From McTaggert to AdS$_5$ geometry

The purpose of this note is to show how an 'AB-series' interpretation of time, given in a companion paper, leads, surprisingly, apparently, to the physicists' important AdS$_5$ geometry. This is not a theory of 2 time dimensions. Rather, it is a theory of 1 time dimension that has both A-series and B-series characteristics.

To summarize the result, a spacetime in terms of (1) the earlier-to-later aspect of time, and (2) the related future-present-past aspect of time, and (3) 3-d space, gives us the AdS$_5$ invariant.

I must assume the reader is already somewhat familiar with the theory of time proposed in the companion paper Merriam (2019). In that paper I called the future/present/past spectrum g, in this paper I'll call it u, in my vain quest to use appropriate letters for the variables.

This is the model:

\[
\begin{align*}
&\uparrow \\
&t_3 \\
&\uparrow \\
&t_2 \\
&\uparrow \\
&t_1 \\
&\downarrow
\end{align*}
\]

The earlier-times to later-times timeline stays in one ordering (of one kind or another), but the whole timeline moves from future to present to past, with the present staying put. (The present does not 'move up the B-series' as in some spotlight theories because ipso facto the presents wouldn't be ontologically privileged.) As later and later B-series times become present, time goes on.

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In 1+1 spacetime, in terms of t and x, in one convention, we have the invariance of

\[(1) \quad \tau^2 = -c^2 t^2 + x^2\]

under Lorentz transformations. The Minkowski 1+3 interval is in terms of (t, x^3). Also, in the same convention,

\[(2) \quad t \rightarrow i ct\]
for the imaginary unit $i$ and the speed of light $c$. We want a generalization to a new invariant $\tau'$ in terms of the A-series and the B-series and $x^3$, $(u, t, x^3)$ and the transformations that leave it invariant. That's because 1 dimension of time has 2 related parameters, in this theory, one for the A-series and one for the B-series. But it's not immediately clear (to me) in what way(s) or if such a generalization is possible, because probability would seem to get involved. Nevertheless, we can try. (And this, also, has to do with whether the future is branching.) In what might be called 1+1+1 spacetime, in terms of $u$, $t$, and $x$, it would be nice if there were some kind of invariant, for Alice,

$$\tau'^2 = |c_1|^2 k^2 u^2 - c^2 t^2 + x^2$$

for some complex number $c_1$, and some new constant $k$ in units of meters per e. We expect $c_1$ to be complex because $i$ appears in (2). $k$ is a new constant, a 'conversion factor' in meters/e, in analogy to the speed of light, which is a constant or 'conversion factor', $c$, in meters/sec. (Yes they can each be rescaled such that, in their respective units, $k = 1$, and $c = 1$, this is discussed in the other paper.) $c$ is the rate the position changes when going from earlier to later times. $k$ is the rate the position changes as it becomes from Alice's future into her present and then into her past. These are, in this theory, not the same thing.

Consider

$$d\tau'^2 = |c_1|^2 k^2 du^2 - c^2 dt^2 + \sum_{i=1}^{3} dx^2$$

(Wu, 2016, Wikipedia, 2019). A minus sign between $t$ and $u$, it was argued in the companion paper, comes from their opposite orientation: earlier-to-later times go into the future while future-present-past times come out of the future. Obviously other ideas are possible, but the simplest thing to try in analogy to (2) is therefore

(5) $u \rightarrow -iku$.

In which case

$$d\tau'^2 = -k^2 du^2 - c^2 dt^2 + \sum_{i=1}^{3} dx^2$$

If I'm not mistaken, (6) is the AdS$_5$ metric. Let's be clear on the interpretation of (6). It does not have 2 dimensions of time (I'm claiming). It is a proposal for a metric on a different kind of 'spacetime'. It has 3 dimensions of space, and it has one dimension of time, but that dimension has related A-series and B-series characteristics. This might be called AB-spacetime. The A-series characteristics are, of course, 'ontologically private', as defined in the companion paper. Thus, (6) is a metric on Alice's 'private' AB-spacetime.

(Another thing to try is $u \rightarrow -ih'u$ for the imaginary unit $i$ and some constant $h'$ based on Plank's constant $h$, or indeed why not Newton's constant $G$, but the dimensions might be off.)
If 'time is going backward' in the sense of the other paper, then \( u \rightarrow iku \), since both the A-series and the B-series come out of the future in this case, and as we saw \( c \rightarrow ict \). And this gives (6) again. For the rate \( r \) defined in the other paper, a real number in units of seconds/e, we have \( u \rightarrow -irku \), and the signature of the metric is preserved.

To summarize again, a spacetime in terms of 1, the earlier-to-later aspect of time (the philosopher's B-series), and 2, a related future-present-past aspect of time (the philosopher's A-series), and 3, 3-d space, appears, surprisingly, to give the AdS\(_5\) metric (6).

**References**

