Logic is Not Universal: An Empirical Perspective on its Origins and Limitations

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1 Abstract

This essay undertakes an examination of the foundational status of logic, rethinking its origins and limitations from an empirical perspective. Drawing upon the original concepts of Conceptual Quarks (CQs)—defined as the fundamental units of knowledge—and the Programmed Conceptual Deconstruction (PCD) algorithm, which I created for this study, I present methodologies to systematically analyze and deconstruct the principles of logic, exposing their empirical roots. Utilizing a custom-built artificial intelligence model, the study proposes that classical logical principles, such as identity and non-contradiction, are emergent phenomena shaped by sensory context rather than immutable absolutes. By integrating philosophical inquiry with technological innovation, this work offers a new perspective on traditional perspectives, including those of Kant and Leibniz, and establishes a novel framework for understanding logic as a contextual and contingent phenomenon rather than an universal truth.

2 Introduction

Is 1 +1 always 2? This rather simple equation symbolizes the certainty and universality that we often attribute to logic. Logic has long been considered the foundation upon which we build our understanding of the universe. Since Ancient Greece (Aristotle 1998), it has served as the standard for rational knowledge (Russell and Whitehead 1910) and is constantly praised by both science and philosophy. Furthermore, from a young age, we are taught to see it as an absolute truth, sometimes perceiving it as an almost unquestionable ideal.

However, this essay explores the idea that logic is not a universal truth but a product of our reality that could fail in a different context. This challenges the views of philosophers like Descartes and Kant, who posited that logic and reason are universal and a priori, independent of empirical experience (Descartes 1998; Kant 1998) as well as those of empiricists like Hume, who considered logic to be a relationship of ideas valid independently of experience (Hume 2000).

To explore this idea, I present two original concepts developed for this work: Programmed Conceptual Deconstruction (PCD) and Conceptual Quarks (CQs).

PCD is specifically designed to dismantle logic into its most fundamental components. It is not only designed as a set of instructions but also comes with a program. PCD will allow us to generate extensive knowledge trees to visualize how logic is deconstructed into increasingly simpler notions, thereby revealing the fundamental principles of logic and their foundation in the sensory world.

Conceptual Quarks (CQs) are the most basic and fundamental units of any notion, the building blocks upon which any tower of knowledge is constructed. CQs will help us pinpoint what are exactly the true origins of logic.

To delve even deeper into the idea, I have designed three different experiments and created an AI model called Vera. Vera has learned from scratch the three fundamental principles of logic and, also from scratch, to count and add, but in a particular way: it has learned without any supervision and solely through visual experience.

These experiments aim to demonstrate that neither logic nor fields that share the same Conceptual Quarks, such as mathematics, represent divine or universal knowledge, rather that they are merely a contingent construction rooted in sensory experience. For anyone interested in replicating the experiments or starting their own research, the code, datasets, and developed programs will be available in the appendix.

3 Programmed Conceptual Deconstruction and Conceptual Quarks: The Search for the Origin

PCD, or Programmed Conceptual Deconstruction, serves as the foundational methodology for this inquiry. It's not just a set of instructions, but a fully realized algorithm that I've designed to systematically dismantle any concept or field into its most fundamental sub-concepts, which I've termed as Conceptual Quarks (CQs). You could imagine it by picturing a squirrel meticulously climbing an oak tree in search of acorns. The trunk of the oak would represent the concept under scrutiny, and the squirrel's task, analogous to the algorithm's, would be to explore every branch, every offshoot, no matter how seemingly insignificant, and to locate the fundamental concepts (CQ's)—the 'acorns' of understanding within that particular field or concept.

For instance, if "Mathematics" were our original concept, then its initial branches could be "Geometry", "Algebra", "Calculus", and "Statistics" as seen in figure one. Each of these branches is then further explored, leading to more granular concepts.

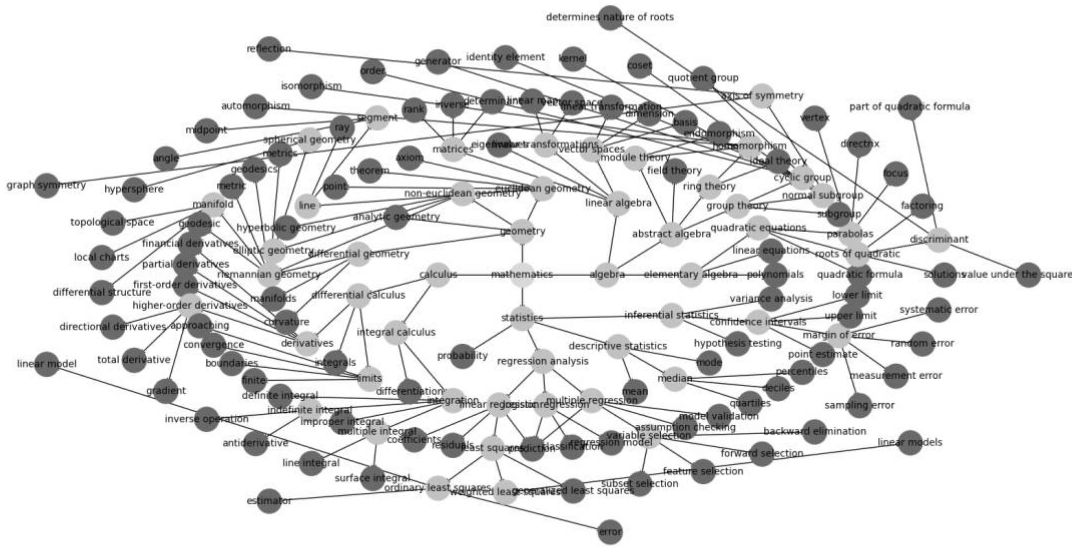

Figure 1: PCD applied to mathematics with a depth limit of 6.

A crucial aspect of PCD is the determination of the relevant sub-concepts into which any concept can be divided into and whether it is possible to do so or not. This is achieved through a combination of methods:

Lexical Analysis: The algorithm analyzes the definition of the current concept, extracting key terms that are themselves concepts and of a lesser complexity.

Referential Significance: It cross-references these terms with authoritative sources (e.g., established encyclopedias, academic databases, or domain-specific lexicons) to assess their conceptual significance within the field.

Complexity Reduction: PCD prioritizes terms that represent a reduction in conceptual complexity compared to the parent concept. This ensures a trajectory towards fundamental elements.

Eventually, Conceptual Quarks (CQs) are reached. A Conceptual Quark (CQs) is determined when PCD cannot deconstruct a concept into more sub-concepts that are relevant, or because it encounters a conceptual loop and it cannot proceed further along a branch. Conceptual Loops are detected using a Conceptual Closure Algorithm (CCA). The CCA identifies instances where a concept's definition refers back to itself, either directly or through a chain of definitions. When a loop is encountered, that branch's exploration is terminated, as further traversal would yield no further insight, and the loop is identified as a QC. An example of a conceptual loop could be {Plato, The Republic}, since the concept of Plato refers to The Republic and the concept of The Republic refers back to Plato, and this loop repeats itself infinitely.

The idea of Conceptual Quark parallels the concept of quarks in physics, which were introduced by Gell-Mann (Gell-Mann 1964) to describe the fundamental constituents of matter. Similarly, we could think of Conceptual Quarks as the 'quarks' of the conceptual world, the most basic and irreducible units upon which any notion is constructed. For the reader's benefit, I'll provide open-source access to the PCD program in the appendix section of this document, within the PCD GitHub repository. I'll also provide the written instructions on how the PCD algorithm works.

The subjectivity inherent in conceptual analysis is acknowledged. To mitigate this, several strategies can be employed. **Triangulation**: consulting multiple authoritative sources to ensure a consensus on the relevance and foundational nature of identified CQs. **Peer Review**: The identified CQs are subjected to review by experts in the relevant field to validate their fundamental nature**. Iterative Refinement**: The PCD process is iterative, allowing for repetition and selection of the most common answer as consensus.

3.1 Determining the Foundations of Logic

What do Conceptual Quarks reveal about the universal validity of logic? Logic is a field that builds on itself. Its most advanced concepts are deduced from its most basic principles, so we need only analyze its most basic concepts to get the CQs and our answers. You can imagine logic as a brick tower, building on its base, which like any other tower, would not stand upright without its base, so the base will be the first thing we analyze.

Principle of Identity: $A = A$

Principle of Non-Contradiction: $\neg(A \wedge \neg A)$

Principle of the Excluded Middle: $A \vee \neg A$

Disjunctive Syllogism: If $A \vee B$, then $\neg A \rightarrow B$

Conjunction: If A and B are true, then $A \wedge B$

Hypothetical Deduction: If $A \rightarrow B$ and $B \rightarrow C$, then $A \rightarrow C$

Modus Ponens: If $A \rightarrow B$ and A is true, then B is also true

Modus Tollens: If $A \rightarrow B$ and $\neg B$ is true, then $\neg A$ is also true

Figure 3: Fundamental principles of logic to be analyzed.

In the definition of all the principles we can find some repeated concepts, such as "true", "be", or "equal", and it is these concepts to which we will apply a PCD of maximum depth in order to find their Conceptual Quarks. For maximum precision, we will prioritize a manual application of the algorithm, following its instructions. Since this application spans seven written pages, I'll make it available through the appendix, inside the PCD GitHub repository.

In this document I will present the complete result of the manual application alongside a visualization from the program, which is shown in Figure 4.

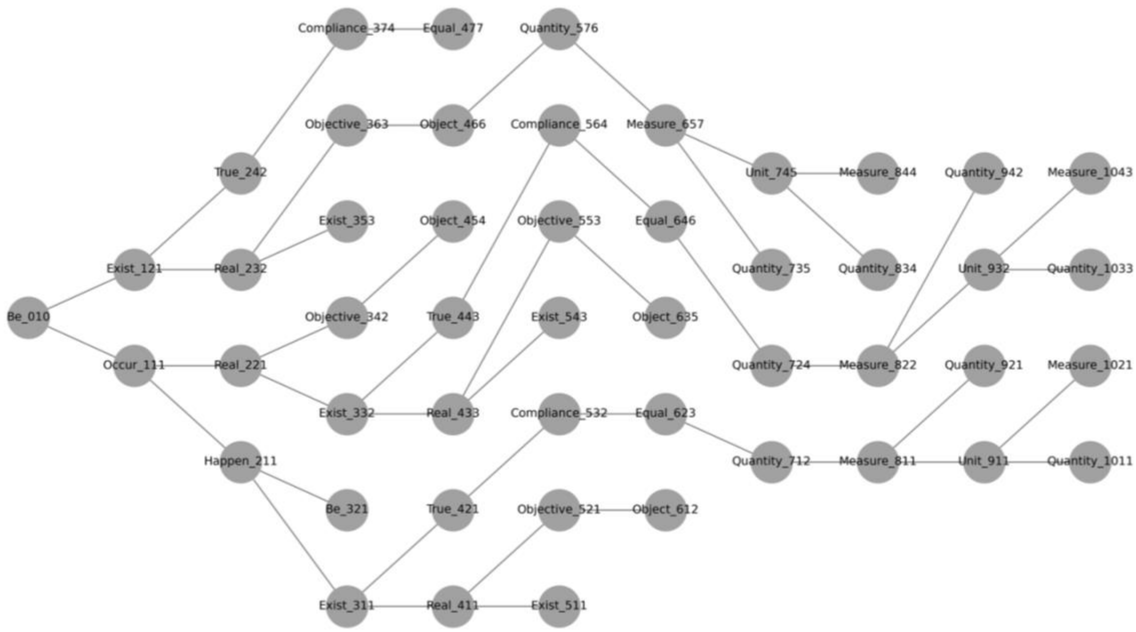

Figure 4: PCD applied in software on "equal", "be" and "true".

The manual application of PCD is interesting because when PCD is applied to "be", we discover during the deconstruction process both "true" and "equal", which are also deconstructed. When deconstructing a concept, the algorithm relies on the official definition for that concept, from which it extracts only terms that are relevant and of lesser conceptual complexity.

The Conceptual Quarks found when applying PCD to the aforementioned principles are as follows:

CQs = [{Exist_1.2.1, Real_2.3.2 ,Exist_3.5.3}, Object_4.5.4, Object_4.6.6, [{Exist_3.1.1 ,Real_4.1.1 ,Exist_5.1.1},{ Exist_3.3.2 , Real_4.3.3 ,Exist_5.4.3}, Object_6.1.2, Object_6.3.5, {Quantity_5.7.6 ,Measure_6.5.7 ,Quantity_7.3.5}, {Measure_6.5.7 ,Unit_7.4.5 ,Measure_8.4.4},{Quantity_5.7.6 ,Measure_6.5.7 ,Unit_7.4.5, Quantity_8.3.4}{Quantity_7.1.2 ,Measure_8.1.1 ,Quantity_9.2.1}, {Quantity_7.2.4 ,Measure_8.2.2 ,Quantity_9.4.2},{Quantity_7.1.2 ,Measure_8.1.1 ,Unit_9.1.1 ,Quantity_10.1.1} , {Measure_8.1.1 ,Unit_9.1.1 ,Measure_10.2.1} ,{Quantity_7.2.4 , Measure_8.2.2 ,Unit_9.3.2,Quantity_10.3.3},{Measure_8.2.2 ,Unit_9.3.2 ,Measure_10.4.3}]

CQs = [{Exist,Real}, Object, {Quantity,Measure,Unit}]

3.2 Analyzing Findings

As we can see, the CQs found for the most fundamental concepts of logic are two conceptual loops and a singular concept.

The first CQ, {Exist, Real}, indicates the condition that something must first exist in order to "be", just as something must exist at least as a proposition for it to be "true", or for two things to be "equal".

Next in the list we find the concept of Object, which has the following definition: "Anything that can be the subject of knowledge or sensitivity on the part of the subject, including the subject itself". Something must first qualify as an Object first in order to "be", or to be "true", or to be "equal" to something else, much like the previous CQ indicates that something must exist at least as a concept to meet these conditions.

Finally, we have the second conceptual loop {Quantity, Measure, Unit}, which pertains to the attributes of everything that can "be" and "is" an object. Interestingly, this last CQ can also be obtained by applying PCD in depth to mathematics.

As we can observe, the Conceptual Quarks of the most basic principles of logic are grounded in what **exists** and is **real**, in what is an **object**, and in what is **measurable**. This allows us to affirm that the building blocks upon which the tower of logic is founded are part of the sensory realm. This perspective resonates with Quine's critique of the analytic-synthetic distinction, implying that logical principles are not entirely independent of empirical content (Lakoff and Núñez 2000; Quine 1951).

This directly contradicts rationalists like Descartes and Kant, and even empiricists like Hume, since PCD refutes both the *a priori* notion of rationalists and the lack of questioning of some empiricists towards their logical structures.

Now, in the next section, we will verify the algorithm's results by teaching Vera the three principles of logic, namely identity, non-contradiction, and the excluded middle, solely through experience and sensory input.

4 Recreating the Principles of Logic Solely Through Experience

This section aims to confirm whether the principles of logic are rooted in sensory reality. To test this, I designed an experiment with a cube and a room, both shown in Figure 5.

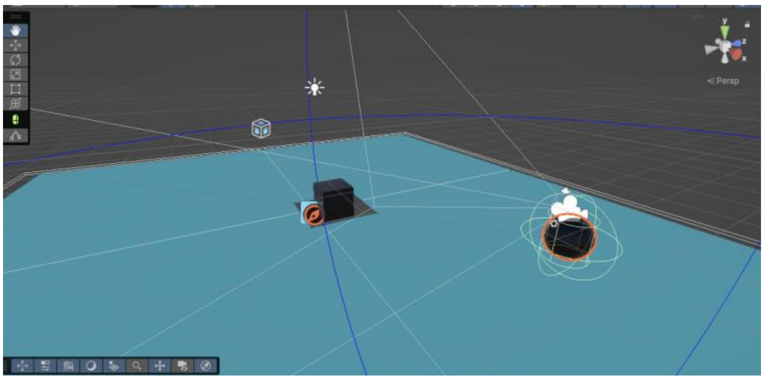

Figure 5: Experiment scenario, with the cube in the center, and Vera on the right.

The insights gained from PCD directly inform the design of our experiments with Vera. The identified CQs—{Exist, Real}, Object, and {Quantity, Measure, Unit}—serve as guiding principles for creating an environment in which Vera can potentially rediscover the fundamental principles of logic.

{Exist, Real}: The experimental setup ensures that Vera interacts with a tangible object (the cube) within a defined space (the room). This provides a concrete manifestation of existence and reality.

Object: The cube is specifically chosen as a distinct, bounded object with consistent properties. This allows Vera to learn the concept of an "object" as something that can be differentiated from its surroundings and possesses stable attributes.

{Quantity, Measure, Unit}: While not explicitly explored in the first experiment, this CQ informs the design of subsequent experiments (Section 5) where Vera will learn about quantities and units through interaction with multiple objects.

In essence, the experimental environment is crafted to embody the CQs, providing Vera with the necessary sensory input to potentially derive logical principles. This is not merely a simulation but a carefully constructed reality designed to test our hypothesis about the empirical origins of logic. Vera will navigate the room randomly while capturing images of its surroundings. This has resulted in a dataset of 4,448 images which will be available alongside the code in the appendix.

Vera uses a neural network capable of processing images, a convolutional autoencoder (Hinton and Salakhutdinov 2006; LeCun et al. 2015), which works in the following way. Imagine you saw a cat earlier in the day and you only remembered its key features such as its size, color, and distinct markings. Later, you try to reconstruct the image of the cat in your mind, based only on those remembered features. Well Vera does something similar; it extracts essential details from an image, compresses them, and then tries to recreate the original image using only the key features it retained. Also, Vera learns in an unsupervised manner, all on her own.

After Vera finished the exploration and processed the images, we end up with the representation that can be seen in Figure 6.

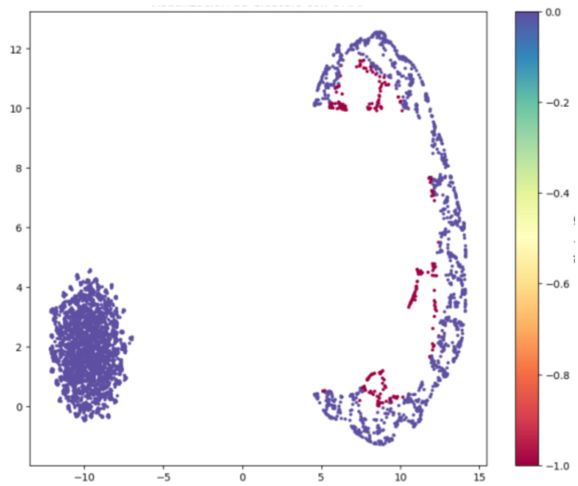

Figure 6: Clusters obtained by Vera.

In the representation we can observe two main clusters: one with an elliptical shape in the lower-left area and another with a horseshoe shape in the right area. The left cluster represents the moments when Vera was not looking at the cube in the center of the room, and the right cluster represents what Vera understands as the cube located in the center of the room.

What does this graph mean? This graph is significant because we can observe the three principles of logic present in it. Let us briefly recap what these principles are.

- **Principle of Identity**: A proposition is equal to itself. This means that if a proposition is true, then it is true: A = A.
- **Principle of Non-Contradiction**: A proposition cannot be both true and false at the same time. It is not possible for A and ¬A to be true simultaneously.
- **Principle of the Excluded Middle**: A proposition must be either true or false; there is no third option. For every proposition A, A is either true, or it is not true.

We can see that Vera applies the **Principle of Identity**, precisely because we can observe two distinct clusters: one in which the cube "is" and another in which the cube "is not". This means that Vera recognizes the existence of the cube and that whenever it has seen the cube, it has equated it to itself, and therefore it has placed it in the cube cluster, thus applying the Principle of Identity: A = A.

This can be observed more visually in Figure 7, where Vera indicates where it places everything it sees within the representation of the data. In the figure, we can see how whenever the cube appears, Vera equates it to itself and recognizes its identity by assigning it to the right cluster.

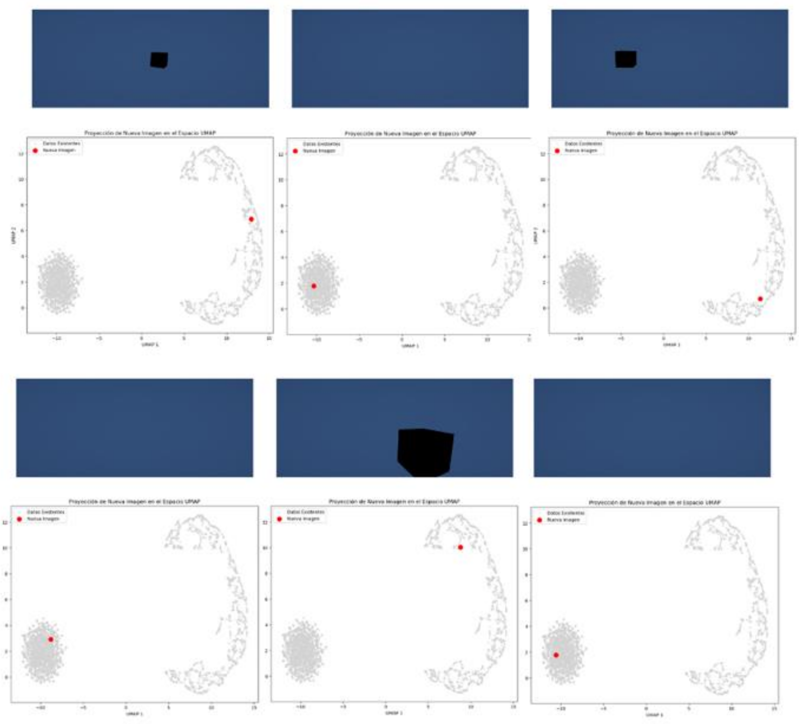

Figure 7: Different moments captured by Vera and their placement.

We can also see how Vera applies the **Principle of Non-Contradiction**. For Vera, there is no such thing as the cube both being and not being at the same time; it either "is" in the right cluster, or it "is not" in the left cluster. Furthermore, regardless of the angle or distance from which Vera observes the cube, it always recognizes it. The cube never stops being the cube for Vera, which is why it consistently applies the Principle of Non-Contradiction.

Finally, Vera also applies the **Principle of the Excluded Middle** because the cube is either present, or it is not present, there is no third option.

Some will think: but what if it reflects logic in the representation only because we humans have created neural networks with logic? This is proven not to be the case in section 6, in which, presenting Vera with a quantum reality, she applies in her representation a logic which breaks with our classical logic, which means that neural nets like Vera's only reflect the underlying logic of the context to which they are exposed.

These results directly refute what Leibniz argued, which was that the principles of identity, non-contradiction, and the excluded middle were universal and eternal truths (Leibniz 1966), independent of experience and part of a logical order preceding the world. This experiment suggests the opposite, namely that these principles are grounded in empirical interaction with the environment.

When Aristotle laid the foundations for these principles in his formal logic, he considered them universal truths, applicable to all beings in any context. However, Vera's learning demonstrates that these principles are only valid in contexts where objects maintain coherent and observable properties, which is not necessarily the case in other realities like in quantum mechanics (we will delve into this in more detail in Section 6).

How is it possible that Vera is applying the three principles upon which logic is based after only exploring a room with a cube for a short time?

The answer is simple: Vera is not thinking, nor is it consciously applying the principles of logic. Vera has only visualized and learned everything about a room, and the fact that it is applying the three principles of logic in its data representation simply reflects that learning logic is a byproduct of learning about a reality where coherent and observable properties are maintained, such as in ordinary human experience.

Some might argue that Vera is an AI, and humans or animals arrive at logic differently, possibly without relying on sensory input. While acknowledging the differences between Vera's artificial neural network and the biological brains of humans and animals, I posit that the fundamental principles governing learning from sensory experience are shared. Vera's ability to derive logical principles from interacting with a simplified environment provides a compelling model for understanding how such principles might emerge in other cognitive systems.

The example of a child learning to walk illustrates this point. The child's understanding of the ground as a stable surface (Identity), the impossibility of the ground simultaneously being and not being (Non-Contradiction), and the binary state of being on or off the ground (Excluded Middle) are not innate but derived from repeated sensory interactions. These interactions, like Vera's, involve encountering a consistent environment where objects maintain stable properties.

Furthermore, consider the behavior of animals. A sheep eating grass implicitly relies on logical principles. The grass is either there or not there (Excluded Middle). It cannot be both in the sheep's grasp and not in its grasp at the same time (Non-Contradiction). The grass also maintains its identity as a distinct entity throughout the process (Identity). These principles, while not consciously articulated, are evident in the animal's actions, suggesting that they are derived from its interaction with the world.

While Vera's learning process may differ in its specifics from that of biological organisms, the underlying principle—that logical principles can emerge from interacting with a structured sensory environment—remains valid. This suggests that logic is not unique to humans or even to biological systems but is a fundamental property of any system capable of learning from experience in a consistent and measurable reality.

5 Arithmetic, Logic Derived from Experience

To test whether Vera can learn or not the remaining CQ that constitutes logic, we will show her a dataset of 1,000 images showing different quantities of squares (examples in Figure 8).

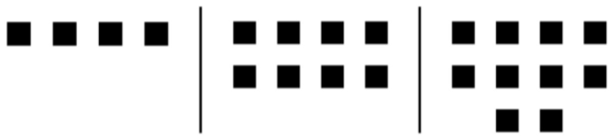

Figure 8: Examples of the dataset.

Using a neural network similar to the one from the previous section. Vera managed to organize these images into 10 different clusters, one for each quantity, effectively learning to count from 1 to 10, just like it is shown in figure 9.

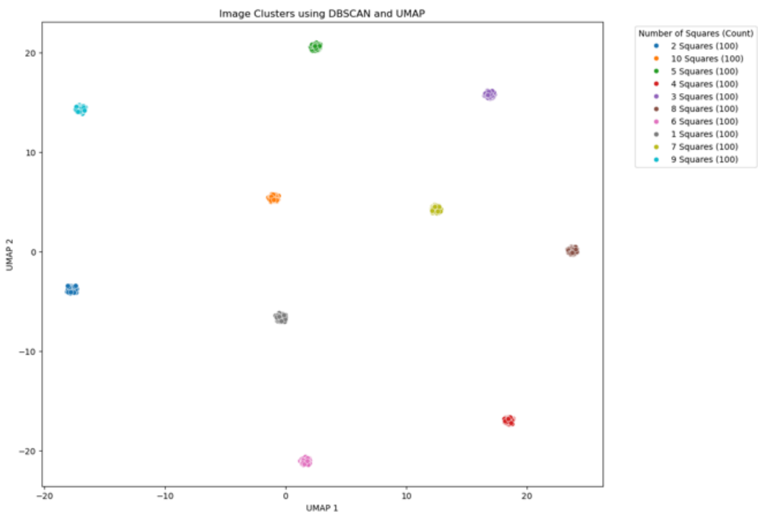

Figure 9: Vera's 10 clusters.

The ability to differentiate between different quantities of units is not only one of the foundations of logic but also of certain fields that use logic, such as mathematics, and it has grasped it solely through sensory experience. This mirrors Jean Piaget's findings that children's logical and mathematical concepts develop from hands-on interactions with their environment, supporting the idea that logic emerges from sensory experience (Lakoff and Núñez 2000; Piaget 1952).

But we will not stop here, as we will also verify if Vera can generalize the learned concept of quantity by performing the arithmetic operation of addition. To test this, Vera will be shown new shapes, such as circles, and we will see if it can perform addition by observing images where different quantities are combined (e.g., two squares and two circles).

As we can see in Figure 10, Vera is also able to successfully recognizes the total quantity of squares and circles, proving it understands addition as well as the concepts of unit and quantity.

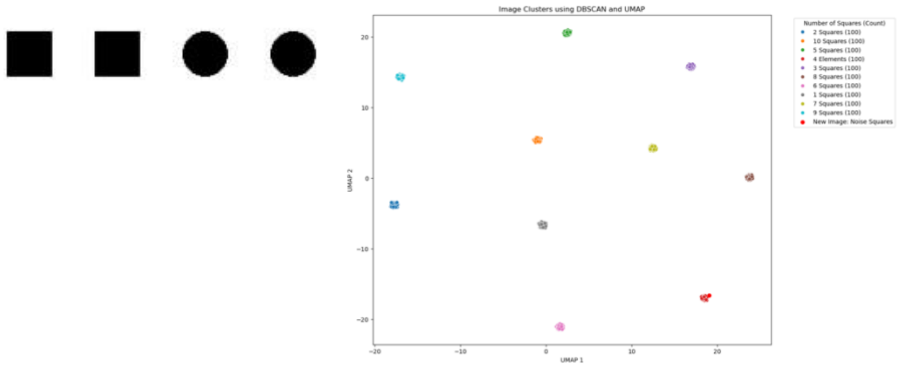

Figure 10: Input and result, marked by the red circle (bottom right corner).

These results challenge the ideas of philosophers like Frege (Frege 1980) and Whitehead (Whitehead 1925). Frege argued that logic and mathematics are independent of experience, while Whitehead believed mathematics were superior to physical reality. Vera's ability to learn these concepts purely from sensory experience suggests the opposite: mathematical ideas like addition are grounded in structured sensory input, not a Platonic realm of universal abstractions.

This experiment also highlights a key flaw in the Platonic view of logic (Kahn 1981; Plato 2004): logic must first be understood as grounded in sensory reality before imagining it as a universal concept.

Furthermore, for example, quantum logic can also exist in a domain independent of the sensory, just as an alternative logical system could exist where adding two units does not equal to two units, and this system could validate itself in such a hypothetical conceptual reality, however, this doesn't mean that such a system would be universally valid in reality, which applies to logic too.

In the next section, we'll explore how logic behaves in contexts vastly different from our everyday experience, such as in quantum mechanics.

6 Logic in Quantum Mechanics

Logic, as derived from our everyday reality, does not hold universally. Quantum mechanics provides a clear example, as it challenges classical logic. For example, Hilary Putnam, as he put it in "*Is Logic Empirical?*" (Putnam 1968), suggests that quantum principles may require revisions to traditional logical frameworks. For instance, in quantum reality, principles like non-contradiction do not apply, as objects can exist in multiple states simultaneously. So, as the validity of a logical system depends on the context in which that logical system is posed, there cannot be a truly universal logical system as long as there are multiple contexts as it is in our case.

To emphasize the point that every logical system arises from a concrete empirical framework, we will take Vera on a trip to the quantum world and expose her to quantum reality in order to compare the logic it discovers with the classical logic that Vera applied in her representation in Section 4.

In this scenario, we will simulate the same room as in Section 4, but with quantum superposition, which implies that an object can exist in multiple states or positions at the same time. The cube will thus exist in two positions at the same time, as shown in Figure 11.

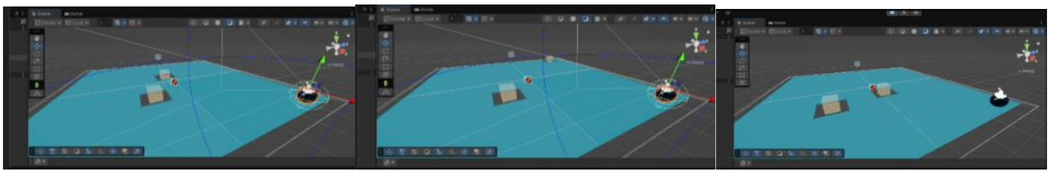

Figure 11: Vera observing the quantum superposition of the cube.

The quantum world is dangerous, so this time Vera will not have the ability to move and will merely act as an observer. The neural network which Vera will use to learn about this reality is exactly the same as the one it used to learn in Section 4. After starting the simulation and spending some time inside this pseudo-quantum reality, Vera captured 2726 different images, which will also be available via link in the appendix.

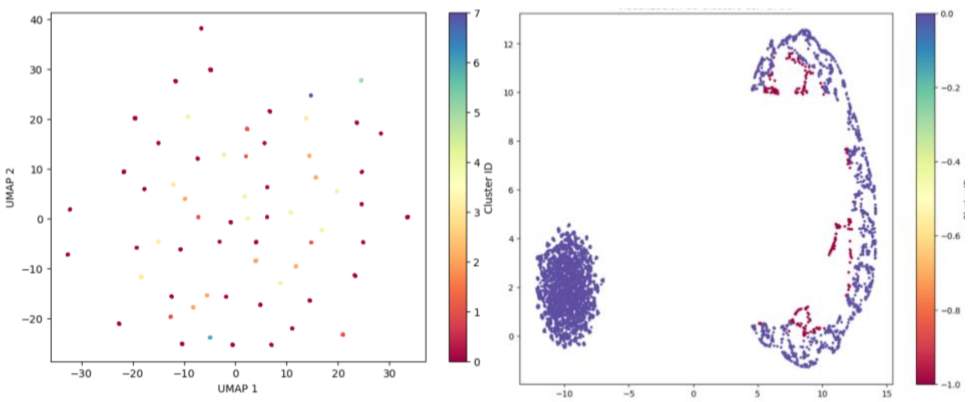

Figure 12: Perception of the cube in quantum reality (left) and in everyday reality (right).

By processing the images obtained from the cube within this quantum reality, we obtain the graph visible on the left side of Figure 12. It is apparent from the graph that the principles of classical logic as we know them are not fulfilled. In the right-hand graph, in which Vera was in our everyday reality, no matter what angle it was at or how far away, Vera always recognized the cube and placed it in the right-hand cluster.

On the other hand, in the left graph, the cube "is" and "is not" at the same time, because at a given time in a given position, Vera places it inside a cluster, indicating the existence of the cube. However, at another time, in another position, and still in view, Vera places the cube in a different cluster. This means that, on the one hand, the cube exists, as Vera places it in a cluster associated with the cube, but at the same time, it does not exist, as Vera does not place it in the other clusters that also represent the same cube. This completely breaks the principle of non-contradiction in logic, which states that something cannot "be" and "not be" at the same time—a principle that was perfectly respected in Section 4. So, what has changed?

What has changed is not Vera's neural network, since it is exactly the same as in Section 4. Nor has the cube itself changed. What has changed is the reality in which Vera and the cube are present, which in this case is the quantum one. This implies an intrinsic logic which is different from that of our everyday reality, as it is reflected in Vera's representation. This is supported by Niels Bohr's principle of complementarity (Bohr 1934) which illustrates how quantum entities exhibit mutually exclusive properties that challenge classical logical principles, reinforcing the notion that logic is context dependent.

That is, if we posit a reality, such as the quantum reality in this case, that alters the Conceptual Quarks of logic, such as [{Exist, Real}, Object, {Quantity, Measure, Unit}], then logic ceases to be fulfilled and ceases to function, that is, it ceases to be true, as we have just demonstrated with this experiment. Bas C. van Fraassen also emphasizes that our logical frameworks must adapt to empirical realities, such as those presented by quantum mechanics, where phenomena defy classical explanations (van Fraassen 1991).

This experiment supports the findings made with Programmed Conceptual Deconstruction, which stipulate that logic is grounded in the sensory reality we inhabit, since we can see a clear change in the logic present in Vera's representation of the data and the only thing that has changed between Section 4 and this Section is the sensory environment Vera is in.

It also supports the claim that no logic is truly universal, as classical logic does not hold in every environment. This could also apply to future contexts that we discover in the future, as Hans Reichenbach said, advances in physics necessitate revisions to our understanding of logical principles (H. Reichenbach 1944, 1958), which further illustrates how different empirical realities can influence the validity of logic.

All of this points to logic not being universal. Different contexts, whether quantum, or hypothetical, produce unique logical systems, each valid only within its own framework.

7 Conclusion

Through tools such as Programmed Conceptual Deconstruction (PCD) we have been able to determine which Conceptual Quarks underlie logic, these being two conceptual loops and an individual concept: [{Exist, Real}, Object, {Quantity, Measure, Unit}].

Our analysis shows that logic is based on reality, it is contingent on the specific contexts that support its foundational CQs and therefore also not universal. This context-dependency is evident in realities like in quantum mechanics and would also apply to any alternate context lacking the essential CQs of our logical system, such as a concept of unit or object.

Rather than seeking universal truths, we should adopt a contextual approach that reflects more the contingent nature of logic and its origins in sensory reality.

8 Glossary of Terms

Conceptual Quark: the most fundamental sub concepts and conceptual loops of any concept or field of knowledge. Conceptual Quarks are obtained by applying Programmed Conceptual Deconstruction to any concept.

Programmed Conceptual Deconstruction: term that describes the algorithm created by the author for this study as a tool to analyze the origins of logic. Programmed Conceptual Deconstruction allows to systematically deconstruct any unit of knowledge into its most fundamental sub-concepts and conceptual loops, that is, into Conceptual Quarks.

Conceptual Loop: concepts or instances that refer to themselves in a circular fashion.

Conceptual Closure Algorithm: an algorithm part of Programmed Conceptual Deconstruction that checks for the existence of conceptual loops.

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10 Appendix

PCD: https://github.com/codermtk/PCD.git

Section 4 Experiment: https://github.com/codermtk/Section-4-Experiment

Section 5 Experiment: https://github.com/codermtk/Section-5-Experiment

Section 6 Experiment: https://github.com/codermtk/Section-6-Experiment