Tolman (1979); for more on the philosophical issues that arise see Sklar (1993) and Albert (2000); for a helpful road-map and source of references regarding the issues, see North (2011).

[10] Following Albert, I assume that it's legitimate to apply statistical mechanics to the world as a whole. Some have resisted this suggestion, arguing that we should restrict the scope of statistical mechanics; for example see Leeds (2003).

[11] For convenience, I'll use the term "macrostate" to refer to the property of a system, to the region of phase space occupied by microstates with that property, and to the proposition that the system has that property; context will make it clear how it's being used. Likewise, I'll use the name of a macrostate $A$ to refer to the property, the corresponding region of phase space, and the proposition of the system having that property.

has been fixed. I describe the somewhat clunkier versions of the central principles and arguments that take these parameters into consideration in the appendix.

The standard statistical mechanical probabilities are given by the Liouville measure, i.e., the Lebesgue measure over the canonical representation of the phase space.[12] In particular, the statistical mechanical probability of the world being in macrostate $A$, given that it's in macrostate $B$, is equal to the proportion (according to the Liouville measure) of $B$'s microstates that are compatible with $A$.

Note that we're assuming deterministic dynamics – given the microstate of the world at one time, the dynamics determine the microstate of the world at any other time.[13] And note that these dynamics preserve the Liouville measure – the Liouville measure of any macrostate $A$ is the same as the Liouville measure of any macrostate $A^*$ that we can get to by evolving $A$ forwards or backwards in time in accordance with the dynamics. Thus we can think of any questions about statistical mechanical probabilities in terms of probabilities over initial conditions: the answer to the question "what is the probability of $A$ given $B$ (at $t$)" is always the same as the answer to the question "what is the probability of the initial conditions having been such that they would evolve into macrostate $A$ at $t$, given that we know they're such that they'll evolve into macrostate $B$ at $t$?"[14]

For the purposes of this paper, I take an *account of statistical mechanics* to be an account which provides at least: (a) a complete specification of the laws, and (b) a characterization of statistical mechanical probabilities. The discussion in this paper will focus on four accounts: the initial formulations of the nomic and indifference accounts (N1 and I1), and the revised formulations of the nomic and indifference accounts that posit a Past Hypothesis (N2 and I2). The discussion of the Reversibility Objection in section 3 will focus on the first pair of accounts, N1 and I1. The discussion of the Meta-Reversibility Objection in section 4 will focus on the second pair, N2 and I2.

My characterizations of the nomic accounts largely follow Albert (2000). I take the two nomic accounts, N1 and N2, to be:

**N1:**

(a) The complete laws are (i) the laws of classical mechanics, and (ii) a *Statistical Postulate*, which assigns chances to the (lawfully-permitted) initial conditions that are proportional to the Liouville measure.

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[12] Or, if we're holding fixed features of the system (such as its total energy) that reduce the dimensionality of the macrostates we're considering, we employ the Lebesgue measure over the canonical representation of the appropriate sub-space.

[13] There are exceptions to the claim that classical mechanics is deterministic (see Earman (1986), Xia (1992) and Norton (2003)), but we can ignore these exceptions for the purposes at hand.

[14] Assuming, of course, that there *are* initial conditions. At worlds in which there are not – say, world which are infinitely temporally extended in both directions – one would have to replace this talk of initial conditions with talk about the conditions of some "early" time slice. (Similar remarks apply with respect to the Past Hypothesis presented below.)

(b) The statistical mechanical probability of $A$ given $B$ is the chance of $A$ given $B$ assigned by the Statistical Postulate.

**N2:**

(a) The complete laws are (i) the laws of classical mechanics, (ii) a *Statistical Postulate*, which assigns chances to the (lawfully-permitted) initial conditions that are proportional to the Liouville measure, and (iii) a *Past Hypothesis*, which permits all and only the initial conditions belonging to some particular low-entropy macrostate.

(b) The statistical mechanical probability of $A$ given $B$ is the chance of $A$ given $B$ assigned by the Statistical Postulate.

I take the two indifference accounts, I1 and I2, to be:

**I1:**

(a) The complete laws are (i) the laws of classical mechanics.

(b) An Indifference Principle constrains rational belief. The statistical mechanical probability of $A$ given $B$ is what this Indifference Principle assigns as the rational initial credence in $A$ conditional on the macrostate $B$ and the laws.

**I2:**

(a) The complete laws are (i) the laws of classical mechanics, and (ii) a *Past Hypothesis*, which permits all and only the initial conditions belonging to some particular low-entropy macrostate.

(b) An Indifference Principle constrains rational belief. The statistical mechanical probability of $A$ given $B$ is what this Indifference Principle assigns as the rational initial credence in $A$ conditional on the macrostate $B$ and the laws.

These characterizations of I1 and I2 are underspecified, since I haven't identified what these Indifference Principles take $\mu$ to be. In order to keep my discussion as general as possible, I won't assume much about what $\mu$ is like. But I will assume that the Indifference Principles these accounts employ yield statistical mechanical probabilities that roughly match those assigned by the corresponding nomic account (so that I1 yields roughly the same statistical mechanical probabilities as N1, and I2 yields roughly the same statistical mechanical probabilities as N2). That is, I will assume that:

$$\mu(A|\text{I1} \wedge E) \approx ch_{\text{N1}E}(A), \tag{5}$$

$$\mu(A|\text{I2} \wedge E) \approx ch_{\text{N2}E}(A). \tag{6}$$

Note that these two constraints on $\mu$ are compatible. I1 and I2 are disjoint, since I2 entails that there's a lawful constraint on the initial conditions beyond those imposed by the dynamics (the Past Hypothesis), while I1 entails that there is not. Thus (5) and (6) impose constraints on different parts of logical space, and so won't conflict.

I assume (5) and (6) in order to focus on the key differences between the nomic and indifference approaches. The core disagreement between nomic and indifference approaches isn't over which probabilities should be assigned in typical statistical mechanical cases, but rather over how these probabilities should be understood. For mere disagreement over which probabilities should be assigned won't pull the two camps apart, since for any probability assignment, one can find both a nomic account and an indifference account that yield that assignment (the nomic account saying that these values obtain as a matter of natural law, the indifference account saying they obtain in virtue of *a priori* constraints on rational belief).

# 3 The Reversibility Objection

Although initially attractive, the first formulations of the nomic and indifference accounts given above, N1 and I1, run into trouble.

Consider N1. This account yields the right predictions regarding what the future will be like. It predicts, for instance, that the partially diffused milk in a cup of hot cocoa will become completely diffused. This is because, according to the Liouville measure, the overwhelming majority of microstates compatible with the partially-diffused-milk-in-cocoa macrostate will evolve into microstates in which the milk is wholly diffused throughout the cocoa. And thus it assigns an overwhelmingly high chance to the milk becoming wholly diffused.

But N1 seems to yield the wrong predictions regarding what the past was like. For instance, it predicts that the partially diffused milk in a cup of hot cocoa was previously completely diffused. This is because, according to the Liouville measure, the overwhelming majority of microstates compatible with the partially-diffused-milk-in-cocoa macrostate evolved from microstates in which the milk is wholly diffused throughout the cocoa. And thus, again, it assigns an overwhelmingly high chance to the milk having been wholly diffused.

The problem is that N1 makes the same kinds of predictions about the future and the past. But this seems wrong – thermodynamic phenomena seem to be temporally asymmetric.

Let's take this worry one step further. We've said that N1 seems to yield the wrong predictions about the past – predictions that don't line up with our memories about the past, our written records about the past, and so on. But why think that our memories about the past, our written records about the past, and so on, are accurate? For, according to the Liouville measure, only a miniscule fraction of the microstates compatible with our beliefs and records evolved from past microstates that line up with our beliefs and records about the past. So if we carry this worry to its logical conclusion, the concern is that if we believe the account is true, we're rationally required to believe the *Skeptical Hypothesis* (*SH*): that our beliefs about the past are wrong.

I1 runs into similar problems. While I1 prescribes beliefs about the future that line up well with what actually happens, it prescribes beliefs about the past that don't line up with our memories, records, etc. So as with N1, I1 requires us to believe that our memories, records, etc., are wrong. Thus if we believe I1 is true, we're rationally required to believe the Skeptical Hypothesis. Call this the *Reversibility Objection* to the initial formulations of the nomic and indifference accounts.

## 3.1 Formalizing the Reversibility Objection

Let's formulate the Reversibility Objection more precisely. To begin, the argument needs a way to move from an account yielding skeptical consequences to the account being false. Thus the argument must assume something like the following:

**Adequacy:** The correct account of classical statistical mechanics $X$ is such that, given evidence like ours, one could rationally believe $X \wedge \neg SH$.

A couple comments. First, the "evidence like ours" clause is intended to restrict our attention to agents who get roughly the same kinds of evidence as we do – evidence from their senses, their memories, and the like. What this is intended to exclude is the logical possibility of agents who *directly* get $X \wedge \neg SH$ as evidence. It's hard to imagine what getting such evidence would be like – perhaps a divine revelation that produces veridical certainty in $X \wedge \neg SH$ would qualify. But in any case, if we don't exclude such agents then Adequacy becomes toothless, since such agents could rationally believe $X \wedge \neg SH$ for any $X$, as long as $X \wedge \neg SH$ is logically coherent. (To avoid cumbersome phrasing, I'll sometimes leave the "evidence like ours" clause implicit in what follows.)

Second, Adequacy entails that the correct account of classical statistical mechanics must be one that we could rationally believe. This is a non-trivial assumption. After all, just because we can't rationally believe something doesn't mean it's false. Still, it seems at least *prima facie* plausible that the correct account of classical statistical mechanics is one that we could rationally come to believe.[15]

Third, assuming that the correct account of classical statistical mechanics is one that we could rationally believe, it's a further assumption to maintain that we could rationally believe both it and that the Skeptical Hypothesis is false. Now, once we grant that the correct account of statistical mechanics $X$ is one we could rationally believe, one could argue that this further assumption is easy to get. For, following Albert (2000), one could maintain that the Skeptical Hypothesis undermines our evidence for

[15] Of course it's somewhat awkward to talk about what the "correct" account of classical statistical mechanics is when we know classical statistical mechanics doesn't actually obtain. One can see the discussion here as in the same vein as discussions about whether the "correct" account of classical spacetime is Newtonian or Galilean. In all of these contexts, "correct" means something like "most attractive given the desiderata of theory choice and some subset of our actual evidence".

believing something like classical statistical mechanics in the first place. So we'd only ever be in a position to believe $X$ if we were also in a position to believe $X \wedge \neg SH$.[16]

Given Adequacy, one can formulate the Reversibility Objection to the initial formulations of the nomic and indifference accounts as follows. Let's understand "believe" as "having a credence of greater than 0.5 in", and let's use "$E$" to stand for any total body of evidence like ours. Then we can reformulate Adequacy as:

**Adequacy:** The correct account of classical statistical mechanics $X$ is such that one could rationally have $cr_E(X \wedge \neg SH) > 0.5$.

Now suppose that $X = \text{N1}$. As we saw above, according to N1, the chance of the Skeptical Hypothesis being true, given that the world is in the macrostate picked out by $E$ is very high (i.e., $ch_{\text{N1}E}(SH) \approx 1$).[17] Given this, we can run the Reversibility Objection against N1 as follows:

**The Reversibility Objection (against N1):**

**P1.** $ch_{\text{N1}E}(SH) \approx 1$

**P2.** Bayesianism (Probabilism and Conditionalization)

**P3.** Chance-Credence Principle

**P4.** Adequacy

**5.** $ic(SH|\text{N1} \wedge E) \approx 1$ (P1+P3)

**6.** $cr_E(SH|\text{N1}) \approx 1$ (P2(cond)+5)

**7.** $cr_E(\neg SH|\text{N1}) \approx 0$ (P2(prob)+6)

**8.** $cr_E(\text{N1} \wedge \neg SH) \lll cr_E(\text{N1})$ (7)

**9.** $cr_E(\text{N1} \wedge \neg SH) \approx 0$ (P2(prob)+8)

**10.** $cr_E(\text{N1} \wedge \neg SH) \not> 0.5$ (9)

**C.** $\neg$N1 (P4+10)

Next suppose that $X = \text{I1}$. Given (5), we can run the Reversibility Objection against I1 using an argument similar to the one we gave against N1:

[16] That is, suppose the Skeptical Hypothesis undermines our evidence for believing something like classical statistical mechanics, in the sense that for any account of classical statistical mechanics $X$, $cr_E(X|SH) \approx 0$. If $cr_E(X|SH) \approx 0$, it follows that $cr_E(X \wedge SH) \lll cr_E(SH)$, and thus that $cr_E(X \wedge SH) \approx 0$. And so it follows that $cr_E(X) = cr_E(X \wedge SH) + cr_E(X \wedge \neg SH) \approx cr_E(X \wedge \neg SH)$. Thus if we're granting that the correct account of statistical mechanics $X$ is such that we could rationally have $cr_E(X) > 0.5$, then it's arguably not much more of an assumption to grant that the correct account of statistical mechanics is such that we could rationally have $cr_E(X \wedge \neg SH) > 0.5$, since $cr_E(X) \approx cr_E(X \wedge \neg SH)$.

[17] I'm abusing notation here slightly. Although the $T$ and $K$ arguments that pick out a chance distribution are given by the conjunction of N1 and $E$, I am not identifying $T$ with N1 (since $T$ is supposed to just be a complete *chance* theory, whereas N1 is a complete description of all of the laws, not just the chancy ones), or $K$ with $E$. Rather, decomposing N1 into a chancy component $A$ and a non-chancy component $B$, the $T$ and $K$ arguments will be $T = A$ and $K = B \wedge E$, respectively.

**The Reversibility Objection (against I1):**

**P1.** $\mu(SH|\text{I1} \wedge E) \approx 1$

**P2.** Bayesianism (Probabilism and Conditionalization)

**P3.** Indifference Principle

**P4.** Adequacy

**5.** $ic(SH|\text{I1} \wedge E) \approx 1$ (P1+P3)

**6.** $cr_E(SH|\text{I1}) \approx 1$ (P2(cond)+5)

**7.** $cr_E(\neg SH|\text{I1}) \approx 0$ (P2(prob)+6)

**8.** $cr_E(\text{I1} \wedge \neg SH) \ll cr_E(\text{I1})$ (7)

**9.** $cr_E(\text{I1} \wedge \neg SH) \approx 0$ (P2(prob)+8)

**10.** $cr_E(\text{I1} \wedge \neg SH) \not> 0.5$ (9)

**C.** $\neg$I1 (P4+10)

## 3.2 Replies to the Reversibility Objection

To escape the Reversibility Objection, we need to either reject one of the premises of the argument, or modify the account under attack. Thus we have five ways to escape the argument:

(i) reject P1, and dispute that N/I1 assigns a high probability to the Skeptical Hypothesis,

(ii) reject P2, Bayesianism,

(iii) reject P3, either the Chance-Credence Principle (given the nomic approach) or the Indifference Principle (given the indifference approach),

(iv) reject P4, Adequacy, or

(v) accept the conclusion, and modify one's account so that it no longer falls prey to the Reversibility Objection.

Option (i) is to reject P1, and dispute that $ch_{\text{N1}E}(SH) \approx 1$ or that $\mu(A|\text{I1} \wedge E) \approx 1$. For example, one might argue that while our various memories considered individually would fail to tell against the Skeptical Hypothesis, the fact that all of them happen to agree with each other suffices to make the Skeptical Hypothesis unlikely.[18] More generally, one might argue that when "evidence like ours" is properly understood, or when the correct way to calculate the statistical mechanical probabilities in question is

[18] See Schulman (1997), p.154-155.

properly understood, we'll find that the chance/indifference measure of the Skeptical Hypothesis small.[19]

For the most part, people have found these kinds of argument to be unpersuasive.[20] And a full examination of these replies would take us too far afield. But let's consider a version of this kind of reply that challenges P1 by disputing what we can take to be evidence.

I've been following Albert (2000) in taking our evidence *E* to be something like our current memories and experiences, or the current macrostate of the world (at the appropriate level of coarse-grained description). But one might challenge this claim. For example, one might take our evidence to include more than what we currently "have access to", and so take *E* to consist of the sum of our memories and experiences we've had throughout our life. Alternatively, one might understand "evidence like ours" more ambitiously, so as to include things that we've taken science to have firmly established, such as "the universe began 14 billion years ago in a particular low entropy state".[21]

One worry for these alternative conceptions of evidence, as we'll see when we discuss option (iii), is that given the skeptical worries in play regarding our memories and records, one can't take them to be evidence. But let's put this worry aside for now.

A bigger worry for the more modest extensions – such as that one's evidence is the sum of one's experiences throughout one's life – is that P1 will still be true. For when we consider events in the distant past – events before we were born – we'll again find that it's overwhelmingly more likely (according to the standard measure) that our records about the distant past coalesced from some high entropy state than that they accurately reflect what the distant past was like. So adopting these modest extensions won't help us to escape P1.

The more ambitious extensions – such as taking "the universe began 14 billion years ago in a particular low entropy state" to be evidence – will allow us to escape P1. But given the Bayesian account we're assuming here, these extensions are difficult to maintain.

Few would want to maintain that a proposition like "the initial conditions of the universe 14 billion years ago were very low entropy" is something we should remain confident about, no matter what further considerations arise. For example, we might want to lower our credence in this proposition if we found reason to think that some kind of dynamical explanation of thermodynamic asymmetries is correct (such as the proposal appealing to the GRW account of quantum mechanics discussed in Albert (2000)), or if we found reason to believe that our astronomical evidence was systematically skewed in some way, and so on. And presumably some ways of changing one's

[19] For example see Parker (2006), who argues that we cannot understand the content of our records in thermodynamic terms, and that considerations involving the contents of our records will allow us to escape the Skeptical Hypothesis.

[20] For example, see Albert (2000) for a reply to the kinds of considerations Schulman raises.

[21] Thanks to Barry Loewer here for pressing me to take this objection more seriously.

credence in propositions like "the initial conditions of the universe 14 billion years ago were very low entropy" in light of such considerations are rational, and some are irrational. And we'd like a story about which are which.

This story is what the Bayesian apparatus is supposed to give us. But we can only use the Bayesian apparatus to help us here if we're taking "the initial conditions of the universe 14 billion years ago were very low entropy" to be a *hypothesis* – something which is up for grabs – not as evidence. If we take such propositions to be *evidence* – something which is settled, taken for granted, epistemological bedrock – then they're beyond rational evaluation.[22]

(Now, one might grant this and try to escape these worries by vacillating about what one takes evidence to be – perhaps treating propositions like "the initial conditions of the universe 14 billion years ago were very low entropy" as evidence in some contexts, but not others, and so on. But such a position isn't intellectually stable. At the end of the day, we're going have to fit everything we want to say into a single comprehensive epistemology, a single story in which everything falls into place, where there's a determinate fact as to whether something is evidence or not.)

So, at the end of the day, the proponent of this more ambitious picture of evidence will face a dilemma. They can either (a) give a comprehensive story in which things like "the initial conditions of the universe 14 billion years ago were very low entropy" do count as evidence, and give up on giving a story about what kinds of belief changes regarding such propositions are rational, or (b) give a comprehensive story in which things like "the initial conditions of the universe 14 billion years ago were very low entropy" don't count as evidence. The first horn of the dilemma requires one to give up many of the epistemological ambitions of the philosophy of science, while the second horn requires one to accept that this way of escaping P1 is untenable.

Option (ii) is to reject Bayesianism – that is, to reject Probabilism, Conditionalization, or both. But I take rejecting Bayesianism as a way to avoid the Reversibility Objection to be a non-starter. Although various reasons have been offered for modifying the standard Bayesian framework – in order to allow for agents with imprecise credences, for example, or to allow for different kinds of updating with respect to certain kinds of belief – none of these modifications will help one avoid the Reversibility Objection.[23] This eliminates option (ii).

[22]Such credal variations are straightforwardly ruled out by the Bayesian account sketched in section 2, since Conditionalization doesn't allow one's credences in one's evidence to vary – one's credence in one's evidence must be (and remain) 1. But one can get around this obstacle by adopting a standard extension of the Bayesian approach, which replaces Conditionalization with Jeffrey Conditionalization, a rule which allows agents to get uncertain evidence. However, the move to Jeffrey Conditionalization does nothing to address the deeper problem with taking "the initial conditions of the universe 14 billion years ago were very low entropy" as evidence. For if we treat our belief changes in such propositions as due to uncertain evidence, then they're beyond rational evaluation – we can't provide a story as to whether these "evidential" changes in belief are rational or not. But surely we *do* want a story here – an account of what our belief in this proposition should come to be, given certain observations and deductions about what the world is like.

[23]See Levi (1985) and Joyce (2005) for discussion and references regarding imprecise credences, and Elga

Option (iii) is to reject either the Chance-Credence Principle (given the nomic approach) or the Indifference Principle (given the indifference approach). Rejecting the Chance-Credence Principle is not a viable way to escape the Reversibility Objection. Virtually everyone holds that some kind of Chance-Credence Principle constrains rational belief. And while there's a strong case to be made in favor of the particular formulation of the Chance-Credence Principle we've employed, the Reversibility Objection would still go through if we replaced it with any of the other proposed formulations of the Chance-Credence Principle.[24]

Rejecting the Indifference Principle isn't a promising way to escape the Reversibility Objection either. Unlike Chance-Credence Principles, Indifference Principles are widely viewed with suspicion.[25] But a proponent of the indifference approach is committed to accepting some kind of Indifference Principle, so rejecting such principles entirely is not an option. And changing the Indifference Principle in order to try to escape the Reversibility Objection is problematic.

One worry is that changing the Indifference Principle for this reason seems methodologically unsound. The Indifference Principle is supposed to be justified by *a priori* considerations regarding what an ideally rational agent would believe in various evidential situations. To modify the Indifference Principle in order to escape something like the Reversibility Objection would be to modify the Indifference Principle in light of *a posteriori* considerations. And since the Indifference Principle is an *a priori* constraint on rational belief, *a posteriori* considerations shouldn't be relevant.

Here's a more severe concern about modifying the Indifference Principle in order to escape reversibility worries. Given I1 and evidence like ours, we want the Indifference Principle to yield the right results in typical cases, and yet avoid the prescriptions which lead to complaints like the Reversibility Objection. But it's not clear that any plausible *a priori* constraint on rational belief can do this.

Let's look at an example. Let $E$ be the macrostate picked out by our evidence, where this evidence includes the fact that there's a half-melted ice cube in a cup of warm water, let $A_f$ be the macrostate containing the microstates in which the half-melted ice cube is completely melted in the near future, and let $A_p$ be the macrostate containing the microstates in which the half-melted ice cube is completely melted in the recent past. We want the statistical mechanical probability of $A_f$ given I1 and $E$ – the probability of the half-melted ice cube evolving to a completely melted state in the near future – to be high. We also want the statistical mechanical probability of $A_p$ given I1 and $E$ – the probability of the half-melted ice cube having evolved from a completely melted state in the recent past – to be low. Thus we want $\mu$ to be such that

(2000) and Arntzenius (2003) for some discussion and references regarding self-locating beliefs.

[24]For the case in favor of this formulation, see Arntzenius (1995), Meacham (2005), Nelson (2009) and Meacham (2010); alternative formulations have been suggested by Vranas (2004), Hoefer (2007), and Ismael (2011), among others.

[25]For criticisms of such principles, see van Fraassen (1989), Howson and Urbach (2005), Weisberg (2011), and the references therein.

$\mu(A_f|\text{I1} \wedge E) \approx 1$ and $\mu(A_p|\text{I1} \wedge E) \approx 0$.

But these prescriptions are temporally asymmetric – future and past events are assigned different credences. Where does this temporal asymmetry come from?

The laws I1 posits are temporally symmetric, so I1 isn't the source of the asymmetry. What about our evidence, $E$? Well, the claim that the ice cube is in a half-melted state isn't temporally asymmetric. And if we take our evidence to be something like the current "time slice" of our experience, or the current macrostate of the world (at a certain level of coarse-grained description), then the rest of our evidence isn't temporally asymmetric either. For such evidence is synchronic, and in order to be temporally asymmetric evidence must have temporal "breadth".

For this reason, one might maintain that our evidence is diachronic rather than synchronic. For instance, perhaps our evidence is something like the sum of all of the experiences we've had throughout our life. And this diachronic evidence is temporally asymmetric.

In some ways, this is a tricky position to maintain. Evidence, in the relevant sense – the notion of "evidence" that appears in Conditionalization – is something you ought to have a credence of 1 in, something you ought to be virtually certain of. And given the skeptical worries in play regarding our memories and our records of the past, our memories and past experiences are not something we can take to be evidence in this sense.

But, more importantly, these kinds of attempts to extend the notion of evidence run into the problems we saw earlier in the discussion of option (i). The modest extensions, like the one just described above will provide us with temporally asymmetric evidence, but this asymmetry seems unlikely to yield the dramatic asymmetry in prescriptions that we want. The ambitious extensions (which take things like "'the initial conditions of the universe 14 billion years ago were very low entropy" to be evidence), on the other hand, will give us temporally asymmetric evidence which yields the asymmetric prescriptions we desire, but as we've already seen, these accounts of evidence are problematic.

So neither I1 nor $E$ can yield the temporal asymmetry we want with respect to our assignments to $A_f$ and $A_p$. Thus the temporal asymmetry must come from the indifference measure itself. But it's hard to see how any kind of *a priori* consideration could justify treating one direction in time differently from the other. Certainly, none of the standard considerations appealed to in order to justify Indifference Principles – symmetry, parsimony, information-theoretic considerations, etc. – provide grounds for such an asymmetry.

Together, these considerations tell against option (iii). What about option (iv), rejecting Adequacy? The role of Adequacy in the argument is to move us from the uncomfortable result that one can't rationally both believe the proposed account of statistical mechanics and reject the Skeptical Hypothesis, to the conclusion that the proposed account of statistical mechanics is not correct. To reject Adequacy and accept the rest of the argument is to concede the uncomfortable result, but hold on to the proposed account of statistical mechanics anyway. While this technically allows one to escape the Reversibility Objection, it's not very satisfying. Certainly, this seems like a lonely position, since not many people are going to be comfortable embracing the skeptical consequences of the account and calling it a day.

This leaves us with option (v): modifying the account under attack, N/I1. Given that we're going to change N/I1, what parts of the account should we change? There are three options: (v.a) we can change what it takes the laws to be, (v.b) we can change what it takes the statistical mechanical probabilities to be, or (v.c) we can add some further claim to the account which does neither of the above. Let's consider these options in reverse order.

The third way of modifying an account, (v.c) – adding a claim to the account that's doesn't change the laws or the statistical mechanical probabilities – won't help us escape the Reversibility Objection. Let $A$ be the claim we're adding to the account, and let our new account, the conjunction of $A$ and N/I1, be N/IA1. Since N/IA1 logically entails N/I1, if follows from the probability axioms that $cr_E(\text{N/I1} \wedge \neg SH) \geq cr_E(\text{N/IA1} \wedge \neg SH)$. Thus if we're rationally required to be such that $cr_E(\text{N/I1} \wedge \neg SH) \approx 0$, we're also rationally required to be such that $cr_E(\text{N/IA1} \wedge \neg SH) \approx 0$. So the Reversibility Objection will apply to N/IA1 as well.

The second way of modifying an account, (v.b) – changing the statistical mechanical probabilities – ends up collapsing into one of the other options. The Reversibility Objection arises due to the constraints on rational belief that the account imposes. So to escape the Reversibility Objection, we need to change the values the account assigns that constrain rational belief, be they the chances (on the nomic approach) or the indifference measures (on the indifference approach). Changing the chances requires changing the laws that assign the chances, so modifying the nomic account in the second way (v.b) amounts to a special case of changing the account in the first way (v.a). And changing the indifference measure is to modify the Indifference Principle, which (on the indifference approach) is just option (iii).

This leaves us with the first way of modifying an account, (v.a) – changing the laws. Unlike the other options, this option seems viable. Suppose we modify N/I1 by adding a lawful constraint that allows all and only initial conditions that belong to some particular low-entropy macrostate. That is, suppose we modify N/I1 by adding the *Past Hypothesis* to the laws. Then we get a new account, N/I2. And N/I2 is immune to the Reversibility Objection.

The Reversibility Objection doesn't apply to N2 because it isn't the case that $ch_{\text{N2}E}(SH) \approx 1$. The nomic accounts of classical statistical mechanics employ a Statistical Postulate that assigns chances to the lawfully-permitted initial conditions that are proportional to the Liouville measure. And if the account includes a Past Hypothesis, as N2 does, then most of the microstates in our initial account are not lawfully permitted – the laws forbid any microstate which, when evolved backwards according to the dynamics, doesn't lead to the particular low-entropy initial condition required by the Past Hypothesis. Once we rule out these possibilities and renormalize, we find that the majority of the remaining microstates compatible with our evidence *do* line up with our memories and records of what the past was like, *contra* the Skeptical Hypothesis. Thus $ch_{\text{N2}E}(SH) \approx 0$.[26]

The Reversibility Objection doesn't apply to I2 because it isn't the case that $\mu(SH|\text{I2} \wedge E) \approx 1$. It follows from (6) that $\mu(A|\text{I2} \wedge E) = ch_{\text{N2}E}(A)$. And since $ch_{\text{N2}E}(SH) \approx 0$, it follows that $\mu(SH|\text{I2} \wedge E) \approx 0$ as well.

So by adopting a Past Hypothesis, and replacing N/I1 with N/I2, we can escape the Reversibility Objection.

There has been some debate over whether one should understand the Past Hypothesis as a law, or as (say) a contingent generalization.[27] Now that we've given a precise characterization of the Reversibility Objection, we can see that regardless of whether we adopt the nomic approach or the indifference approach, the Past Hypothesis has to be a law in order to address the Reversibility Objection. For if the Past Hypothesis is only a contingent generalization, then adding it to one's account amounts to adopting option (v.c), not (v.a). And, as we've seen, adopting option (v.c) doesn't help us escape the Reversibility Objection.[28]

# 4 The Meta-Reversibility Objection

Adopting the Past Hypothesis addresses the Reversibility Objection as posed. But one might wonder whether we can raise the problem in another way.

Here is one way to see why N/I1 is susceptible to the Reversibility Objection. Conditional on a particular low-entropy initial condition obtaining, N/I1 assigns the right values to past events. But conditional on some high-entropy initial condition obtaining, N/I1 assigns the wrong values to past events. And since N/I1 unconditionally assigns a much higher value to high-entropy initial conditions than to low-entropy initial conditions, the assignments given high-entropy initial conditions will swamp those given low-entropy initial conditions. Thus, when all is said and done, N/I1 assigns the wrong values to past events.

Now, as we saw in the previous section, we can fix this by lawfully requiring the

[26] For a more detailed description of this argument, see Albert (2000). Although the claim that the Past Hypothesis allows us to get around the Reversibility Objection is wide-spread, it is at least mildly contentious. For some worries regarding this claim, see Uffink (2002), Winsberg (2004*a*), Winsberg (2004*b*), Parker (2005), Earman (2006) and Callender (2008).

[27] See Callender (2004) and North (2011) for discussion and references.

[28] Here's another way to see why this is so. Call the contingent version of the Past Hypothesis "$PH_c$", and call the account we get by adding a contingent Past Hypothesis to N/I1, "N/I$1_c$". A little thought shows that $cr_E(\text{N/I1}_c) = cr_E(\text{N/I1} \wedge PH_c) \approx cr_E(\text{N/I1} \wedge \neg SH)$. Thus, since we're rationally required to not believe N/I1$\wedge \neg SH$, we're rationally required to not believe N/I$1_c$.

What makes N/I$1_c$ seem like it might help with the Reversibility Objection is the observation that, unlike N/I1, believing N/I$1_c$ doesn't commit you to believing the Skeptical Hypothesis. This is true. But since we're already committed to not believing N/I$1_c$, this doesn't help us satisfy Adequacy.

low-entropy initial condition to obtain – that is, by adding the Past Hypothesis (*PH*) to the laws. If we do that, then the resulting account, N/I2, ends up assigning the right values to past events.

At this point, however, one could raise a different reversibility worry. To begin, note that there are other lawful constraints on initial conditions one could impose. For example, one could lawfully require that some *high*-entropy initial condition obtain; call this the Past Hypothesis* (*PH**). If we do that, then the resulting account, call it N/I2*, ends up assigning the wrong values to past events.

Now suppose we're rationally required to have credences over lawfully required initial conditions that mirror our credences over contingently obtaining initial conditions, so that we assign a much higher unconditional credence to lawful high-entropy initial conditions like *PH** than to lawful low-entropy initial conditions like *PH*. Then, since we'll unconditionally assign a much higher credence to the account that employs *PH** (N/I2*) than to the account that employs *PH* (N/I2), the credences assigned by N/I2* will swamp those assigned by N/I2. So, when all is said and done, we'll be required to adopt the wrong credences about past events. Call the worry that an account will commit one to this result the *Meta-Reversibility Objection*.

The Meta-Reversibility Objection shouldn't worry proponents of nomic accounts. The objection only gets traction if one's account requires one to assign credences over lawful constraints on initial conditions in a certain way. And nomic accounts don't impose any requirements on how one assigns credences over lawful constraints on initial conditions. Nomic accounts require only that one's credences line up with the chances. And there aren't any chances over different lawful constraints on initial conditions. After all, the Statistical Postulate won't give us chances until we know what the lawful constraints on initial conditions (if any) are!

Indeed, the proponent of nomic accounts is perfectly free to assign, in good conscience, a high initial credence to N2 and a low initial credence to N2*. This may not be egalitarian, but so what? We've known that we need to do something like this in order to ground our inductive biases since Goodman's (1954) famous grue cases. If we want to end up believing the world is like we think it is, our initial credences will have to be biased towards the theories we actually believe in certain ways.

But the Meta-Reversibility Objection *is* a worry for proponents of indifference accounts. For the Indifference Principle makes prescriptions about everything. Thus it makes prescriptions about lawful constraints on initial conditions.[29] And there is substantial pressure on the proponent of the indifference approach to maintain that I2* should be assigned a much higher credence than I2.

I'll discuss this more in section 4.2, but briefly, the worry is this. In order to yield

[29] One might wonder whether one could resist this worry by simply adopting a weaker Indifference Principle, which only makes prescriptions in some limited range of domains. As we'll see, this isn't a promising option. The typical rationales used to justify the Indifference Principle's prescriptions in these limited domains will apply just as well to wider domains. And, as we'll discuss in section 4.2, imposing arbitrary constraints on these rationales would be fatal to the Indifference Principle project.

the expected prescriptions given I1, the indifference measure must assign values to initial conditions that are proportional to the Liouville measure. But if this is how we ought to be indifferent with respect to which initial conditions contingently obtain, it's hard to see why we shouldn't be indifferent in the same way with respect to which initial conditions lawfully obtain. After all, the structure of our ignorance over the lawfully required initial conditions mirrors the structure of our ignorance over the contingent initial conditions. And the standard rationales offered to justify indifference measures – symmetry considerations, information-theoretic considerations, etc. – apply equally well to both cases. If this is right, then the Indifference Principle will prescribe a much higher credence to I2* than I2. So the Meta-Reversibility Objection poses a real threat to indifference accounts.

## 4.1 Formalizing the Meta-Reversibility Objection

Let's formulate the Meta-Reversibility Objection more precisely. To begin, let's spell out the motivating worry a bit more carefully.

Let $\mathrm{I2}^-$ be the proposition that the correct account of classical statistical mechanics is some modified version of I1 which adds a lawful constraint on boundary conditions. Thus I2 is equivalent to $\mathrm{I2}^- \wedge PH$, and $\mathrm{I2}^*$ is equivalent to $\mathrm{I2}^- \wedge PH^*$. Let $PH_c$ be the contingent version of the Past Hypothesis – i.e., the claim that, as a matter of contingent fact, the initial conditions came from some particular low-entropy macrostate – and let $PH_c^*$ be the contingent version of the Past Hypothesis$^*$.

The worry sketched above is that the Indifference Principle will require one to assign credences to $PH$ and $PH^*$ given $\mathrm{I2}^-$ and $E$ that mirror those it requires one to assign to $PH_c$ and $PH_c^*$ given I1 and $E$. That is, the worry is that if the indifference measure is such that:

$$\mu(PH_c|\mathrm{I1} \wedge E) \ll \mu(PH_c^*|\mathrm{I1} \wedge E), \tag{7}$$

it will also be such that:

$$\mu(PH|\mathrm{I2}^- \wedge E) \ll \mu(PH^*|\mathrm{I2}^- \wedge E). \tag{8}$$

And this entails that, given evidence like ours $E$, our credence in $\mathrm{I2}^*$ should be much larger than our credence in I2.[30]

This puts us in an unhappy situation. If our credence in $\mathrm{I2}^*$ is much larger than our credence in I2, then we can't rationally believe that I2 obtains. Thus I2 can't satisfy Adequacy. Of course, we may be able to rationally believe something like $\mathrm{I2}^*$. But $\mathrm{I2}^*$ is subject to the original Reversibility Objection – since almost all of the microstates compatible with our evidence and the Past Hypothesis$^*$ are ones which

[30] We have $\mu(PH|\mathrm{I2}^- \wedge E) \ll \mu(PH^*|\mathrm{I2}^- \wedge E)$. Multiplying both sides by $\mu(\mathrm{I2}^- \wedge E)$ gives us $\mu(PH \wedge \mathrm{I2}^- \wedge E) \ll \mu(PH^* \wedge \mathrm{I2}^- \wedge E)$, or, equivalently, $\mu(\mathrm{I2} \wedge E) \ll \mu(\mathrm{I2}^* \wedge E)$. Dividing both sides by $\mu(E)$ gives us $\mu(\mathrm{I2}|E) \ll \mu(\mathrm{I2}^*|E)$. Then applying the Indifference Principle gives us $ic(\mathrm{I2}|E) \ll ic(\mathrm{I2}^*|E)$, and applying Conditionalization gives us $cr_E(\mathrm{I2}) \ll cr_E(\mathrm{I2}^*)$.

fail to line up with our memories and records of the past, our credence in $I2^* \wedge \neg SH$ will be very low. Thus $I2^*$ won't satisfy Adequacy either. Either way, it looks like the indifference accounts of classical statistical mechanics fail to satisfy Adequacy. They are either susceptible to the Reversibility Objection described in section 3, or they are susceptible to the worry just described above – the Meta-Reversibility Objection.

Here's another way to think about these concerns. Proponents of the indifference approach face a choice between whether or not to take there to be lawful constraints on boundary conditions; i.e., a choice between accepting I1 (which entails that there are no lawful constraints on boundary conditions) or $I2^-$ (which entails that there are). If they accept I1 then they're rationally required to believe the Skeptical Hypothesis – that their memories and records about the past are false. If they accept $I2^-$, then they're also rationally required to believe the Skeptical Hypothesis. For although they'll believe that *if* I2 is true then it's likely that their memories and records about the past are reliable, their credence in I2 will be so low compared to its competitors that this will have little impact on their overall credence. The laws that will dominate their credence will be the high entropy initial condition ones like $I2^*$, which predict that it's much more likely that we coalesced from a high entropy state than that we evolved from a low entropy state. So, when all is said and done, they'll again be rationally required to believe the Skeptical Hypothesis.

Here is a formal characterization of the argument. Given (8), we can set up the Meta-Reversibility Objection against I2 as follows:

**The Meta-Reversibility Objection (against I2):**

**P1.** $\mu(PH|I2^- \wedge E) \ll \mu(PH^*|I2^- \wedge E)$

**P2.** Bayesianism (Probabilism and Conditionalization)

**P3.** Indifference Principle

**P4.** Adequacy

**5.** $ic(PH|I2^- \wedge E) \ll ic(PH^*|I2^- \wedge E)$ (P1+P3)

**6.** $cr_E(PH|I2^-) \ll cr_E(PH^*|I2^-)$ (P2(cond)+5)

**7.** $cr_E(PH \wedge I2^-) \ll cr_E(PH^* \wedge I2^-)$ (P2(prob)+6)

**8.** $cr_E(I2) \ll cr_E(I2*)$ (7)

**9.** $cr_E(I2) \approx 0$ (P2(prob)+8)

**10.** $cr_E(I2 \wedge \neg SH) \approx 0$ (P2(prob)+9)

**11.** $cr_E(I2 \wedge \neg SH) \not> 0.5$ (10)

**C.** $\neg I2$ (P4+11)

This gives us the Meta-Reversibility Objection against I2. What happens if we try to run the Meta-Reversibility Objection against N2?

To run the argument against N2, we would have to replace P1 with $ch_{\mathrm{N2}^- E}(PH) \ll ch_{\mathrm{N2}^- E}(PH^*)$, replace P3 with the Chance-Credence Principle, and replace "I2" and "I2$^-$" with "N2" and "N2$^-$" throughout (where N2$^-$ is the nomic analog of I2$^-$ – the proposition that the correct account of classical statistical mechanics is some modified version of N1 which adds a lawful constraint on boundary conditions). But this argument won't work, because this version of P1 is false. P1 is false because N2$^-$ and $E$ don't suffice to pick out a chance distribution. It's true that N2$^-$ includes the Statistical Postulate, and the Statistical Postulate posits chances. But what chances the Statistical Postulate assigns depend on what lawful constraints there are on the boundary conditions, and N2$^-$ and $E$ don't tell us what these lawful constraints are. So the $ch$ terms on both sides of P1's inequality will be undefined, and P1 will be false.

## 4.2 Replies to the Meta-Reversibility Objection

As with the Reversibility Objection, we can divide the potential replies to the Meta-Reversibility Objection into five options:

(i) reject P1, that $\mu(PH|\mathrm{I2}^- \wedge E) \ll \mu(PH^*|\mathrm{I2}^- \wedge E)$,

(ii) reject P2, Bayesianism,

(iii) reject P3, the Indifference Principle,

(iv) reject P4, Adequacy, or

(v) accept the conclusion, and modify one's account so that it no longer falls prey to the Meta-Reversibility Objection.

One can try to do this by either: (v.a) changing the laws the account specifies, (v.b) changing the statistical mechanical probabilities the account assigns, or (v.c) adding some further claim to the account that does neither of the above.

The first option is to reject P1, and dispute that $\mu(PH|\mathrm{I2}^- \wedge E) \ll \mu(PH^*|\mathrm{I2}^- \wedge E)$. As before, one way to do this is argue that when "evidence like ours" is properly understood, or when the correct way to calculate the probabilities in question is properly understood, we'll find that P1 is false. But if this kind of reply fails in the context of the Reversibility Objection, as we're assuming, it seems it will fail here as well. So this way of rejecting P1 does not look promising.

There's another way to reject P1 here that wasn't available against the Reversibility Objection. Namely, one can reject P1 by arguing that this result comes from using the wrong Indifference Principle.[31] Given this way of pursuing option (i), options (i),

[31] This way of rejecting P1 was not available to opponents of the Reversibility Objection because the value I1 assigns to the relevant proposition (the value of $SH$ given $\mathrm{I1} \wedge E$) is fixed. We stipulated that I1 assigns values that line up with N1's chances, so the value of $SH$ given $\mathrm{I1} \wedge E$ must be equal to the chance of $SH$ given $\mathrm{N1} \wedge E$. And the chance of $SH$ given $\mathrm{N1} \wedge E$ is fixed by our stipulation that the chances given N1 be proportional to the Liouville measure.

By contrast, this way of rejecting P1 *is* available to opponents of the Meta-Reversibility Objection because

(iii) and (v.b) are all versions of a single reply – they're all ways of rejecting the Indifference Principle the argument employs, and replacing it with a principle that makes different prescriptions. This is the most natural reply to the Meta-Reversibility Objection. For the argument hangs on the claim that, given $I2^-$ and $E$, the indifference measure of $PH^*$ is much greater than that of $PH$. And, one might say, it's not obvious why the proponent of the indifference approach should accept this claim.

But this claim turns out to be hard for proponents of the indifference approach to reject.[32] One reason is that they presumably want the Indifference Principle to still yield the standard probability assignments given accounts like I1 and I2. That is, they want the indifference measure to be such that:

$$\mu(PH_c|I1 \wedge E) \ll \mu(PH_c^*|I1 \wedge E). \tag{9}$$

But if the indifference measure assigns those values, it's hard to see why it won't assign similar values to the lawful versions of the Past Hypothesis and the Past Hypothesis* given $I2^-$:

$$\mu(PH|I2^- \wedge E) \ll \mu(PH^*|I2^- \wedge E). \tag{10}$$

After all, the two cases are structurally analogous. In one case we're assessing what the initial macrostate of the world is, as a matter of contingent fact, while in the other we're assessing what the initial macrostate of the world is, as a matter of lawful fact. In all other respects the two cases are the same. And all of the standard considerations that are appealed to in order to justify indifference measures – symmetry, parsimony, informational entropy, and so on – apply in the same way to both cases.

To get a feel for why this is so, let's go through an example in detail. The most popular version of the indifference approach to statistical mechanics, suggested by Jaynes (1983), maintains that the indifference measure of a macrostate should track how uninformative it is to be told that the system is in that macrostate, so that more informative macrostates get smaller indifference measures. And Jaynes proposes that we cash out the relevant notion of informativeness here by appealing to the notion of Shannon entropy, i.e., information entropy.

Here's one way to flesh out the details of such a proposal in a Boltzmannian framework. The *Shannon Entropy $S$* of a random variable $R$ is:

$$S(R) = -\sum_i p(R_i) \cdot \ln p(R_i), \tag{11}$$

the values $I2^-$ assigns to the relevant propositions (the values of $PH$ and $PH^*$ given $I2^- \wedge E$) are not fixed. It's true that the "matching values" stipulation we made in section 2.2 implies that the values of $PH$ and $PH^*$ given $I2^- \wedge E$ should line up with the chances of $PH$ and $PH^*$ given $N2^- \wedge E$, if there are such chances. But, as we just saw in section 4.1, there are no such chances – the chances of $PH$ and $PH^*$ given $N2^- \wedge E$ are not well-defined.

[32]As we saw in section 3.2, there are also concerns regarding whether this way of replying to reversibility worries is methodologically sound, since reversibility worries seem *a posteriori*, while the Indifference Principle is supposed to be an *a priori* constraint. But, these worries are of secondary interest relative to the main concerns I discuss in the text, so I'll put them aside.

where $p(R_i)$ is the probability of $R$ yielding the $i$th possible outcome. Since a macrostate is not a random variable, we cannot apply $S$ to a macrostate $A$ directly. Instead, we need to find some way to construct a canonical random variable associated with $A$. Then we can take the Shannon entropy of that random variable to be the Shannon entropy associated with $A$.

Let a $\Delta x \Delta v$-*partition*, $\gamma$, be a partition of phase space into hypercubes which uniformly have sides of dimension $\Delta x$ (for the spatial dimensions) and $\Delta v$ (for the velocity dimensions), in standard units. Call these hypercubes the $\gamma$-*macrostates*.

Consider some macrostate $A$, and some $\Delta x \Delta v$-partition, $\gamma$, that is fine-grained enough so that we can characterize $A$ as a disjunction of $\gamma$-macrostates. Let "$\gamma A$" be the random variable that has an equal probability of yielding each of the $\gamma$-macrostates that compose $A$.

Let the Shannon entropy associated with $A$ (with respect to $\gamma$) be the Shannon entropy of $\gamma A$. So if $A$ can be characterized as a disjunction of $n$ $\gamma$-macrostates, the Shannon entropy associated with $A$ (with respect to $\gamma$) will be:

$$S(\gamma A) = -\sum_i p(\gamma A_i) \cdot \ln p(\gamma A_i) = n \cdot -\frac{1}{n} \cdot \ln \frac{1}{n} = -\ln \frac{1}{n}. \tag{12}$$

Given this, we can impose the desired constraint on $\mu$ as follows:

**$\mu$-Constraint:** Let $L$ be some (possibly incomplete) description of the laws, which includes the dynamics, and is such that the phase space associated with a system can be given a $\Delta x \Delta v$-partition. Let $A$ and $B$ be arbitrary macrostates in this phase space. Finally, for any macrostate $X$, let $XL$ be the macrostate composed of the microstates compatible with $X$ and the lawful constraints on boundary conditions imposed by $L$, if any. Then for any $\Delta x \Delta v$-partition, $\gamma$, such that $AL$ and $ABL$ can be characterized as disjunctions of $\gamma$-macrostates, $\mu$ must be such that:

$$\mu(A|B \wedge L) \propto \frac{e^{S(\gamma ABL)}}{e^{S(\gamma BL)}}. \tag{13}$$

A couple comments. First, note that while the values we get for $e^{S(\gamma ABL)}$ and $e^{S(\gamma BL)}$ depend on our choice of $\gamma$, the ratio between $e^{S(\gamma ABL)}$ and $e^{S(\gamma BL)}$ do not. So this constraint does not depend on our choice of $\gamma$. Second, note that $e^{S(\gamma A)} = e^{-ln(1/n)} = \frac{1}{e^{ln(1/n)}} = \frac{1}{1/n} = n$. So for any given $\gamma$, the indifference measure of a macrostate is proportional to the number of $\gamma$-macrostates it's composed of. Third, note that for any $\gamma$, each $\gamma$-macrostate has the same Liouville measure. So for any given $\gamma$, the Liouville measure of a macrostate $A$ is proportional to the number of $\gamma$-macrostates it's composed of. Thus this constraint requires the indifference measure to be proportional to the Liouville measure. Fourth, this constraint is incomplete, since it does not fully specify $\mu$. A complete version of this constraint would also tell us how to apply information-theoretic considerations to determine how to be indifferent about everything, and would yield the above constraint as a special case.

This constraint on $\mu$ yields the values we want given $L = \text{I1}$ and $L = \text{I2}$. But it also yields the values that make P1 of the Meta-Reversibility Objection true given $L = \text{I2}^{-}$. And this is what we should expect given a Shannon entropy approach to indifference. The Shannon entropy of a macrostate $A$ can be seen as a measure of how much more information you need, after being told that a system's in macrostate $A$, to know what the system's microstate is.[33] And the only thing relevant to *that* is the *size* of the macrostate. The bigger the macrostate, the bigger the range of microstates it contains, and the more you have to say to pick out a particular microstate. The smaller the macrostate, the smaller the range of microstates it contains, and the less you have to say to pick out a particular microstate. But these information considerations are the same regardless of whether we're assigning credence to the initial conditions contingently being in macrostate $A$, or assigning credence to the initial conditions lawfully being in macrostate $A$. The only thing that matters, information-wise, is the size of the macrostate $A$. Whether $A$ obtains contingently or lawfully is irrelevant.

Here is a related concern for trying to escape reversibility worries by modifying the Indifference Principle. Given I2 and evidence like ours, we want the Indifference Principle to yield the desired prescriptions in typical cases, and yet avoid the prescriptions which lead to complaints like the Meta-Reversibility Objection. But it's not clear that any plausible *a priori* constraint on rational belief can do this.[34]

[33] Or, if we divide up $A$ into a number of small macrostates, a measure of how much more information you need to know a system's small macrostate after being told that it's in $A$.

[34] I raise the temporal asymmetry worry as a general worry for attempts to avoid the Meta-Reversibility Objection by modifying the Indifference Principle. But one can also use temporal symmetry considerations to argue directly for P1.

To make things manageable, let's assume that the only possible lawful constraints on boundary conditions are pairings of one of three constraints on initial conditions (the Past Hypothesis, the Past Hypothesis*, and the Past Hypothesis$^{-}$, which imposes a vacuous constraint on initial conditions), and one of three constraints on final conditions (the Future Hypothesis (*FH*), which requires the world to end in a particular low-entropy state, the Future Hypothesis* ($FH^*$), which requires the world to end in some particular high-entropy state, and the Future Hypothesis$^{-}$ ($FH^{-}$), which imposes a vacuous constraint on final conditions).

Now further assume that: (1) our evidence $E$ is temporally symmetric, (2) the indifference measure is temporally symmetric, (3) given the lawful constraints on boundary conditions, the measure assigns values proportional to the Liouville measure, (4) the measure prescribes the desired values to future events given $\text{I2}^{-}$ and $E$, and (5) the measure assigns a small value to $PH^{-}$ given $\text{I2}^{-}$ and $E$ (which, given (3), is a necessary condition in order for the measure to yield the right prescriptions regarding past events, given $\text{I2}^{-}$ and $E$). Then one can argue for P1 as follows:

**L1.** (3) and (4) entail that the measure must assign a high value to $FH^* \vee FH^{-}$, and a low value to $FH$, given $E$ and $\text{I2}^{-}$.

**L2.** Since E, I2- and the measure are all temporally symmetric (from (1) and (2)), it follows from L1 that the measure must assign a high value to $PH^* \vee PH^{-}$, and a low value to $PH$, given $E$ and $\text{I2}^{-}$.

**L3.** L2 and (5) entail that the measure must assign a high value to $PH^*$, and a low value to $PH^{-} \vee PH$, given $E$ and $\text{I2}^{-}$.

**C.** Thus (from L3) the measure must assign a greater value to $PH^*$ than to $PH$, given $E$ and $\text{I2}^{-}$.

Consider again the example of the half-melted ice cube in warm water, from section 3.2. Given that our evidence ($E$) includes that there is a half-melted ice cube in warm water, we want our credence that it will be fully melted in the near future ($A_f$) given I2 and $E$ to be high, and our credence that it was fully melted in the recent past ($A_p$) given I2 and $E$ to be low. Thus we want $\mu$ to be such that $\mu(A_f|\text{I2} \wedge E) \approx 1$ and $\mu(A_p|\text{I2} \wedge E) \approx 0$. And if we're proponents of I2, we would like it to be permissible to both be confident in I2, and to have the appropriate credences in propositions like $A_f$ and $A_p$ given $E$. Since I2 entails I2$^-$, this won't be permissible unless $\mu$ is such that $\mu(A_f|\text{I2}^- \wedge E) \approx 1$ and $\mu(A_p|\text{I2}^- \wedge E) \approx 0$.[35]

These prescriptions are temporally asymmetric. What is the source of this asymmetry?

The laws I2$^-$ posits are temporally symmetric. It's true that I2$^-$ also entails that there are lawful constraints on boundary conditions, and many of the possible lawful constraints are temporally asymmetric. And conditional on some such asymmetric constraint, $\mu$ will presumably make asymmetric prescriptions. But for each temporally asymmetric constraint, there is a constraint that is its temporal inverse. And without a bias in $\mu$ that favors some asymmetric constraints over their temporal inverses, the weighted average of these contributions to $\mu$ conditional on I2$^-$ cancel out. So I2$^-$ is not the culprit.

What about our evidence $E$? $E$ cannot be the culprit either, for as we saw in section 3.2, our evidence does not suffice to ground the desired asymmetry of our prescriptions.

So the temporal asymmetry must come from the indifference measure itself, either from temporally biased assignments over the different lawful constraints on boundary conditions, or temporally biased assignments given lawful constraints on boundary conditions, or both. But it's hard to believe that any kind of *a priori* constraint on rational belief could yield this kind of temporal asymmetry. Any reasons we could come to have for believing the world is temporally asymmetric would seem to be purely *a posteriori*.

It's worth taking a step back to say a bit more about the dialectical status of these problems. The characterization of the Indifference Principle we gave in section 2.1 imposed few substantive constraints on what the principle was like. So it's logically possible to adopt an Indifference Principle that yields pretty much any prescriptions one wants. One could, for example, adopt an Indifference Principle that only makes prescriptions regarding how to assign values to initial macrostates that might *contingently* obtain. Or one could adopt an Indifference Principle that employs information-theoretic considerations to assign values to the initial macrostates that might contingently obtain, but employs entirely different considerations to assign values to the

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[35]Since I2 entails I2$^-$, $cr_E(\text{I2}^-) \geq cr_E(\text{I2})$. So if we're confident in I2 (e.g., $cr_E(\text{I2}) \approx 1$), it follows that we're confident in I2$^-$ (e.g., $cr_E(\text{I2}^-) \approx 1$). Thus $1 \approx \mu(A_f|\text{I2} \wedge E) \approx \mu(A_f|E) \approx \mu(A_f|\text{I2}^- \wedge E)$; likewise, $0 \approx \mu(A_p|\text{I2} \wedge E) \approx \mu(A_p|E) \approx \mu(A_p|\text{I2}^- \wedge E)$.

initial macrostates that might be lawfully required. More generally, one could simply jerry-rig an indifference measure so that it yields the desired statistical mechanical probabilities given I1 and I2, and doesn't yield the problematic assignments that lead to the Meta-Reversibility Objection.

But the mere logical possibility of such a response is cold comfort. The Indifference Principle is supposed to be an *a priori* constraint on rational belief. For it to be credible that such a principle is correct, one needs to make a convincing case that *a priori* plausibility considerations will pick out this particular constraint on rational belief. And it had better *not* be the case that one can provide a plausible rationale for any logically possible set of prescriptions. For if one could, then the entire project of justifying one Indifference Principle over the others would be a non-starter.

So insofar as one is on board with the project of finding the right Indifference Principle, one can't just choose whatever principle gets one out of trouble, and expect to be able to provide a plausible rationale for it. Rather, one has to try to determine what the plausible constraints on rational belief are, and hope that these considerations deliver an Indifference Principle which avoids the problematic results. And what the discussion above suggests is that if we employ the kinds of considerations people have so far used to justify Indifference Principles – symmetry, Shannon entropy, etc. – and we accept the constraints that they impose on rational belief – e.g., temporal symmetry – then we won't able to do this.

So proponents of I2 can, if they like, see this as a challenge: provide a plausible *a priori* rationale for the Indifference Principle they want to use to recover statistical mechanical probabilities, and demonstrate that these same considerations don't lead to the Meta-Reversibility Objection.

Thus options (i), (iii) and (v.b) – i.e., modifying the Indifference Principle – look like unpromising ways to escape the Meta-Reversibility Objection. What about the other options?

I take option (ii), rejecting Bayesianism, to be untenable, for the same reasons as before. While there may be compelling reasons to modify the Bayesian framework, these changes to the Bayesian framework won't help the proponent of the indifference approach escape the Meta-Reversibility Objection.

Likewise, option (iv), rejecting Adequacy, is unsatisfying for the same reasons as before. To reject Adequacy and accept the rest of the argument is to just accept the uncomfortable consequences the account is alleged to have, and then bite the bullet and hold on to the account anyway.

And again, option (v.c), conjoining some further claims to the account, won't help us escape the argument. Since I2 fails to satisfy Adequacy, any account that logically entails I2 – such as the conjunction of I2 and some further claim $A$ – will fail to satisfy Adequacy as well.

This leaves us with (v.a): changing the laws I2 posits. This option provided a promising reply to the Reversibility Objection. But in this case, it's not clear how changing the laws can help. Since we're restricting our attention to accounts of *clas-*

*sical* statistical mechanics, we're holding the dynamics fixed. So if we want to change I2's laws, it seems we need to reject or modify the lawful constraints it imposes on boundary conditions.[36]

But here the proponent of the indifference approach faces a dilemma. For accounts that impose lawful constraints on boundary conditions that get assigned a high value by the standard Indifference Principles, like I2*, assign high values to the Skeptical Hypothesis, and so don't satisfy Adequacy. And accounts that assign low values to the Skeptical Hypothesis, like I2, impose lawful constraints on boundary conditions that get assigned a low value by the standard Indifference Principles, and so don't satisfy Adequacy. So whichever way one goes, it looks like the resulting account won't satisfy Adequacy.

# 5 Conclusion

The initial formulations of the nomic and indifference accounts are subject to the Reversibility Objection. And proponents of both approaches can escape the Reversibility Objection by modifying their accounts to include the Past Hypothesis. But this move doesn't free the indifference approach of reversibility worries. For by adopting the Past Hypothesis to escape one reversibility worry, indifference accounts become subject to another – the Meta-Reversibility Objection. And modifying indifference accounts again in order to avoid this objection leads us back to accounts that are susceptible to the Reversibility Objection. So, taken together, the Reversibility Objection and the Meta-Reversibility Objection pose a steep challenge to the viability of the indifference approach.[37]

# Appendix

In the text I make the simplifying assumption that the phase space we're working with has been fixed. Here is how the central principles and arguments go if we relax that assumption.

Let $P$ stand for the parameters needed to pick out a particular phase space, given the laws. Once we explicitly take $P$ into account, the chances described in the text

[36]One could also keep all of the laws I2 posits and add some further laws as well. (This move doesn't run into the same worry as (ii.c) because it doesn't just *conjoin* something to I2, it *contradicts* I2. I2 asserts that the complete laws are given by classical mechanics and the Past Hypothesis, while this alternative account would not.) But given that the dynamics are deterministic, there's only room for laws that either (i) pose lawful constraints on the boundary conditions or (ii) pose chancy constraints on the boundary conditions. The former is the case being considered in the text, and the latter moves us out of the realm of indifference approaches.

[37]I'd like to thank Phil Bricker, Maya Eddon, Nina Emery, Barry Loewer and Travis Norsen for comments and discussion.

will now all include $P$ as a subscript; e.g., $ch_{\mathrm{N1}E}(SH)$ will become $ch_{\mathrm{N1}PE}(SH)$. Likewise all the Indifference Principles discussed in the text will all need to take $P$ into consideration. Thus part (b) of indifference accounts I1 and I2 will read:

(b) The statistical mechanical probability of $A$ given $B$ is what the Indifference Principle assigns as the rational initial credence in $A$ conditional on the macrostate $B$, the parameters needed pick out the phase space $P$, and the laws.

And the desired constraint on $\mu$ will be:

**$\mu$-Constraint:** Let $L$ be some (possibly incomplete) description of the laws, which includes the dynamics, and is such that the phase space associated with a system can be given a $\Delta x \Delta v$-partition. Let $P$ be a specification of the further parameters needed to pick out a phase space. Let $A$ and $B$ be arbitrary macrostates in this phase space. Let $AL$ ($ABL$) be the macrostates composed of the microstates compatible with $A$ ($A$ and $B$) and the lawful constraints on boundary conditions imposed by $L$, if any. Then for any $\Delta x \Delta v$-partition, $\gamma$, such that $AL$ and $ABL$ can be characterized as disjunctions of $\gamma$-macrostates, $\mu$ must be such that:

$$\mu(A|B \wedge P \wedge L) \propto \frac{e^{S(\gamma ABL)}}{e^{S(\gamma BL)}}. \qquad (14)$$

Let's turn to the Reversibility Objection and Meta-Reversibility Objection. Let $\forall_e$ be the restricted quantifier that ranges over all and only the $P$s that are compatible with our evidence. To simplify things, I'll assume that these $P$s are finite in number. Then we can run the Reversibility Objection against N1 as follows:

**The Reversibility Objection (against N1):**

**P1.** $\forall_e P \; ch_{\mathrm{N1}PE}(SH) \approx 1$

**P2.** Bayesianism (Probabilism and Conditionalization)

**P3.** Chance-Credence Principle

**P4.** Adequacy

**5.** $\forall_e P \; ic(SH|\mathrm{N1} \wedge P \wedge E) \approx 1$ (P1+P3)

**6.** $\forall_e P \; cr_E(SH|\mathrm{N1} \wedge P) \approx 1$ (P2(cond)+5)

**7.** $\forall_e P \; cr_E(\neg SH|\mathrm{N1} \wedge P) \approx 0$ (P2(prob)+6)

**8.** $\forall_e P \; cr_E(\mathrm{N1} \wedge P \wedge \neg SH) \ll cr_E(\mathrm{N1} \wedge P)$ (7)

**9.** $\sum_{P_e} cr_E(\mathrm{N1} \wedge P \wedge \neg SH) \ll \sum_{P_e} cr_E(\mathrm{N1} \wedge P)$ (8)

**10.** $\sum_P cr_E(\mathrm{N1} \wedge P \wedge \neg SH) \ll \sum_P cr_E(\mathrm{N1} \wedge P)$ (P2(prob)+9)

**11.** $cr_E(\mathrm{N1} \wedge \neg SH) \ll cr_E(\mathrm{N1})$ (P2(prob)+10)

**12.** $cr_E(\mathrm{N1} \wedge \neg SH) \approx 0$ (P2(prob)+11)

**13.** $cr_E(\mathrm{N1} \wedge \neg SH) \not\approx 0.5$ (12)

**C.** $\neg$N1 (P4+13)

The Reversibility Objection against I1 will be similar; we just replace P1 with $\forall_e P\ \mu(SH|\text{I1} \wedge P \wedge E) \approx 1$, replace P3 with the Indifference Principle, and substitute "I1" for "N1" throughout.

Finally, the Meta-Reversibility Objection against I2 will be:

**The Meta-Reversibility Objection (against I2):**

**P1.** $\forall_e P\ \mu(PH|\text{I2}^- \wedge P \wedge E) \ll \mu(PH^*|\text{I2}^- \wedge P \wedge E)$

**P2.** Bayesianism (Probabilism and Conditionalization)

**P3.** Indifference Principle

**P4.** Adequacy

**5.** $\forall_e P\ ic(PH|\text{I2}^- \wedge P \wedge E) \ll ic(PH^*|\text{I2}^- \wedge P \wedge E)$ (P1+P3)

**6.** $\forall_e P\ cr_E(PH|\text{I2}^- \wedge P) \ll cr_E(PH^*|\text{I2}^- \wedge P)$ (P2(cond)+5)

**7.** $\forall_e P\ cr_E(PH \wedge \text{I2}^- \wedge P) \ll cr_E(PH^* \wedge \text{I2}^- \wedge P)$ (P2(prob)+6)

**8.** $\forall_e P\ cr_E(\text{I2} \wedge P) \ll cr_E(\text{I2*} \wedge P)$ (7)

**9.** $\sum_{P_e} cr_E(\text{I2} \wedge P) \ll \sum_{P_e} cr_E(\text{I2*} \wedge P)$ (8)

**10.** $\sum_P cr_E(\text{I2} \wedge P) \ll \sum_P cr_E(\text{I2*} \wedge P)$ (P2(prob)+9)

**11.** $cr_E(\text{I2}) \ll cr_E(\text{I2*})$ (P2(prob)+10)

**12.** $cr_E(\text{I2}) \approx 0$ (P2(prob)+11)

**13.** $cr_E(\text{I2} \wedge \neg SH) \approx 0$ (P2(prob)+12)

**14.** $cr_E(\text{I2} \wedge \neg SH) \not> 0.5$ (13)

**C.** $\neg$I2 (P4+14)

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