Logic, Essence, and Modality

A Critical Review of Bob Hale's *Necessary Beings*

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Introduction

Bob Hale's distinguished record of research places him among the most important and influential contemporary analytic metaphysicians. In his deep, wide ranging, yet highly readable book *Necessary Beings*, Hale draws upon, but substantially integrates and extends, a good deal his past research to produce a sustained and richly textured essay on — as promised in the subtitle — ontology, modality, and the relations between them. I've set myself two tasks in this review. First, to provide a reasonably thorough (if not exactly comprehensive) overview of the structure and content of Hale's book and, second, to a limited extent, to engage Hale's book philosophically. I approach these tasks more or less sequentially: Parts I and 2 of the review are primarily expository; in Part 3 I adopt a somewhat more critical stance and raise several issues concerning one of the central elements of Hale's account, his essentialist theory of modality.

1   Hale's Basic Ontology and Its Logic

Hale's Ontology

Unsurprisingly in a book on necessary *beings*, Hale begins his study by addressing "the central question of ontology" (p. 9), namely: What kinds of things there?[1] His answer (developed and defended in §§1.2-1.6) is "broadly Fregean", both methodologically and

[1]Free-standing chapter, section, and page number references should all be understood as indicating the relevant parts of Hale's book.

substantively — methodologically, like Frege, Hale takes the kinds of things there are to be a direct reflection of a certain logical categorization of natural language expressions; substantively, both his chosen lexical categories and the corresponding ontological kinds are distinctively Fregean. More specifically, the fundamental categories of expressions are *sentences* and *singular terms*, i.e., names and other "devices of singular reference" (p. 11). *Predicates* fall into a hierarchy of derivative categories. Specifically, first-level predicates result from removing one or more singular terms from a given sentence and replacing them with variables; second-level predicates (including, notably, first-order quantifiers) from the removal of one or more first-level predicates (and perhaps one or more singular terms); and so on. A predicate is *n-place*, for natural numbers $n \geq 1$, if it results from the removal of exactly *n* referring expressions. (I'll often use standard λ-notation $[\lambda v_1...v_n \varphi]$ to indicate an *n*-place predicate and its relevant variables.[2]) *Object*s, then, are the kind of thing that can (only) be referred to by a singular term. *Properties* are the kind of things that can be referred to by meaningful (though perhaps quite complex and gerrymandered) predicates, and they fall into a corresponding hierarchy of types. A *first-order* property is the kind of thing that could be referred to by a first-level predicate; a *second-order* property the kind of thing that can be referred to by a second-level predicate; and so on. A property is *n*-place if it can be referred to by an *n*-place predicate and a *relation* is simply an *n*-place property, for $n \geq 2$.[3] So properties for Hale, while perhaps not identical to the meanings of possible predicates, are at least strongly correlated with them and, hence, are to be individuated intensionally (pp. 189, 193) — predicates with different meanings can obviously have the same extensions and so, likewise, predicates referring to different properties. (*Functions* are often singled out for special attention in Hale's account but, metaphysically, can be thought of as many-one relations — though as noted below, some functions are not the values of any predicates and their existence must be inferred by other means.) Hale refers to his conception of properties as an *abundant* conception, as it "takes as sufficient for the existence of a property what one might reasonably see as the bare minimum required to distinguish properties from entities of other categories."[4] Importantly, though, properties for Hale are "by their very nature finitely specifiable" (p. 194), that is, they must be expressed by a *finite* (though perhaps quite complex and gerrymandered)

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[2]Hale does not avail himself of λ-predicates or the like but, as his underlying logic includes comprehension principles (see Ch. 8) their addition would constitute a conservative extension to his logical framework.

[3]Hale himself is not quite this explicit; I'm generalizing from the fact that he refers to (1-place) properties as "non-relational properties" and relations as "relational properties" on p. 12.

[4]As Hale emphasizes, his notion of abundance differs starkly from that of Lewis (1986, pp. 50ff), for whom an abundant conception is one on which every set is (or is the extension of) a property. Neither, however, is Hale's conception *sparse*. For Lewis, sparse properties "carve nature at its joints" and Hale's criterion for property existence can obviously deliver many more properties than that.

predicate in some possible language.

Somewhat curiously, despite the fact that *propositions* feature prominently in Hale's overall theory (as I will discuss in much more detail below) and that there seems little question that he takes them to exist as robustly as properties and relations, he does not include them in his Ch. 1 inventory. Moreover, it seems there is a natural place for propositions in his Fregean ontology that is well-suited to how he usually speaks of them and the uses to which he puts them. Specifically, were Hale simply to define an *n*-predicate generally to be the result of removing *n* referring expressions from a sentence, for *any* natural number $n \geq 0$, and an *n*-place property as the kind of thing that can be referred to by an *n*-place predicate $[\lambda v_1...v_n \varphi]$, then propositions would fall out straightaway as the referents of 0-place predicates $[\lambda \varphi]$.[5] Nevertheless, in a footnote, Hale suggests that, rather than being 0-place properties (hence higher-order entities), propositions are a kind of object (p. 151, fn 14), although it seems to me that nothing in his theory obviously hinges on this point at all and, indeed, that his theory would benefit from classifying them as 0-place properties. Be that as it may, as will become rather more apparent below, somewhat more attention to the nature and theoretical properties of propositions appears to be in order if they are to bear the load Hale puts upon them at several points in his account.

Two other elements of Hale's account of his basic ontology are worth noting. First, the use of 'only' in the definition of the *object* category provides Hale with a simple solution to Frege's problem of 'the concept *horse*' (§§1.7-1.11). Frege held that objects alone can be the referents of singular terms, a view that led him famously to claim that the definite description 'the concept *horse*' does not in fact refer to a concept but to an object. Without the qualification 'only', Hale would face a similar problem with regard to gerunds like 'being a horse' and other forms of nominalization: while '*x* is a horse' would refer to a property, its nominalized counterpart would by definition have to refer to an object. However, with the qualification 'only', Frege's difficulty is neatly sidestepped, as the property ostensibly referred to by the gerund — viz., of course, the property *being a horse* — is *also* referred to by the first-level predicate '*x* is a horse' and, hence, is not an object.

Second, the modal character of the criterion for the existence of properties and relations is particularly important to Hale's account. Obviously, there are objects and relations that are not, as a matter of contingent fact, the referents of any actual expressions. Thus, to be an object is to be the kind of thing for which there is or *could be* a singular term in some possible language (and for which there could be no other kind of referring expression); to be a relation is to be the kind of thing for which there is or *could be* a meaningful predi-

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[5]See Zalta 1983, Ch. III for a very detailed account of a higher-order modal language and semantics with such predicates.

cate (p. 20, fn 29). Additionally, however, by appealing to the thesis defended in §5.4 that modal truths are necessary, Hale parlays his criterion for the *de facto* existence of properties and relations into an ingenious sort of ontological argument for the *necessary* existence of purely general properties and relations (§7.2). That argument is then extended (§7.3) to an argument for the necessary existence of "basic" functions, that is, functions that exist (only) in virtue of being implicitly definable via abstraction principles — the *number-of* function on properties given by Hume's Principle being the obvious paradigm. And, having secured their necessary existence it is argued in turn (§7.4) that the *values* of such functions must themselves be necessary as well — notably, the cardinal numbers exist as the values of the *number-of* function and, hence, as necessarily existing *objects*. Hale thus secures, not simply the existence, but the *necessary* existence of prominent members of each of his central ontological categories — thus delivering on the promise implicit in the title of his book.

Hale's Logical Framework

The absence of propositions from Hale's Ch. 1 inventory seems to reflect a corresponding divide in the logical foundations of his theory. In Ch. 8 Hale develops a non-modal higher-order logic with full impredicative comprehension (with, as we'll see, a novel semantics) and, in Ch. 10, a propositional modal logic with propositional quantifiers. In practice, however, Hale makes use of an amalgam of the two — he freely expresses his theses and arguments using various combinations of modal operators and propositional, first-order, and higher-order quantifiers.

Higher-order Logic. The first several sections of Ch. 8 offer a series of responses to arguments against second- (and higher-) order logic chiefly from Quine (1969; 1986) and Parsons (1983, pp. 165-67). (For simplicity, like Hale, I'll restrict my focus primarily to second-order logic here, as both the formal machinery of second-order logic and the concomitant philosophical issues generalize straightaway to the broader higher-order logic he avails himself of in practice.) All of these arguments, Hale notes, presuppose a "standard" semantics for a second-order language L in which (a) second-order quantifiers range over sets and (b) the range of the n-place second-order quantifiers $\forall F/\exists F$ in a model $M = \langle D, V \rangle$ of L is the full power set of the nth Cartesian product D^n of the domain D. In particular, the domain of a second-order monadic quantifier is the full power set $\wp(D)$ of the set D of (first-order) objects in I.[6] Hale, of course, takes second-order quantifiers to range over

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[6](b) seems to be the main source of Quine's (1986, p. 66) characterization of higher-order logic as "set theory in sheep's clothing", as it renders, in particular, the second-order comprehension principle $\exists F \forall x(Fx \leftrightarrow \varphi(x))$ valid (where F is not free in φ). Comprehension can thus be made to appear as if it is simply the

properties-in-intension but considers (a) in itself largely unproblematic as a fact about the *formal* semantics of second-order languages — a formal semantics is a model of language and meaning and needn't reflect every aspect of one's metaphysics, and for formal purposes properties-in-intension can be modeled as sets (see, e.g., §8.5, p. 193). (b) however is "fundamentally at odds" with Hale's conception of properties. For, on the reasonable assumption that there are infinitely many objects, it follows from (b) that there are uncountably many properties and, hence, given Hale's criterion of property existence, we would need to assume that there are "uncountably many mutually untranslatable languages". Moreover, intuitively, given a denumerable domain D, $\wp(D)$ (i.e., $\wp(D^1)$ in particular) contains subsets $\{a_1, a_2, ...\}$ that are utterly *arbitrary* and, hence, that could only be expressed by an infinitary monadic predicate of the form $[\lambda x \, x = a_1 \vee x = a_2 \vee ...]$.

But (b), of course, is not inevitable. As Hale notes, so-called "general" model theory, or *Henkin semantics*, for second-order languages provides an important alternative. Specifically, in Henkin semantics, given a model $\langle D, V \rangle$ for a language L, it is required only that the domain of the n-place second-order quantifiers contain the subsets of D^n that are *definable* in L (see, e.g., Enderton 2001, §4.4 and Manzano 1996, Ch. 3).[7] The definability condition here, of course, gets us much closer to Hale's conception of properties. But it still misses the mark, on two counts. First, the "full" models of standard second-order semantics are still legitimate Henkin models — obviously, for a standard second-order model $\langle D, V \rangle$ of a language L, the set of *all* subsets of D^n contains all the *definable* subsets of D^n. Hence, there are Henkin models that still contain utterly arbitrary subsets of D^n. A semantics that is faithful to Hale's conception of properties would ideally rule these out of court and require that the second-order quantifiers range over all and *only* definable properties. Henkin semantics is thus in this regard too permissive vis-à-vis Hale's conception. Second, though, Henkin semantics is also, in another regard, too conservative. Recall that a sufficient condition for a property to exist for Hale is the mere *possible* existence of a predicate that expresses it. Hence, the restriction to properties that are definable *in L alone* is too restrictive; it doesn't reflect the fact that it is always possible to identify further properties by increasing our expressive resources. Consequently, for a given model $\langle D, V \rangle$, Hale takes the definable subsets of D^n over which the n-place second-order quantifiers

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result of an innocuous application of a valid inference rule, existential introduction (the sheep's clothing!), to a first-order logical truth, $\forall x(\varphi(x) \leftrightarrow \varphi(x))$, instead of a substantive set theoretic hypothesis with significant ontological commitments. See ibid., p. 68.

[7]In fact, Hale builds upon a slightly different notion of Henkin semantics found in Shapiro 1991, §3.3, which stipulates that the domain D_n of the n-place second-order quantifiers for a model $\langle D, V \rangle$ of L can be *any* nonempty subset of D^n. Models in which D_n contains all the L-definable subsets, for all n, are said to be *faithful* (ibid., §4.3).

range to be those "definable in some specified language [which] need not be the formal language with the interpretation of which we are concerned." For example, one might take them to be those that are definable in some extension of L, or even those expressible in the metalanguage itself.[8] This definition of definability, then, allows one to identify ever more definable properties over a given "base" model $\langle D, V \rangle$ of a given language L by increasing expressive resources beyond those of L. Hale's "modest semantics" is therefore a more faithful formal reflection of his philosophical conception of properties.

One important formal note. Hale claims in his book that his second-order semantics is, like Henkin semantics, still expressively first-order and, hence, can be given a complete proof theory. However, in the course of more recent research, Hale has discovered that this claim was in error; he details the issues in the forthcoming article listed in the references for this review.[9]

Propositional Modal Logic. In a number of places, Hale makes free use of a standard Kripke-style possible world semantics for propositional modal logic, particularly in discussions pertaining to his counterfactual definition of necessity (see below). In fact, however, Hale is not a realist about possible worlds — total ways things could have been — but is comfortable speaking in terms of standard possible world semantics because he believes that all the formal advantages of possible world semantics remain available in his preferred semantics (Ch. 10) wherein full-blown worlds are replaced by *possibilities*, i.e., partial, finitely specifiable ways things could have been. (Humberstone (1981) seems to have been the first to pursue this idea.[10])

That Hale's finitary notion of a possibility is redolent of his conception of properties is not an accident. Following Stalnaker (2012, p. 8), Hale in fact *identifies* possibilities with properties of a certain sort and so, ideally, a theory of possibilities would be an application

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[8]Though in this latter case, as definability is a semantic notion, the metalanguage would presumably have to be powerful enough to express its own model theory. ZF, notably, has this capability; see, e.g., Drake 1974, Ch. 3, §5.

[9]In more detail, as is well known, standard Henkin semantics is compact and has the downward Löwenheim-Skolem property and, hence, by Lindström's famous theorem (1969), from a model theoretic perspective it is expressively first-order. Hence, it can be outfitted with a complete proof theory. However, upon further investigation, Hale discovered that his semantics — at least where the language L^+ specified for definability is the metalanguage — shares a critical feature with standard second-order semantics, namely, that, the domains of the second-order n-place quantifiers for a given model $M = \langle D, V \rangle$ of a language L are *fixed* relative to D — they consist of *exactly* the subsets of D^n that are definable in L^+. As detailed in the indicated article (to which Professor Hale himself alerted me), this feature apparently enables one to recreate versions of the usual proofs of the categoricity of second-order arithmetic and analysis with the standard second-order semantics (see Shapiro 1991, §4.2). If so, Hale's semantics, like the standard semantics, will not have the first-order features of Henkin semantics and will, in particular, not have a complete proof theory.

[10]Though he acknowledges "considerable" similarities to ideas of Kamp (1975) and Fine (1975) in their work on vagueness (ibid, p. 316).

of a (presumably modal) extension of his higher order logic (p. 231). This is not really how things play out in Hale's semantics, however, or even in his broader theory. The identification of "ways" with properties is on the face of it natural enough: a *way* for a thing to be, after all — blue-eyed, metallic, divisible by 2 — is usually expressed by means of an adjectival phrase (or corresponding nominalization) that is naturally understood to signify a property of the thing in question. However, a way for "things" or "the world" to be is not typically expressed adjectivally, but by means of a complete sentence (or corresponding nominalization): we speak simply of the possibility of Aristotle's being a philosopher, or the possibility that WWII never occurred. And so it is with every example of a possibility that Hale raises (notably, in §9.4, as discussed below). And, indeed, so it is in his modal semantics where, it turns out, possibilities are represented by, in effect, finite consistent sets of formulas.[11] Thus, as Hale regularly speaks of the propositions expressed by sentences (e.g., pp. 120-1, 138-9, 147, 246), it would seem that, both in practice and in theory, *propositions*, rather than properties, play the role of possibilities in Hale's account — specifically, a possibility is simply a proposition that could be true. Granted, Hale could conceivably push the issue and claim that possibilities are "propositional properties" of the form *being such that* φ, for sentences φ, that are true (or not) of situations or states of affairs or the like (as in, e.g., Zalta 1993, §§4-5). But Hale does not include any such entities to serve as the bearers of such properties in his ontology and, hence, this gambit seems unavailable.

Regardless of how this matter might ultimately play out in Hale's theory, it seems clear that the propositional modal semantics of Ch. 10 is intended as a sketch of how the Henkin semantics of Ch. 8 would have to be extended to accommodate the introduction of modal operators. And although Hale does not explain this bifurcated presentation of his logical foundations, I surmise that the reason for it is simply that the focus of each case framework is so different and each has its own technical and expository challenges. In his higher-order semantics, the focus is on properties and the challenge is the introduction of a new notion of definability. In his possibility semantics, the focus is on a novel notion of a possibility that introduces a number of tricky complications, even for the basic normal modal system K it is defined for in Ch. 10. The central challenge is to deal with the inherent *incom-*

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[11] More exactly, a model of a modal language L is a 4-tuple $\langle W, R, S, v \rangle$ where W is a set of indices, R is an accessibility relation, S a similarity relation, and v a partial function on $\mathcal{F} \times W$ into $\{0, 1\}$, where $\mathcal{F}$ is the set of formulas of L. Fixing w, then, v_w ($= v \upharpoonright \mathcal{F} \times \{w\}$) is just a partial function on $\mathcal{F}$ and, hence, can be identified with the set $\{\varphi : v_w(\varphi) = 1\} \cup \{\neg\psi : v_w(\psi) = 0\}$. Of course, not every such set — e.g., $\{p \rightarrow q, p, \neg q\}$ — will represent a genuine possibility. This issue is resolved by means of certain "coherence" conditions, as noted in the following paragraph. I should note, too, that the finitude of possibilities is not strictly imposed in Hale's formal semantics, but it is clear that this is just an artifact of his theory and that finitude is at the heart of his philosophical conception; thus p. 231: "Possibilities are just ways the world might be or have been, each of which is specifiable by a finite description...".

pleteness of possibilities (§§10.4-5) given their partial nature — a disjunction, for instance, might be true at a possibility even though neither disjunct is assigned a truth value. This challenge is met (§10.6, pp. 237-8), quite cleverly in my view, by defining a set of formulas to represent a possibility just in case it is a member of a so-called *coherent* set P that meets certain consistency, closure, and completeness conditions. Closure in particular ensures that any incompleteness in any given possibility in such a set P is always "grounded" in some other more "determinate" possibility in P. Hale spares us the details of a semantic completeness theorem but assures us that such theorems are provable, not only for K but, in suitably adjusted semantic frameworks, for S4 and S5. However, as noted above, Hale's higher-order semantics is apparently not first-order, so a completeness theorem will obviously not be possible for the full amalgam of higher-order logic and propositional modal logic in which he does the bulk of his philosophical work.

2   Hale's Theory of Modality

Modality: Indispensable, Fundamental, and Irreducible

Chapters 2 and 3 of Hale's book are devoted to showing that modality is indispensable, fundamental, and irreducible. By the indispensability of modality Hale means that there are certain modal truths that we cannot but acknowledge. The thesis is argued first for the special case of logical necessity in Chapter 2 and extended to non-logical necessities in Chapter 4. Specifically, drawing upon an argument by McFetridge (1990), Hale argues that the intuitive notion of validity presupposes the legitimacy of certain basic rules of inference that are necessarily truth preserving (§2.2). McFetridge's own argument proceeds *a priori* and does not single out any particular rules as necessary. Hale argues more boldly that we can do so simply by thinking carefully about what it means to understand the notion of a derivation in a given logical system — regardless of whether or not it is the system one accepts as one's own (§2.6). For example, regardless of one's views about the objectively correct inferential principles governing negation, it is uncontroversial true that, *in classical logic*, 'p' follows from '$\neg\neg p$' (in a language L containing 'p' as a sentence letter). But to *know* that is true requires one to know (1) that it is a rule of classical logic that, for any A, if A is a statement of L, then A follows from $\ulcorner\neg\neg A\urcorner$; (2) that, by universal instantiation from (1), if 'p' is a statement of language L, then 'p' follows from '$\neg\neg p$' in classical logic; (3) that 'p' is indeed a statement of L; and, hence, (4) that, by *modus ponens*, 'p' follows from '$\neg\neg p$' in classical logic. Hence, at a bare minimum, one must acknowledge the validity — i.e., the *necessarily* truth preserving character — of universal instantiation and *modus ponens* in order simply to understand the notion of a derivation in a logical

system.

Having established that at least some logical necessities are indispensable, Hale moves on in Chapter 3 to argue that necessity and related alethic modalities are "fundamental and irreducible", that is, "they are neither reducible to nor derivable from" non-modal facts (p. 63). Hale focuses upon *combinatorialism* (§§3.1-3.3), of which he counts Lewis's extreme realism to be a species, arguing in particular that combinatorialist accounts of what possibility *is* are committed to a controversial theory of what possibilities *there are*, viz., that *any* mathematically possible combination of fundamental entities (perhaps subject to certain restrictions[12]) is *genuinely* possible. As that rules his brand of essentialism (described below) *a priori* out of court — along with most any account that takes *de re* modality seriously — Hale charges combinatorialism with question-begging vis-á-vis what many consider to be the most important questions of modal metaphysics. Combinatorialism "cannot, therefore, command general assent...[and] will commend itself at best only to those who are already persuaded that the only absolute necessities are logical" (p. 80).

The Definition of Necessity

Having made room for non-logical necessities, Hale takes another cue from McFetridge and offers a general *definition* of necessity in Chapter 4. McFetridge understands the validity of a rule of inference to consist in its preserving truth relative to any set of presuppositions (1990, p. 153). Hale takes this to be a significant insight into the nature of necessity generally. Specifically, a rule 'p, so q' is valid just in case its corresponding conditional '$p \rightarrow q$' is necessary. So, following McFetridge, for '$p \rightarrow q$' to be necessary is for it to hold relative to any set of presuppositions, which Hale takes to be best spelled out counterfactually: it is necessary that $p \rightarrow q$ just in case it would have been true "come what may", i.e., no matter what had been the case; more formally, $\Box(p \rightarrow q)$ if and only if $\forall s(s \:\Box\!\!\rightarrow (p \rightarrow q))$, where '$\Box\!\!\rightarrow$' is of course the counterfactual conditional operator.

Truth come what may, Hale notes, is the mark of unrestricted, or *absolute*, necessity (p. 47).[13] The necessity of the conditionals corresponding to valid rules of inference is thus a type of absolute necessity. Obviously, however, there is no reason to restrict the idea here to such conditionals, or to conditionals generally. Consequently, Hale proposes to extend McFetridge's proposal to all propositions and, indeed, to *define* absolute necessity in general as truth come what may:[14]

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[12]Lewis (1986), in particular, famously adds the proviso "size and shape permitting" (p. 89).

[13]Hale borrows "come what may" from Quine (1951:38), though Quine himself used it only in his characterization of (the "dogmatic" conception of) analytic statements, not of unrestricted necessities generally.

[14]In §4.1 (p. 98) Hale only says that our *beliefs* in the necessity of certain propositions can be explained in

Df□ $\Box p =_{df} \forall q(q \,\Box\!\!\rightarrow p).$ [15]

Accordingly (though Hale doesn't actually make this explicit), understanding a proposition to be *possible* as usual just in case its falsity is not necessary, $\Diamond p =_{df} \neg\Box\neg p$, the account yields a corresponding definition of possibility:

Df◇ $\Diamond p =_{df} \exists q\neg(q \,\Box\!\!\rightarrow \neg p).$

That is, p is possible just in case some proposition fails to counterfactually imply that it is false, i.e., roughly, in terms of the standard Lewisian account of counterfactuals, just in case p is true in some closest q-world.[16]

It is important to note that Hale takes modal truths themselves to be *necessary* and, hence, that he accepts the characteristic principles of the modal systems S4 and S5, viz.

4 $\Box p \rightarrow \Box\Box p$

5 $\Diamond p \rightarrow \Box\Diamond p,$

propositions that he appeals to at several points in the development of his theory.[17] Unpacking **4** and **5** in terms of **Df□** and **Df◇**, then, Hale's commitment to the necessity of the modalities amounts to his commitment to

4C $\forall q(q \,\Box\!\!\rightarrow p) \rightarrow \forall q(q \,\Box\!\!\rightarrow \forall r(r \,\Box\!\!\rightarrow p))$

5C $\neg\forall r(r \,\Box\!\!\rightarrow \neg p) \rightarrow \forall q(q \,\Box\!\!\rightarrow \neg\forall r(r \,\Box\!\!\rightarrow \neg p)).$[18]

And, in fact, Hale constructs a lively *argument* for the truth of **4C** and **5C** in §5.4; it is worth sketching the argument, as it bears on issues raised in Part 3 below. Briefly, Hale first argues for a key lemma, viz., that his counterfactual definition **Df□** of necessity yields the standard possible world truth condition for $\Box p$, viz.,

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counterfactual terms but he explicitly refers to **Df□** as a definition in §5.4 (p. 130).

[15]Williamson (2010) investigates the extent to which standard modal logic can be embedded in counterfactual logic.

[16]This truth condition presupposes the "Limit Assumption", roughly, the assumption that for any proposition q and any world w, there is always a set of q-worlds closest (i.e., most similar) to w; that there is no infinite sequence of q-worlds that are increasingly similar to w (Lewis 1973, §1.4). Hale seems to accept the Limit Assumption in all his applications of possible world semantics for counterfactuals.

[17]For Hale's dependence on necessity of modality, see pp. 136, 140, 145-46, 167 and §§6.2 and 9.2.

[18]Instead of **5C** proper, Hale uses the counterfactual version of the **5** equivalent, $\Diamond\Box p \rightarrow \Box p$, i.e., if p could have been necessary, then it is necessary. I assume the proof of **5C** in this form is somehow easier or more intuitive than that of its equivalent counterpart. I've stuck with **5C** simply because it corresponds exactly to **5**.

(1) $\Box p$ is true at w iff, for every w-accessible world w', p is true at w'.[19]

Given (1), Hale then argues that **Df□** yields an unrestricted accessibility relation, hence one validating **4C** and **5C** and, consequently, axioms **4** and **5**:

> When we define $\Box p$ as a universally quantified counterfactual, $\forall q(q \;\Box\!\!\rightarrow p)$, the quantifier $\forall q$ is to be understood as absolutely unrestricted — as ranging over all propositions whatever. This gives it a kind of *modal* strength additional to that carried by a singular counterfactual. Since no proposition which is not actually true is excluded from the range of its quantifier, the claim is effectively equivalent to the claim that no matter how things might have been, it would still have been true that p — expressed in terms of worlds, that it is true that p at every possible world without restriction.

However, the quantifiers in the embedded counterfactuals in **5C**, at least, give rise to complications. Presumably, the "additional" modal strength provided by the unrestricted range of the propositional quantifier is exactly the strength needed to yield **5C**. But to say that the range of the quantifiers is "absolutely unrestricted" is to say nothing more than that their intended range is all of the propositions there are — that is, all of the propositions that *in fact* exist. It does not obviously follow that the propositions there *are* are all the propositions there *would be* under different counterfactual circumstances. Indeed, Hale himself appears to be committed to ontological principles that entail that some propositions that are, in fact, possible exist only contingently and, as a consequence, would have failed to be possible in other circumstances, thereby threatening to undermine the argument above and, with it, the validity of, at least, the modal system S5. This matter will be taken up in some detail below.

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[19]Professor Hale has acknowledged (in generous correspondence with the author) that there appears to be a flaw in the argument given in the book for (1). Briefly, the problem lies in showing that the right-hand side of (1), viz.,

(1) For every w-accessible world w', p is true at w'.

is equivalent to

(2) For every proposition q, and every nearest w-accessible world w' at which q is true, p is true at w'.

That (1) entails (2) is trivial. In the other direction, given (2) and an arbitrary w-accessible world w', Hale argues that there is bound to be a proposition q such that w' is the nearest q-world accessible from w and, hence, by assumption, that p is true at w'. The existence of such a proposition q is of course trivial if, following standard possible world semantics for languages with propositional quantifiers, propositions are identified with arbitrary sets of possible worlds. (Notably, let $q = \{w'\}$.) But, while we've noted some unclarity regarding Hale's conception of propositions, he in no wise conceives of them as sets of worlds and he nowhere specifies existence conditions that would guarantee anything like the variety of propositions needed to ensure the existence of the requisite proposition q in the proof. I should note, however, that, in the correspondence noted, Hale sketched a promising alternative proof of the desired lemma in terms of the possibility semantics laid out in Ch. 10.

Hale's Essentialist Theory of Necessity and Possibility

The core of Hale's overall vision is his essentialist theory of necessity, laid out chiefly in chapters 5 (esp. §5.5) and 6. Just to be clear before continuing: Hale's essentialist theory is not an *alternative* to **Df□**; assertions of the form $\square p$ are still understood to *mean* that p would have been true no matter how things might have been. Rather, the essentialist theory purports to *explain* them, to identify what it is in virtue of which any given necessary truth p is necessary.

Hale lays the initial groundwork for his theory in Ch. 5, whose focus in the source of *logical* necessities. In §§5.2-5.3, drawing on arguments from Dummett and Quine, Hale develops a sustained, detailed, and sophisticated critique of conventionalism. In §5.5 Hale takes the first step toward his central thesis, arguing that the explanation and source of logical necessity is to be found in the *natures* of appropriate logical objects, e.g., the various truth functions on propositions that ground the necessity of propositional tautologies. The thesis is presented in full flower in Ch. 6, where Hale generalizes from logical necessities to all absolute necessities. Specifically, Hale holds that all things have *essences*, or (synonymously) *natures* (p. 151), where "[a] thing's nature or essence is what is given by its definition." The definition of a thing, in turn, is a true sentence (possibly in an ideal but still finitary logical language) that "stat[es] what it is to be that thing" (pp. 152-3) and hence what it is that "distinguishes it from everything else" (p. 151).[20] Essences themselves are *properties* (p. 153, fn 17; p. 159). Although he does not explicitly say so, given Hale's Fregean methodology for extracting predicates from sentences by the removal of singular terms, I assume that, if the sentence $\ulcorner \forall x(x = a \leftrightarrow \varphi) \urcorner$ is a correct definition of object a, then a's essence is the property $[\lambda y \, \forall x(x = y \leftrightarrow \varphi)]$ or, more simply, the necessarily equivalent property $[\lambda x \varphi]$.[21] Moreover, importantly, Hale intends that "a statement of an individual's nature will be genuinely informative or explanatory." Hence, in contrast to, notably, Adams (1979) and Plantinga (1979), Hale does *not* consider identity properties, or *thisnesses*, i.e., properties of the form $[\lambda x \, x = a]$, to be essences.

Many modal metaphysicians are essentialists of one stripe or another, of course, but in Hale's comprehensive vision, essences play a critical foundational role: they both serve as the source and explanation of necessity (hence possibility) and establish the link between modality and ontology — they provide "the relations between them" alluded to in the subtitle of the book. It is useful once again to contrast Hale's view with Plantinga's. For

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[20]Hale says that definitions are "true propositions" in this passage but it is clear he means "true sentences". E.g., in the same paragraph (p. 153) he notes that "the same sentence may serve both as a definition of [a] thing and of a word for the thing defined."

[21]Equivalent because a's nature can only be true of a.

Plantinga, a property F is an essence just in case there could have been something such that (i) it couldn't have existed without exemplifying F and (ii) nothing other than it could have exemplified F; that is, more succinctly:

PE   Property F is an *essence* $=_{df} \Diamond \exists x \Box \forall y (Fy \leftrightarrow y = x)$

Hence, for Plantinga, the alethic modalities are fundamental and to be an essence is simply to have the right higher-order alethic modal property; a property's being an essence, as we might put it, is *explained* in terms of the alethic modalities.

In stark contrast, for Hale, the order of explanation goes in the other direction. More specifically, on Hale's account, for *every* proposition p, p is necessary if and only if p is true *in virtue of* the essences of some finite number of entities $X_1, ..., X_n$. Adapting notation introduced by Fine (1995), Hale symbolizes "p is true in virtue of the essences of $X_1, ..., X_n$" as "$\Box_{X_1,...,X_n} p$" and thus formalizes the essentialist theory of necessity as (p. 150):

ETN $\Box p \leftrightarrow \exists X_1 ... X_n \Box_{X_1...X_n} p.$

Whenever we can identify the entities $X_1, ..., X_n$ in virtue of whose essences p is true, Hale counts it as an *explanation* of p's necessity; p is necessary *because* $\Box_{X_1,...,X_n} p$.[22] Of course, it may well be that we are inherently incapable of *identifying* the entities involved in any given explanation, but whether or not we can do so in any given case, according to Hale, such explanations exist for all necessities.

As he takes $\Diamond p$ to be definable as usual as $\neg \Box \neg p$, Hale derives a corresponding essentialist account of possibility (§9.4):

ETP $\Diamond p \leftrightarrow \neg \exists X_1 ... X_n \Box_{X_1...X_n} \neg p.$

A proposition p is possible, that is, just in case its negation fails to be true in virtue of the essences of any objects; it is possible *because* there are no essences in virtue of which it is false, no essences that *rule it out* (p. 219). Thus, since there is, for Hale, an essentialist explanation for every necessity and every possibility, modality in general is explained by essences.

Hale illustrates his theory with a number of examples. The logical necessity *If $A \wedge B$ is true, then A is true* is explained by the nature of conjunction; "conjunction just *is* that binary function of propositions the value of which is a true proposition iff both its arguments are true propositions" (p. 132). A bit more formally expressed: $\Box((A \wedge B) \rightarrow A)$ *because* $\Box_\wedge((A \wedge B) \rightarrow A)$. Likewise, it is 0's nature to be the number of the concept *not*

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[22]Hale also claims that the more general biconditionals of the form given in **ETN** can also be expressed as $\Box p$ *because* $\exists X_1 ... X_n \Box_{X_1...X_n} p$ (pp. 219-20), although explanations of these forms are of course not nearly as informative as those in which particular entities $X_1, ..., X_n$ have been identified.

equal to itself and it is 1's to be 0's successor; and it follows from the nature of the *less-than* relation $<$ that every number is less than its successor.[23] Hence, $\Box(0 < 1)$ *because* $\Box_{0,1,<}0 < 1$, i.e, the necessity of $0 < 1$ is explained by the natures of 0, 1, and the *less-than* relation. Finally, Aristotle's essence is to be "a human being which is distinguished from every other human being by having such-and-such an origin" (p. 151). The metaphysical necessity *Aristotle is a man* is explained by Aristotle's essence (presumably, in particular, because his origin involves his having a Y chromosome). Thus, $\Box$Man(Aristotle) *because* $\Box_{\text{Aristotle}}$Man(Aristotle). Likewise, *Aristotle is a philosopher* is possible, not simply because he is, in fact, a philosopher, but *because* no essence — his own, most relevantly — rules it out, i.e., we have, for any number n of things, $\neg\exists X_1...X_n\Box_{X_1...X_n}\neg$Philosopher(Aristotle). By contrast, *Aristotle is a frog is* ruled out by his essence and, hence, $\neg\Diamond$Frog(Aristotle) *because* $\Box_{\text{Aristotle}}\neg$Frog(Aristotle) and hence because $\exists X\Box_X\neg$Frog(Aristotle).

Hale is careful to avoid any misunderstandings of his thesis. Notably, the thesis is not *reductive*; Hale is not purporting to *reduce* necessity to some non-modal notion. Essences are themselves intrinsically modal and, in contrast once again to Plantinga, who *defines* essences to be properties that are essential to their bearers, for Hale, their modal character is itself fundamental and irreducible. Moreover, since, as we've seen, Hale believes he has shown that necessities are themselves necessary, the explanans $\Box_{X_1,...,X_n}p$ in an essentialist explanation is *itself* necessary. Importantly, however, it is not the *necessity* of the explanans that is doing the explaining — that would of course render the explanation circular. Rather the explanation is supplied solely by the essences of the relevant entities $X_1,...,X_n$. Essences thus provide what Hale calls a "*non-transmissive*" explanation of necessity (§5.5), that is, "an explanation of the form '$\Box p$ because q' in which the *explanans*, q, is indeed necessary...but in which what explains the necessity of the explanandum is not q's necessity, but its *truth simpliciter*" (p. 131). The non-transmissive character of essentialist explanations, I take it, is what gives substance to Hale's claim that, in addition to their explanatory role, essences also serve as the *source* of modality — modality, as it were, "emerges" from *de facto* (albeit necessary) truths about essences.

It follows from Hale's theory of modality that *knowledge* of modal truths is fundamentally knowledge of essences. How such knowledge is possible is addressed in the final chapter of Hale's book. (As I am focused on logical and metaphysical issues in this review, I will not offer detailed exposition or commentary on this element of Hale's theory.)

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[23]Hale is a neo-logicist and, hence, is able to define the *less-than* relation in well-known ways in second-order logic. That $sn = m$ implies $n < m$ follows easily in that framework plus Hume's Principle. See, in particular, §6.3 and the Appendix to Ch. 7.

3 A Closer Look at Hale's Essentialism

Hale's theory is remarkable in both scope and power. As one might expect of any theory that offers such a detailed, comprehensive, and provocative philosophical vision, concerns can be raised about certain elements. By my lights, the bulk of these concerns arise with regard to Hale's essentialist theory of modality. In the remainder of this review, I will discuss what strike me as the most interesting and significant.

On the Formal Expression of Hale's Essentialist Theory

Hale makes generous use of the tools of mathematical logic to express and clarify his views and his arguments, and overall he wields them effectively. It is in the main a virtue that he is not overly taken with formality — philosophy is always center stage and, but for a few relatively short technical sections, formal details receive attention only to the extent that they bear on the substantive philosophical matters at hand.

At the same time, I think there are a few points where somewhat more attention to formal detail is warranted, particularly with regard to his essentialist theory. First, Hale's use of (only) second-order variables notwithstanding in the notation $\Box_{X_1,\dots,X_n} p$, the examples above make clear that the entities involved in an essentialist explanation can be first-order, higher-order, or a mix thereof. It seems to me that this can be dealt with in either of two ways. First, Hale could use metavariables $\chi_1, \dots, \chi_n$ ranging over variables of all types instead of object language variables when talking about the general form of essentialist explanations. However, in light of the point I will make in the following paragraph, this option is probably not feasible. A more promising approach would be for Hale to introduce a general type of variable that can take any type of entity as value — perhaps restricted only to contexts of the form $\Box_{X_1,\dots,X_n} p$. Although the prospect of such variables would be pretty starkly out of keeping with Hale's highly typed conception of properties, as far as I can see, there is nothing in principle that would prevent their introduction into his logical framework.[24] For simplicity, I will assume this latter approach for the remainder of this review but, for uniformity with the text, will continue to use the (strictly speaking, higher-order) variables $X_1, \dots, X_n$.

Second, note that **ETN** is not really adequate as a formal statement of Hale's essentialist theory in its full generality. Rather, **ETN** is simply a schema expressing the general form of a biconditional that, according to the theory, holds for each proposition p. But it is

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[24]In fact, Hale has in recent correspondence reported that in as yet unpublished work he argues that such variables are needed to give adequate expression to his broadly Fregean account of the connections between logico-syntactic and ontological categories.

not itself an expression of the theory proper; indeed, the most natural, non-schematic expression of what Hale actually has in mind by his schema seems to require an infinitary language. To see this, note that the number n of entities $X_1, ..., X_n$ and, hence, the requisite number of existentially quantified variables, involved in essentialist explanations can vary arbitrarily from one explanation to another. Thus, the full, non-schematic expression of the essentialist theory of necessity in its full generality is this: for all propositions p, p is necessary just in case there is *some finite number n* of entities in virtue of whose essences p is true, i.e., either one, or two, or three, or ... . And that ellipsis is of course naturally expressed by means of an infinitary language (where ω^+ is $\omega \setminus \{\varnothing\}$):

ETN$_\infty$ $\forall p(\Box p \leftrightarrow \bigvee_{n \in \omega^+} \exists X_1...X_n \Box_{X_1...X_n} p)$.[25]

Likewise, p is possible just in case its negation fails to be true in virtue of the essences of *any* finite number of things, i.e., in virtue of the essence of any one thing, or the essences of any two things, or ... . Instead of **ETP**, then, we have:

ETP$_\infty$ $\forall p(\Diamond p \leftrightarrow \neg \bigvee_{n \in \omega^+} \exists X_1...X_n \Box_{X_1...X_n} \neg p)$.

On the face of it, given his deep commitment to the finitary nature of properties, the use of an infinitary language and its concomitant logic might seem out of the question for Hale. However, as he's not skeptical of the infinite *per se*, this commitment is not on the face of it incompatible with his adopting an infinitary language — he could make use of the full resources of such a language for the purpose of expressing his theory formally and simply stipulate that legitimate *definitions* (of properties, individuals, etc) have to be finitely expressible.[26]

That said, I suspect Hale would prefer to avoid the use of an infinitary language[27] and the obvious move for him to make, *qua* neo-Fregean, would be to express his theory by

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[25]To see the difficulties of expressing the theory without general variables, consider what **ETN$_\infty$** looks like in a standard *monadic* second-order infinitary language:

$$\forall p(\Box p \quad \leftrightarrow \quad \bigvee_{n \in \omega^+} \exists x_1...x_m \Box_{x_1...x_m} p \vee \bigvee_{n \in \omega^+} \exists F_1...F_n \Box_{F_1...F_n} p \vee$$
$$\bigvee_{n \in \omega^+} \bigvee_{m \in \omega^+} \exists x_1...x_m F_1...F_n \Box_{x_1...x_m F_1...F_n} p)$$

That is, p is necessary iff it is true in virtue of the essences of some finite number of objects, or some finite number of properties, or some finite mixture of objects and properties. In a general second-order language containing n-place predicate variables for arbitrarily large n, the problem would be how to generalize the third infinitary disjunct above to the case where p is true in virtue of some finite mixture of objects and *relations of arbitrary arity*. Even if it can be done, the result will rather spectacularly complicated; reason enough to go with the option of general variables instead.

[26]Infinitary logic is of course not expressively first-order but, as already noted above, the loss of a complete proof theory and the other oft-touted virtues of first-order logic appears to be a cost Hale has already paid for his preferred higher-order semantics.

[27]He has subsequently confirmed this in correspondence.

quantifying explicitly over the natural numbers. But the implementation of the idea is not straightforward. As noted above, Hale borrowed the idea of a "true in virtue of the essences of" operator from Fine. However, Fine's operators are all of the form $\Box_\pi$ for a single monadic predicate π ranging over properties of individuals. With this operator, it is possible to express a restricted version of **ETN** as follows (where 'N' is the *number of* operator and N is the property of being a natural (hence finite) number[28]):

ETN$_N$ $\forall p(\Box p \leftrightarrow \exists F \exists n(\mathsf{N}xFx = n \wedge Nn \wedge \Box_F p)$

But this of course is not sufficiently general for Hale, since entities of any type, not just individuals, can figure into an essentialist explanation. Assuming there are infinitely many types, the full expression of Hale's theory would once again require us to generalize **ETN$_N$** by adding infinitely many new disjuncts on the right of the biconditional, where each new disjunct, in place of 'F', would have a predicate of some higher type.

But even *that* would not be sufficiently general, since, as noted, Hale allows for "mixed" combinations of entities of various types to figure into a single essentialist explanation — an individual and a property, for example. So as far as I can see, the only way for Hale to express his essentialist theory in a finitary language would be to add an explicit, Zermelo-style (hence, highly non-Fregean) theory of sets to his framework, one that would permit the existence of arbitrary finite sets of entities of any type:

ETN$_Z$ $\forall p(\Box p \leftrightarrow \exists s \exists n(|s| = n \wedge Nn \wedge \Box_s p)$

This, in turn, in order to express membership in a set s, would require him to introduce either a type-free membership predicate whose first argument can take a referring expression of any type (or a general variable), or else, for each of the infinitely many types, a new membership predicate whose first argument is restricted to referring expressions of that type.

Whatever mechanisms Hale chooses for dealing with these issues, the bottom line here is that it seems there is still a non-trivial amount of (for the most part, straightforward) work to be done on the theory's formal foundations. (For simplicity, in what follows I will continue using **ETN** and **ETP**, as they suffice for purposes here and can, if necessary, be thought of as placeholders for more complete principles.)

Essences, Modal Logic, and Contingent Beings

The fraught and unsettled relationship between certain principles of quantified modal logic and a number of strong intuitions about contingent existence is well known and I'll

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[28]Hale defines N in accordance with the standard neo-Fregean series of definitions based on a number-of operator and Hume's Principle. See in particular §§7.3-4.

not rehearse the details here. (See Menzel 2008 for a reasonably comprehensive overview.) In a nutshell, if we extend the propositional modal system S5 — Hale's preferred modal system, as noted above — with classical first-order quantification theory, a number of propositions that are starkly incompatible with strong intuitions about contingency — notably the Barcan formula $\Diamond\exists x\varphi \rightarrow \exists x\Diamond\varphi$ and its converse — turn out to be logical theorems.[29] Here I note only that, in order to deal with this particular problem, Hale opts for a common workaround (see, e.g., Fine 1978, §3, and Turner (2005), §6), namely, to abandon classical quantification theory in favor of a free quantification theory (§9.2).

As I've already hinted above, rather more novel tensions with implications for S5 arise with regard to Hale's essentialist theory of modality — specifically, his essentialist theory of possibility **ETP** — and another of his firm philosophical commitments. As he discusses extensively in Chapter 9, Hale is a committed *contingentist*. That is, unlike *necessitists* like Linsky and Zalta (1994) and Williamson (2012), Hale accepts both that there are contingent beings — beings like Aristotle that exist but might not have — and that there could have been *other* contingent beings, i.e., things other than the ones that happen in fact to exist (p. 206).[30] But Hale is also a *strict* contingentist or, in the somewhat unfortunate but now fairly widespread terminology of Plantinga 1983, an "existentialist". Some contingentists — Plantinga himself, notably — believe that all properties and propositions exist necessarily. Consequently, on this view, even if Aristotle and all the objects (if any) involved in his essence had failed to exist, his essence would still have existed; at the least, there would still have been his thisness, *being Aristotle*. By contrast, Hale holds that, had Aristotle (and at least some of the objects (if any) involved in his essence) failed to exist, then Aristotle's essence would also have failed to exist (§9.4.3). More generally, following the likes of fellow strict contingentists Prior (1968, pp. 71-2), Adams (1981, §3.4), Fitch (1996), and Nelson (2009, §6),[31] under those unfortunate Aristotle-free circumstances, there would have been no property — whether a substantive essence or a mere thisness — or any other sort of "proxy" (Bennett 2006) associated uniquely and essentially with Aristotle alone. Likewise, had Aristotle (and his essence) not existed, there would also have been no singular *propositions* directly about Aristotle, as there would have been "nothing for them to be about" (p. 225). Hence, assuming, as we have, that a possibility is just

[29]Prior (1956) was the first to demonstrate this for the Barcan Formula. Its converse is derivable if we extend only the most basic "normal" modal system K with classical quantification theory. For detailed proofs, see Menzel (ibid.), §2.1.

[30]I borrow the terms "necessitism" and "contingentism" (and their cognates) from Williamson (ibid., p. 2), who defines the latter as the thesis (slightly weaker than I've expressed it above) that it is *possible* that there are contingent beings, $\Diamond\exists x\Diamond\neg E!x$.

[31]Prior (1957, p. 114) also cites Peirce in this regard; see Hartshorne and Weiss 1933, p. 147.

a proposition that could have been true, there would have been no singular *possibilities* about Aristotle either.

Hale appeals to strict contingentism to fend off the following objection (§9.4.5). If Aristotle's nature hadn't existed, then there would have been no natures jointly *ruling out* his being a frog and, hence, by Hale's essentialist theory of possibility **ETP**, it would have been possible that Aristotle be a frog. Thus he writes (p. 226, my emphasis):

> If Aristotle's nature had not existed, say because certain objects involved in his nature—such as his biological parents—had not existed, then Aristotle would not have existed, and *there would have been no possibilities concerning Aristotle*—it would not have been possible ... that Aristotle should have been a frog, because, had Aristotle not existed, there would have been no singular possibilities [or impossibilities[32]] concerning Aristotle at all. There would only have been general possibilities, to the effect that there might have existed a man with such-and-such specific characteristics, etc.

Now, Hale appears not to notice that, since he is rejecting an apparent implication of **ETP**, the above argument is actually a *reductio* of the principle in its current form. For, assuming the principle is meant to hold in modal contexts, it *does* appear to entail that the proposition *Possibly, Aristotle is a frog* holds in counterfactual situations in which Aristotle's essence (and, let us suppose for good measure, any other essence that would rule out Aristotle's being a frog) fails to exist. And Hale's response is that the objection fails because, in such counterfactual situations, there exists no such possibility as *Aristotle is a frog*. More generally put, his response is that a proposition must *exist* in a counterfactual situation if it is to be considered *possible* there, or expressed a bit more innocently,

P   For all propositions p, necessarily, p is possible only if p exists.

ETP therefore requires two revisions if it is to incorporate principle **P** properly so that it can be used to rebuff the above objection. First, to incorporate principle **P** we need to be able to express that a proposition exists. Accordingly, let us add an existence operator $E!$ to Hale's framework and assume it is appropriately axiomatized.[33] Second, the principle,

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[32]It is clear from the context that Hale has this more general claim in mind — he is denying, after all, that the *de facto* impossibility *Aristotle is a frog* could have been possible, on the grounds that it doesn't exist in Aristotle-free counterfactual situations.

[33]The most natural way of doing so would be to allow identity statements for propositions and adapt the usual definition: $E!p =_{df} \exists q\, q = p$. However, Hale's modal language doesn't include propositional identities and it is not clear they could even be accommodated in his possibility semantics as it stands, as the propositional quantifiers are not objectual. However, if Hale is dubious about attributing literal existence to propositions or about this method of doing so, I believe he could follow Prior (1968, Ch. XIII) and identify $E!$ with

so revised, needs to be explicitly necessitated, since the thesis concerns what is possible in modal contexts — in particular, in the case at hand, it is being applied to counterfactual situations in which Aristotle wouldn't have existed, so we can't understand it as a simple *de facto* biconditional. Incorporating both points, we have:

ETP′ $\Box[\Diamond p \leftrightarrow (E!p \wedge \neg \exists X_1...X_n \Box_{X_1...X_n} \neg p)],$[34]

that is, necessarily, a proposition p is possible if and only if it *exists* and is not ruled out by any existing essences. By **ETP′**, Aristotle's being a frog now rightly turns out *not* to be possible in counterfactual circumstances in which Aristotle's essence and, hence, all singular propositions about Aristotle, fail to exist.

But there are a couple of problems with this response. First, it is unclear how Hale can account for the necessity of **ETP′** in terms of his essentialist theory **ENT**, as it is unclear in virtue of what actually existing essences the embedded proposition $\Diamond p \leftrightarrow (E!p \wedge \neg \exists X_1...X_n \Box_{X_1...X_n} \neg p)$ might be true, for any given p. Second, and rather more interestingly from a logical point of view, Hale's response to the objection above is to deny that intuitive *im*possibilities like Aristotle's being a frog would have been possible if Aristotle's nature hadn't existed on the grounds that "there would have been no singular possibilities [or impossibilities] concerning Aristotle at all". But that seems equally to serve as grounds for denying that intuitive *possibilities* like Aristotle's being a philosopher would still have been possible if Aristotle's nature had failed to exist. It follows straightaway that the characteristic S5 axiom **5** fails.

In more detail, let p^* be the proposition *Aristotle is a philosopher* and let q^* be any proposition that is incompatible with the existence of Aristotle's essence — the proposition *there are no humans*, say. Then, according to Hale, had q^* been true, Aristotle's essence (hence Aristotle himself) wouldn't have existed and, hence, p^* wouldn't have existed, i.e., $\neg E!p^*$, and so, by **ETP′**, p^* wouldn't have been possible, $\neg \Diamond p^*$. So, succinctly put, $q^* \Box\!\!\rightarrow \neg \Diamond p^*$. It follows that it's not the case that, if q^* had been true, then p^* would have been possible, $\neg(q^* \Box\!\!\rightarrow \Diamond p^*)$. So, by a simple generalization, $\exists q \neg(q \Box\!\!\rightarrow \Diamond p^*)$ i.e., $\neg \forall q(q \Box\!\!\rightarrow \Diamond p^*)$. So by **Df□** (indeed, apart from **Df□**, simply by basic principles of modal logic with counterfactuals), $\neg \Box \Diamond p^*$. But, by **ETP′**, p^* is possible *simpliciter*, $\Diamond p^*$, because it exists and, obviously, as noted earlier, no essences rule out Aristotle's being a philosopher. Hence, we

Prior's "statability" operator S, where Sp, evaluated with regard to a counterfactual situation w, indicates, intuitively, that all of the properties and individuals that p is about or otherwise "involves" exist in w. As far as I can see, this move does not require reified propositions or objectual propositional quantifiers and the logical effect of the operator vis-à-vis the issue at hand is indistinguishable from that of the existence predicate.

[34]More exactly, if we were to make use of the infinitary solution to the issues noted above concerning **ETP**,

ETP′$_\infty$ $\Box \forall p \Box[\Diamond p \leftrightarrow (E!p \wedge \neg \bigvee_{n \in \omega^+} \exists X_1...X_n \Box_{X_1...X_n} \neg p)].$

have $\Diamond p^* \wedge \neg\Box\Diamond p^*$, i.e., $\neg(\Diamond p^* \to \Box\Diamond p^*)$. So it appears that axiom **5** fails in general in Hale's theory, given his strict contingentism and the modified essentialist theory of possibility **ETP**′ that his contingentism requires.

It is illuminating to reflect on Hale's situation vis-à-vis that of his fellow strict contingentists Prior, Adams, Fitch, and Nelson. All four arrived at arguments similar to the one above and, hence, bit the bullet and abandoned the S5.[35] Prior, however, argued that strict contingentism entails far more radical revisions to classical modal logic: not only does S5 fail, so too does the usual modal equivalence

□/◇ $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$.

Specifically, Prior argued (1957, pp. 48-9) that, not only is it not possible that Aristotle fail to exist, $\neg\Diamond\neg E!a$, but that its impossibility is a *logical* truth. The crux of the argument is a metaphysical principle — hence, for Prior, a logical principle — even stronger than **P**, viz.,

T   For all propositions p, necessarily, p is true only if p exists.

Suppose then, in particular, that the proposition $\neg E!a$ that Aristotle doesn't exist is true with respect to a world w. Then by principle **T**, it exists at w. But, by strict contingentism, since it's a singular proposition about Aristotle, it can exist at w only if Aristotle does as well, i.e., only if $E!a$, contradicting our assumption that $\neg E!a$. Hence, $\neg E!a$ is not true at any world, $\neg\Diamond\neg E!a$ However, one cannot infer from this that Aristotle's existence is necessary, $\Box E!a$, i.e., that $E!a$ is true at all worlds. So we must abandon the equivalence □/◇.[36]

Adams (1981) side-stepped Prior's argument by introducing his influential distinction between truth *in* a world and truth *at* a world.[37] Specifically, Adams resolved the apparent clash between strict contingentism and our strong modal intuitions about possible nonexistence by distinguishing between two different "perspectives" — *internal* and *external* — from which a proposition's truth value can be determined with respect to a possible world: from a perspective *within* an Aristotle-free world w, the proposition *Aristotle doesn't exist* does not itself exist and, hence, from that perspective, cannot be true; it is not true *in w*; this was Prior's perspective. However, from an *external* perspective in the actual world, the proposition in question both exists and quite correctly *characterizes w*; it is true *at w*.

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[35]See Prior (1957, Ch. V; 1968, Ch. XIII), Adams (1981, p. 30), and Nelson (2009, pp. 143-5); Nelson, I should note, in as yet unpublished work, has recanted on this point and no longer thinks strict contingentism is incompatible with **5**. Fitch (op. cit., p. 68) does not reject **5** *per se* but, rather, the corresponding semantic condition (given reflexivity) of symmetry on worlds.

[36]For a fairly detailed critical examination of Prior's modal logic Q, see Menzel 1991, §2.

[37]This is also essentially the distinction between the *inner* and *outer* senses of truth introduced in Fine 1985, p. 163.

Cashing modal truth in terms of truth *in* a world, then, Prior's principle **T** is valid; in terms of truth *at* a world, though, it is not. So by basing the determination of truth values for propositions on truth *at* a world, the strict contingentist can in good faith reject principle **T**, at least with regard to negative existentials like $\neg E!a$, and the equivalence $\Box/\Diamond$ is restored.[38]

Interestingly, though, despite having introduced this distinction, Adams and, following him, Fitch and Nelson (albeit for different reasons[39]) did not apply the distinction to *modal* propositions; rather, for them, the truth value of a modal proposition for a world w is still determined from a perspective within w, i.e., for them, principle **T** still held for modal propositions. It follows immediately that a proposition must exist at a world in order to be possible there.[40] Hence, all three philosophers continued to accept principle **P**. Accordingly, in particular, for them, all singular modal propositions about Aristotle — notably, *Aristotle could have been a philosopher*, $\Diamond p^*$ — are false with respect to Aristotle-free worlds in virtue of their nonexistence there. So, although they salvage Aristotle's nonexistence vis-á-vis Aristotle-free worlds via the truth-in/truth-at distinction, not so his possible career as a philosopher. The characteristic axiom **5** of the system S5 is thus still invalid; we have $\Diamond p^*$ but not $\Box\Diamond p^*$.

However, as Turner (2005, p. 205) cogently points out, there seems no more reason for Adams, Fitch, and Nelson to adopt an internal perspective with respect to modal propositions than there is for doing so with respect to negative existentials. For, from the perspective of the actual world, just as we "see" that $\neg E!a$ characterizes an Aristotle-free world w in virtue of Aristotle's absence there, so too we see that $\Diamond p^*$ characterizes w simply in virtue of the truth *simpliciter* of p^* *here* and, hence, at some world.[41] The strict contingentist can thereby sail neatly between the Scylla of necessitism (or Plantingian haecceitism) and the

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[38]It's *necessitation* in fact remains invalid in Adams' system; see Menzel 2008 §4.2.2.2 for details.

[39]In Adams' case, at least, the reason for this appears simply to be that he took the logical form of modalized atomic propositions like *Possibly, Aristotle is a philosopher* to be *predications* (see Adams 1981, pp. 28-9) and, hence, given the "serious actualism" principle that, necessarily, exemplification entails existence, can't be true in worlds in which the subjects of the predication fail to exist. (For more on serious actualism, see (Plantinga 1983, Hinchliff 1989 Bergman 1996, and Hudson 1997.) In Menzel 1993 (fn 29) it is suggested that Adams might not have taken this course if he'd made use of a language with complex predicates in which is to possible to distinguish the logical form of a modalized atomic proposition like *Possibly, Aristotle is a philosopher*, $\Diamond Pa$, from that of the corresponding predication *Aristotle is such that, possibly, he is a philosopher*, $[\lambda x \Diamond Px]a$. Fitch (ibid., pp. 67ff), by contrast, seemed to base his own choice to evaluate modal propositions in term of truth *in* a world on the idea that what is possible at a given world "depends" on what is actual there.

[40]In somewhat more detail: presumably, a proposition p is possible at a world only if the modal proposition $\Diamond p$ is true there and hence, given **T** for modal propositions, only if $\Diamond p$ exists there. But, by strict contingentism, $\Diamond p$ exists there only if p does. Hence p is possible at a world only if it exists there.

[41]Menzel 1991, fn 33 provides a succinct version of Turner's argument and its logical implications are developed explicitly in Menzel 1993, pp. 130-33.

Charybdis of a crippled modal logic. She can acknowledge that, if Aristotle's nature hadn't existed, there would have been no singular possibilities about Aristotle, in the sense that no singular propositions about him would have *existed*; at the same time, by specifying the truth conditions for propositions in terms of truth *at* a world in accordance with Turner's insight, one can assert that singular propositions about Aristotle would nonetheless have been *possible* — notably all those, like p^*, that are true *simpliciter*.

Now, importantly, it seems to me that Hale could incorporate the "external" perspective into his possibility semantics[42] and, under that semantics, he could quell the threat that contingent possibilities raised for his argument (sketched in Part 1 above) for the validity of S5 under his counterfactual definition **Df□** of necessity. However, his essentialist theory of possibility appears to *force* the internal perspective. For, on his essentialist theory, what is possible with respect to a world (or, more generally, counterfactual situation) w is always determined entirely by the essences that happen to exist *in w*. The addition of the existence requirement in **ETP′** in order to avoid the original problem — that Aristotle's being a frog would have been possible if Aristotle's essence hadn't existed — thus appears unavoidable. But, as pointed out above, this revision appears to entail the invalidity of the **5** axiom.

The existence requirement could be avoided, of course, if one were allowed, in any modal context, to quantify, not just over all of the essences there are in that context but all "possible" essences as well, i.e., more exactly, if, instead of **ETP′**, we replaced **ETP** with

ETP″ $\Box[\Diamond p \leftrightarrow \Box\neg\exists X_1...X_n\Box_{X_1...X_n}\neg p]$.

But, as Hale himself recognizes (§9.4.1), this seems to undermine the essentialist theory altogether, as it would seem that any attempt to explain the new necessity operator in terms of existing essences (in accordance with **ETN**) would be bound to fail. So the joint incompatibility of Hale's strict contingentism, his essentialist theory of possibility **ETP′**, and the modal system S5 appears to me to be unavoidable.

I cannot see that Hale has any clear path to the resolution of this incompatibility. But how serious, after all, is it? Although Hale seems to be quite firmly attached to S5 and appeals to the necessity of the alethic modalities at several points in his presentation, his overall theory would likely remain largely intact if the **5** axiom were not available to him in its full generality. Perhaps then he should simply follow the example set by Prior here.

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[42]Specifically: in Hale's semantics, as noted in fn 11, possibilities are, in effect, partial truth value assignments on the set of formulas that meet certain conditions. Although Hale only defines the semantics explicitly for propositional modal languages in his book, I cannot see that there would be any problem in baking the notion of truth *at* a possibility into the semantics in a way that verifies S5 by allowing, e.g., conjunctions of the form $\neg E!a \wedge \Diamond Pa$ — expressing that object a doesn't exists but is nonetheless possibly P — to take a value of 1 at some worlds (and 0 at none in which $\neg E!a$ is true) if Pa is true at any world.

For Prior, logic was the servant of metaphysics, not the other way around. Uncomfortable as the implications of the latter might be for the former, they must simply be met with a stiff upper lip (and, preferably, some arguments to suggest that the resulting implications aren't so bad after all). Hale's attitude about the loss of S5 — assuming, as it seems to me, it is not essential to his theory — should perhaps be the same: the necessities and possibilities there would have been under different circumstances are determined entirely by the *essences* there would have been under those circumstances. A consequence of this conceptual truth is that not every actual possibility would have been possible had things been otherwise, and our logic must therefore simply be made to reflect this remarkable metaphysical discovery.

Closing Remark

Despite my reservations about a number of the finer points of Hale's vision, I would be utterly remiss not to express how much I admire his book, how much I enjoyed studying it, and how very much I learned from it; wrestling with its sophisticated and erudite arguments was a deeply rewarding experience for me. And to say that I wrestled with it is by no means to imply that the book is tough going. To the contrary, it is a scintillating read; arguments are laid out with remarkable clarity, style, and rigor, proposals are developed and defended in assiduous and sparkling detail, and nearly every page is marked by Hale's seemingly limitless philosophical energy and ingenuity. I cannot recommend the book highly enough; indeed I consider it required reading for anyone whose work extends at all into ontology, the philosophy of language, or the philosophy of logic and mathematics.

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