

**LANGUAGE AND LOGIC IN WITTGENSTEIN'S
*TRACTATUS LOGICO-PHILOSOPHICUS***

Daniele Mezzadri

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Department of Philosophy

University of Stirling

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Abstract

This thesis discusses some central aspects of Wittgenstein's conception of language and logic in his *Tractatus Logico-Philosophicus* and brings them into relation with the philosophies of Frege and Russell. The main contention is that a fruitful way of understanding the *Tractatus* is to see it as responding to tensions in Frege's conception of logic and Russell's theory of judgement. In the thesis the philosophy of the *Tractatus* is presented as developing from these two strands of criticism and thus as the culmination of the philosophy of logic and language developed in the early analytic period.

Part one examines relevant features of Frege's philosophy of logic. Besides shedding light on Frege's philosophy in its own right, it aims at preparing the ground for a discussion of those aspects of the *Tractatus'* conception of logic which derive from Wittgenstein's critical response to Frege. Part two first presents Russell's early view on truth and judgement, before considering several variants of the multiple relation theory of judgement, devised in opposition to it. Part three discusses the development of Wittgenstein's conception of language and logic, beginning with Wittgenstein's criticism of the multiple relation theory and his early theory of sense, seen as containing the seeds of the picture theory of propositions presented in the *Tractatus*. I then consider the relation between Wittgenstein's pictorial conception of language and his conception of logic, arguing that Wittgenstein's understanding of sense in terms of bipolarity grounds his view of logical complexity and of the essence of logic as a whole. This view, I show, is free from the internal tensions that affect Frege's understanding of the nature of logic.

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Introduction

It is perhaps trite to remark that Wittgenstein's *Tractatus Logico-Philosophicus* was influenced by Frege and Russell. Wittgenstein himself acknowledges this in the Preface to the *Tractatus* by claiming that he is "indebted to Frege's great works and to the writings of my friend Bertrand Russell for much of the stimulation of my thoughts" [TLP: p. 3]. Frege's and Russell's impact on Wittgenstein's thought is, however, *twofold*; on the one hand, the *Tractatus* develops some important philosophical insights which it owes to Frege and Russell; on the other hand, it can be seen as a critical reaction to some crucial aspects of Frege's and Russell's philosophy.

In the thesis I concentrate on the latter point; the guiding thought is that the *Tractatus'* conception of the nature of language should be understood as the culmination of Wittgenstein's critical insights about Russell's theory of judgement, while the conception of logic as tautologous, which derives from the pictorial understanding of the nature of the proposition, is to be seen as embedding a rejection of some aspects of Frege's conception of the nature of logic.

The outcome of the thesis is that, while Wittgenstein's pictorial conception of language stands in a fundamental opposition to Russell's conception of judgement, seen as involving a misapprehension of the nature of propositional articulation, the *Tractatus'* conception of logic should not be thought as implying a rejection of Frege's views on logic *as a whole*. The conception of logic presented in the *Tractatus* is better understood as dispelling tensions in Frege's thought on

logic, and thus as rejecting some of Frege's views on the matter while retaining and developing others.

The structure of the thesis implements this general idea as follows; in part one I discuss some internal tensions in Frege's conception of logic that bear a direct relation to Wittgenstein's understanding of the nature – and the essence – of logic and of logical propositions. In the second part I address Russell's views on the nature the proposition and the development of his views on judgement from 1903 to 1913, when his work was abandoned in consequence of Wittgenstein's criticism. Part one and two should thus be conceived of as two independent beginnings of the thesis, aimed at analysing those views of Frege's and Russell's that are rejected in the *Tractatus*.

In part three I discuss Wittgenstein's conception of language and logic in the *Tractatus*, showing how his mature view of the nature of the proposition (the picture theory) arises – via the early account of sense presented in the *Notes on Logic* – out of Wittgenstein's criticism of Russell's theory of judgement. I then consider how Wittgenstein's pictorial account of language grounds his understanding of the nature of logical constants and of logical propositions. I argue that the philosophy of logic that derives from it helps to resolve the tensions present in Frege's conception of logic, discussed in the first part of the thesis.

Part three thus analyses Wittgenstein's answer, in the *Tractatus*, to the problems in Frege's and Russell's philosophy, and thus connects the discussion of Frege's and Russell's conceptions of logic, language and judgement dealt with separately in the previous two parts of my work.

In the first part of the thesis, after a preliminary discussion of some of Frege's basic ideas on the essence of logic, I address, in chapter 2, Frege's conception of logic as a normative and descriptive science. On Frege's view, logic has an irreducibly *normative* character. Logical laws are laws of thought, in the sense that they prescribe how one ought to think – as opposed merely to describing how one does think; logic – for Frege – teaches us how to think in order to think *truly*. Frege, however, also thinks that logic is *descriptive*, in essentially the same sense in which natural sciences are; but logic has a wider subject-matter than any of the particular sciences, because logical laws are the most general laws of truth. The normativity of logic is for Frege grounded in its laws' being descriptive laws of truth, and so it is dependent upon its being a descriptive science, for the descriptive laws of logic provide the theoretical foundation of its normative role *vis-à-vis* thought. Any law, Frege thinks, is normative, for any law should guide thought with respect to its particular subject-matter, but logical laws have a wider normative force than any other particular science, because they are the most general laws of truth.

I then explore, in chapter 3, the question whether on Frege's perspective logic is a formal science. Frege's understanding of logic as the most general science of truth – whose laws encompass those of more particular sciences – suggests that he conceives of logical laws as being contentful and general, as being the most general laws there are. But, particularly around the time of the composition of *Begriffsschrift* (1879), Frege often envisages a different role for logic, which makes it different from other sciences not as regards its subject matter, but as regards its very nature. According to this strand in Frege's philosophy of logic, the difference between logic and other sciences is not a matter of *degree*, but is a difference in

kind. Logic, on this view, more than having a (general) content of its own, represents the framework within which the content of particular sciences can be expressed; the function of logic, on this perspective, is thus not that of giving the most general laws of truth, but that of providing the structural scaffolding which, once implemented with a content and thus a particular vocabulary, can form a language where sense can be perspicuously expressed.

In chapter 4 I isolate another strand in Frege's conception of logic, which I call the *constitutive* one, according to which logic is internal to thinking and thus constitutive of the possibility of thought. In this sense the laws of logic prescribe how one *must* think, to use Frege's words, 'in so far as one is to think at all'. For Frege, in fact, the notions of thought, judgement, and inference seem to be at least partly defined in terms of acceptance of the laws of logic – laws which constitute a framework of rationality only within which thinking can be exercised.

However, while Frege's adherence to the descriptive and normative character of logic is relatively uncontroversial, his endorsement of the idea that logic is – in some way – constitutive of thought is less clear, and bears a contrast with the normative strand in his philosophy of logic. Given logic's normative status regarding thought, not complying with the laws of logic would qualify as bad thinking, not as failing to think at all. In this context, Frege's discussion – in *The Basic Laws of Arithmetic* (1893) – of the possibility of illogical thought, fleshed out in terms of the possibility of beings that reject the basic laws of logic, is particularly relevant. According to the normative strand in Frege's conception of logic, those who reject the laws of logic are simply wrong thinkers, or judgers. But if the idea is to be taken seriously that the laws of logic form the background

within which judgements can be made at all – the idea that logic is constitutive of the possibility of thought – then this goes against the possibility of illogical thought.

In the second part of the thesis I discuss the development of Russell's views on the nature of the proposition and judgement from 1903 to 1913. The main focus is on Russell's multiple relation theory of judgement (developed in the years 1910-1913), a theory that was devised in direct opposition to the view on propositional unity, truth and judgement put forward in *The Principles of Mathematics* (1903), and discussed in chapter 5.

The core idea of the multiple relation theory is that judgement is not a dual relation between a judging mind and a proposition (as was previously thought by Russell), but a multiple relation between the mind and several entities. A judgement is thus a multiple-relational complex. I analyse (in chapters 6-7) the modifications the multiple relation theory undergoes through the years 1910-1913, arguing that the reasons for these changes have to do with the role of the subordinate relation in a judgement complex, namely the relation other than the judging one. Russell's official position, in all the variants of the multiple relation theory, is that the subordinate relation is one of the *terms*, one of the *objects*, of the complex, and is thus not meant to provide the unity of it (it is not, to use Russell's terminology, the *cement* of the complex). But Russell, at the same time, also seems to grant that the subordinate relation has a different status than the other judgemental components.

In *Theory of Knowledge* (1913) Russell enriches the multiple relational account of judgement by adding the *logical form* of the judged complex as one of the constituents of a judgment. This modification, I examine in chapter 7, is motivated by the problems with the previous accounts of judgement earlier analysed. I conclude by considering whether the multiple relation theory presented in 1913 is successful and argue for a negative answer, claiming that the account of judgement there is not free from the problems that confront all versions of Russell's treatment of the subordinate relation in a judgement complex.

In the third part of the thesis I analyse Wittgenstein's conception of language and logic in the *Tractatus* and link it to Russell's theories of judgement and Frege's reflection on logic, presented and explored in the previous two parts.

Wittgenstein's pictorial conception of language is approached historically through an analysis of Wittgenstein's early criticism of Russell's theory of judgement, and the account of sense in the *Notes on Logic*, where the seeds of the mature picture theory are to be found. I discuss the former in chapter 8, by adjudicating some interpretations of it put forward in the literature; my claim is that Wittgenstein's objection to Russell (that he did not rule out the possibility of nonsense judgements) amounts to a criticism of Russell's account of the subordinate relation in a judgement complex; Wittgenstein criticises Russell's official position that the subordinate relation occurs in a judgement in the same way in which other constituents do. Wittgenstein's critique thus concerns Russell's idea that the subordinate relation is an object, a term, in the judgement structure.

In chapter 9 I analyse Wittgenstein's account of the structure of a proposition in the *Notes on Logic* and show how the elements of his objection to Russell's treatment of judgement are elaborated into a general account of symbolism and linguistic significance. A significant feature of it is the logical difference between the components of a proposition, *names* and *forms*. For Wittgenstein the form of a proposition, the relational element that ties together its other elements – is of a *different* logical kind than that of its names; the two cannot be conceived as being on a par; in this sense Wittgenstein's conception of a proposition in the *Notes* can be seen as implementing his criticism of Russell's theory of judgement, where the judgement components (with the exception of the judging relation) are considered as being all objects, and thus as being all on the same level. The division of propositional constituents into names and forms, further, requires that a proposition has a structural similarity with what it represents, a distinctive feature of the *Tractatus'* picture theory.

In the theory of the nature of the proposition proposed in the *Tractatus* the idea of a structural similarity between the dimension of language and that of reality is deepened into the idea that a proposition has something *identical* with what it represents – the same mode of combination of elements – and is by virtue of this a *picture* of reality. However, the picture theory presents a significantly different account of the nature of propositional constituents from the one given in the *Notes*; in contrast to the latter, in the *Tractatus* propositions are seen as constituted by names alone, in immediate combination. In chapter 10 I discuss the picture theory and consider how the idea that a proposition and its corresponding reality share the same relations among their elements can help one to understand

Wittgenstein's conception of the nature of propositional constituents. Chapters 8-10, therefore, explain the development of Wittgenstein's conception of language as originating from his critique of Russell's conception of judgement.

The idea that a proposition has sense (and is thus true or false) by virtue of picturing reality, applies *directly* only to elementary propositions. Complex, or molecular, propositions (among which, notably, propositions of logic are to be found) are in the *Tractatus* accounted for by arguing that they are *truth-functions* of elementary ones, namely propositions whose truth-value is determined by the truth-value of the propositions occurring in them. We thus seem to be presented with a *bipartite* account; on the one hand we have elementary propositions, explained in terms of pictures, on the other we have molecular propositions, explained in terms of truth-functions. The bipartite account gives a picture of the relation of language and logic in the *Tractatus* according to which the former is somehow *prior* to the latter. Only when the question of how a proposition expresses its sense is settled can one address the question how propositions enter into logical relations with one another; only when an account of (elementary) sense is reached can one enquire into the nature of logical interconnectedness and thus into the nature of logic.

In chapter 11 I reject the bipartite account by showing that, for Wittgenstein, a correct understanding of the nature of logic is implied – and exhausted – by an understanding of the nature of language. The whole of logic, essentially, is given as soon as propositions with sense (propositions that are true or false) are given. To understand how a proposition expresses its sense (is true or false) is all that is needed to give an account of the nature of logic. What

Wittgenstein gives, in the *Tractatus*, is thus an account of the internal unity of language and logic. This general idea is spelled out in the *Tractatus* in terms of elementary propositions' already containing all logical constants in themselves.

In chapter 12 I contend that a way to understand why logical complexity is already implied in elementary propositions lies in Wittgenstein's conception of the sense (bipolarity) of a proposition, the possibility for a proposition to be true or false; a proposition's having sense (picturing reality) in fact amounts to its having two poles (the true and the false) – or truth-possibilities. Wittgenstein extends this account to molecular propositions (and thus to the propositions of logic) by considering logical constants as *operations* on elementary propositions' true-false poles, which give combinations of true-false poles (propositions) as result. Logical complexity is thus explained as the result of operations on proposition's truth-poles, which are in turn given as soon as propositions representing (depicting) reality are given.

On this account, propositions of logic (tautologies and contradictions) are conceived of as by-products of the procedure by means of which propositions can be obtained from elementary ones. Wittgenstein's conception of sense and truth, thus, makes him able to explain the relation and the difference between contingent and logical truth. A proposition of logic (tautology) is true independently of how things are in the world, as opposed to contingent propositions, whose truth and falsity can be decided only upon comparison with reality.

In the last chapter of the thesis I relate Wittgenstein's conception of logic to Frege's, and show how it is able to resolve the tensions in Frege's conception of logic discussed in the first part. I begin by analysing Wittgenstein's criticism of

Frege's conception of logic as a general science. Both Frege and Russell thought of logical propositions as being the most general laws of truth, and, accordingly, they thought logic to be the most general science of truth. Wittgenstein's account, however, shows that generality is not the mark of the logical. For a proposition to belong to logic, its truth must be independent of reality, must depend only on the internal articulation of the proposition. A proposition of logic (tautology) is true no matter what, because the senses of its constituent propositions are, by means of the application of logical operations, brought into equilibrium with each other, so that the resulting proposition conveys no information and has no content; a tautology is true only by virtue of its structure. But this implies that Wittgenstein rejects the idea that logic is the most general descriptive science, a distinctive feature of Frege's conception of logic.

Frege's idea that logic is the most general descriptive science, and thus that logical propositions have maximally general content is in tension with his idea (envisaged at least at the time of *Begriffsschrift*) that logic is formal, that it does not have a content but deals with the relations between contents. Wittgenstein, I argue, by rejecting the idea that logic is descriptive and general can endorse the formality thesis. Logic is formal, for the *Tractatus*, in the sense that logical operations do not contribute to propositions' representational content. They are instead operations on propositions' truth-conditions. No material information is added by logical constants, and for this reason they are not representatives.

I then move on to consider Wittgenstein's stance towards the idea that logic is normative for thought. For Frege the normative status of logic is consequent upon its being the most general descriptive science. In Frege the normative role

logic has for thought stems from its role of being the most general science of truth; any law is for Frege normative, but laws of logic have more right to be called normative for thought (to be called 'laws of thought'), because they are the most general laws. But by rejecting the idea that logic is descriptive, that logical propositions have general content, Wittgenstein is in a position to reject the idea that logic is normative in Frege's sense. Logic cannot be normative for thought if the reason for this is that it is the most general science of truth. There is, however, a sense in which logic is normative on Wittgenstein's conception also; logical (internal) relations between propositions are normative in the sense that they provide the justification for inferring a proposition from another (or others), and so they license inferences.

Lastly, I show that Wittgenstein's view of logic is in agreement with the constitutive strand in Frege's conception, discussed in the first part of the thesis, namely with the idea that logic provides the preconditions for thought, and is thus internal to thinking. A thought, insofar as conceived of as a proposition with a sense, is already logically adequate; this amounts to the idea that logical complexity is already given at the level of elementary propositions. Thus there cannot be something like an illogical thought, or an illogical proposition. A condition for a proposition to say what it does (for being a picture) is in fact that it shares logical form with reality, and thus logic forms a condition on a proposition's expressing its sense. Logic forms the background within which thoughts can be expressed. It is thus given as soon as propositions describing reality are given, and it constitutes, therefore, the essence of any description, the essence of representation as such.

Part One

Frege's Conception of Logic

Chapter One

Logic: Thoughts and the Predicate 'True'

[1.1] The Predicate 'True'

An investigation of Frege's conception of logic must, primarily and above all, explain what Frege thinks the nature of logic to be, and what, in his opinion, logic is about and aims at. Logic, we are told by Frege, is a science, and the word 'true' can be used to indicate its goal [see L2: 128]. Logic, for Frege, is especially concerned with the study of the predicate 'true'. "[T]he laws of logic are nothing other than an unfolding of the content of the word 'true'. Anyone who has failed to grasp the meaning of this word [...] cannot attain to any clear idea of what the task of logic is" [L1: 3]. Thus, Frege thinks that the distinctive feature of his conception of logic consists in its "giving pride of place to the content of the word 'true'" [NLD: 253].

Does this insistence on the relation between logic and truth mean that logic aims, for Frege, at discovering truths? Not quite. In fact, all the sciences have that as their goal; logic, we might say, is *internally* related to the word 'true', it is concerned with that predicate "in way analogous to that in which physics has to do with the predicates 'heavy' and 'warm' or chemistry with the predicates 'acid' or 'alkaline'" [L2: 128]. To put it in even clearer terms, Frege says that the word 'true' indicates the goal and the aim of logic as 'good' and 'beautiful' indicate those of ethics and aesthetics.

The analogy between logic on the one hand and physics and chemistry on the other (in particular the analogy between the relation they bear on predicates

such as 'true', 'heavy' and 'acid'), though useful to understand the *central* role played in logic by the predicate 'true', should not be pushed too far; physics and chemistry are particular sciences, specific branches of knowledge, with a peculiar subject-matter. They aim at discovering truths with respect to their subject-matter, truths about physical objects and chemical relations between elements, whereas logic, abstracting from any specific subject-matter, aims at discovering the laws of truth in their utmost generality; specific sciences are therefore more particular than logic; the latter, Frege says, is "the science of the most general laws of truth" [L2: 128].¹

Even the analogy between logic and aesthetics soon reveals its limitations: 'true' and 'beautiful' do not have, so to speak, the same *grammar*; "what is beautiful admits of degree, but what is true does not" [L2: 131]; of course, things can be more or less beautiful, and something can be more beautiful than something else; but things cannot be more or less true, and a thing cannot be more true than another true thing. Secondly, and more importantly, Frege argues that what is true is true independently of recognising it as such, whereas something is beautiful only insofar as somebody recognises it to be beautiful; something is beautiful for me as long as I recognise it as beautiful, but what is true is not true by virtue of my thinking it as true. This implies that there cannot arise a dispute about what is considered beautiful, there cannot be any error. Only where truth is involved does the possibility of error and of contradiction arise. Frege explains this point by saying that while 'true' is *objective*, 'beautiful' is radically *subjective*.

¹ Ricketts [1986: 80] glosses this passage by saying that, for Frege, "[t]he laws of physics are the laws of truth about mass, motion, heat, and light. In mastering these laws we come to understand *physical* truth. The laws of logic generalize over every thing [...]. It is then in studying logic that we learn whatever there is to be learned about truth *simpliciter*".

We will come back to this distinction, as it constitutes one of the cornerstones of Frege's conception of logic.

We observed that Frege begins his investigation of the nature and goal of logic by saying that it is a science intrinsically concerned with truth, and, consequently, with the predicate 'true'. We have also seen that logic has a peculiar status, which distinguishes it from particular sciences as well as from disciplines such as ethics and aesthetic; the parallels between the subject-matter of logic, the predicate 'true', and those of physics, chemistry and aesthetics, should only go so far; it should be taken as a means to understand the crucial role played by the predicate 'true' in logic; as Frege says, the previous discussion is only of use for "giving a rough indication of the goal of logic" [L2: 128].

Given the utmost importance of the predicate 'true' in defining the nature and the aim of logic, we would expect Frege to give some definition of the predicate, but, according to Frege, this is exactly what cannot be done. As he says: "[I]t would be futile to employ a definition in order to make it clearer what is understood by 'true'" [L2: *ibid.*]. Frege's argument is that such a definition would presuppose the very same thing that is to be defined: "If, for example, we wished to say 'an idea is true if it agrees with reality' nothing would have been achieved, since in order to apply this definition we should have to decide whether some idea or other did agree with reality" [L2: *ibid.*]. Frege's conclusion is that truth is "something so primitive and simple that it is not possible to reduce it to anything still simpler" [L2: 129], and that truth is *sui generis* and *indefinable*.²

² See, for example, 'Thoughts': "[A]ny [...] attempt to define truth [...] breaks down. For in a definition certain characteristics would have to be specified. And in application to any particular case the question would always arise whether it were *true* that the characteristics were present. So

[1.2] Thoughts

In order to clarify the nature of the predicate 'true' it is useful, Frege thinks, to consider to what that predicate can be applied. According to Frege, it is "for the most part, ascribed to sentences" [L2: *ibid.*]; not, however, by virtue of their being arrays of sounds or of graphic signs, but by virtue of their possessing a *sense*. Moreover, the predicate true is not applied to all sentences, it is not applied, for instance, to questions, commands and the like, but only to declarative, or, as Frege says, *assertoric* sentences, those by means of which "we communicate facts and propound mathematical laws or laws of nature" [L2: *ibid.*]. The sense of an assertoric sentence is, for Frege, a *thought* [see L2: 131]. As he says, "thoughts are senses of sentences [...]. The thought, in itself imperceptible by the senses, gets clothed in the perceptible garb of a sentence, and thereby we are enabled to grasp it. We say a sentence *expresses* a thought" [T: 354]. For Frege, then, the predicate 'true' (and thus truth) is primarily applied to the *senses* of *assertoric* sentences, which in turn can be defined as *thoughts*. Frege, in fact, means by a 'thought' "something for which the question of truth can arise at all" [T: 353].

In an assertoric sentence, Frege says, two components should be distinguished: the *content* of the sentence and its *assertoric force*. The former is the sense of the sentence, namely, as we have seen, the thought it expresses.³ The

we should be going round in a circle. So it seems likely that the content of the word 'true' is *sui generis* and indefinable" [T: 353]. For a discussion of Frege's thesis of the indefinability of truth see Sluga [2002].

³ In the early phase of his work (comprising writings like the *Begriffsschrift* and *The Foundations of Arithmetic*) Frege had not yet distinguished the two semantic notions of sense [*Sinn*] and reference/meaning [*Bedeutung*] of a sentence. In the *Begriffsschrift*, for instance, he speaks of the 'assertible content' of a sentence, [see CN: § 2] or of its 'conceptual content', [see CN: § 8]. These notions are interchangeable. In the essay 'On Sense and Meaning' Frege split that notion in those of

notion of assertion, on the other hand, depends for Frege on another notion, that of *judgement*, which should be understood as the recognition of the truth of a thought; the assertion, then, is the expression of such recognition. Three components must thus be kept distinct when analysing the notion of a thought:

- (1) The grasp of a thought – thinking
- (2) The acknowledgement of the truth of a thought – the act of judgement
- (3) The manifestation of this judgement – assertion [T: 355-356, see also IL: 185].

Grasping a thought means thus understanding the sense of an assertoric sentence; the recognition of its truth is a judgement, and, finally, an assertion is the expression of such recognition, the manifestation of the judgement.

The use made by Frege of the verb ‘grasping’, with regard to thoughts, makes it clear that, for him, thinking cannot be regarded as a process that generates thoughts (as well as it would be wrong to identify a thought with an act of thinking); thoughts are *grasped* and not *created* by thinking. “What is grasped [...] is already there and all we do is take possession of it” [L2: 137]. This also implies that thoughts, according to Frege, are *objective*, and do not depend, for their existence, on a person’s grasping them; thoughts are independent of our thinking altogether. In this context, the phraseology of ‘grasping’ should be taken quite seriously; as Frege says in the introduction to *The Basic Laws of Arithmetic*:

The picture of grasping is very well suited to elucidate the matter. If I grasp a pencil, many different events take place in my body [...]. But the totality of these events neither is the pencil nor creates the pencil; the pencil exists independently of them. And it is essential for grasping that something be there which is grasped [...]. In the same way, that which we grasp with the mind also exists independently of this activity [...]; and it is neither

sense of a sentence [the thought expressed] and its reference/meaning [its truth-value]. See [CO: 187].

identical with the totality of these events nor created by it as a part of our own mental life [BLA: 23-24].⁴

Thoughts are thus essentially different from ideas, mental pictures, mental states, which are dependent on someone's having them; it would be nonsense, in fact, to speak of a state of pain, for instance, without implying that it is someone's state of pain. Ideas and mental states are for Frege, due to their intrinsic *subjectivity*, private and incommunicable. "[I]t is impossible to compare my sense-impression with someone else's. For that, it would be necessary to bring together in one consciousness a sense-impression belonging to one consciousness and a sense-impression belonging to another consciousness" [T: 361]. But that, as we have seen, is not the case with thoughts. Frege's favourite examples of thoughts are generally laws of nature or mathematical laws, and of course such thoughts cannot be private and incommunicable.

If other people can assent to the thought I express in the Pythagorean theorem just as I do, then it does not belong to the content of my consciousness. I am not its owner, yet I can acknowledge it as true. However, if what is taken to be the content of the Pythagorean theorem by me and by somebody else is not the same thought at all, we should not really say '*the* Pythagorean theorem' but '*my* Pythagorean theorem', '*his* Pythagorean theorem' and these would be different [...] [T: 362].

A thought therefore does not belong specially to the person who grasps it, because "everyone who grasps it encounters it in the same way, as the same thought" [L2: 133];⁵ if this were not the case, Frege argues, and thoughts were ideas, then there would be "no science common to many on which many could work" [T: 362].

⁴ See Burge [1992: 639].

⁵ Of course Frege does not deny that different people can attach to the same thought, for instance the Pythagorean theorem, different ideas or mental images. But this does not affect the *objectivity* of the thought itself, and so does not make it something subjective and private.

While Frege sharply distinguishes thoughts from ideas, he also recognises that there are similarities between them; both “cannot be seen, or touched, or smelled, or tasted, or heard” [T: 360]; namely, both are *imperceptible*, and this common characteristic marks a difference between them and physical objects, which are *actual, spatial, and handleable*, [see FA: 34-35]. But thoughts share with physical objects the feature of being objective (independent of our consciousness and thinking) and hence the possibility of being *grasped*. Thus thoughts can be said to belong neither to the external world of physical things, nor to the inner world of private ideas. As Frege says:

A third realm must be recognized. Anything belonging to this realm has it in common with ideas that it cannot be perceived by the senses, but has it in common with things that it does not need an owner so as to belong to the contents of his consciousness. Thus for example the thought we have expressed in the Pythagorean theorem is timelessly true, true independently of whether anyone takes it to be true. It needs no owner. It is not true only from the time when it is discovered; just as a planet, even before anyone saw it, was in interaction with other planets [T: 363].

As well as the picture of ‘grasping’, the picture of the ‘third realm’, I believe, is not only a picturesque image Frege uses to highlight features of thoughts. Items in the third realm are for Frege “genuinely existing entities”,⁶ objective entities which timelessly exist independently of human thinking.

The characteristics we have been elucidating in Frege’s conception of a thought can help us to understand why he thinks that the predicate ‘true’ applies to thoughts. As seen, the predicate ‘true’ does not admit of degree, and what is true is so independently of one’s recognising it as such; finally, it can only be applied to what is independent of one’s consciousness (otherwise contradictions

⁶ Burge [1992: 638], who continues saying: “[Frege] opposed thinking of such entities as having some derivative status. He inveighs against any suggestion that they are product of the mind, mere symbols, or otherwise dependent on events in time” [ibid.].

and disagreements could not even in principle arise); suitable candidates to which the predicate 'true' can be applied are thus thoughts, namely senses of assertoric sentences.

The notion of 'assertion' (the expression of the acknowledgement of the truth of a thought, or, in other words, the manifestation of a judgement) can also clarify one of the 'peculiarities' of the predicate 'true'; while that predicate seems to express a *property* of thoughts (their being true), the content of the property expressed seems to be, after all, altogether *empty*. Frege explains this point by saying that "the sentence 'I smell the scent of violets' has just the same content as the sentence 'It is true that I smell the scent of violets'. So, it seems, then, that nothing is added to the thought by my ascribing to it the property of truth" [T: 354]. The truth of a thought is not expressed by ascribing to it the property 'true'. Thus "[t]he word 'true' is not an adjective in the ordinary sense" [MBLI: 251], for it "has a sense that contributes nothing to the sense of the sentence [thought] in which it occurs as a predicate" [MBLI: 252]. For Frege, the ascription of truth to a thought is expressed, instead, by expressing the thought by means of an *assertoric* sentence.

This, in turn, gives a new sense to Frege's idea that the predicate 'true', as he puts it, can give only "a rough indication of the goal of logic" [L2: 128]. The reason is that the predicate 'true' "only makes an abortive attempt to indicate the essence of logic, since what logic is really concerned with is not contained in the word 'true' at all, but in the assertoric force with which a sentence is uttered" [MBLI: 252]. For Frege, then, the truth of a thought cannot be expressed by means

of the predicate 'true'; this adds nothing to the thought itself. But a thought is put forward as true when it is expressed assertively.

In Frege's conceptual notation, his Begriffsschrift, the role of assertion, and its relation with truth, is expressed by the 'judgement (or assertion) stroke', represented by the (vertical part of the) symbol " \vdash " [see CN: § 2], which Frege puts before any formula of his system, when the formula is intended to be put forward as *true*.⁷ The judgement stroke (let us recall that a judgement is for Frege the recognition of the truth of a thought) is the first symbol to be introduced in Frege's formal system, suggesting then that logic has a fundamental relation, rather than with the predicate 'true', with the notion of *truth*, which is expressed in the assertoric force of a sentence.

⁷ Only the vertical stroke, in Frege's system, has the function of *asserting* what follows it. The horizontal stroke, called the 'content stroke', "ties the symbols which follow it into a whole" [CN: § 2], namely into, using Frege's later terminology, a single *thought*.

Chapter Two

Is Logic Normative or Descriptive?

[2.1] Laws of Truth and Laws of Thought

The aim of our introductory discussion in the previous chapter has been that of clarifying what, according to Frege, truth applies to. The answer, as we have seen, is: thoughts. Does this mean that the laws of logic, in their being laws of truth [see T: 351, L2: 145], are thus laws of thought too? Frege discusses this question at length in several writings of his, and he says that before an answer to it is possible, it is necessary to clarify the sense in which we use the word 'law'. In *The Basic Laws of Arithmetic* Frege writes:

Our conception of the laws of logic is necessarily decisive for our treatment of the science of logic, and that conception in turn is connected with our understanding of the word 'true'. It will be granted by all at the outset that the laws of logic ought to be guiding principles for thought in the attainment of truth, yet this is only too easily forgotten, and here what is fatal is the double meaning of the word 'law'. In one sense a law asserts what is; in the other it prescribes what ought to be. Only in the latter sense can the laws of logic be called 'laws of thought': so far as they stipulate the way in which one ought to think [BLA: 12].

Frege here emphasises that the word 'law' is ambiguous, and this ambiguity can be fatal for a correct understanding of what the nature of logic and its aim are. In one sense, Frege says, a 'law' 'asserts what is'; such a law, in other words, asserts how things are. Laws of nature, the laws of physics and chemistry, for instance, describe and explain natural phenomena, and their necessary regularities.⁸ A law, understood in this way, is thus *descriptive*. But there is another sense in which one can conceive of a law: a law can in fact not merely describe how things are, but

⁸ See Cerbone [2000: 295].

prescribe how things ought to be. It is in this latter sense of the word 'law' that we conceive of ethical (or juridical) laws: "[T]hese do not describe how human beings in fact behave, but how they ought to behave in order to be morally or ethically good".⁹ In this second sense of the term 'law' we speak of a law as being *normative*, or *prescriptive*.

Frege intends the laws of logic to be normative in the sense in which ethical laws are; as he puts it: "Like ethics, logic can also be called a normative science" [L2: 128]. We saw that Frege understands the laws of logic to be 'laws of truth', and therefore the aim of logic is to unfold the (most general) laws of truth [see L2: *ibid.*]. Truth also defines the 'ought' contained in the normative character of the laws of logic: "How must I think in order to reach the goal, truth?" [L2: *ibid.*]. This is the question which, according to Frege, logic answers; logic prescribes "guiding principles for thought in the attainment of truth" [BLA: 12].

The normative aspect of logical laws might be overlooked if we think of laws of logic as being 'laws of thought', because of the *descriptive* sense that that word can have. Laws of thought can, in fact, be understood as describing how thinking is actually performed by human beings: this would be an understanding of the laws of logic as "laws in accordance with which thinking actually takes place and by whose means we could explain a single thought process in a particular person in a way analogous to that in which we explain, say, the movement of a planet by means of the law of gravity" [L1: 4]. Such an interpretation of the expression 'laws of thought' would lead, according to Frege, to a *psychologistic* conception of logic, according to which laws of logic are

⁹ Cerbone [2000: *ibid.*].

psychological laws, namely descriptive laws of how thinking *is* performed and *takes* place; such a conception would not allow for the possibility that thinking is not always in accordance with the laws of logic, and it would therefore jettison the *normative* sense of the word 'law' altogether.¹⁰ Against this conception (one of Frege's main polemical targets throughout his whole career) Frege reaffirms the normative nature of logic and its connections, under this respect, with ethics. Since a misinterpretation of the expression 'laws of thought' can lead to a psychological conception of logic, Frege proposes avoiding that expression in logic.

We can also think of [...] [laws of truth] as prescriptions for making judgements; we must comply with them in our judgements if we are not to fail of the truth. So if we call them laws of thought, or, better, laws of judgement, we must not forget we are concerned here with laws which, like the principles of moral or the laws of the state, prescribe how we are to act, and do not, like the laws of nature, define the actual course of events. Thinking, as it actually takes place, is not always in agreement with the laws of logic any more than men's actual behaviour is in agreement with the moral law. I therefore think it better to avoid the expression 'laws of thought' altogether in logic, because it always misleads us into thinking of laws of thought as laws of nature [L2: 145].¹¹

Baker and Hacker hold that the idea that logic is intrinsically normative was very widespread among logicians and philosophers in the nineteenth century, so much that it can be defined as the 'established tradition'.¹² That tradition can in fact be traced back to (at least) Kant, according to whom in logic "the question is [...] not how we think, but how we ought to think. [...] In logic we do not want to know how the understanding is and thinks, but how it ought to proceed in thinking".¹³

¹⁰ See Baker, Hacker [1984: 37-41].

¹¹ Russell [PP: 40-41] makes a similar point when he says, in regard to some fundamental laws of logic: "The name 'laws of thought' is [...] misleading, for what is important is not the fact that we think in accordance with these laws, but the fact [...] that when we think in accordance with them, we think *truly*".

¹² See Baker, Hacker [1984: 37, 43].

¹³ Kant [1974: 16].

[2.2] From Laws of Thought to Laws of Truth: Logic as Normative and Descriptive

Given Frege's characterisation of logic as prescribing how to think in order to reach the truth, and not merely describing how thinking takes place, Frege's discussion of the same topic at the beginning of 'Thoughts' may be seen as surprising:

To discover truths is the task of all sciences; it falls to logic to discern the laws of truth. The word 'law' is used in two senses. When we speak of moral or civil laws we mean prescriptions, which ought to be obeyed but with which actual occurrences are not always in conformity. Laws of nature are general features of what happens in nature, and occurrences in nature are always in accordance with them. It is rather in this sense that I speak of laws of truth [T: 351].

Does not Frege here completely reverse the argument he developed in the other passages we have previously quoted? We have seen that he often associates logic to ethics in order to stress the normative character of logical laws, while, at the same time, he draws attention to the differences between the normative laws of logic and the merely descriptive laws of nature. In 'Logic', we saw, he explicitly says that the laws of logic, "like the principles of morals or the laws of the state, prescribe how we are to act, and do not, like the laws of nature, define the actual course of events" [L2: 145]. In this passage from 'Thoughts', however, he associates the laws of logic to the laws of nature, in their being descriptions of what happens.

In order to solve the puzzle, we should understand that, for Frege, normativity alone cannot *exhaust* a characterisation of logic. As Sullivan puts it: "It would be wrong [...] to say that for Frege logic is a prescriptive rather than descriptive discipline. That is the best one could get if one insisted on sticking

with the phrase ‘laws of thought’, but that is only to make the best of a bad job. Prescriptions must always rest on a basis of descriptive theory: dietary advice, for instance, is given on the basis of nutritional theory. Similarly logic in its prescriptive aspect rests on its more fundamental characterization as comprising *laws of truth*”.¹⁴ Logic has a *prescriptive* nature, namely it prescribes how one ought to think if one wants to reach truth, because its laws are, at bottom, *descriptive* (and general) laws of truth. It is possible, for logic, to prescribe how to think in order to reach truth only because laws of logic are laws of truth. Normativity, therefore, must rely on a descriptive basis.¹⁵ Frege, in fact, argues that “[f]rom the laws of truth there follow prescriptions about asserting, thinking, judging, inferring. And we may very well speak of laws of thought in this way too” [T: 351].

The contrast between the normative aspect of logic and its descriptive one can arise because the expression ‘laws of thought’, as we saw Frege arguing, is highly misleading. Interpreting laws of logic as laws of thought can easily encourage the mistake of thinking of them as descriptive laws of how thinking actually takes place, of what, in other words, happens to be the case in one’s mind. If so, then, laws of logic would be psychological laws, namely laws describing mental processes. But in that case it would be nonsense to speak of them as ‘laws of truth’, since there would be no thought, which, just by virtue of its being a thought, would be in disagreement with them. It is in order to prevent this kind of misunderstanding that Frege stresses the fact that our thinking is not always in

¹⁴ Sullivan [2004: 683].

¹⁵ Dummett [1991: 225] similarly argues that attributing to Frege the view that logic is ‘normative’ is “no more than a half-truth”. Dummett connects this view to Husserl, who claimed that “each normative [...] discipline presupposes one or more theoretical disciplines as its foundations” [1970: 87].

accordance with the laws of logic, and thus stresses the fact that those laws teach us how we *ought* to think in order to reach the truth.

It is, in fact, only because the laws of logic are laws of truth that they can have a prescriptive role in guiding one's thinking. Accordingly, from their descriptive nature there follow prescriptions about judging and inferring. For this reason Frege sometimes calls the laws of logic 'laws of judgement' or of 'valid inference' [see L2: 145, L1: 3]. This is an important component of Frege's polemic against psychologism in logic; in being laws of truth, laws of logic are laws of judgement and of valid inference, for from their role of descriptive laws of what *is* true, there follow prescriptions about how judgements and inferences *ought* to be performed if one wants to attain truth. This amounts to a conception according to which logic is concerned with the *justification* we have for holding judgements and for drawing inferences:

The grounds on which we make a judgement may justify our recognizing it as true; they may, however, merely give rise to our making a judgement, or make up our minds for us, without containing a justification for our judgement. Although each judgement we make is causally conditioned, it is nevertheless not the case that all these causes are grounds that afford a justification. [...] Now the grounds which justify the recognition of a truth often reside in other truths which have already been recognized [...]. Logic is concerned only with those grounds of judgements which are truths. To make a judgement because we are cognisant of other truths as providing a justification for it is known as *inferring*. There are laws governing this kind of justification, and to set up these laws of valid inference is the goal of logic [L1: 2-3].

Frege is here drawing a sharp line of demarcation between the laws of logic which justify judgements and inferences and psychological laws which merely describe (or explain) the causal chain of mental events that brings about a judgement or the conclusion of an inference. The distinction closely follows that between laws of truth and laws of taking-to-be true. It is precisely this distinction that a

psychologistic conception of logic does not recognise, mistaking the logical grounds of a judgement, or an inference, for their psychological ones [see L2: 147].

The notion of (logical) justification is crucial in Frege's logic, for his insistence that judgements and inferences are to be logically justified has motivations that go beyond his polemic against psychologism, and reach the heart of his logical system.¹⁶ Frege's *Begriffsschrift*, in fact, is primarily conceived by Frege as a means to express, in a totally unambiguous way, the content of assertoric sentences,¹⁷ and for proving the validity of inferences. The notion of content (of conceptual content, as it is called in the *Begriffsschrift*) is considered by Frege insofar as it "influences its *possible consequences*. Everything necessary for a correct inference is fully expressed" [CN: § 3]. Frege thus works with a notion of content that is wholly functional to the possibility of drawing valid inferences, logically *justified* ones. The laws of valid inference are therefore, in converse, also laws of logical proof, (the "firmest method of proof" [see CN: Preface]), which is Frege's core concern in the *Begriffsschrift*.

[2.3] Normativity, Descriptivity, Generality

The status of logic is thus, for Frege, twofold; logic is *both* descriptive and normative; the normative status of logic (prescribing norms for thinking is so far as truth is concerned) stems from its descriptive status (being laws of truth).

[L]aws of truth [...] must provide the norm for holding something to be true. And these will be the laws of logic proper [L2: 146].

¹⁶ See Goldfarb [2001: 33-34].

¹⁷ *In primis*, of course, those of mathematics.

From the laws of truth there follow prescriptions about asserting, thinking, judging, inferring [T: 351].

So it is the descriptive status of logic – as the science of truth – that grounds for Frege its normative authority over one’s thinking in so far as it aims at truth. But how exactly does Frege conceive of this dependency?

William Taschek has recently pointed out that “Frege held that from *any* law, any general truth, there follows a prescription that one ought to think in accordance with it. In this case, laws of logic are not distinctive *in virtue of their issuing in prescriptions about how one ought to judge*, but are distinctive only in virtue of the maximal generality of their subject matter and, so, only in the maximal *scope* of their prescriptive influence”.¹⁸ This is so because – as we said above – Frege thinks that any descriptive law is, by virtue of being descriptive, also normative.

As Frege puts it:

Any law that states what is can be conceived as prescribing that one should think in accordance with it, and is therefore in that sense a law of thought. This holds for geometrical and physical laws no less than for logical ones [BLA: 12].

For Frege, however, laws of logic have – at least in virtue of their scope – more right to be called prescriptive (or normative) than laws of other sciences. As Frege says, logical laws “have a special title to the name ‘laws of thought’ only if we mean to assert that they are the most general laws, which prescribe universally the way in which one ought to think” [BLA: 12]. It seems, then, that for Frege any descriptive law is at the same time normative. But is there any sense in which descriptive laws of logic have – as it were – a *higher degree* of normativity than laws belonging to other sciences? On this Taschek comments:

¹⁸ Taschek [2008: 382].

[I]t would seem that Frege is suggesting that the normative status of the laws of logic, their normative authority over our thinking, is immediately *consequent* upon their status for us as general truths. In this respect, the normative authority that the laws of logic have over our thinking is not different in *kind* from that which derives from our acceptance of any law, whether in physics, geometry, or psychology.¹⁹

Now, for Taschek, this account of the normativity of logic – seen as being not qualitatively different from the normativity belonging to any other science – is problematic. As he argues:

To be sure, if I do acknowledge the truth of a law of physics – or, for that matter, of any other special science – I *do* thereby find myself under an obligation to ‘think in accordance with it’. But what can this mean except that I am under a *logical* obligation to have my other judgements be consistent with this one and, in particular, to be willing to acknowledge as true all instances of the general claim? This, though, is an essentially logical obligation; it does not in any sense derive from the law of physics itself, independently of the normative demands of logic. The normative demands of logic must, so to speak, already be in place for there to be any obligation, once one has committed oneself to the truth of a law of physics, to ‘think in accordance with’ that law. The latter obligation presupposes wholly general logical obligations.²⁰

The idea is that the normativity that pertains to logic is somehow independent of – and surely *prior* to – the normativity of special sciences. In order for the laws of special sciences to have normative force upon thought, the normativity of logic has to be already in place. In fact, it seems, the normative force upon thought that a law of – for instance – physics has, consists in setting forth an obligation to have judgements *logically* consistent with that law. This seems to mean that the normativity of logic is thus different in *kind* – and not merely in scope – from the normativity of special sciences. But what does this distinctive logical obligation consist in? What does it mean that acknowledging a truth means to be under a (logical) obligation to have other judgements be consistent with it? According to Taschek that amounts to acknowledging a rule of

¹⁹ Taschek [2008: 383].

²⁰ Taschek [2008: 387].

inference, that is, to commit oneself “to a logical norm concerning judgements [...] where this commitment cannot itself consist in (or be grounded upon) our accepting as true some further substantive law. If this is right, then the prescriptive force of any *substantive* logical law must itself be viewed as grounded in our prior commitment to some fundamentally non-descriptive normative logical principles, which themselves are not grounded in our acceptance of any more fundamental substantive claim”.²¹

Taschek’s claim is thus that Frege has to accept the view that logic has a *distinctive* normative dimension, which grounds (but is not to be confused with) the prescriptive force pertaining to other special sciences. His claim is thus that the normative character of logic is shown by non-descriptive principles – laws of inference – that guarantee that thinking proceeds in accordance with principles already accepted (laws of truth); however, Taschek argues, these normative principles are independent of – and do not rest on – general principles of truth, general laws of truth.

It seems, however, that Frege did not have an understanding of the notion of an *inference* independent of that of *truth*; on the contrary, Frege seems to think that non-descriptive normative logical principles (laws of inference) *are* grounded in one’s acceptance of substantive claims (laws of truth). As seen, Frege discusses inference as follows:

Logic is only concerned with those grounds of judgement that are truths. To make a judgement because we are cognisant of other truths as providing justification for it is known as *inferring*. There are laws governing this kind of justification, and to set up these laws of valid inference is the goal of logic [L1: 3].

²¹ Taschek [2008: 388].

For Frege inference has an intrinsic connection with truth. This explains why he thought that logic is normative because it prescribes norms for thought in so far as it is aimed at truth. Logic is normative because it sets forth how one must think in order to reach the goal, truth. A criticism of this position is advanced by Simon Evnine: “[T]hinking logically helps us to think truly if we start with true beliefs from which we logically derive others. So how can logic as such help us achieve the goal of thinking truly? Would it not make more sense to hold that logic is normative for our thought if we want to think consistently, or rationally, or coherently?”²² Now, the problem with this criticism – as Evnine is of course well aware of – is that it begs the question against Frege’s notion of inference. For Frege to infer means to “recognize a truth on the basis of other previously recognized truths according to a logical law” [PMC: 17]. It seems, therefore, that the normative status of logic – which surely for Frege has to do with the acceptance of laws of inference – cannot be divorced from the acknowledgement of substantive general truths (primitive laws of logic); and both – primitive logical laws and rules of inference – are considered by Frege self-evident and thus not in need of proof or justification.²³ This is the sense in which, I think, Frege conceives the *normativity* of logic as resting upon its being a *descriptive* science of truth.

²² Evnine [unpublished].

²³ See Burge’s excellent discussion in Burge [1998]. See also section [4.4] below.

Chapter Three

Is Logic Formal?

[3.1] Formality of Logic and the Begriffsschrift

We saw in the previous chapter that Frege retains a twofold role for logic. Logic is a normative and descriptive science, prescribing how one should think, because its laws are, at bottom, descriptive laws of truth. Intrinsic to Frege's conception of logic as descriptive is the idea that logic is *general*, that its laws are the most general laws of truth; whereas particular sciences aim at discovering truths with regard to their subject-matter (physics, for example, aims at discovering truths about mass and motion, chemistry about the composition of elements, and their reciprocal reactions), logic aims at discovering *the* laws of truth, which hold for any science whatsoever. "[W]e do not demand of [...] [logic] that it should go into what is peculiar to each branch of knowledge and its subject-matter. On the contrary, the task we assign to logic is only that of saying what holds with the utmost generality for all thinking, whatever its subject-matter" [L2: 128]. Therefore, in Frege's conception, logic is the most general science, whose general laws of truth are more general than those peculiar to any particular science.

This feature of Frege's conception of logic has recently been highlighted in order to show that, for Frege, logic is *contentful* and *general*, and, therefore, not formal. Logic is for Frege not *intrinsically* different from particular sciences, for it is simply more general than any of them. It has a wider range of application, so wide that it can be said to encompass everything. Logic's laws of truth are therefore

laws which hold for everything whatsoever.²⁴ For this reason, Frege's conception of logic has been labelled 'universalist', and contrasted with contemporary (so-called 'schematic') conceptions of logic, according to which logic is formal, because its laws are not contentful, but are metalinguistic representations (schemata) of the form of sentences.²⁵

According to Warren Goldfarb, the idea that logic is for Frege not formal, but on the contrary contentful and general, "comes out early in his work, in the contrast he makes between his *begriffsschrift* and the formulas of Boole".²⁶ Øystein Linnebo, on the contrary, argues that Frege retained (in the early phase of his thought) the idea that logic is formal; moreover, he justifies this by relying on the same source, namely on Frege's polemic against Schröder (and not only Schröder), who held Frege's conceptual-notation to have the same aim as Boole's logical calculus. As Linnebo says:

Some interpreters have taken Frege's distinction between a calculus and a language as evidence that Frege regarded logical truths as contentful. But a careful reading of the relevant passages shows that what Frege had in mind wasn't this strong claim that the purely logical part of the *Begriffsschrift* has content all by itself, but the weaker claim that this language allows content to be expressed *when contentful signs from the special sciences are added to it*.²⁷

The solution to the controversy can perhaps be found in a careful reading of those texts in which Frege, attempting to clarify the misunderstanding of assimilating his conceptual notation to that developed by Boole, provides the

²⁴ See, in particular, Goldfarb [2001: 28] and [1979: 352], Ricketts [1986: 76] and [1996: 59-61], Van Heijenoort [1967: 325].

²⁵ See especially Goldfarb [2001].

²⁶ Goldfarb [2001: 27].

²⁷ Linnebo [2003: 241].

reader with important insights about the nature and aim of his own symbolism.²⁸

Goldfarb gives pivotal importance, in attributing to Frege the ‘universalist’ conception of logic with its embedded idea that logic is not formal, to the following passage from ‘On the Aim of the ‘Conceptual Notation’’:

I did not wish to present an abstract logic in formulas, but to express a content through written symbols in a more precise and perspicuous way than is possible with words. In fact, I wished to produce, not a mere *calculus ratiocinator*, but a *lingua characteristic* in the Leibnizian sense [ACN: 90-91].

This passage does not, in my opinion, constitute enough evidence for the claim that, according to Frege, logical truths are contentful and thus that logic is not formal. What it shows is the overarching preoccupation Frege had that his *Begriffsschrift* must enable us to unambiguously express the content of propositions.²⁹ This is also testified by Frege’s conviction that a true conceptual notation “must have simple modes of expression for the logical relations which, limited to the necessary, can be easily and surely mastered. These forms must be suitable for combining most intimately with a content” [SJC�: 88]. Boole’s calculus is, in Frege’s words, only “a clothing of abstract logic in the dress of algebraic symbols. It is not suited for the rendering of a content, and that is also not its purpose. But that is exactly my intention” [ACN: 93].

What Frege has in mind here, I think, is not a distinction between his symbolism, with its alleged *contentful* logical truths, and the alleged *formal* symbolism devised by Boole. To interpret the matter in this way is to be misled by the expression ‘abstract logic’, as if it meant merely ‘formal logic’. What Frege

²⁸ The texts in question, besides the *Begriffsschrift* itself, are [ACN], [BCCS], [BFLCS], and [SJC�]. See, on Frege’s criticism of Boole’s concept-script, Sluga [1987].

²⁹ Frege’s primary interest, as already recalled, was in the content of arithmetical propositions; but his *Begriffsschrift* is not limited to that.

intends to say is that his concept-script is able to express the content of propositions, and to unambiguously show the logical relations between them, whereas Boole's symbolism is not. Boole's logic, according to Frege, is "a technique for resolving logical problems systematically [...] [in which] there is no concern about content whatsoever" [BCCS: 12]. In order to explain this point, it could help to look at what Frege says of his symbolism.

In § 1 of the *Begriffsschrift* Frege says that he will use, in his book, two kinds of symbols: those which have a completely *fixed sense*, and those which one can take to *signify various things*. Frege discusses this distinction in 'On the Aim of the 'Conceptual Notation'', where he writes that in his *Begriffsschrift* he put together the existing symbols of mathematics with new symbols, in order "to form a single formula language"; and he goes on saying that "[i]n it, the existing symbols [of mathematics] correspond to the word-stems of [ordinary] language; while the symbols I add to them are comparable to the suffixes and [deductive] formwords {*Formwörter*} that logically interrelate the contents embedded in the stems" [ACN: 93].

In the first part of the *Begriffsschrift*, the symbols 'which can take to signify various things' are capital Greek letters, such as *A*, *B*, *Γ*, whereas (some of) the symbols 'which have a completely fixed sense' are:

The judgement stroke: , which will be to the left of the symbols 'signifying various things', namely capital Greek letters.

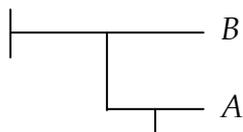
The conditional stroke: 

The negation stroke: 

The latter two symbols are symbols for sentential connectives, in particular for conditionality and negation (by means of which all the other sentential connectives can be obtained). We see then how these two different kinds of symbol can work together. The formulae ' $A \rightarrow (B \rightarrow C)$ ' and ' $(A \rightarrow B) \rightarrow C$ ' are expressed in Frege's bidimensional symbolism in the following way:



The disjunction between A and B can be obtained by the joint use of the conditional stroke and the negation stroke in the following way:



This expression can be translated in modern symbolism as ' $\neg A \rightarrow B$ ', which has the same truth-conditions as ' $A \vee B$ '.

It is interesting to note that Frege explicitly explains the nature of the two different symbols he uses by resorting to the dichotomy formal/material. He says that every highly developed language must consist of two parts:

[W]e may distinguish the *formal* part which in verbal language comprises endings, prefixes, suffixes, and auxiliary words, from the *material* part proper. The signs of arithmetic correspond to the latter [BCCS: 13, my emphasis].

Accordingly, the Begriffsschrift is supposed to provide the *logical scaffolding* (through the signs which have a completely fixed sense) necessary to express the content of (mathematical) propositions. The signs of arithmetic, the material part of the language, are thus to implement the formal part of the Begriffsschrift. Once the Begriffsschrift is implemented by the material part, the one that expresses a content, it becomes a totally unambiguous *language*.

As it is, however, the Begriffsschrift *is* formal, in the sense that the content provided by the material part of the language is still lacking; the place of the material part (actual propositions, of arithmetic, for instance) is taken by the signs ‘which can signify various things’, namely the capital Greek letters A, B, Γ . These symbols can signify various things in the sense that they can be taken as undetermined placeholders for which actual sentences can be substituted. Frege explicitly acknowledges this when he says that the formulae of his conceptual notation “are actually only *empty schemata*; and in their application one must think of whole formulae in the places of A and B – perhaps extended equations, congruences, projections” [ACN: 97, my emphasis].

One can object to this conclusion by suggesting that all the formulae of the Begriffsschrift follow the content-stroke (which is the horizontal part of the judgement stroke), and what follows the content-stroke, as Frege himself says, “must always have an assertible content” [CN: § 2]. This seems to suggest that what follows the content stroke, thus any formula of the Begriffsschrift, must have a content. But I do not see much force in this suggestion. What Frege is saying here, is that what follows the content stroke must be a *proper thought*, and thus that the schematic letters A, B, Γ , must be replaced by thoughts, something, as we

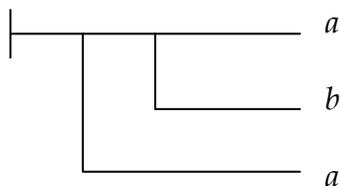
recall, “for which the question of truth can arise at all” [T: 353]. Frege is merely saying, therefore, that not everything can follow the judgement stroke, but only what can be *judged* or *asserted*, that is, thoughts. The idea ‘house’, for instance, cannot follow the content stroke [see CN: § 2], namely cannot be substituted for one of the capital Greek letters. But this in no way compromises the formality of the Begriffsschrift; according to contemporary ‘schematic’ conceptions of logic, likewise, what can be substituted for the schematic letters is to be proper sentences, and not merely names or predicates. Such a substitution would not be an *interpretation* of a schema, but it would make it simply nonsense.

We have thus seen that Goldfarb’s claim that for Frege logic is not formal cannot be supported by resorting to his polemic against Schröder about the similarities and differences between his own conceptual-notation and Boole’s. On the contrary it emerges, from that polemic, that Frege took the Begriffsschrift as providing the formal part of a language (the prefixes, suffixes, and auxiliary words) which should be joint together with the material part (that provided with a content) represented by the formulae of arithmetic. In other words, it seems as though Frege took the Begriffsschrift as being a schematic logic to be implemented by contentful sentences, in order to produce a language in which the logical relations between sentences could be wholly and perspicuously expressed.³⁰

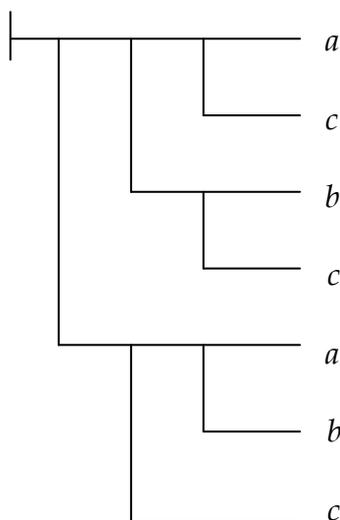
It would be premature, however, to dismiss the question of the formality of logic so quickly. The question is that what we have been saying about the formality of Frege’s concept-script only holds for the *first part* of the *Begriffsschrift*

³⁰ I think the texts where Frege criticises Schröder and Boole, here discussed, provide the best evidence for Putnam’s [2000: 220] claim that “the frequently heard statement that for Frege the laws of logic are like ‘most general laws of nature’ is not the whole of the story”.

(that concerned with the 'definition of the symbols'). In the second part (where Frege develops 'some judgements of pure thought'), the capital Greek letters A, B, Γ , used as indefinite symbols of assertible contents, and thus as empty schemata, disappear. In the second part we find Frege's exposition of the axioms of his system, namely those propositions from which it is possible to derive all other theorems; and Frege's axioms, besides the logical signs, such as the judgement stroke, the content stroke, the conditional stroke, the negation stroke, and the sign for quantification and identity of content (and the function/argument distinction), contain only italic letters, such as ' a ', ' b ', ' c ', like in the following two:



to be read as ' $a \rightarrow (b \rightarrow a)$ ', and:



to be read as ' $(c \rightarrow (b \rightarrow a)) \rightarrow ((c \rightarrow b) \rightarrow (c \rightarrow a))$ '.

Frege says that italic letters express generality, in a different way from how German letters do; the latter express generality when the scope of generality is

limited to a part of a formula [see CN: § 11], the former, on the other hand, express generality when the scope of it is the whole of an asserted formula.³¹ This means that those formulae cannot be read as *schemata*, but as *fully quantified statements*, where the letters are quantified variables. In this sense, therefore, Frege's axioms are not schemata that await interpretation, namely that need to be interpreted by means of substituting their schematic letters with sentences, for they are "not schemata, but fully interpreted, genuine truths";³² they state laws about everything whatever. This provides evidence against Frege's adoption of the idea that logic is formal.

Yet, a reading of the explanation Frege gives of axiom 1 (the first formula represented above) of the *Begriffsschrift* casts some doubts about the interpretation we have been just given: Frege says that that axiom should be read as saying that the case in which *a* is denied, *b* is affirmed and *a* is affirmed is excluded, and that this is obvious since *a* cannot be at the same time be affirmed and denied. But he also says that it is possible to express the judgement in the following way:

'If a proposition *a* holds, it holds also in the case an arbitrary proposition *b* holds'. For example, let *a* stand for the proposition that the sum of the angles of the triangle *ABC* is two right angles and *b* stand for the proposition the angle *ABC* is a right angle. Then we obtain the judgement: 'If the sum of the angles of the triangle *ABC* is two right angles, this holds also for the case in which the angle *ABC* is a right angle'[CN: § 14].

According to Frege's reading of the axiom in question, the italic letters in the axiom seem to stand for *propositions*, namely *linguistic items*. And this seems to suggest a metalinguistic (schematic) reading of the axiom, which, however, seems

³¹ See Bynum [1972: 68-69]. See also [CN: § 11].

³² Sullivan [2004: 680].

hard to reconcile with the idea (distinctive of ‘universalist’ conceptions of logic) that logical truths are genuine and wholly general statements about reality.

There seem to be tensions, therefore, in Frege’s attitude towards the idea of the formality of logic. On the one hand he seems to reject the idea of a *formal generality*, namely the idea that logical laws are not about anything substantial, but about logical forms, empty schemata. Frege’s idea of generality, on the contrary, seems to be tied up to the role of quantifiers; as Van Heijenoort says: “The universality of logic expresses itself in an important feature of Frege’s system. In that system the quantifiers binding individual variables range over all objects”.³³ But, on the other hand, we have seen that Frege himself discusses his *Begriffsschrift* in terms that seem to make its logical system coherent with the idea that logic is formal. In describing the *formal* and the *material* part of a language, Frege seems to retain for the former the function of providing a logical scaffolding, to be implemented by contentful sentences belonging to the material part. And in discussing this formal aspect of his logical system, Frege explicitly claims the formulae of the (first) part of the *Begriffsschrift* to be empty schemata. At the same time, in explaining his axioms (for which, apparently, the schematic function does not hold), he seems to offer a ‘metalinguistic’ interpretation of them, claiming that we can interpret the italic letters in them as standing for *propositions*.

Thus, while Frege’s conception of generality clearly points towards a ‘universalist’ understanding of logic, thus towards reading his italic letters as universally quantified variables, other passages of Frege’s, in particular those concerned with his polemic against Schröder, point towards an opposite direction,

³³ Van Heijenoort [1967: 325].

namely towards a distinction between a formal (logical) part of language and a material (contentful) one, and thus towards an interpretation of his logical formulae as empty schemata.

[3.2] Frege and Kant on the Formality of Logic

In order to better understand Frege's attitude towards the idea of the formality of logic, it is perhaps helpful to look at Kant, who, as we mentioned at the end of section [2.1], shared with Frege the idea that logic is normative for thought. Kant explains the idea that logic is formal in the following terms:

If [...] we set aside all cognition that we must borrow from *objects* and reflect solely upon the use of the understanding in itself, we discover those of its rules which are necessary throughout in every respect and regardless of any special objects [...]. Insights into these rules can therefore be gained a priori and *independently of any experience*, because they contain, *without discrimination between objects*, merely the conditions of the use of the understanding itself [...]. And it also follows from this that the universal and necessary rules of thought in general can concern solely its *form*, and not in any way its *matter*.³⁴

This conception of logic as general *because* formal is explained by Kant through a comparison with grammar; logic is similar to a *general grammar*, "which contains nothing beyond the mere form of a language in general, without words, which belong to the matter of language".³⁵

This sounds interestingly similar to Frege's discussion of his symbolism, where he says that "we may distinguish the formal part which in verbal language comprises endings, prefixes, suffixes, and auxiliary words, from the material part proper." [BCCS: 13]. The formal part of a language, as we have seen, is that with

³⁴ Kant [1974: 14-15]. Logic can thus be seen, on Kant's perspective, as providing the *form of coherent thought*, as Putnam [2000: 220] observes. The relation between Frege and Kant with regard to the idea of the formality of logic is discussed in depth in MacFarlane [2002]. In the remainder of this section, I will often refer to this work.

³⁵ Kant [1974: 15].

which Frege's Begriffsschrift is concerned, that which is to be implemented by the material part of the language, for instance, the signs of arithmetic. The formal part of a language, therefore, is the one that logically inter-relates contents by providing a grammar, or syntax, while these contents are in turn provided by the material part of language.

But to say that logic is formal, in Kant's sense, has consequences that Frege would not accept. In fact, as MacFarlane glosses Kant: "To say that logic is [f]ormal [...] is to say that it is completely indifferent to the semantic contents of concepts and judgements and attends only their forms. For example, in dealing with the judgement that some cats are black, logic abstracts entirely from the fact that the concept *cat* applies to cats and the concept *black* to black things, and considers only the way in which the two concepts are combined in the thought: the judgement's *form* [...]"³⁶

According to this conception, logic deals with the *logical forms* of judgements, determined by the *logical signs* which occur in them, while it abstracts from the *content* of judgements themselves; logic is thus only concerned with judgements' form, and not with their content.³⁷ However, there is an explicit passage from Frege's 1906 essay 'On the Foundations of Geometry: Second Series', which shows that he rejected this conception. If logic were formal in this sense, Frege argues,

then it would be without content. Just as the concept *point* belongs to geometry, so logic too, has its own concepts and relations; and it is only in virtue of this that it can have a content. *Toward what is thus proper to it, its relation is not at all formal.* No science is completely formal; but even gravitational mechanics is formal to a certain degree, in so far

³⁶ MacFarlane [2002: 28-29].

³⁷ This position was also held by Bolzano, who saw logic as dealing with the 'forms of judgement'. MacFarlane sees Bolzano as inheriting a Kantian view; see MacFarlane [2002: 45].

as optical and chemical properties are all the same to it. [...] To logic, for example, there belong the following: negation, identity, subsumption, subordination of concepts [FG2: 338].

Whereas for Kant (and the contemporary 'schematic' conceptions) logical signs, like the quantifiers and the sentential connectives are an indication of the form of a judgement, or of a statement, and do not have a content themselves, for Frege they do have a content, and logic, therefore, must be concerned with such a content.

MacFarlane explains this difference between Kant and Frege as following:

Whereas on Kant's view the 'some' in 'some cats are black' is just an indicator of form and does not itself have a semantic content, Frege takes it (or rather its counterpart in his Begriffsschrift) to have its own semantic content, to which logic must attend. The existential quantifier refers to a second-level concept, a function from concepts to truth values. Thus logic, for Frege, cannot abstract from all semantic content: it must attend, at least, to the semantic contents of the logical expressions, which on Frege's view function semantically just like non-logical expressions.³⁸

Frege viewed quantifiers as standing for genuine concepts, in particular as second-level concepts, namely as functions from (first-level) concepts to truth-values. Sentential connectives, as well, Frege argues, stand for functions, and thus are expressions whose reference is a function; in the case of connectives, they refer to functions from truth-values to truth-values. But then, according to Frege, quantifiers and connectives are not mere indicators of the *form* of a judgement, for these expressions do have a semantic *content*, which, in turn, is seen by Frege as what logic must attend to. If so, then, logic cannot be considered *completely formal*, because logic cannot abstract from the content of some concept-expressions (among others, connectives and quantifiers). Logic, Frege seems to imply, has its proper 'notions', or 'concepts', such as negation, identity, conditionality etc., and for this reason it is not completely formal.

³⁸ MacFarlane [2002: 29-30].

But then, the difference between *logical* and *non-logical* vocabulary seems to be blurred, for both logical expressions (quantifiers and connectives) and non-logical ones (names and predicates) have semantic content. That distinction is on the contrary wholly retained under Kant's perspective (and the 'schematic' conceptions); for Kant logical expressions function as indicators of logical form, and not as expressions provided with content. But if logical expressions have, for Frege, a semantic content as well as do non-logical expressions, then one must assume that the distinction between the *formal* and the *material* part of a language (the distinction that Frege put forward in his polemic essay against Boole) is one that Frege rejected later in his career.

[3.3] Concluding Remarks

There is a significant chronological difference, in fact, between the essays in which Frege seems to endorse the idea that logic is formal (all belonging to his early period, and in particular, to the years 1879-1883) and the essay from which we quoted the passage in which Frege rejects the idea that logic is 'unrestrictedly formal' (1906). Linnebo argues that Frege was, in his early period, committed to the Kantian view that logic is formal, but that, later on, due to his increasingly commitment to logicism, he abandoned it. In *The Foundations of Arithmetic*, the work in which Frege firstly put forward the idea that arithmetic is (or, better, can be) part of logic, there is no sign of the idea that logic is formal. For Linnebo, the idea of logic as a formal science "was a rather short-lived part of Frege's view of logic. It conflicts with Frege's view in [...] [*The Foundations of Arithmetic*] that the numbers are logical objects. If logical objects exist, then logic will be a science with

its own realm of objects, in which case logic can no longer be said to ‘disregard the particular characteristics of objects’’.³⁹

MacFarlane, on the other hand, does not put Frege’s rejection of the formality of logic in chronological terms, but he seems to implicitly assume that Frege never held that logic is formal; he agrees that the view according to which logic is formal “is evidently incompatible with Frege’s view that logic can supply us with substantive knowledge about objects – for example, the natural numbers”;⁴⁰ but he goes on saying that “Frege has reasons for rejecting it that are independent of his commitment with logicism and logical objects”.⁴¹ We have in fact seen that the reasons Frege adduces for his rejection of the idea that logic is formal, in the passage quoted, have nothing to do with logicism. He claims that logical expressions, such as quantifiers and connectives, have a content of their own, for they refer to concepts. And since these concepts are peculiar to logic, logic’s relation to them is not formal; in other words, logic cannot abstract from all the content of judgements, for quantifiers and sentential connectives are part of the content of a judgement, and not mere indicators of its logical form.

This position, however, is in contrast with Frege’s discussion of the aim of his *Begriffsschrift* in the essays earlier analysed. For there Frege says that his major preoccupation, the unambiguous expression of a content, could be met by means of a separation between different symbols, with different functions; symbols ‘that can signify various things’, which express a content when implemented with actual sentences, and signs with ‘fixed sense’, whose function is to logically

³⁹ Linnebo [2003: 244].

⁴⁰ MacFarlane [2002: 29].

⁴¹ MacFarlane [2002: *ibid.*].

interrelate the former signs. In that distinction the function of expressing a content seems entirely to be left to the 'indeterminate' signs, those which, once implemented, constituted the *material* part of a language, as opposed to the *formal* one, represented by the logical signs. But if, as we saw, Frege thought that logical signs, belonging to the formal part of a language, have semantic content of their own (for they refer to concepts), then the distinction between formal and material part of a language vanishes, and logical signs will appear, as MacFarlane says, as functioning semantically just like non-logical ones.

We can thus find a tension in Frege's thought, between a view of logic as formal and a view of it as contentful and general; this is highlighted by Hilary Putnam, who claims that "[Frege's] writing reflects a tension between the pull of the Kantian view [according to which logic is formal] and the pull of the view that the laws of logic are simply the most general and the most justified views we have".⁴² I do not think that a straight solution is ready at hand. I noted, in fact, that it is not completely plausible to dismiss this question by assuming that Frege, after having endorsed the idea that logic is formal, later rejected it, as a consequence of his logicism. This is due to the fact, earlier noted, that the grounds for Frege's rejection of the idea that logic is 'unrestrictedly formal' (in the 1906 excerpt) do not resort to logicism, but simply to a characterisation of logic as having its own peculiar concepts, and thus of logical expressions as having a particular semantic content.

⁴² Putnam [2000: 220].

Chapter Four

Is Logic ‘Constitutive’ of Thought?

[4.1] The Idea of ‘Constitutivity’, and its Problems

We have discussed, in section [2.2], the twofold role (normative and descriptive) of logic, on Frege’s conception. In their being descriptive laws of truth, laws of logic set forth at the same time prescriptions about how one ought to think in order to reach the truth, because they establish the standards of valid inference. This, now rather familiar, line of thought is developed in an already quoted passage from *The Basic Laws of Arithmetic*, but with a striking conclusion:

Any law asserting what is, can be conceived as prescribing that one ought to think in conformity with it, and is thus in that sense a law of thought. This holds for laws of geometry and physics no less than for laws of logic. The latter have a special title to the name ‘laws of thought’ only if we mean to assert that they are the most general laws, which prescribe universally the way in which one ought to think *if one is to think at all* [BLA: 12, my emphasis].

Here Frege does not merely reaffirm the compatibility between descriptive and normative aspects of the laws of logic, he goes as far as saying that laws of logic prescribe how thinking is to be performed if it is to be thinking *at all*. This is much stronger a claim, which suggests an altogether new role for laws of logic, that of being ‘constitutive’ of thinking, in the sense of being “constitutive of the possibility of thought”.⁴³ We noted earlier that Frege shared with Kant the idea of the normative nature of logic. In the passage from Kant’s *Logic* partially quoted at the end of section [2.1], it also seems to emerge the conception of logic as constitutive of thought. Kant writes in fact that “in logic we do not want to know

⁴³ Conant [1991: 134]. This is what Linnebo [2003: 244-246] calls Frege’s ‘constitutivity thesis’ with regard to logic; I will follow him in using this expression.

how the understanding is and thinks, but how it ought to proceed in thinking. Logic shall teach us the right use of the understanding, i.e. *the one that agrees with itself*".⁴⁴ Logic, for Kant, provides the rules "which are necessary throughout in every respect and regardless of any special objects, because without them we would not think at all";⁴⁵ in the *Critique of Pure Reason* we find the similar claim that logic, insofar as it is "the science of the rules of the understanding in general, [...] contains the absolutely necessary rules of thought without which there can be no employment whatsoever of the understanding".⁴⁶

But what does it mean to say that the laws of logic are *constitutive* of thought? Some indications towards an answer come from § 14 of *The Foundations of Arithmetic*, where Frege warns the reader against overestimating the degree of similarity between geometry and arithmetic; according to Frege "arithmetic lies deeper than any of the empirical sciences, and even than that of geometry" [FA: 21].⁴⁷ The laws of geometry hold good of what is spatially intuitable (conceivable), whereas the laws of arithmetic extend more widely, for they "govern all that is numerable", and this, Frege says, "is the widest domain of all; for to it belongs not only the actual, not only the intuitable, but everything thinkable" [FA: *ibid.*]. Laws of logic, then, in being even more basic than those of arithmetic, have as their domain 'everything thinkable'. But then, if all that is thinkable lies within the laws of logic, what contradicts those laws, what is outside the realm of logic, will lie

⁴⁴ Kant [1974: 16, my emphasis].

⁴⁵ Kant [1974: 14].

⁴⁶ Kant [1929: 93]. According to Putnam [2000: 219], in Kant's lectures on logic and the *Critique of Pure Reason*, we find "the repeated insistence that illogical thought is not, properly speaking, thought at all"; this, I take it, is an ascription of the 'constitutivity thesis' to Kant.

⁴⁷ Again, we can take what Frege says about arithmetic, in this context, as holding for logic too. Frege holds in fact that "it is impossible to effect any sharp separation of [mathematics and logic]" [FA: IX].

outside of the realm of the thinkable too. Frege, in fact, says that “we have only to try denying any of [...] [the laws of arithmetic and logic], and complete confusion ensues. Even to think at all seems no longer possible” [FA: *ibid.*].

We have seen that Frege defines the laws of logic as (general) ‘laws of truth’, and says that from this it follows that the laws of logic have a prescriptive role with regard to how one ought to think, namely that they prescribe how judgements and inference must be carried out if the goal is attaining truth. But now we see that Frege affirms something much stronger than that; he says that laws of logic ‘prescribe universally the way in which one has to think if one is to think at all’, and that, therefore, if one denies those laws, he cannot be said to *think* at all, because the result is no more than ‘complete confusion’.

Can this conception account for the possibility of logical mistakes? Of course, and Frege recognises this several times in his writings, “[w]e can make mistakes” [L1: 2], and we can draw incorrect inferences [see L1: 2-4]; that is exactly why laws of logic cannot be reduced to psychological laws. How can we be said, when contradicting the laws of truth, not to be thinking at all? In a somehow Cartesian fashion, we can say that of course if I take something false to be true I am *thinking*, in particular, I am making an incorrect *judgement*; this can be the result of a wrong inference. But it does not at all seem that my thinking is nothing but ‘complete confusion’. But now, if the laws of logic are defined as what *constitutes* thought, then a logical mistake, or an incorrect inference, would seem not to be thinking altogether, and this is absurd. Conversely, we can say that since it seems evident that I am *thinking*, then, according to the ‘constitutivity thesis’, my thinking is, just by virtue of being such, in accordance with the laws of logic; but,

again, this is absurd since I am making incorrect judgements or incorrect inferences. Such a position, in other words, would seem not to allow for the possibility of logical mistakes, and this is strikingly odd, for Frege resorts to this very possibility in motivating his anti-psychologistic conception of logic.

We have briefly noted that Kant held something similar to the ‘constitutivity thesis’, for he thought that a logical mistake would be “a form of thinking contrary to the understanding”,⁴⁸ namely contrary to the faculty of thought, and, hence, contrary to itself. In fact, Kant acknowledges that, according to such a position, “[h]ow an error [...] is possible is difficult to comprehend; as indeed how any force should deviate from its own essential laws”.⁴⁹ But Kant has his story to tell in order to give a solution to this seemingly paradoxical account of logical mistakes, a story that presupposes his theory of the mind and in particular its distinction between the different faculties of *sensibility* and *understanding*.⁵⁰ This story, however, is not available to Frege, and so his solution to the puzzle (if there is any) must lie elsewhere.⁵¹

Before discussing Frege’s stance on this question, however, it is indispensable to investigate the matter more closely, for, while some passages of Frege’s seem to suggest that he held the idea that logic is constitutive of thought, others seem to show that he did not. Frege’s commitment to the ‘constitutivity thesis’, therefore, seems not to be as straightforward as his commitment to the idea that the laws of logic are normative and descriptive. In the next section I will

⁴⁸ Kant [1974: 59].

⁴⁹ Kant [1974: *ibid.*].

⁵⁰ See, for a brief description of Kant’s solution, Linnebo [2003: 239-240].

⁵¹ For Kant, besides, the idea of constitutivity of logic is by no means independent of the idea that logic is formal. On the contrary, the formality of logic can be seen as being a consequence of its being constitutive of thought. See MacFarlane [2002: 46-48].

highlight some controversial passages from Frege's writings which deal with this question, and I will then discuss some interpretations that have been put forward in order to explain them.

[4.2] Logical Aliens

In the Introduction to *The Basic Laws of Arithmetic*, in the context of a sustained criticism of the psychologistic account of logic, Frege explicitly discusses the possibility of illogical thought, represented by the possibility of beings that judge and think in contradiction with the laws of logic. Following a standard terminology, I shall call these beings 'logical aliens'.⁵²

[W]hat if beings were [...] found whose laws of thought flatly contradicted ours and therefore frequently led to contrary results even in practice? The psychological logician could only acknowledge the fact and say simply: those laws hold for them, these laws hold for us. I should say: we have here a hitherto unknown type of madness. Anyone who understands laws of logic to be laws that prescribe the way in which one ought to think – to be laws of truth, and not natural laws of human beings' taking a thing to be true – will ask, who is right? Whose laws of taking-to-be-true are in accord with the laws of truth? The psychological logician cannot ask this question; if he did he would be recognizing laws of truth that were not laws of psychology [BLA: 14].

Frege is here criticising the psychologistic logician, according to whom the laws of logic are laws of thought, namely empirical laws that govern the process of thinking, and of taking-to-be-true. If they were so, Frege argues, "the possibility remains of men or other beings being discovered who were capable of bringing off judgements and contradicting the laws of logic" [BLA: *ibid.*]. This possibility seems, *prima facie*, to corroborate the psychologistic logician's claim that laws of logic are neither objective nor eternally valid. But then Frege asks: who is *right*? Are we right, whose laws of taking-to-be-true follow from the laws of logic (the

⁵² See Ricketts [1986: 68], Conant [1991: 142], and Cerbone [2000: 296].

laws of truth), or are perhaps they right in doing exactly the opposite, in holding to be true what contradicts those very laws? The psychologistic logician, Frege says, cannot answer (and cannot even ask) this question, for in doing so he would recognise laws of truth that are independent of empirical, psychological, laws of taking-to-be-true. Frege's point is therefore that the psychologistic account of logic is defective, for it does not allow for the normative aspect of logical laws, that one ought to think in accordance with the laws of truth. The point of Frege's story about logical aliens is therefore showing that the psychologistic account of logic is incapable of accounting for any question of *correctness* of inferences.⁵³ And such an account of logic, for Frege, cannot be a satisfactory one.

However, Frege does not only say that, even if we find logical aliens, there cannot be any doubt as to who is right, he also says that, contrary to the psychological logician, according to whom the 'illogical' laws governing the aliens' thinking are only laws among others, for him logical aliens manifest 'a hitherto unknown type of madness'; the 'constitutivity thesis' seems to surface in this phrase, namely the idea that not only do laws of logic govern how one ought to think if one is to attain truth, but they constitute the possibility of thought, to the extent that denying those laws means to fall prey to a 'kind of madness'; and this claim seems decidedly stronger than just saying that logical aliens are merely wrong in their judgements and inferences.

Later on in his discussion, Frege seems to confirm his adoption of the 'constitutivity thesis' with regard to logical laws (he mentions, in particular, the law of identity); as he says:

⁵³ See Ricketts [1986: 69] and Conant [1991: 144-145].

If we step away from logic, we may say: we are compelled to make judgments by our own nature and by external circumstances; and if we do so, we cannot reject this law – of Identity, for example; we must acknowledge it unless we wish to reduce our thoughts to confusion and finally renounce all judgment whatever [BLA: 15].

But, rather surprisingly, Frege expresses agnosticism about this claim:

I shall neither dispute nor support this view; I shall merely remark that what we have here is not a logical consequence. What is given is not a reason for something's being true, but for our taking it to be true. Not only that: this impossibility of our rejecting the law in question hinders us not at all in supposing beings who do reject it; where it hinders us is in supposing that these beings are right in so doing, it hinders us in having doubts whether we or they are right [BLA: *ibid.*].

With this last piece of argumentation, Frege seems to withdraw his assent to the 'constitutivity thesis', for he says he will not support the claim that, in order to make judgements and inferences, we are compelled to accept the laws of logic. He does not oppose that opinion either. His argument is that the impossibility of our rejecting the laws of logic (if we want to make judgements or inferences) does not testify for those laws to be true. What this impossibility shows is that we have reasons to hold them to be true. But this is entirely different a matter; Frege never tires of stressing the difference between the laws of being true and the laws of taking-to-be-true. Taking something to be true happens in accordance with psychological laws, but, as we saw in chapter [1], something is true wholly independently of anyone's recognising it as such; laws of logic, the laws of being true, are "boundary stones set in an eternal foundation, which our thought can overflow, but never displace" [BLA: 13]. This last sentence, which reaffirms Frege's conviction that laws of logic are absolutely objective (eternally and unconditionally valid), seems at the same time to leave room for the possibility of a thought that contradicts them, for, as he says, 'our thought can overflow them',

even though it can by no means ‘displace’ them. What is in question is thus our *taking them to be true*, and not their *being true*.

The impossibility of our rejecting the laws of logic, for Frege, shows that we are hindered from supposing the logical aliens to be right in rejecting them. In fact, if we take those laws to be true, we cannot at the same time agree with those who reject them. We are compelled, therefore, to say that they are *wrong*, but this is very different from saying that they are *mad*, or that they have renounced ‘all judgement whatever’; the claim is surely weaker than the one the ‘constitutivity thesis’ puts forward.

[4.3] Some Interpretations

In facing this aspect of Frege’s reflection on logic, we seem to be pushed in different directions, for Frege does not present us with a clear position on whether logic is constitutive of thought. As a consequence, the question is much debated among scholars, whose interpretations have reached different conclusions. In this section I intend to analyse some of them.

Øystein Linnebo argued that Frege, in the early phase of his thought (the one comprising writings such as the *Begriffsschrift* and *The Foundations of Arithmetic*), shared Kant’s view that “without the laws of logic there could be no thought at all”;⁵⁴ gradually, however, Frege would have ceased to endorse that thesis, as, according to Linnebo, *The Basic Laws of Arithmetic* shows; the evidence for this is the excerpt from that book where Frege refrains from either disputing or supporting that view, quoted above. One of the reasons for Frege’s agnosticism

⁵⁴ Linnebo [2003: 239].

about the 'constitutivity thesis', according to Linnebo, was the necessity he felt to "guarantee the logicity of the axiom system he needed for his logical reduction".⁵⁵ Since Basic Law V cannot be said to be a law which, if denied, would cause complete confusion in our thought, Frege was somehow forced to abandon the idea that logical laws are constitutive of thought.

Another reason for Frege's abandonment of the 'constitutivity thesis' – on Linnebo's reading – has to do with his anti-psychologistic attitude. Given his constant (for Linnebo, increasing) anti-psychologism, Frege would have found himself more and more uncomfortable with an explanation of the validity of logical laws in terms of the constitution of human thought and mind. This emerges, in particular, in Frege's idea that the impossibility of our rejection of the laws of logic is not a reason for those laws *to be* true, but merely a reason for our *taking them to be true*.

There is a simple aspect that prevents me from accepting Linnebo's conclusion that Frege stopped from endorsing the 'constitutivity thesis' in *The Basic Laws of Arithmetic*. The point is simply that there is not enough textual evidence to support that conclusion. We have in fact seen that, among remarks that actually suggest such a rejection [see the already quoted argument in BLA: 15], there are others (which Linnebo does not mention) that push in the opposite direction. I cannot help considering the passage in which Frege defines logical laws as laws 'which prescribe universally the way in which one ought to think if one is to think at all', or the one in which he assesses the logical aliens' rejection of the logical laws as a 'kind of madness', or, finally, the passage where he claims

⁵⁵ Linnebo [2003: 248].

that acknowledging and doubting a law in the same breath is like “an attempt to jump out of one’s own skin” [BLA: 15] as confirmations of the view that logic is constitutive of the *possibility* of thought; and Frege explicitly refrains from rejecting that opinion.

I want now to turn to James Conant’s reading of Frege’s attitude towards the ‘constitutivity thesis’, and in particular to his reading of Frege’s discussion of logical aliens.⁵⁶ Conant holds, contrary to Linnebo, that Frege’s is an argument “against the possibility of [...] logical aliens”, which, “read in its stronger form, amounts to an argument against the very intelligibility of this scenario”.⁵⁷ Conant, as well as Ricketts, conceives of Frege’s discussion of logical aliens as an argument against the idea that laws of logic are empirical laws of taking-to-be-true; for Frege, as we saw, they are instead objective laws of truth, completely independent of someone’s taking them to be true or false. In fact, Frege observes, “[t]here is no contradiction in something’s being true which everybody takes to be false” [BLA: 13].

Frege highlights the tensions in the psychologistic conception of logic by showing that the psychologistic logician, when presented with the possibility of beings whose judgements flatly contradict the laws of logic, must acknowledge this fact without posing the question as to who is right in their inferences and judgements. Since, for the psychologistic logician, laws of logic are psychological laws of taking-to-be-true, both we and they are justified in the respective judgements and inferences. This implies, as we have seen, that no question of

⁵⁶ Conant [1991: 134-150]. His reading, as he himself acknowledges, is much indebted to that offered by Ricketts [1986]. I will then take both into consideration. See also Weiner [1990: 73-76].

⁵⁷ Conant [1991: 142].

correctness of inferences can arise within the psychologistic conception, for the psychological logician does not recognise any laws of truth apart from those of taking-to-be-true. But this, for Frege, is to conflate what is objective (what is true) with what is merely subjective (what is taken-to-be-true). In turn, this implies that, under the psychologistic conception, the very idea of a *disagreement* between us and logical aliens disappears:

If we could not grasp anything but what was within our own selves, then a conflict of opinions [based on] a mutual understanding would be impossible, because a common ground would be lacking, and no idea in the psychological sense can afford us with such a ground. There would be no logic to be appointed arbiter in the conflict of opinions [BLA: 17].

Frege's idea is that *disagreement*, *incompatibility*, or *conflict* of opinions, are available notions only insofar as "the principles of logic provide us with a shared background, context, or framework that enables us, first, to recognize disagreement, and second, to arbitrate the consequent debate".⁵⁸ Logic is thus conceived of by Frege as an 'arbiter' because it provides the background necessary to make sense of the notion of a 'conflict of opinions'. Logic can provide such a background because its laws are *objective* laws of truth, of which the (subjective) laws of taking-to-be-true are independent. Conflicting opinions can thus be recognised as conflicting only because they refer to something which is objective (laws of truth). Only objective laws of truth can thus determine the *correctness* and the *incorrectness* of opinions or inferences (correct inferences or opinions are those made in accordance with them, incorrect are those made in conflict with them), and, therefore, only objective laws of truth can sanction the possibility of disagreement.

⁵⁸ Ricketts [1986: 69].

If, on the other hand, no objective laws of truth are recognised at all, but laws of logic are identified with psychological (and thus subjective) laws of taking-to-be-true, then “the psychologistic logician is treating disagreement in judgement as merely a species of psychological difference. As such, the recognition of disagreement no more raises an issue of correctness than does the acknowledgement of any other personal idiosyncrasy”.⁵⁹ Not only, as Ricketts observes, does the psychologistic conception not allow for any question of correctness, but, in refusing objective (non-psychological) laws of truth, it does not allow for any ‘recognition of disagreement’ at all, for, as we have said, such a recognition is only possible insofar as the laws of logic are recognised to be objective laws of truth, which form an independent and objective context, or background, in which opinions and judgements can stand in some sort of (objective) relation to one another. If logic is not recognised as being such an *arbiter*, then, no objective relations between judgements can be identified, and *accordance* or *conflict* between judgements turns out to be mere (psychological) *similarity* or *difference*.

Conant nicely summarises Frege’s critique of the psychologistic conception of logic as being a two-horn dilemma:

Frege’s strategy [...] is to present the psychologistic logician with a dilemma: either 1) he can claim that his account reveals that the judgements of the aliens conflict with ours, in which case his idea of one judgement’s conflicting with another can be shown to tacitly rely upon the idea of their logical incompatibility (that is, upon a non-psychological notion of incompatibility), or 2) he can refrain from telling us anything about the logical relation in which their judgements stand to ours, in which case he can tell us nothing about their thought whatsoever.⁶⁰

⁵⁹ Ricketts [1986: *ibid.*].

⁶⁰ Conant [1991: 146-147].

The first horn of the dilemma (accepting the idea that our judgements and logical aliens' are in conflict with each other) presupposes that they disagree upon something which is true independently of our - or their - judgements (thus something which is not merely psychological), and thus presupposes the idea of logic as an *arbiter*, as a shared background which constitutes the relations in which different judgements can stand to each other (agreement, disagreement, entailment, independence, etc.). This, as we have seen, implies the refutation of the psychologistic conception of logic altogether, for it would imply the existence of 'laws of truth that are not laws of psychology'.

But the second horn of the dilemma is not a better one for the psychologistic logician. In fact, if disagreement is reduced to mere psychological difference, then our judgements and those of the logical aliens cannot be said to stand in any *logical* relation to each other; but since the psychological logician is supposed to share *our* laws of logic (he in fact says that "those laws hold for them, these laws hold for us" [BLA: 14]) then he "is in no position to tell us anything about the thought of logical aliens. For he has banished from his account the resources for discerning *any* sort of logical structure in the utterances of the aliens".⁶¹ Logical aliens' thought would be completely incomprehensible to us, and this is, arguably, the reason why Frege calls it a 'kind of madness'.

From the story of the logical aliens Conant draws the following conclusion:

Frege's ultimate aim in the thought-experiment is [...] to try to get his interlocutor to see the force of the [...] point that there isn't any sense to be made of the idea of undertaking to disagree with a principle of logic - that it is these principles which make both agreement and disagreement possible. What we are left with, if deprived of these principles - is not the possibility of agreement of *another kind*, but rather simply the *absence* of the possibility

⁶¹ Conant [1991: 147].

of agreement altogether. The ultimate point of Frege's thought-experiment therefore is to highlight the special role that logic has in constituting the possibility of rational discourse.⁶²

The second horn of Frege's dilemma, therefore, commits us to saying that we would not be even in principle able to recognise logical aliens' judgements as judgements, and their whole thought as thought; if laws of logic were mere psychological laws of taking-to-be-true, then any logical relations between our judgements and theirs would be reduced to a psychological difference, and the logical aliens' thought would be something of which we could not make any sense. It would not be (to us), therefore, thought at all. This conclusion, then, is perfectly in agreement with the idea that logic is 'constitutive' of thought. In fact, as Conant says, "if the laws of logic prescribe how one ought to think if one is to think at all, then Frege *must* say that what has been proposed here [logical aliens' thought] is not a kind of thought".⁶³

The problem with Conant's reading is the following: even if he rightly says that, according to the argument just developed, Frege *must* say that logical aliens' is not thought at all, but simply an *illusion* of thought, Frege does *not* say so. Conant draws conclusions, from Frege's discussion, that Frege himself does not, at least explicitly, endorse. We have seen, in fact, that Frege does not, in the end, jettison the *intelligibility* of the logical aliens scenario, as Conant reads him as doing. Frege does not reject the hypothesis of logical aliens as impossible, and not even as implausible. Frege's conclusion, after having considered the possibility of logical aliens, is far less radical than Conant's. As he says, in fact, the "impossibility of our rejecting the [...] [laws of logic] hinders us not at all in

⁶² Conant [1991: 147-148].

⁶³ Conant [1991: 149, my emphasis].

supposing beings who do reject [...] [them]; where it hinders us is in supposing that this beings are right in so doing, it hinders us in having doubts whether we or they are right" [BLA: 15].

I am not here criticising Conant's, and Ricketts', reading as implausible. On the contrary, I am rather sympathetic to it. Its weakness is not, as it were, *internal*, but merely *external*. It is simply not enough supported by Frege's discussion, for, while it is in agreement with some of Frege's arguments (his idea of logic as an *arbiter*, which constitutes a shared background of rationality), it is in conflict with others, such as the one we have just quoted, which, on the other hand, stress the *normativity* of logic, and thus merely sanction the *wrongness* of judgements made in conflict with logical laws. In conclusion, then, both Linnebo's proposal, according to which Frege rejected the idea of the constitutivity of logic in *The Basic Laws of Arithmetic*, and Conant's, according to which Frege seriously adopted and put forward that very idea, cannot be said to be wholly correct interpretations of Frege's conception of logic, for both overlook some aspects of his thought on the matter.

These conclusions, and this criticism of Conant's reading, are expressed in a paper by David Cerbone.⁶⁴ Cerbone argues that in Frege's conception of logic two different (and to a certain extent opposing) strands are present: the 'normative', and the 'constitutive' one. On the first strand, according to which logic is normative in prescribing how to think in the attainment of truth, the possibility of logical aliens is not rejected. Logical aliens, in their making judgements and drawing inferences contrary to the laws of logic, are simply wrong. This, however,

⁶⁴ Cerbone [2000: 295-299, 310-311].

does not exhaust Frege's conception of logic. The second strand, the 'constitutive' one, pushes in the opposite direction. According to it, in fact, logic provides a shared framework which constitutes the possibility of agreement and disagreement. In this latter sense logic is constitutive of the very possibility of judging and thinking in general, as opposed to merely venting one's inner mental states. It is thus constitutive of what we would define as *rationality*.

These two strands, Cerbone argues, put forward two different conceptions of logic and of its role in human thinking, which, in the end, cannot be reconciled.

As he writes:

Frege's strategy of appealing to the possibility of logical aliens, then, has the effect of bringing to light tensions in his own conception of logic between the notion that logic provides laws in accordance with which one *ought* to think and the notion that logical laws are internal to thinking. That is, on first blush, logical aliens appear to present a kind of possibility, namely the possibility of illogical thought [...]. As one tries, however, to take seriously this kind of possibility, one begins to lose sight of its being a possibility at all.⁶⁵

Whereas Conant and Ricketts take the constitutive strand in Frege's conception of logic as being the *whole* of his conception of logic,⁶⁶ Cerbone acknowledges that Frege's conception of logic was not so straightforward and transparent, and that two different strands, the 'normative' and the 'constitutive', are both present and in tension with each other.

According to Cerbone, this tension in Frege's philosophy of logic was one of the targets of Wittgenstein's critique, who rejected the idea of logic as

⁶⁵ Cerbone [2000: 298].

⁶⁶ Conant acknowledges that there are tensions in Frege's conception of logic (see Conant [1991: 137-141]), but he does not question the idea that logic, for Frege, is constitutive of the possibility of thought. The same can perhaps be said of Burge [1992: 648-649], according to whom when Frege says that we must accept the laws of logic unless we want reduce our thought to confusion, and renounce all judgement whatever, "he is speaking in his own voice", because "the nature of reason – and even non-degenerate judgment – is partly constituted by the prescription that one acknowledge at least the simple and basic laws of truth."

'normative science' while retaining a conception of it as constitutive of thought. In the *Tractatus*, in fact, we find the idea that logic is constitutive of thought clearly expressed:

Thought can never be of anything illogical, since, if it were, we should have to think illogically [TLP: 3.03].

What makes logic *a priori* is the *impossibility* of illogical thought [TLP: 5.4731].

In a passage from the later *Remarks on the Foundations of Mathematics* Wittgenstein writes:

The propositions of logic are 'laws of thought', 'because they bring out the essence of human thinking' – to put it more correctly: because they bring out, or shew, the essence, the technique, of thinking. They shew what thinking is and also shew kinds of thinking [RFM: 90].

Wittgenstein is here putting forward his conception of logic in opposition to Frege's; this is revealed by a subsequent passage, where he explicitly mentions Frege and his discussion of logical aliens' rejections of the laws of logic. "Frege says in the preface to the *Grundgesetze der Arithmetik*: '...here we have a hitherto unknown kind of insanity' – but he never said what this 'insanity' would really be like" [RFM: 95]. This passage, to my ears, is highly reminiscent of another excerpt from the *Tractatus*, which reads as follows:

It used to be said that God could create anything except what would be contrary to the laws of logic. – The truth is that we could not *say* what an 'illogical' world would be like [TLP: 3.031].

Wittgenstein challenges Frege's acceptance of the possibility of logical aliens, of beings whose laws of thought flatly contradict ours, as not being a possibility at all. Since laws of logic constitute the possibility of human thought,

for they are a necessary prerequisite of our capacity of bringing off judgements, logical aliens' thought, which is supposed to contradict those laws, cannot be said (by us) to be thought at all, but, at least, only an *illusion* of thought. Since we would not be able to identify any logical relation between our judgements and theirs, we would not be able to identify their judgements and assertions as judgements and assertions, and, thus, their thought would not be acknowledged by us as being thought altogether.

In this sense, I think, Wittgenstein is criticising Frege's notion of *madness*. It seems as though Wittgenstein is reading Frege as assuming that logical aliens' 'madness' is somehow a distorted form of thought, but still thought after all; Wittgenstein seems to be reading Frege along the following lines: "In saying that we would have here a kind of madness, Frege does not go so far as to suggest that these beings, whose thought would be, by hypothesis, illogical, would not really have thought at all".⁶⁷ Wittgenstein thus pushes Frege to show what this kind of madness would look like, what this distorted form of thought would look like, challenging his wavering over the 'constitutivity thesis'; if logic is indeed constitutive of the possibility of thought, then logical aliens' 'thought' cannot be considered thought at all, not even a deformed – distorted – form of thought.

⁶⁷ Eynine [unpublished]. A similar interpretation seems offered by Conant [1991: 148]. "Madness' is the notion Frege reaches for in an attempt to meet the psychologistic logician halfway. It is a notion one might reach for when confronted by beings whose capacities for rational thought appear deformed – whose processes of thought remain opaque to us. Frege does not reach for in the service of an attempt to characterize the Other of reason, but rather in the service of trying to find a sense for his interlocutor's words".

[4.4] Logic, Thought and Rationality

When discussing the ‘constitutivity thesis’ in section [4.1] we noted the problems related to an account that makes logic internal to thinking, namely a necessary condition of thinking as such. How is it possible, under this perspective, to account for logical mistakes and wrong inferences without being forced to deny that they are instances of thinking at all? We have also seen that Frege recognises the possibility of logical mistakes indeed, for this very possibility motivates his conception of logic as normative, as a science teaching us how we *ought* to think, and not merely how we *do* think. In this section I will attempt to dispel, at least to a certain extent, the paradox.

An initial step in this direction consists in distinguishing between *basic logical laws* and logical laws that can be *derived* from the former by means of laws of inference, and to which they also can be *reduced*. We have earlier stressed that Frege argues against psychologism because it confuses the grounds that logically justify inferences and judgements with the psychological grounds that simply describe the causal chain of mental events responsible for a judgement, or for the conclusion of an inference. These latter grounds are not grounds that logically justify anything. They can explain why we made some particular judgement, but not if we were logically justified in making it. For Frege a judgement can be said to be logically justified if it can be shown to be logically reducible to others. This is the form a logical proof of a judgement assumes. As Frege writes in *The Basic Laws of Arithmetic*, “[t]he question why and with what right we acknowledge a law of

logic to be true, logic can answer only by reducing it to another law of logic. Where that is not possible, logic can give no answer" [BLA: 15].

We are justified in acknowledging a law of logic to be true, if we can show that it can be logically reduced to another law of logic. Of course this latter law of logic will be in need of justification, and if a justification is given that shows its dependence on another law, then it will be this last one in need of justification. When no justification of this kind can be given, Frege says, 'logic can give no answer'. We can find the same kind of reasoning in a passage from 'Logic', where Frege asks how a judgement is logically justified: "If it is not justified in terms of other truths, then logic doesn't need to bother itself with it any further. If, on the other hand, a law of logic can be reduced to other laws by a process of inference, then it is evidently the task of logic to carry out this reduction." [L1: 6]; and again: "We justify a judgement either by going back to truths that have been recognized already or without having recourse to other judgements. Only the first case, inference, is the concern of Logic" [KSL: 175].

But such a justificatory process must end at some point, on pain of starting an infinite regress that cannot justify anything. If a truth can be logically justified at all, then the process must end with the reduction of that truth to some *primitive*, or *basic*, logical laws. These truths are primitive, or basic, for there cannot be any proof of them, in that they need *no* justification; Frege explains this point as following:

The *axioms* are truths as are the theorems, but they are truths for which no proof can be given in our system, and for which no proof is needed. It follows from this that there are no false axioms, and that we cannot accept a thought as an axiom if we are in doubt about its

truth; for it is either false and hence not an axiom, or it *is* true but stands in need of proof and hence is not an axiom [LM: 205].⁶⁸

Basic logical laws are therefore, for Frege, laws “which [...] neither need nor admit of proof” [FA: 4]; they are self-evidently true. No law about whose truth we are in doubt can be a basic truth, for, if this law is doubtful then either it is false (and therefore not a law) or it needs logical justification. We then see that for Frege the process of logical justification of judgements can end in two ways: the judgement in question relies on some other judgement which is neither a basic logical law nor reducible to it; in this case that judgement is logically *unjustified*, and it is not a sufficient ground (from a logical point of view) for taking something to be true; with judgements like this logic need not ‘bother itself’ in any way, if not to show their lack of logical grounds. In the other case, the judgement can be shown to rely on some basic logical laws, which in turn, need not be justified in that self-evident. The judgement in question is thus justified. The notion of a primitive law of logic is a cornerstone of Frege’s philosophy of logic. In fact, for Frege, it is our capacity to acknowledge the truth of basic logical laws that grounds our being *rational* agents, and it is primarily with regard to the primitive laws of logic (and basic laws of inference, such as *modus ponens*) that logic can be said to be *constitutive* of our thought.

Although in the discussion of logical aliens Frege entertains the possibility of beings that reject *basic* logical laws, elsewhere he makes it clear that such a

⁶⁸ Here Frege speaks of ‘axioms’ and ‘theorems’, instead of basic logical laws and derivative laws, but this is not very important in this context. A basic logical truth is an axiom only relatively to a particular formal system; namely as Burge [1998: 311] says: “whatever a basic truth is an axiom depends on its being used as an axiom, as starting point in a system of derivation. (This differentiates the notion *axiom* from the notion *basic truth*.)”.

rejection would be 'absurd', if not nonsensical altogether. In 'Compound Thoughts' he writes:

The assertion of a thought which contradicts a logical law can indeed appear, if not nonsensical, then at least absurd; for the truth of a logical law is immediately evident of itself, from the sense of its expression [CT: 405].

We have already seen that Frege's discussion of logical aliens points towards a conception of logic according to which logic constitutes a shared background which makes rational communication possible. It is logic's being an *arbiter* of different opinions that gives it this special role. As Ricketts writes:

Once two of us recognize that we have made contradictory judgements, we each put forth reasons for our opposed claims. Application of the principles of logic enables us to measure the relevance of these further claims to the original dispute. It also enables us to locate the source of our controversy within the bodies of belief we each bring to bear on the dispute. Once we have done this, characteristically, we will be in a position to use the basic laws and methods of the particular subject matter under consideration to resolve the difference.⁶⁹

We cannot use this procedure with logical aliens; not only would it be impossible to resolve a dispute between us and them using logical laws (because they do not recognise them to be true), but even the possibility of a dispute would be barred from the beginning, for there would be no logical relation between our statements or judgements, and theirs. The difference between our statements and theirs would merely be a question of psychological difference. But, then, as Ricketts continues, logical aliens are "beings with whom we cannot reason".⁷⁰

Frege's speaking of logical aliens' thought as a kind of 'madness', thus, shows his inclination "to believe that a disposition to acknowledge basic logical

⁶⁹ Ricketts [1986: 69].

⁷⁰ Ricketts [1986: *ibid.*]. See also Weiner [1990: 76]: "given Frege's view, our ability to communicate with someone is dependent on our ability to succeed in understanding her or his assertions as assertions that do not obviously contradict primitive logical laws".

truths and inferences [...] is a condition not only for being rational but for being a judge or thinker at all".⁷¹ Thus basic logical laws can be said to be *descriptive* (and objective) laws of truth whose acceptance turns out to be a requisite for the possibility of our making judgements and therefore for the possibility of being rational agents (for the possibility of sharing opinions, solving disputes, and so on). If so, basic logical laws are laws of thought in a deep and non-psychological sense, for they are *constitutive* of the very possibility of our thought.

We can also see that Frege does not imply that logical mistakes, erroneous judgements and wrong inferences, are impossible. What is impossible (or at least absurd), for Frege, is that we judge a *basic* law of logic to be false. That would not be a logical mistake, but, as it were, a sign of 'madness'. Frege's theory, of course, does not require that we recognise all the countless laws of logic to be true; but we have a way to determine their truth, which consists in carrying out a proof of the law in question, showing that the law in question relies on some primitive law, whose truth, in turn, is self-evident. But in the case of many logical truths, their conceptual content can be so complicated that "[o]ur minds are simply not comprehensive enough" [LM: 209], and for this reason we can make mistakes.⁷²

Secondly, the difference between *non-acknowledging* a law of logic (or rule of inference), in the sense of *not being aware* of it, and *rejecting* it, should be kept distinct. Frege's idea that logic is constitutive of thought does not require that one should be aware of all (basic) laws of logic. What it does require, though, is that one should accept basic logical laws upon reflection, for only such acceptance puts

⁷¹ Burge [1992: 647].

⁷² Frege discusses this point with reference to the content of the word 'integral'; one might be unaware of everything appertaining to its sense. This is one of the reasons why one can make mistakes in working with integrals. See [LM: 209] and Weiner [1990: 76-77].

one in a position to be a rational agent and to engage in rational discourse. So, as long as one is engaged in communicating with others (exchanging opinions and so forth), one seems to be under a logical obligation to conform to the laws of logic.

This conclusion, however, while able to salvage Frege's conception of logic from obvious criticisms against the idea that logic is constitutive of thought, contrasts with Frege's agnosticism about the supposition that there may be beings that reject the laws of logic. It also contrasts with Frege's *weak* claim that the impossibility of our rejection of the basic laws of logic only hinders us from supposing that these beings are right in doing so. If the acceptance of basic laws (and in the passage in which Frege discusses logical aliens he is dealing with the law of identity, which is indeed a basic logical law) is a necessary condition for being rational, and even 'for being judge and thinker at all', then logical aliens are not simply mistaken in their judgements and inferences, they are not thinkers at all. What is at stake in the logical aliens' scenario, in fact, is the possibility of beings that overtly reject basic laws of logic, a possibility that Frege's own general conception of logic seems committed to rule out.

[4.5] Conclusion

We discussed Frege's conception of logic and highlighted some tensions, consisting in the fact that Frege's reflections on logic seem to point to different, and indeed opposite, directions. Let us summarise the tensions:

We observed in section [3.1] that in the early phase of his career Frege was inclined to think that logic is *formal*, because it provides the scaffolding within which thoughts (of whatever nature) can be perspicuously expressed. In this sense

logic is not on the same level as other sciences. Particular sciences are not formal, because they have their own vocabulary; they use concepts with a particular content. The task of logic, and of the Begriffsschrift in particular, is for Frege that of providing the structure within which the content of particular sciences can be expressed, to provide the “logical cement” [BCCS: 13] that ties up together the contentful concepts used in specific sciences. Logic thus abstracts from the content of specific sciences and only attends to the form of their expression, to the way in which their concepts are put together.

This view, however, contrasts with other central tenets of Frege’s conception of logic; one is the view that logical expressions work semantically like non-logical ones, that they have a content of their own, a view that we presented in section [3.2]. Another is Frege’s repeated idea – discussed in section [2.3] and at the beginning of section [3.1] – that logic is the most general science, whose domain extends more widely than those of particular sciences. If so, then, the difference between logic and other sciences should not be drawn by relying on the formal nature of the former and the contentful character of the latter, but on their different scope of application. Logic has a universal applicability whereas particular sciences have a more restricted field. Their difference is thus one of *scope* and not of *kind*.

We then moved on – in chapter [4] – to consider Frege’s attitude towards the idea that logic is *constitutive* of the possibility of thought, and we noticed that while the general trend of Frege’s discussion on the nature of logic points towards the conclusion that ‘illogical thought’ is not thought at all, other passages – where Frege is more sensitive to the *normative* role logic has for thought in so far as it

aims at truth (discussed in sections [2.1] and [2.2]) – seem to allow for that possibility.

Some commentators, we saw in section [4.3], claim that there is no paradox here; but this comes at the price of downplaying, or overlooking at all, some strands in Frege's reflection on logic; so Linnebo, in arguing against Frege's adoption of the constitutivity thesis in *The Basic Laws of Arithmetic*, does not mention the passages where the thesis clearly seems to be retained. Conversely, Conant and Ricketts convincingly put forward arguments in support of the 'constitutivity thesis', but at the price of not taking into enough consideration Frege's 'agnosticism' towards the possibility of beings that reject the laws of logic.

Despite presenting internal tensions, Frege's philosophy of logic strongly influenced successive philosophers, among them, notably, Wittgenstein. The latter, in particular, as we briefly observed at the end of section [4.3], often confronted with Frege on questions of logic, in both his early and late phase of his philosophical career; his attitude towards Frege was twofold; while acknowledging Frege as one of the major inspirations for his own philosophical insights in the *Tractatus*, he often took Frege, and in particular his conception of logic, as a critical target. In the third part of this thesis, after having analysed the development of Wittgenstein's reflection on language and logic in the *Tractatus*, I discuss how his deep engagement with Frege made him able to dispel the tensions that affect Frege's philosophy of logic while remaining faithful to some of its main tenets.

Part Two

Russell on the Nature of Judgement

Chapter Five

Propositional Unity, Truth and Judgement in 1903-1905

[5.1] Terms and Propositions

Russell's view of judgement in *The Principles of Mathematics* is part of his (and Moore's) revolt against idealism at the beginning of the twentieth century. It can be summarised, in a nutshell, using Hylton's words:

If we can say that Idealism is a view according to which the mind (in some sense of 'mind') is *active*, and plays a role in the constitution of reality, then we can equally say that [...] [Russell's view in the *Principles*] is a view according to which the mind is completely passive, and in no way creative.⁷³

Judgement, understood as the *act* of judgement, therefore, plays no significant role in Russell's metaphysics in the *Principles*; what is more important from a philosophical point of view is the *object* of judgement, which, according to Russell, is an objective and mind-independent complex entity; Russell calls such an entity a *proposition*. Propositions, for Russell, are by no means linguistic entities (Russellian propositions should not be thought of as being sentences; instead they are what is expressed by sentences),⁷⁴ but are worldly entities, constituents of reality; they are, as Hylton says, "*out there*".⁷⁵ The act of judgement is, for Russell, the *recognition* (as true or false) of a proposition; as Russell puts it, "all knowledge must be recognition, on pain of being mere delusion" [PoM: 451]. The notion of judgement is then, according to this picture, derived from the notion of a

⁷³ Hylton [1990: 110-111].

⁷⁴ As Russell famously remarked: "a proposition, unless it happens to be linguistic, does not itself contain words: it contains the entities indicated by words" [PoM: 47]. Russell's position is due to his anti-psychologistic attitude; as he says, in fact, the idea that propositions have words as constituents "is due to the notion that propositions are essentially mental and are to be identified with cognitions" [PoM: *ibid.*], a view Russell rejects.

⁷⁵ Hylton [1984: 381].

proposition, which in turn is, for reasons to be discussed later, intimately related to the notion of truth. A proposition is in fact defined by Russell as “anything that is true or that is false” [PoM: 12-13]. In order to investigate Russell’s conception of judgement, therefore, we have to analyse his conception of a proposition in some detail.

We said that Russell conceives of propositions as objective, mind-independent complex entities: this last qualification implies that propositions have constituents. The constituents of propositions are *terms*. The notion of a term is explained by Russell as follows:

Whatever may be an object of thought, [...] or can be counted as *one*, I call a *term*. This, then, is the widest word in the philosophical vocabulary. I shall use as synonymous with it the words unit, individual, and entity [...]. A man, a moment, a number, a class, a relation, a chimaera, or anything else that can be mentioned, is sure to be a term; and to deny that such and such a thing is a term must always be false [PoM: 43].

A term is thus an entity,⁷⁶ a constituent of the universe; besides, a term is for Russell “immutable and indestructible” [PoM: 44] because, he argues, “[w]hat a term is, it is, and no change can be conceived in it which would not destroy its identity and make it another term” [PoM: *ibid.*]. Terms, we said, constitute propositions, in the sense that propositions are complex entities structured by the interplay of different kinds of terms. According to Russell, terms can be divided into two main categories, *things* and *concepts*: “The former are the terms indicated by proper names, the latter those indicated by all other words” [PoM: *ibid.*]. Things, Russell warns, must be understood in a wide sense, “as embracing all

⁷⁶ ‘Term’, as well as ‘proposition’, may seem to have a linguistic ring to it; the same applies to the different kinds of terms we will discuss later, for example ‘concept’ and ‘predicate’. These are examples of what Linsky calls Russell’s attitude to ‘ontologize grammar’. See Linsky [1988: 625].

particular points and instants”, and “bits of matter, particular states of mind, and particular existents generally”, and, further, “points in a non-Euclidean space and the pseudo-existents of a novel”, plus classes, numbers, men, spaces [see PoM: 44-45]. A thing is thus something that should be conceived of as generally *object-like*. Among concepts, in turn, one can further distinguish “those indicated by adjectives and those indicated by verbs” [PoM: 44]; this latter distinction amounts to the distinction between *predicates* and *relations*. Both things and concepts, for Russell, should be conceived of as objective mind-independent entities; they differ because, as we will see, they have different ways of occurrence into propositions.

A proposition is thus a complex entity whose structure is constituted by the combination of *things* with *concepts* or of *concepts* with other *concepts*.⁷⁷ In order to clarify his usage of the notion of a term, and, in turn, of a proposition, Russell says:

I shall speak of the *terms* of a proposition as those terms, however numerous, which occur in a proposition and may be regarded as subjects about which the proposition is. It is a characteristic of the terms of a proposition that any one of them may be replaced by any other entity without ceasing to have a proposition [PoM: 45].

According to this characterisation of the notion of a term, then, “not all of the terms which are the constituents of the proposition occur as terms of the proposition”;⁷⁸ in fact, despite the proposition expressed by the sentence ‘Socrates is mortal’ has as constituents two terms, Socrates and mortality, only *one* term, Socrates, occurs as a term ‘of the proposition’, for Socrates is the subject of that proposition, namely what the proposition is about. In the proposition expressed by the sentence ‘Socrates is older than Plato’, on the other hand, two terms occur as terms of the proposition, Socrates and Plato.

⁷⁷ See Griffin [1980: 119].

⁷⁸ Linsky [1992: 245].

In the second part of the above quotation Russell says that in a proposition any term of the proposition may be replaced by any entity (term) without ceasing to have a proposition (though, of course, not the same proposition); in the proposition expressed by the sentence 'Socrates is mortal' the subject may be replaced by any entity whatsoever without ceasing to have a proposition; this means that the subject of that proposition, which is a *thing*, does not have to be replaced by another *thing* in order to have another (different) proposition; for Russell the substitution may be carried out with any term; since predicates are terms, Socrates in 'Socrates is mortal' may be substituted by the predicate 'mortality' without ceasing to have a proposition: 'Mortality is mortal' is thus a proposition. Since relations are terms as well, 'Socrates' in 'Socrates is mortal' may be substituted by the relation 'to the left of' without ceasing to have a proposition: 'To the left of is mortal' is therefore a proposition.⁷⁹

As regards the occurrence of terms in a proposition, Russell writes:

In 'Socrates is human' the notion expressed by *human* occurs in a different way from that in which it occurs when it is called *humanity* [like in 'humanity belongs to Socrates'], the difference being that in the latter case, but not in the former, the proposition is *about* this notion. This indicates that humanity is a concept, not a thing [PoM: 45].

Russell explains the difference between *things* and *concepts* by saying that while concepts can occur either as subjects or as part of the assertion in a proposition (what is said about its subject), things can *only* occur as subjects. The concept 'human' occurs as part of the assertion about Socrates in the proposition expressed by the sentence 'Socrates is human'; in particular, it occurs as a predicate; thus, Russell says, a predicate is a concept, other than a verb, which occurs in a

⁷⁹ See Griffin [1980: 121].

proposition having only one term or subject.⁸⁰ But that very same concept can occur as subject in the equivalent (though different) proposition ‘Humanity belongs to Socrates’. It is the possibility of a concept’s *twofold* occurrence in a proposition that distinguishes concepts from things. A thing, on the other hand, cannot occur in the position in which a predicate can occur; “With the sense in which *is* has in this proposition [‘Socrates is human’], we no longer have a proposition at all if we replace *human* by something other than a predicate” [PoM: 45]. This implies, thus, that ‘Humanity is Socrates’, for instance, is *not* a proposition. “Socrates is a thing, because Socrates can never occur otherwise than as a term [namely as subject] in a proposition: Socrates is not capable of that curious twofold use which is involved in *human* and *humanity*” [PoM: *ibid.*].

Russell insists on the fact that the difference between a concept’s occurrence as predicate and its occurrence as term in a proposition is *not* a difference in the concept itself; the difference does not lie “in the intrinsic nature of the terms” [PoM: 46], which is the same in both cases, but is a mere “grammatical difference” [PoM: *ibid.*].⁸¹ As he writes, the difference can be explained by saying that “[i]n the first case, the concept in question is used as a concept, that is, it is actually predicated of a term or asserted to relate two or more terms; while in the second case, the concept is itself said to have a predicate or a relation” [PoM: *ibid.*]. Russell then considers, and rejects, the idea that the difference between a concept

⁸⁰ And, as Linsky [1992; 245] notes, “[b]y extension, verbs (relations) are concepts other than predicates which occur in propositions having more than one term or logical subject”.

⁸¹ The fact that Russell leaves room for a grammatical difference which is not reflected in reality contrasts with his view that language, to use Hylton’s phrase, is simply a “transparent medium” through which reality can be analysed. See Hylton [1990: 171]. In the *Principles*, in fact, Russell generally takes language as reflecting the structure of reality with perfect symmetry. This is the reason why he says that the study of grammar is capable of shedding light on philosophical questions, and therefore that grammar, though not as a master, should at least be taken as guide in philosophical investigations [see PoM: 42].

used predicatively and used as subject is an irreducible difference. This amounts to a rejection of Frege's view that concepts (particular kinds of functions) can never be made the subject of a proposition, as objects can.⁸² According to Frege, a concept has a predicative nature (by virtue of being an unsaturated – or incomplete – entity) and cannot be made the subject of a proposition; of course, Frege realises, we can turn the predicate 'is human' into the noun 'humanity' and therefore turn it into the logical subject of a proposition; but this implies that we have replaced the *concept* denoted by the predicative expression 'is human' by an *object* denoted by the name 'humanity'; an incomplete entity by a complete one.

This view, however, generates a somehow paradoxical situation:

By making concepts and functions essentially incomplete, Frege disables them from playing the role of logical subjects. [...] But a [...] consequence of the incompleteness of functions is that we cannot talk *about* them since they cannot play the role of logical subjects.⁸³

This is known as the 'paradox of the concept horse': when we use the expression 'the concept horse' as a logical subject, as something *about* which we say something, we are actually speaking about an object, and not a concept; we are speaking about the object denoted by the complete (or saturated) expression 'the concept horse'. Thus, according to Frege, the expression 'the concept horse has a predicative nature' must be false, since in it 'the concept horse' denotes an object and not a concept. Frege considers such a paradox as being due to "a kind of necessity of language" [CO: 193], and thus as being unavoidable. For Russell, on the other hand, a theory, like Frege's, containing "propositions which according to

⁸² According to Linsky [1992: 249], Russell was unaware of Frege's views when he wrote this section of the *Principles*. In fact, Russell discusses and criticises Frege's conception only in the Appendix A of the *Principles*, written after the completion of the book.

⁸³ Linsky [1992: 248].

that very theory cannot be true means, quite simply, that the theory cannot be true and must be rejected".⁸⁴

[I]f there were any adjectives which could not be made into substantives without change of meaning, all propositions concerning such adjectives (since they would necessarily turn them into substantives) would be false, and so would the proposition that all such propositions are false. But this state of things is self-contradictory [PoM: 46].

Therefore, Russell concludes that "on the whole, the doctrine of concepts which cannot be made subjects seems untenable" [PoM: 510].

Russell's conclusion is that "every constituent of every proposition must, on pain of self-contradiction, be capable of being made a logical subject" [PoM: 48]. Not only predicates, as we have seen, but verbs as well [see PoM: 47-48] can be used as logical subjects in propositions.⁸⁵ A verb, for Russell, can be used as *verb* or as *verbal noun*; this difference parallels the difference between a predicate used *predicatively* (indicated by an adjective) and used as *noun*; the difference as regards verbs can be exemplified by contrasting, for instance, the different occurrence of the concept 'killing' in 'Brutus killed Caesar' and 'Killing is a crime'. For Russell, "the concept which occurs in the verbal noun is the very same as that which occurs as verb" [PoM: 48]. This, again, follows from the rejection of Frege's distinction between functions and objects. Even in regard to verbs, Russell argues, the difference between the two occurrences of the concept is merely grammatical and in "external relations" [see PoM: *ibid.*].

⁸⁴ Hylton [1990: 176].

⁸⁵ Russell is not rigorous in his use of terminology, and often conflates the ontological and the linguistic level; while this is not particularly worrying given Russell's conception of the 'transparency of language', it is sometimes confusing. In the present case, one should note that 'verb' is a linguistic entity, and that Russell should have used the term 'relation', namely what is indicated by a verb. In general, among terms which are concepts, Russell distinguishes *predicates* and *relations*, in turn indicated – respectively – by the linguistic entities of *adjectives* and *verbs*. [See PoM: 44]. I will, however, allow myself to speak of verbs as occurring in propositions, or as constituents of propositions, in order to remain close to Russell's actual discussion.

[5.2] The Unity of the Proposition

The question of the twofold nature of verbs is by Russell considered to be “inherent in the very nature of truth and falsehood” [PoM: 48]. Investigating the nature of truth, however, is not one of Russell’s main concerns in *The Principles of Mathematics*, and this is the reason why he decides to leave the question unsettled: “The nature of truth [...] belongs no more to the principles of mathematics than to the principles of everything else. I therefore leave the question to the logicians” [PoM: 49]. But the twofold nature of the verb is important for the question of the unity of the proposition, question which occupies § 54 of the *Principles*.

The problem of the unity of the proposition is presented as a problem concerning the process of analysing a proposition into its constituent parts. This involves treating the relation (the verb) in a proposition as not actually relating the terms of the proposition but, as Russell says, *in itself*; Russell illustrates this point in a well known passage of the *Principles*:

Consider [...] the proposition ‘*A* differs from *B*.’ The constituents of this proposition, if we analyze it, appear to be only *A*, difference, *B*. Yet these constituents, thus placed side by side, do not reconstitute the proposition. The difference which occurs in the proposition actually relates *A* and *B*, whereas the difference after analysis is a notion which has no connection with *A* and *B*. [...] A proposition, in fact, is essentially a unity, and when analysis has destroyed the unity, no enumeration of constituents will restore the proposition [PoM: 49-50].

Analysis implies that every component of a proposition is turned into a substantive, in a way that makes the relating-relation, expressed by the verb and responsible for the unification of all the constituents of a proposition, be simply another term among terms; its role of being a kind of ‘propositional glue’ disappears. The outcome of analysis is thus that a proposition is turned into a

collection of terms, no longer unified by the verb, and hence no longer capable of assertion and truth.

The verb – Russell claims – “when used as a verb, embodies the unity of the proposition” [PoM: 50]. The notion of *unity* is, in the *Principles*, a technical one; in particular, it represents one of the possible relations between whole and part, as described in chapter XVI of the *Principles*. To begin with, Russell claims that, provided that certain qualifications hold, “[w]henever we have any collection of many terms [...] there the terms [...] together form a whole” [PoM: 138-139] of which the terms are parts. Russell distinguishes, roughly, two kinds of wholes, *aggregates* and *unities*; an aggregate is a whole which “is completely specified when all its simple constituents are specified” [PoM: 140]; in an aggregate its constituent parts occur as terms in a collection, without “any direct connection *inter se*” [PoM: *ibid.*]; the connection among them is merely due to their “being parts of one and the same whole” [PoM: *ibid.*]. But “other wholes occur, which contain relations or what might be called predicates, not occurring simply as terms in a collection, but as relating or qualifying. Such wholes are always propositions. These are not completely specified when their parts are all known” [PoM: *ibid.*]. This creates the problem of unity which arises from the analysis of the proposition; as Russell goes on to say:

Take, as simple instance, the proposition ‘*A differs from B*’, where *A* and *B* are simple terms. The simple parts of this whole are *A* and *B* and difference; but the enumeration of these three does not specify the whole, since there are two other wholes composed of the same parts, namely the aggregate formed of *A* and *B* and difference, and the proposition ‘*B differs from A*’ [PoM: *ibid.*].

The result of analysis is thus an *aggregate* while the starting point was a proposition, a *unity*. When the verb does relate other terms, we then have a unity

and therefore a proposition; when it does not so relate, but is considered as a mere term of a collection, we have no unity and thus, in the end, no relation whatsoever with truth and falsehood.

Analysis thus destroys [see PoM: 50] the unity of the proposition; but the problem, however, is embedded in Russell's own account of propositions as constituted by terms; Russell's account seems unable to vindicate a feature Russell himself wants to grant for propositions, namely their being complex *unified* entities. The unity of the proposition, as Russell sees it, is due to the relation (the verb) uniting the other constituent terms of the proposition; but relations are terms as well, and terms, as we have previously said, are to be conceived, according to Russell's general account, as being object-like; they are *units, individuals, entities* [see PoM: 43]. How can something object-like be responsible for the unification of other object-like entities into a unity (the proposition)? As Hylton puts it:

If everything is, so to speak, object-like, what *could* be the source of the unity of the proposition? Anything one might put forward as an answer would turn out to be just one more item in need of unification. [...] Russell's attitude [...] [in the *Principles*] was that any component of the proposition would be – well, just one more component with the same status as the others.⁸⁶

Russell attempts to solve this problem by arguing that the relation (verb) is indeed a term, but that relations have the capacity of being either relating relations or relations in themselves (likewise verbs can either function as verbs or as verbal nouns). When relations do actually relate, they embody the unity of the proposition, but this is exactly what Russell, given his general account of terms, cannot explain: "Russell's problem is that he has no explanation whatever for what it is for the verb to be used as a verb rather than as inert verbal noun and

⁸⁶ Hylton [1984: 382].

logical subject. How do relations relate?"⁸⁷ Russell cannot, in fact, resort to an *essential* difference between relations as actually relating and relations in themselves; he cannot say that a verb used as verb can relate other terms because it is *intrinsically different* from the verb used as verbal noun; this would mean to embrace Frege's distinction between functions (predicates and relations) and objects; he sometimes seems on the brink of accepting it, for instance when he writes that "[t]he fact seems to be that a relation is one thing when it relates and another when it is merely enumerated as a term in a collection" [PoM: 140]; such a solution, on the other hand, would entail embracing the paradox originating from Frege's account. This Russell was not willing to accept;⁸⁸ he insisted, as we saw, on the fact that "every constituent of every proposition must, on pain of self-contradiction, be capable of being made a logical subject" [PoM: 48], and therefore, that "the concept which occurs in the verbal noun is the very same as that which occurs as verb" [PoM: *ibid.*]; thus the relation which occurs as relating the constituents of a proposition must, likewise, be the very same relation which is other times considered in itself.

This view, according to which there is *nothing* which is not a term makes it difficult to explain what the source of the unity of the proposition is; as Hylton writes:

Only something with a quite different status [from that of a term], Russell implied, could play the role of unifying the components into a proposition. But Russell's metaphysics rules out the possibility of there being anything with this kind of different status. [...] The constraints within which Russell was working in the years immediately following his

⁸⁷ Linsky [1992: 245].

⁸⁸ He wrote to Frege on 24 June 1902: "I know very well what good reasons there are to be found for this [Frege's] view; yet it is self-contradictory" [PMC: 134].

rejection of idealism make the problem of the unity of the proposition in principle unsolvable for him at that time.⁸⁹

Russell in fact concludes his discussion of the unity of the proposition by admitting his inability to find a plausible account of how propositions are essentially unified: “The verb, when used as a verb, embodies the unity of the proposition, and is thus distinguishable from the verb considered as a term, though I do not know how to give a clear account of the precise nature of the distinction” [PoM: 50]. Russell’s problem is that his own metaphysical account, with its all-embracing category of a term, points towards the denial of any actual distinction between a verb used as verb and a verb used as verbal noun (or as term), or, equivalently, between a relation actually relating and a relation in itself.

[5.3] Truth and Judgement

Propositions, we have seen, are for Russell complex entities, constituted by different kinds of terms, and capable of being true or false. A proposition is a particular kind of whole, namely, as seen, a *unity*. But a unity, in turn, is itself a term, and thus a proposition is a term, made up of simpler constituent terms.⁹⁰ In the end, therefore, the idea that a term is, in Russell’s metaphysics in the *Principles*, an all-embracing category must be taken quite literally. There is nothing, for Russell, which is not a term. This, however, may raise a question. We have seen that Russell regards propositions as what is either true or false [PoM: 12-13]; why, given that propositions are terms, is it only propositions, and not other terms, like

⁸⁹ Hylton [1984: 382]. A brief diagnosis of Russell’s inability to deal with the problem of the unity of the proposition is given by Davidson [2005: 102-106].

⁹⁰ As Russell says: “It is important to realize that a whole is a new single term, distinct from each of its parts and from all of them” [PoM: 141].

things, concepts, or aggregates, which are either true or false? Why do only propositions possess this characteristic?

We saw that, for Russell, propositions can be the result of the articulation of concepts with concepts or of things with concepts; things alone cannot form a proposition, but they can form another kind of whole that Russell calls an 'aggregate'. But neither simple terms nor aggregates can possibly be true or false. What makes propositions have this peculiar property (truth) is the fact that in them, one of their constituents is a verb, and relates the other constituents together: "the true logical verb in a proposition may be always regarded as asserting a relation" [PoM: 49]. Besides, we also saw that Russell insists on the requirement that every term in propositions must be made capable of being a logical subject. At the same time, Russell notes that when the verb of a proposition is turned into the subject the proposition is about, namely is turned into a verbal noun, the characteristic assertive function of a proposition disappears:

By transforming the verb, as it occurs in a proposition, into a verbal noun, the whole proposition can be turned into a single logical subject, no longer asserted and no longer containing in itself truth or falsehood. [...] [N]either truth nor falsity belongs to a mere logical subject [PoM: 48].

Russell's example is the difference between the proposition (expressed by the sentence) 'Caesar died' and the single logical subject 'The death of Caesar'. Russell argues that while the former has a logical complexity that makes it express an *assertion* about its subject, and therefore makes it *internally* related to truth and falsehood, the latter is a single logical subject, lacking a verb expressing an assertion. As Russell says, the difference between the two "seems to be that the death of Caesar has an external relation to truth or falsehood (as the case may be),

whereas ‘Caesar died’ in some way or others contains its own truth or falsehood as an element” [PoM: *ibid.*]. It seems, therefore, that Russell makes the notion of a *proposition* essentially related to that of an *assertion*, and thus to that of a *verb* used as verb. It is in fact the verb – Russell claims – “used as a verb, [which] embodies the unity of the proposition” [PoM: 50]; when the verb is actually used to relate the terms of a proposition, then the proposition itself can be seen as a logically *complex* unity expressing an assertion, and thus capable of being true or false. It seems thus that for Russell it is a proposition’s making an assertion that makes it internally related to the notion of truth.⁹¹

However, a clear account, or a definition, of the nature of *propositional complexity* is something that Russell says he is not able to provide. It is therefore useful to briefly come back to Russell’s treatment of the problem of unity, bearing in mind that this problem is crucial for Russell’s conception of truth. The notion of assertion, for Russell, can be explained by saying that it is “everything that remains of a proposition when the subject is omitted” [PoM: 83]. Therefore the assertive nature of the proposition expressed by the sentence, say, ‘Socrates is mortal’ can be highlighted by omitting the subject; ‘...is mortal’ will thus express the assertion; but this notion, as Hylton remarks, is contradictory given Russell’s

⁹¹ It is unclear the extent to which Russell intends to mark a difference between a (asserted) proposition and what he calls ‘a single logical subject’. Sometimes he seems to imply that both are propositions. Griffin [1980: 120] interprets Russell in this way. And at § 135 of the *Principles* Russell in fact *appears* to say that, for instance, ‘A’s difference from B’ is as much a proposition as ‘A differs from B’, the difference between them being that while the latter is an asserted proposition, the former is an unasserted one [see PoM: 139]. At § 52 Russell more explicitly says that “there seems to be no possibility of maintaining that the logical subject which results [from turning the verb of a proposition into a verbal noun] is a different entity from the proposition” [PoM: 48]. But, at the same, time, by declaring a proposition “anything that is true or false” [PoM: 12-13], and arguing that “neither truth nor falsehood belongs to a mere logical subject” [PoM: 52], he seems to be committed to rule out that a logical subject can be a proposition after all. Secondly, the claim that the verb used *as verb* embodies the unity of the proposition seems to compel Russell to deny that when the verb is used as verbal noun, as in ‘the death of Caesar’, we have a unity and thus a proposition.

metaphysics,⁹² and would imply that there are, after all, entities which cannot be made the subject of a proposition, contrary to Russell's insistence that nothing can be prevented from being a logical subject. What, in fact, does the notion of an assertion amount to? That notion simply amounts to the notion of a verb used as verb, namely to the notion of a relation as actually relating. As soon as that notion is in place, then it is clear that the term working as a relating relation (the verb) in a proposition cannot be turned into a logical subject without losing its characteristic of providing the assertive nature of the proposition, namely its unity.

There appears to be an ultimate notion of assertion, given by the verb, which is lost as soon as we substitute a verbal noun, and is lost when the proposition in question is made the subject of some other proposition. [...] Thus the contradiction which was to have been avoided, of an entity which cannot be made a logical subject, appears to have here become inevitable [PoM: 48].

When the relation works as relating relation in a proposition, then, it works in it as assertion; when, on the other hand, the relation is turned into a subject, or is considered as a relation in itself, is a mere *inert* propositional element. It seems, therefore, that Russell is not able, within the resources of his metaphysics, to account for propositional complexity and therefore for the relation a proposition has to the notion of truth and falsehood, relation given by its being a *unity*, and possessing an *assertive* nature. Russell is ready to recognise the difficulty posed by the notion of an assertion, "with which I do not know how to deal satisfactorily" [PoM: 48]. This difficulty, as we have said, is part of the difficulty of explaining propositional complexity. In the paper 'The Nature of Truth' (1905) he reaffirmed his inability to handle the problem. "Propositions are complexes of a certain kind,

⁹² See Hylton [1990: 177].

for some complexes are not propositions – for example, ‘the cow with the crumpled horn’ ‘Charles’ I’s execution’, etc. Propositions are distinguished, as a rule, in language, by the presence of a verb; and verbs seem to be used to express just that particular kind of complexity which propositions have and other complexes do not have. But I do not know how to describe this kind of complexity” [NT: 503].

Given the fact, already stressed, that the problem of the unity of the proposition is the problem of explaining how the internal complexity of a proposition makes it internally related to truth and falsehood, Russell’s failure to solve that problem will imply his inability to give an account of truth and falsehood altogether.⁹³ And this explains Russell’s reticence about the latter issue in the *Principles*, and his willingness to leave the question to the logician.

In a paper titled ‘Meinong’s Theory of Complexes and Assumptions’, published in 1904, however, Russell discusses the question of the nature of truth more extensively; his attitude to the problem of truth is twofold: on the one hand he seems to acknowledge difficulties about propositional unity [see MTCA: 436-437]; on the other hand, he tends to think that the complexity of a proposition (its being a *unity*) cannot be defined, so he describes it as being of a “special and apparently indefinable kind” [MTCA: 437], and as being “apparently incapable of

⁹³ If I am in the right in stressing the relatedness of these two issues in Russell’s view on propositions and truth, then Pelham’s account of Russell’s conception of truth in the *Principles* must be rejected. Pelham [1993: 338] writes: “Propositions are the bearers of truth on [...] [Russell’s and Moore’s] account but propositions are simply complex concepts which may be seen to be true when the elements of the proposition fit together in the right way”. This explanation does not work, precisely because Russell is not able to give an account of *how* the elements of proposition fit together (let alone whether they do it in the *right* or *wrong* way). This criticism, I think, can also be advanced against Ricketts [2001: 108], who claims that “Russell’s picture [...] seems to be that there are two ways in which the relation of loving can join two terms so as to form a proposition, the true way and the false way”.

definition" [MTCA: 463]. Russell sets thus aside in this way the problem of propositional unity, and reaffirms the position he held in the *Principles* about the nature of propositions; propositions are mind independent entities, which can be either true or false [see MTCA: 457, 472, NT: 494, 496].

Truth is seen as "a simple and undefinable property, which some propositions have and some lack. No further explanation is possible".⁹⁴ As Russell says:

It may be said – and this is, I believe, the correct view – that there is no problem at all in truth and falsehood; that some propositions are true and some false, just as some roses are red and some white [MTCA: 473].

One may wonder whether this simple view of truth⁹⁵ amounts to a *theory* of truth altogether. In fact, it amounts to the recognition, by Russell, that no account of truth can be seen as tenable. In 'The Nature of Truth', in fact, Russell claims he has no positive account of truth to advocate, and contents himself to criticise other accounts [see NT: 492].

To sum up: in the *Principles* Russell claims that propositions are objective complex entities; it is their complexity (in particular their internal articulation given by the presence of a relating relation) that makes them *unities* internally related to the notion of truth; but Russell is unable to provide an adequate account of propositional complexity, and this prevents him from giving an account of truth and falsehood (as this latter account must depend on the former); in the 1904-1905 papers we mentioned, Russell concludes that propositional complexity is

⁹⁴ Hylton [1990: 280].

⁹⁵ Called by Griffin [1980: 120] the 'intuitional theory of truth'.

indefinable, and therefore adopts the view that truth is likewise an indefinable property of propositions.

This account of truth is presupposed by Russell's account of judgement, to which I want now briefly to turn my attention. Judgement, or belief, is a relation between a subject, a judging mind, and a proposition; propositions, in fact, "may be assumed, believed or disbelieved" [MTCA: 472]. We said, at the beginning of this chapter, that the notion of a judgement is notably absent from the *Principles* because of Russell's anti-idealist stance; according to him the activity of the mind is confined to the mere recognition or passive apprehension of propositions.⁹⁶ So understood, a judgement is a *binary*, or *dual*, relation (of awareness) between a judging mind and a complex single entity, a proposition, whose truth or falsehood, in turn, determines the truth or falsehood of the corresponding judgement; as Russell says, a judgement or belief consists in "a certain attitude towards propositions, which is called knowledge when they are true, error when they are false" [MTCA: 473]. Therefore, an instance of *knowledge* is a belief in a true proposition, an instance of *error* is a belief in a false proposition.

Russell's position on truth is unsatisfactory. As Linsky said, it "really amounts to a declaration of inability to account for the truth of propositions at all";⁹⁷ and his account of judgement, since based on the former, is no better off. Russell's account simply does not explain the difference between truth and falsity; secondly, as, Russell himself acknowledges his account leaves without explanation

⁹⁶ See Hylton [1984: 381].

⁹⁷ Linsky [1992: 253]. As Hylton [1990: 340] rhetorically asks: "Surely there should be something to be said about [...] the difference between truth and falsehood? And can we really take it as a matter of brute fact that every proposition is either true or false, as it would be a brute matter of fact that every rose is either red or white, if it were true at all?"

the fact that true judgements are preferred to false ones, because we aim at truth and not at falsehood. Both false and true judgements, on Russell's account, have objective objects (propositions) in reality; so why should one aim at truth instead of falsity, given that both truth and false propositions are genuine and objective constituents of reality? Russell acknowledges that his theory "*seems* to leave our preference for truth a mere unaccountable prejudice, and in no way to answer to the feeling of truth and falsehood" [MTCA: 473]. Russell does not have a good answer to the problems posed by his conception of a proposition and its consequences for truth and falsehood, and even tries, very implausibly, to make our preference for truth over falsehood a matter of ethics:

What is truth, and what falsehood, we must merely apprehend, for both seem incapable of analysis. And as for the preference which most people [...] feel in favour of true propositions, this must be based apparently upon an ultimate ethical proposition: 'It is good to believe true propositions, and bad to believe false ones' [MTCA: 474].

In the next chapter we will consider how Russell modifies his account of judgement in consequence of the problems here discussed.

Chapter Six

The Multiple Relation Theory of Judgement in 1910-1912

[6.1] Truth and the 1910 Multiple Relation Theory of Judgement

Russell came to be more and more dissatisfied with his account of truth and judgement; his doubts about it were first put forward in an essay written in 1906, titled 'On The Nature of Truth', the third part of which was significantly revised and eventually published as a single paper in 1910 under the title 'On the Nature of Truth and Falsehood'. This paper, to which we will now turn our attention, contains Russell's own criticisms of his early account of truth and judgement, and a new theory of both. In short, what Griffin calls the *intuitional* theory of truth, is replaced by a *correspondentist* account of truth and falsehood, and the *dual* theory of judgement replaced by the *multiple relation* theory.

The correspondence theory of truth and the multiple relation theory of judgement in Russell's new account are by no means independent of each other. In fact, they are meant to be one and the same (this is so because, as we will see, truth, according to the new perspective, applies primarily to judgements). There is a priority, however, in Russell's motivations for changing his perspective on truth and judgement, and it was Russell's increasing dissatisfaction with his previous account of truth.⁹⁸ Once Russell realised that that account was defective, he was then forced to change his account of judgement (for, as we know, Russell's early theory of judgement was fully built upon the intuitional theory of truth); the new account, the multiple relation theory of judgement, was in fact seen as the only

⁹⁸ See Hylton [1984: 383-384].

account of judgement capable of cohering with the correspondence theory of truth.

The fact that Russell's adoption of the multiple relation theory of judgement was motivated by concerns about the nature of truth emerges clearly in 'On the Nature of Truth and Falsehood'. In his 1903-1905 account, Russell claimed that propositions are objective mind-independent entities, and that judgements have propositions as objective ground. True propositions are the objective ground of true judgements, while false propositions are the equally objective ground of false judgements. The truth or falsehood of Objectives, or propositions, on Russell's 1903-1905 account, is thus the source of the truth and falsehood of the corresponding judgements. Russell now finds such an account implausible, on the grounds that it is subject to a 'fatal' objection, which he presents as follows:

If we allow that all judgements have Objectives, we shall have to allow that there are judgements which are false. Thus there will be in the world entities, not dependent upon the existence of judgements, which can be described as objective falsehoods. This is in itself almost incredible: we feel that there could be no falsehoods if there were no minds to make mistakes. But it has the further drawback that it leaves the difference between truth and falsehood quite inexplicable [NTF: 119].

Russell now realises that his previous view, according to which the truth or falsehood of propositions is an ultimate and unaccountable feature of them, is implausible. There can be truth and falsehood at all, Russell now suggests, only in consequence of the fact that judgements are made upon reality, not because bits of reality are intrinsically true or false. Moreover, and more importantly, the difference between truth and falsehood is left, on that account, without explanation, and so, as we saw, is our preference for truth over falsehood.

In 'On the Nature of Truth and Falsehood' one can see the emergence of the correspondentist account of truth:

[I]t is difficult to abandon the view that, in some way, the truth or falsehood of a judgement depends upon the presence or absence of a 'corresponding' entity of some sort. And if we do abandon this view, and adhere to the opinion that there are both true and false Objectives, we shall be compelled to regard it as an ultimate and not further explicable fact that Objectives are of two sorts, the true and the false. This view, though not logically impossible, is unsatisfactory, and we shall do better, if we can, to find some view which leaves the difference between truth and falsehood less of a mystery [NTF: 119].

But as soon as the correspondence account of truth is in the picture, then there is no room any longer for the idea of objectively true and false propositions. The bearers of truth and falsehood, on Russell's new account, are in fact *judgements*, and propositions, as we will see more extensively later, will be disposed of. According to the view I have presented, therefore, the main reason of Russell's dissatisfaction with his early account of propositions lies in its inability to give an explanation of the nature of truth. But this, we saw in section [5.3], was due to Russell's inability to deal with the problem of propositional complexity. Russell's inability to provide an account of what makes a proposition a unity internally related to truth and falsehood is therefore the main reason behind his abandonment of the notion of a proposition.⁹⁹

In 'On the Nature of Truth and Falsehood' Russell asks: "If I judge (say) that Charles I died on the scaffold, is that a relation between me and a single 'fact', namely Charles I's death on the scaffold, or 'that Charles died on the scaffold', or

⁹⁹ In many of his writings Russell had the tendency of presenting his rejection of propositions as primarily due to ontological motivations, namely as due to the impossibility of believing that *there are* (false) propositions. In the lectures on *The Philosophy of Logical Atomism*, delivered in 1918, he wrote: "Time was when I thought there were propositions, but it does not seem to me very plausible to say that in addition to facts there are also these curious shadowy things going about such as 'That today is Wednesday' when in fact it is Tuesday. I cannot believe they go about in the real world. It is more than one can manage to believe, and I do think no person with a vivid sense of reality can imagine it. [...] To suppose that in the actual world of nature there is a whole set of false propositions going about is to my mind monstrous" [PLA: 196-197].

is it a relation between me and Charles I and dying and the scaffold" [NTF: 118]? The theory of judgement endorsed by Russell in 1910 can be seen as fully embracing the second alternative. A judgement is not a *dual* relation between a mind and a proposition, but a *multiple relation* between a mind and what were previously considered the constituents of a proposition (its terms). Russell provides a succinct account of his theory:

The theory of judgement which I am advocating is, that judgement is not a dual relation of the mind to a single Objective, but a multiple relation of the mind to the various other terms with which the judgment is concerned. Thus if I judge that *A* loves *B*, that is not a relation of me to '*A*'s love for *B*', but a relation between me and *A* and love and *B*. If it were a relation of me to '*A*'s love for *B*', it would be impossible unless there were such a thing as '*A*'s love for *B*', i.e. unless *A* loved *B*, i.e. unless the judgement were true; but in fact false judgements are possible. When the judgement is taken as a relation between me and *A* and love and *B*, the mere fact that the judgment obtains does not involve any relation between its objects *A* and love and *B*; thus the possibility of false judgements is fully allowed for [NTF: 122].

Our aim in this section is to explain exactly what this account of judgement amounts to. The core of the multiple relation theory of judgement, as Russell explains it, is that there is no single object of judgement, either when the judgement is true, or when it is false. The objects involved in a judgement are no longer propositions, *Objectives* or *unities*, but what was previously thought to be the constituents of propositions, namely *particulars* and *universals*, or, to use the jargon of the *Principles*, things and concepts (relations or predicates). The relation involved in a judgement is therefore one between a judging mind and several objects: the plurality of the latter is what determines the relation to be multiple (and not dual).¹⁰⁰ Now, it should be noted that Russell's rejection of the idea that judgement has as objective ground a proposition (Objective) does not imply that a

¹⁰⁰ A multiple relation is in fact defined by Russell as a relation which requires more than two terms [see NTF: 121], whereas a relation is dual when it involves only two terms [see NTF: 122].

judgement has not an objective ground: the new position does not entail a retreat from the claim that what makes a judgement true or false is a purely objective matter: in fact, the objects involved in the multiple relation constituting judgement (both when the judgement is true and when it is false) “are not fictions” [NTF: 120], but are still considered actual mind-independent objects.

Russell’s primary reason for adopting a multiple relational account of judgement is that it provides an explanation of the difference between truth and falsehood. As Russell writes:

We may [...] state the difference between truth and falsehood as follows: Every judgement is a relation of a mind to several objects, one of which is a relation; the judgment is *true* when the relation which is one of the objects relates the other objects, otherwise it is false [NTF: 122].

The difference between truth and falsehood implied by the multiple relation theory of judgement is due to the different role of the relation involved in the judgement, the relation other than the multiple relation of judging. Both when one’s judgement, for instance ‘*A loves B*’, is true or false, the objects of the judgement are several distinct entities, namely *A*, love, and *B*. When the judgement is true, Russell argues, the relation ‘love’, one of the objects of the judgement, actually relates the other objects, but when the judgement is false, the relation does not relate the other objects, namely *A* and *B*.

The difference between truth and falsehood, on Russell’s multiple relational account of judgement, can therefore be explained by saying that when the judgement is true the objects with which the judgement is concerned (namely the particulars and the relation other than the judging one) form a complex unity (or a complex object), whereas they do not form any such complex object when the

judgement is false. "If A loves B , there is such a complex object as ' A 's love for B ', and vice versa; thus the existence of this complex object gives the condition for the truth of the judgement ' A loves B '" [NTF: 123].

This account is said by Russell to entail a correspondence theory of truth because the complex which obtains, or exists, in the case the judgement is true is said to correspond to the judgement complex (the judgement is in fact a complex on its own, a complex formed by several objects related by the multiple relation of judging). In a paper written in 1910 (later included in the first volume of *Principia Mathematica*) Russell discusses the notion of a corresponding complex, whose existence is responsible for the truth of a judgement, as follows:

[W]e may define *truth* [...] as consisting in the fact that there is a complex *corresponding* to the [...] judgement. That is, when we judge ' a has the relation R to b ', our judgement is said to be *true* when there is a complex ' a -in-the-relation- R -to- b ' and is said to be *false* when this is not the case [TLT: 10].

And again:

When a judgement occurs, there is a certain complex entity, composed of the mind and the various objects of the judgement. When the judgement is *true* [...] there is a corresponding complex of the *objects* of the judgement alone. Falseness [...] consists in the absence of a corresponding complex composed of the objects alone [TLT: 11].¹⁰¹

To sum up Russell's theory so far: the multiple relation theory of judgement differs from Russell's previous conception because it conceives of judgement as a multiple relation holding between a mind and several objects; Russell's early

¹⁰¹ One should bear in mind that Russell's multiple relation theory of judgement applies to what he calls *elementary judgements*, where an elementary judgement is one which "merely asserts such things as ' a has the relation R to b ', ' a has the quality q ' or ' a and b and c stand in the relation S '" [TLT: 11]. The definition of truth as correspondence, likewise, only applies to elementary judgements. As Russell notes, in fact: "It is evident [...] that the definition of *truth* is different in the case of general judgements from what it was in the case of elementary judgements" [TLT: 12]. Since the truth which applies to elementary judgements is called '*elementary truth*', the correspondence theory of truth is a theory of elementary truth.

account of judgement, on the other hand, saw the objective ground of a judgement as being constituted by a single complex objective, a proposition. According to the early account, then, it was the truth or falsehood of this latter unified complex which determined the truth or falsehood of the corresponding judgement. According to the new conception, truth is a property of judgements [see NTF: 124]; a judgement is true when the objects related by the multiple relation of judgement (with the exception of the judging mind) form a complex unity, namely when the relation other than the judging one holds between the objects the judgement is about. If the relation does not relate them, then the judgement is false, for there is no complex unity corresponding to it. This conception of judgement and truth has, as noted, important ontological consequences. The previous ontology of objective propositions is now replaced by an ontology of *facts*, which no longer requires the world to be composed of actual and non actual complexes.¹⁰²

However, Russell realises that the account of judgement thus sketched cannot be said to be definitive; towards the end of 'On the Nature of Truth and Falsehood', he proposes an important amendment to the theory, in consequence of the difficulty posed by *asymmetric* relations, namely those relations which have a *direction*. Russell discusses the problem in the following way:

Let us take the judgement 'A loves B'. This consists of a relation of the person judging to A and love and B, i.e. to the two terms A and B and the relation 'love'. But the judgement is not the same as the judgement 'B loves A'; thus the relation must not be abstractly before the mind, but must be before it as proceeding from A to B rather than from B to A. The 'corresponding' complex object which is required to make our judgement true consists of A related to B by the relation which was before us in our judgement [NTF: 123].

¹⁰² See Hylton [1990: 337], and Griffin [1985: 216].

The problem lies in the fact that it is not sufficient to characterise a judgement as true when the relation relates the other objects (the particulars – other than the judging mind – involved in the judgement): such a characterisation in fact would not mark a difference between the truth of the judgement that *A* loves *B* and the judgement that *B* loves *A*. More specifically, such a conception of judgement must be fallacious because it would qualify the judgement that *A* loves *B* as true even when, as a matter of fact, *A* does not love *B*, but *B* loves *A*.

Russell amends the theory by claiming that in the judgement complex the relation other than the judging one should not occur as a mere object or – to use the language of the *Principles* – as a relation in itself (this seems what Russell means by saying that ‘the relation must not be abstractly before the mind’), but with a *sense*, or *direction*; this amounts to saying that if the judgement in question is ‘*A* loves *B*’, one of the objects related by the multiple relation of judging must be the relation ‘loves’ as proceeding from *A* to *B* and not vice versa. Only if the relation ‘loves’ does relate *A* and *B* in the *corresponding* complex ‘*A* loves *B*’, thus in the *same way* as it does in the judgement (with the same *direction*), is the judgement true, otherwise it is false. Russell thus provides the following modification of his account of judgement and truth, taking into account the possible direction of the relation:

We may distinguish two ‘senses’ of a relation according as it goes from *A* to *B* or from *B* to *A*. Then the relation as it enters into the judgement must have a ‘sense’, and in the corresponding complex it must have the same ‘sense’. Thus the judgement that two terms have a certain relation *R* is a relation of the mind to the two terms and the relation *R* with the appropriate sense: the ‘corresponding’ complex consists of the two terms related by the relation *R* with the same sense. The judgement is true if there is such a complex, and false when there is not. The same account, *mutatis mutandis*, will apply to any other judgement [NTF: 123-124].

The judgement that *A* loves *B* involves a multiple relation between a mind and several objects (*A*, loves, and *B*), and the judgement is true when, and only when, there is a corresponding complex formed by the objects of the judgement (*A* and *B*) related by the relation (loves); otherwise it is false. However, since some relations are *asymmetrical*, the direction or sense of the relation matters for identifying the resulting complex. Thus Russell argues that the relation in the judgement must have the same sense as the relation in the corresponding complex.

But Russell's account of judgement is problematic. Peter Geach notices this in a passage of his *Mental Acts*, and poses the following question about Russell's theory:

[I]f the relation *R* is before the mind, not as relating *a* and *b*, but only as a term of a judging relation that holds between the mind, *a*, the relation *R*, and *b*, how can there be any talks of the relation *R*'s 'proceeding' from *a* to *b* rather than from *b* to *a*? How can a relation that occurs not as relating things, but as one of the things related by another relation, occur with one or the other 'sense'?¹⁰³

Geach is here stressing a tension in Russell's formulation of the multiple relation theory. We have seen that the theory requires the relation to occur in the judging complex as an *object*. It is necessary for Russell to rule out the possibility that such a relation actually holds together the other constituents of the judgement: if this were so, the constituents of the judgement would form a unity not different from a proposition (an Objective) and then the multiple relation theory would collapse into the dual theory. On the other hand, the possibility of asymmetric relations forces Russell to admit that the relation should enter into the judgement complex not merely as an *object*, but with a *sense*, or *direction*. But this raises the dilemma

¹⁰³ Geach [1957: 51].

highlighted by Geach: if the relation is an *object*, it cannot have a *sense*, and if it has a sense, it cannot be an object alongside the other components of the judgement.

Russell's discussion of the multiple relation theory in 'On the Nature of Truth and Falsehood' is rather sketchy. In particular, Russell never explains clearly what he means by saying that a relation has a *sense*: he seems to be implying that it proceeds from an object to another. But a relation cannot proceed from an object to another without *relating* the two objects, without, thus, being a relating relation. But this would have consequences that Russell cannot accept; in fact, as soon as one accepts that the relation R , in the judgement that aRb , is a relating relation, then one must conclude that the objects with which the judgement is concerned acquire a *unity* independent of the judging relation: the objects of the judgement, so related by the sense of the relation, would constitute a genuine *fact* in the world.¹⁰⁴ If so, then, there would be no room altogether for a correspondence theory of truth, for when the judgement is true the judgement has in itself, as sub-propositional part, the judged fact.¹⁰⁵

Secondly, if the relation R , in the judgement that aRb , is a relating relation, then the theory would make false judgements impossible, because *any* judgement, just by virtue of occurring, would *create* the fact with which it is concerned. Any judgement, in fact, is a multiple relation between a mind and several objects (among which a relation must occur); but if these objects are related by the relation in an appropriate *order* or *direction*, this would imply that any judgement, by

¹⁰⁴ This is due to the fact that, as Hylton [1990: 335] notes, "[o]ne point of continuity between the old view [that of the *Principles*] and the new [the multiple relation theory] is that it is the objects themselves with which we are in contact when we judge – not ideas or concepts of the objects".

¹⁰⁵ See Candlish [1996: 113] and Baldwin [1991: 47].

simply occurring as judgement, is a *true* judgement. A way of putting the problem is Candlish's, according to whom such a theory would make "false judgement impossible [...] because any coherent judgement at all will make itself true in the act of formulation. [...] [F]alse judgement is impossible because any judging creates the fact that makes the judgement true.¹⁰⁶ So, if by saying that the relation (other than the judging one) has a *direction*, Russell means that it actually is a relating relation, then his multiple relation theory would collapse, for it would be shown to be flawed with regard to one of the reasons of its formulation, particularly the need to provide an account of truth and allow for false judgements without assuming objective falsehoods (objectively false propositions).

On the other hand, if Russell does not mean to regard the relation as relating, then it is not at all clear what could be meant by the claim that a relation – one of the objects in the judgement structure – has a direction or sense. It is not clear that he can give a coherent explanation of the idea that an *object* proceeds (or goes) from an object to another, or that such an explanation can be given at all. As we will see in the next section, Russell soon provides a modification of the multiple relation theory which can be seen as attempting, even though Russell is not explicit about it, to solve this problem, the problem of the status of the relation other than the judging one.

¹⁰⁶ Candlish [1996: 114-115].

[6.2] The 1912 Multiple Relation Theory of Judgement

The new multi-relational account of judgement, presented in chapter 12 of Russell's 1912 book *The Problems of Philosophy*, is briefly presented as follows:

The relation involved in *judging* or *believing* must, if falsehood is to be duly allowed for, be taken to be a relation between several terms, not between two. When Othello believes that Desdemona loves Cassio, he must not have before the mind a single object, 'Desdemona's love for Cassio', or 'that Desdemona loves Cassio', for that would require that there should be objective falsehoods, which subsist independently of any minds; and this, though not logically refutable, is a theory to be avoided if possible [PP: 72].

This is in accordance with what Russell says about the nature of judgement and its relation with truth and falsehood in 'On the Nature of Truth and Falsehood'. But, as briefly anticipated, the multiple relation theory presented in *The Problems of Philosophy* seems to be modified in order to amend precisely the difficulty we have identified in Russell's previous account, due primarily to the relation's occurrence with a *sense*, or *direction*, in the judging complex.

Russell first of all defines his terminology: he calls the judging mind the 'subject' of a judgement, and all the other items involved in the judgement its 'objects'; in turn, the subject and the objects of a judgement are called the 'constituents' of the judgement. Russell then says:

It will be observed that the relation of judging has what is called a 'sense' or 'direction'. We may say, metaphorically, that it puts its objects in a certain *order*, which we may indicate by means of the order of the words in the sentence. [...] Othello's judgement that Cassio loves Desdemona differs from his judgement that Desdemona loves Cassio, in spite of the fact that it consists of the same constituents, because the relation of judging places the constituents in a different order in the two cases. [...] This property of having a 'sense' or 'direction' is one which the relation of judging shares with all other relations [PP: 73].

This passage shows that the main point of difference between the 1910 account and the new one lies in the fact that while on the former account it was the relation other than judgement, namely the relation we may call the 'subordinate' or

‘embedded’ relation, as providing the *sense* or *direction* of the judgement, the 1912 account transferred the onus of being responsible for the direction of the judgement to the judging/believing relation itself, namely to the multiple relation constituting judgement. Othello’s judgement that Desdemona loves Cassio, then, differs from Othello’s judgement that Cassio loves Desdemona, not because in the first judgement the relation ‘loves’ goes from Desdemona to Cassio, whereas in the second judgement it goes in the opposite direction, but because in the two judgements the objects are put by the relation of judging (or believing) in a different order.

Transferring the bearer of *direction* from the subordinate relation to the judging relation, however, implies that the embedded relation is merely a term among other terms, an object of acquaintance.

When an act of believing occurs, there is a complex, in which ‘believing’ is the uniting relation, and subject and objects are arranged in a certain order by the ‘sense’ of the relation of believing. Among the objects [...] one must be a relation [...]. But this relation, [...] as it occurs in the act of believing, is one of the objects – it is a brick in the structure, not the cement. The cement is the relation ‘believing’ [PP: 74].

The metaphor of *bricks* and *cement* is useful; while on the 1910 account the subordinate relation was itself functioning (at least implicitly) as *cement* in the sub-complex judged, because it was to provide its *sense*, on the 1912 account the embedded relation is just another term or, to use an expression of Linsky’s, “an inert constituent of the judgement”.¹⁰⁷ The only relation actually functioning as *cement*, is, on the 1912 account, the multiple relation of judging. As Russell says, evidently criticising his previous account of judgement:

¹⁰⁷ Linsky [1992: 256].

As regards the sense of R in aRb, [...] [t]here must never, so I now perceive, be any relation having sense in a complex except the relating relation of that complex; hence in the act of judging, the sense must be confined to the judging, & must not appear in the R".¹⁰⁸

We may thus sketch the difference between the two accounts in the following way: according to the 1910 multiple relation theory of judgement, a subject *S*'s judgement (*J*) that *A* loves *B* can be represented as '*J* (*S*, *A*, love from-*A*-to-*B*, *B*)' whereas a subject *S*'s judgement (*J*) that *B* loves *A* can be rendered as '*J* (*S*, *A*, love from-*B*-to-*A*, *B*)'. According to the 1912 account the former judgement would be '*J* (*S*, *A*, love, *B*)', whereas the latter would be '*J* (*S*, *B*, love, *A*)', where the judging relation puts the constituents of the judgement in a certain order.

When discussing the 1910 account of the multiple relation theory I said that, for Russell, a judgement has *unity*. According to the *Principles* the object of a judgement is an already unified complex, a proposition; the judgement itself does not contribute in any way to the unity of the object of judgement. The 1910 account, at first sight, seems to propose an opposite picture. There is no *unity* in the object of judgement, for there is no single object of judgement, but several distinct entities to which the judging mind is related by the multiple relation of judgement. But, in fact, one of the objects of the judgement, namely the subordinate relation, was actually meant to enter in the judgement structure with a *sense*, and if this is understood as meaning that the relation *relates* the other objects of the judgement according to a certain direction, then the 1910 account is at bottom, we might perhaps say, a *disguised* form of dual relation theory of judgement, for it sees judgement as a relation between a mind and the unified

¹⁰⁸ Quoted in Candlish [1996: 116], and Griffin [1985: 220].

complex made up by the objects of the judgement related by the *sense* or *direction* of the subordinate relation.

The characteristic feature of the version of the theory of judgement presented in *The Problems of Philosophy*, as we have sketched it, is that it withdraws the role of providing a sense or direction (to the judgement) from the subordinate relation. The subordinate relation is now squarely a *brick* in the judgement structure, not its *cement*; it is a mere object-relation, related (together with the objects of the judgement) by the multiple relation of judgement. But now it seems as though Russell's notion of a judgement can hardly be differentiated from the sum of the several dual relations of 'awareness', or, 'acquaintance', a judging mind has with the entities about which the judgement is concerned. As Candlish puts it, commenting on Russell's 1912 account of judgement:

When I judge *A* loves *B*, the relation *love* appears purely as a relation in itself [...] not as a relating one. What then makes this into a *judgement*? Suppose my mind leaps from *A* to love to *B*, or groups them in that order: this is not a judgement that *A* loves *B*. It will not be a judgement unless *love* is allowed to appear as a relating relation itself. And once that is allowed, all the old difficulties of the 1910 theory re-emerge.¹⁰⁹

And Pears argues, on similar lines: "Russell maintains that the only acquaintance needed [...] [in order to judge that aRb] is separate acquaintance with each of its constituents, *a*, *R*, *b*. However, [...] [t]he subject still has to combine these constituents in thought, and when he does this, how does he know that the combination is meant for the possibility of aRb and not for the possibility that each of its constituents exist separately?"¹¹⁰ On the 1910 account a *judgement* that aRb (as opposed to bRa) was explained by the subordinate relation's having a direction,

¹⁰⁹ Candlish [1996: 117].

¹¹⁰ Pears [1989: 174].

going from *a* to *b*. But relying exclusively on the relation between the mind and the items of a judgement, in order to characterise the structure of the judgement does not seem to explain *how* a judgement is at all different from several instances of acquaintance with objects; it does not explain how a judgement is a *combination* of elements instead of a mere *grouping* of them.

A related problematic aspect to Russell's 1912 account of judgement is the following; we saw that on the 1910 account Russell grants a difference between particulars and universals (especially relations). Unlike particulars (objects), relations can occur in complexes with a direction or sense. On the 1912 account, however, the subordinate relation is explicitly treated as a *brick* in the judgement structure, as an *object*, not different from the other constituents of the judgement. But then Russell does not seem to be in a position to give an account of a *relational* brick [see PP: 74] or of what is meant by an *object-relation* [see PP: 74-75]. By abandoning the idea that the subordinate relation (in a judgement complex) has a sense or direction, Russell seems to lose the resources to distinguish it from a mere object, a particular.

In the next chapter we will be explaining and discussing the version of the multiple relation theory of judgement Russell develops in the 1913 manuscript *Theory of Knowledge*, and show that the new theory was, at least partly, motivated by the problematic aspects of his account of judgement discussed above.

Chapter Seven

The Multiple Relation Theory of Judgement in 1913

[7.1] Propositional Thought, Understanding and Judgement

One of Russell's aims, in *Theory of Knowledge*, is to explain the difference between sensation, imagination, perception on the one hand, and what he calls 'propositional thought' (understanding and judgement) on the other. The former all involve a dual relation of acquaintance with particular objects (sense-data); the latter, by contrast, is "fundamentally different from acquaintance: the logical form of the occurrences involved, whatever it may be, must be unlike that of [...] dual relations" [TK: 110]. Propositional thought is explained by Russell in terms of a multiple relational account; such an account, however, is significantly different from both the 1910 and the 1912 ones. In this chapter we will discuss Russell's notions of 'propositional thought', 'understanding' and 'judgement', and then consider the features that separate the 1913 account from its predecessors.

A 'propositional thought' is defined by Russell as "one which involves a 'proposition' in its meaning" [TK: 110], and it may be subdivided into different categories; believing, disbelieving, doubting are all examples of propositional thought. Understanding, moreover, is the most comprehensive form of propositional thought, the one that presupposes all others. But before we follow Russell's discussion of the notions of understanding and judgement, however, it is necessary to briefly consider his usage of the notion of a proposition, which occurs in the *Theory of Knowledge* manuscript, but which was notably absent from the 1910 and the 1912 multiple relational accounts of truth and judgement. It is in fact fairly

obvious to say that the multiple relation theory of judgement stands in overt opposition to the notion of a proposition as it was used in the *Principles*, and so it seems odd to find that very notion embedded in a discussion of the notion of understanding and judgement which presupposes a multiple relation.

Russell, in *Theory of Knowledge*, considers a proposition to be what is common between assertions, questions, commands, having the same *content*; to use Russell's own example [see TK: 107]: consider the sentences 'beggars are riders', 'are beggars riders?', 'beggars shall be riders'; in all of these sentences the relation between beggars and riders is the same, but the first sentence expresses that relation as the object of an assertion, the second as the object of a question, the third as an object of a volition. Russell concludes from this that "[w]e should not say that all these phrases 'have the same meaning', yet they all have something very important in common. The word 'proposition' is a natural one to use for expressing what they all have in common: we may say that they express different attitudes toward the same 'proposition'" [TK: *ibid.*].

A proposition, Russell goes on to say, can be regarded as being an 'incomplete symbol' [TK: 109]. What Russell means is that the *phrase* expressing a proposition is an incomplete symbol, namely an expression which requires a context before it can express a complete meaning [see PM: 44].¹¹¹ This, Russell claims, can be seen if we consider the proposition as what is expressed by the expression 'beggars being riders', from which any reference to assertion, question, or command has been omitted. A judgement is thus understood as an *act* providing the context for a sentence expressing a proposition to acquire a

¹¹¹ See Hylton [1990: 342].

complete meaning. A proposition, at any rate, is not considered as a single entity; both true and false propositions must be considered as “non-entities” [TK: 110], expressed by incomplete symbols. Propositional thought, namely any mental act directed towards propositions so understood, does not therefore involve a dual relation between a mind and a single proposition, a single Objective.

Since Russell thinks understanding to be the most comprehensive form of propositional thought, he concentrates on it, and therefore in *Theory of Knowledge* we find a multiple relational analysis of *understanding* instead of one of *judgement*. This does not matter much, because Russell considers understanding and judgement “to be closely akin” [TK: 108]; “[b]oth [...] have not a single object, the ‘proposition’, but have a plurality of objects, united with the subject in a *multiple* relation” [TK: 137]. Moreover, as we said, belief presupposes understanding, for, as Russell says, “[i]t is obvious that we cannot believe or disbelieve or doubt a proposition without understanding it” [TK: 110]. Generally speaking, therefore, what Russell says about understanding in *Theory of Knowledge* can be applied, *mutatis mutandis*, to judgement as well.

We can now turn our attention to Russell’s multiple relational account of understanding, considering in which ways it differs from his previous accounts of judgement. A general statement of the account Russell adopts in 1913 is to be found in the following passage:

Let us take as an illustration some very simple proposition, say ‘*A* precedes *B*’, where *A* and *B* are particulars. In order to understand this proposition it [...] is obviously necessary that we should know what is meant by the words which occur in it, that is to say, we must be acquainted with *A* and *B* and with the relation ‘preceding’. It is also necessary to know how these three terms are meant to be combined; and this [...] requires acquaintance with the general form of a dual complex [TK: 110-111].

We know that both variants of the multiple relation theory hitherto considered saw it required that, in order to judge, one should be acquainted with the *objects* of the judgement (with the things the judgement is about). Therefore, on both the 1910 and the 1912 account, in order to understand 'A and B are similar', one has to have a relation of acquaintance (or 'awareness') with A, B, and similarity. But now Russell says that a further component must be introduced in order to understand a proposition, and this new component is the *form* of the proposition, the *form* of the *complex* in question: In order to understand 'A and B are similar', we must be acquainted with A and B and similarity, and with the general form of (symmetrical) dual complexes [see TK: 112]. Our next task will then be to give a brief account of Russell's notion of the form of a complex.

[7.2] Logical Form

As Russell explains, the form of a complex is the way in which the constituents of the complex are ordered and put together:

[W]hen *all* the constituents of a complex have been enumerated, there remains something that may be called the 'form' of the complex, which is the way in which the constituents are combined in the complex [TK: 98].

Acquaintance with the form of a complex is in turn necessary for understanding the complex:

[I]n order to understand 'A and B are similar', we must know what is supposed to be done with A and B and similarity, i.e., what it is for two terms to have a relation; that is, we must understand the form of the complex [...] [TK: 116].

Russell's discussion of the notion of form in *Theory of Knowledge* is rather muddled.

Here, instead of providing a full account of it, we will simply highlight some of its

relevant features, necessary for explaining Russell's conception of understanding and therefore of judgement.¹¹² Russell explains why the form of a complex is a necessary requisite for *understanding* the complex in question as follows:

Let us suppose that we are acquainted with Socrates and with Plato and with the relation 'precedes', but not with the complex 'Socrates precedes Plato'. Suppose now that some one tells us that Socrates precedes Plato. How do we know what he means? It is plain that his statement does not give us *acquaintance* with the complex 'Socrates precedes Plato'. What we understand is that Socrates and Plato and 'precedes' are united in a complex of the form ' xRy ', where Socrates has the x -place and Plato has the y -place. It is difficult to see how we could possibly understand how Socrates and Plato and 'precedes' are to be combined unless we had acquaintance with the form of the complex [TK: 99].

Russell thus understands the form of a complex as the way in which its constituents are related in the complex. This conception of the logical form of a complex entails, for Russell, that the form itself is *not* a constituent of the complex. If the form of a complex were itself a constituent of the complex, Russell argues, "there would have to be a new way in which it and the [...] other constituents are put together, and if we take this way again as a constituent, we find ourselves embarked on an endless regress" [TK: 98]. While the constituents of a complex are, loosely speaking, "things" (particulars are *objects* while universals, *predicates* and *relations*, are something like "platonian ideas" [TK: 92]), the form of a complex is "not a 'thing'" [TK: 98], but the way in which objects and universals are arranged together.

Besides, the notion of form is, in *Theory of Knowledge*, a *logical* notion; Russell claims that forms are *logical objects* [see TK: 97]. Although Russell admits that acquaintance can have a somehow different meaning when logical notions such as forms are concerned, from the meaning it has when things are concerned, he

¹¹² For a more detailed account of the notion of form in Russell's 1913 *Theory of Knowledge* manuscript see Pears [1977].

grants that acquaintance with logical objects is possible, in which case it can take the name of “logical experience” or “logical intuition” [TK: 97, 101]. Understanding (and judging) a complex thus presupposes logical intuition, acquaintance with its logical form. Russell is also committed to the claim that logical forms are *simple* [TK: 129]. However, his characterisation of the logical form of a complex as the way in which its constituents are united makes it difficult to see how such a form can be a *simple* logical object. To see this, we must briefly take into account Russell’s idea that every logical notion involves a generalisation:

Every logical notion, in a very important sense, is or involves a *summum genus*, and results from a process of generalization which has been carried to its utmost limit. This is a peculiarity of logic, and a touchstone by which logical propositions may be distinguished from all others [TK: 97].

We have already seen that Russell considers the logical form of a proposition to be what is left when all its constituents have been enumerated; what he means is that the form of a complex like ‘*A* is similar to *B*’ is the result of a process of *generalisation* or abstraction which involves the constituents of the complex. As Russell says, “[t]he natural way to symbolize a form is to take some phrase in which actual entities are put together in that form, and replace all these entities by ‘variables’, i.e. by letters having no meaning” [TK: 98]. Thus the symbol ‘ xRy ’ can be used to symbolise the form of any dual complex, namely of any complex involving two constituents united by a relation.

Apparently, however, ‘ xRy ’ is still not a completely generalised symbol: as Russell says:

If we take some particular dual complex xRy this has three constituents, x , R , and y . If we now consider ‘something has the relation R to y ’, we get a fact that no longer contains x , and has not substituted any other entity for x , since ‘something’ is nothing. Thus our new

fact contains only y and R . For similar reasons, 'something has the relation R to something' contains no constituent except R ; and 'something has some relation to something' contains no constituent at all. It is, therefore, suitable to serve as the 'form' of dual complexes [TK: 114].

An obvious difficulty seems to be involved in this: how can 'something has some relation to something' be *simple*? Russell admits that, at first sight, "it seems to have a structure, and therefore to be not simple; but it is more correct to say that it is a structure" [TK: *ibid.*]; he concludes, however, that the difficulty lies in language, for it "is not well adapted for speaking of such objects" [TK: *ibid.*]. As Russell goes on:

If there is such a thing as acquaintance with forms [...] then a form must be a genuine object; on the other hand, such absolutely general 'facts' as 'something is somehow related to something' have no constituents, are unanalyzable, and must accordingly be called simple [TK: 129].

Russell defines acquaintance with logical forms as being a particular case of *understanding*: understanding and believing, as regards logical forms, are in fact both dual relations. Understanding a logical form is thus different from, and simpler than, understanding a non-logical complex, such as 'A is to the right of B'. As we will see more fully later, the understanding of the latter must be analysed, unlike a relation of acquaintance, as a multiple relation.

Before considering how the logical form of a complex is supposed to enter in a multiple relational account of understanding, we should notice some unclarities involved in the notion of logical form as it is expounded in *Theory of Knowledge*. In particular, Russell does not provide a univocal account of it, but seems to change its meaning several times in his discussion. We are firstly told that the logical form of a complex is not a thing, but the *way of combination* of its elements, namely the *structure* of the complex; this structure is, in turn,

understood as a *simple object* with which one has to be acquainted in order to understand the complex in question; its simplicity is then based by Russell on the fact that such an object is actually a completely generalised *fact*. Forms are thus simultaneously described as *structures*, *simple logical objects*, and *absolutely general facts*.¹¹³

[7.3] Logical Form and Judgement

In this section we will explain the reason why Russell introduces the notion of the form of a complex as a component of the judging complex. The multiple relation theory in *Theory of Knowledge* seems to be presented in direct opposition to its predecessors. Russell says:

I held formerly that the objects alone sufficed, and that the 'sense' of the relation of understanding would put them in the right order; this, however, no longer seems to me to be the case [TK: 116].

I think the account Russell here says he formerly held is the 1912 one. If my interpretation – presented in section [6.2] – is correct, that account was devised in order to overcome the problem posed by granting that the subordinate relation in a judgement complex has a sense or direction, a feature of the 1910 account. This, as noted, was partly motivated by the need to account for judgements about asymmetrical complexes. On the 1912 account, by contrast, judgements about asymmetrical complexes were accounted for by claiming that the judging relation puts the objects the judgement is about in the right order. In the above passage,

¹¹³ Wittgenstein famously remarked that logical forms, on Russell's account, "were to have the useful property of being compounded, and were to combine with this the agreeable property that they could be treated like 'simples'" [NL: 100-101]. See, for critical appraisals of Russell's discussion of the notion of logical form, Griffin [1980: 167-168], Pears [1977: 179-181], and Hylton [1990: 344-348].

however, Russell says that this solution does no longer seem to him correct. It is tempting to see the introduction of logical forms as Russell's way out of this difficulty, and therefore to consider the 1913 account as aiming to solve the problems posed by asymmetrical complexes.

Despite being rather widespread among commentators,¹¹⁴ this interpretation is mistaken. Russell's introduction of logical forms in the 1913 account is not intended to solve the problem of the direction (or sense) of the judged complex.¹¹⁵ In the passage cited above, where Russell criticises his previous account, he does not refer to an asymmetrical complex, like 'A precedes B', but to a symmetrical one, namely 'A is similar to B'; and, of course, it is only with regard to asymmetrical complexes that the problem of direction arises. Russell, however, considers it an unnecessary complication in the attempt to provide an account of understanding. The difficulties posed by asymmetrical complexes are "not an essential part of the difficulty of discovering what is meant by 'understanding a proposition'. We shall do well, therefore, to take examples which do not introduce 'sense'" [TK: 112]. Moreover, Russell's explanation of the notion of logical form implies that two dual complexes such as 'A precedes B' and 'B precedes A', have the *same* form; so the notion of form cannot play a role in explaining in what respect the two complexes differ:

A complex has a property which we may call its 'form', and the constituents must have what we call determinate 'position' in this form. 'A precedes B' and 'B precedes A' have the same form as well as the same constituents, they differ only as regards the 'position' of the constituents [TK: 81, see also 145].

¹¹⁴ A recent example is Hanks [2007: 127].

¹¹⁵ See Hylton [1990: 344-345] and Candlish [1996: 119].

This provides evidence for the conclusion that the problem of the sense or direction of a complex is not supposed to be accounted for by the introduction of the notion of logical form in the account of judgement. Since, moreover, this is the element of novelty in Russell's 1913 account, one can say that the 1913 account, as a whole, is not meant to provide a solution to that problem.

If so, then, what are Russell's reasons for introducing logical form in his account of judgement and therefore for modifying his previous theories? Here I want to discuss Hylton's proposal, according to which the notion of logical form, in Russell's 1913 account, is motivated by two main reasons. The first reason is identified by Hylton as being expressed by the following excerpt from *Theory of Knowledge*, a passage we have already quoted when discussing Russell's notion of logical form:

Let us suppose that we are acquainted with Socrates and with Plato and with the relation 'precedes', but not with the complex 'Socrates precedes Plato'. Suppose now that some one tells us that Socrates precedes Plato. How do we know what he means? It is plain that his statement does not give us *acquaintance* with the complex 'Socrates precedes Plato'. What we understand is that Socrates and Plato and 'precedes' are united in a complex of the form ' xRy ', where Socrates has the x -place and Plato has the y -place. It is difficult to see how we could possibly understand how Socrates and Plato and 'precedes' are to be combined unless we had acquaintance with the form of the complex [TK: 99].

Here Russell marks a difference between perceptual knowledge of a complex, for instance 'Socrates precedes Plato', and different instances of acquaintance with 'Socrates', 'Plato', and 'precedes'. Neither amounts to the judgement that 'Socrates precedes Plato'. A judgement does not provide acquaintance with a complex, for the judgement may be false, and, therefore, there may be no such complex. At the same time, how can acquaintance with 'Plato', 'Socrates', and 'precedes' put one in the position to understand that 'Socrates precedes Plato'? The solution is that

judgement proceeds via the inclusion of the logical form of the complex which corresponds to the judgement.

[T]he crucial point about a judgement and its 'corresponding' fact is that the judgement is to be true if (but only if) there is such a fact; and this requires that the judgement should not merely have the same objects as the fact, but should also represent them as combined in the way they are combined in the fact (if there is one). So the logical form which figures in the judgement is the form of the corresponding fact.¹¹⁶

We can thus see that the inclusion of the logical form of the (judged) complex can be at least partly motivated by Russell's realisation of the shortcomings of the 1912 account of judgement (highlighted at the end of section [6.2]), according to which the objects with which the judgement is concerned (and the judging mind) were the only elements of a judging complex. Now Russell thinks that that account is mistaken, and that the judgement complex should also include, besides the objects of the judgement, the way in which the objects are *combined* in the corresponding complex if the judgement is true. In a judgement, thus, objects are essentially *combined* with one another; but on the 1912 account, judgemental constituents do not have the *internal* resources to combine, because they are all considered as objects (bricks). Russell's idea, that the judgement relation itself puts the judgement object in a certain order does not seem to enable him to distinguish a judgement (where object are *combined*) from merely *grouping* objects in thought. In the next chapter we will see that this reason Russell offers for modifying his previous account of judgement is connected to Wittgenstein's criticism.

Hylton then presents what he takes to be Russell's second reason for introducing logical forms in his account of judgement. Russell presents it as follows:

¹¹⁶ Hylton [1990: 345].

Suppose we wish to understand 'A and B are similar'. It is essential that our thought should, as is said, 'unite' or 'synthesize' the two terms and the relation; but we cannot actually 'unite' them, since either A and B are similar, in which case they are already united, or they are dissimilar, in which case no amount of thinking can force them to become united. The process of 'uniting' which we can effect in thought is the process of bringing them into relation with the general form of dual complexes [TK: 116].

Russell acknowledges that uniting objects in thought (as is required by judgement) cannot entail uniting them in reality: a judgement, in other words, cannot create the fact that was said to correspond to it. This, in fact, would have the consequence that truth cannot, after all, be explained in terms of correspondence between a judgement and reality. This acknowledgement, then, amounts to a recognition of the shortcomings of the 1910 account of judgement. As Hylton comments on the last excerpt quoted: "If the judgement [...] [represents the objects as combined] simply by bringing it about that the objects are so combined, then this no longer appears to be uniting them in thought only. So we seem to be in danger of losing the crucial distinction between uniting the objects in thought and uniting them in reality".¹¹⁷ This is exactly the problem with Russell's 1910 account's treatment of the subordinate relation, discussed in section [6.1], now fully acknowledged by Russell himself.

Russell argues that the unification of the elements of a judgement does not amount to creating the corresponding fact, because, in a judgement, objects are brought into relation with each other and with the general form of the corresponding complex; the general form is thus understood as a further element in the judging complex. Hylton says:

[T]he judgement represents the constituents as combined in the right way not by so combining them but by including 'the way they are to be combined' as a further entity, the logical form, which the judging mind combines with the others. And the mode of

¹¹⁷ Hylton [1990: 346].

combination of all these entities (*including* the form) clearly need not to be (and in fact cannot be) the same as that of the fact corresponding to the judgement, so we are in no danger of having to identify uniting in thought with bringing about the corresponding fact.¹¹⁸

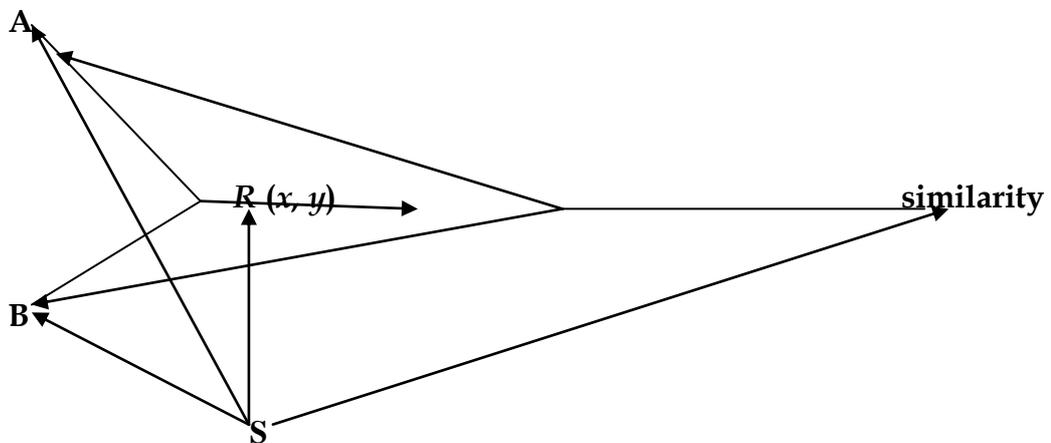
The inclusion of the logical form of the (judged) complex in the judgement structure can thus be seen as Russell's attempt to provide a solution to the problems that affected *both* his previous accounts of the multiple relation theory. If my interpretation is correct, the main problems with those two accounts had to do with the role of the *subordinate relation* (the relation other than judging/believing/understanding) of the judgement complex.

But nothing has yet been said about how the subordinate relation is discussed in Russell's 1913 account. Russell conceives the relation of 'understanding' between the subject and the constituents of a complex plus its logical form as being the *relating relation* of the understanding complex: "[W]hen a subject *S* understands 'A and B are similar', 'understanding' is the relating relation, and the terms are *S* and A and B and similarity and $R(x,y)$, where $R(x,y)$ stands for the form [...] [of a dual complex]" [TK: 117]. The subordinate relation is not, therefore, considered to be a *relating* relation at all. It is, as was on the 1912 account, a mere *object*, namely a *brick* in the understanding complex. This is also confirmed by another passage of Russell's:

[I]f we call the subject *S*, and the relating relation (of which 'understanding' is the one presupposed by all the others) *U*, and the objects *x*, *R*, *y* (taking the case of a proposition asserting a dual relation for the sake of illustration), and γ the form of a dual complex, the total complex which occurs when the subject has the relation *U* to the objects in question may be symbolized by $U(S, x, R, y, \gamma)$ [TK: 115].

¹¹⁸ Hylton [1990: 346].

U [the relation of ‘understanding’] is the relating relation in the complex, whereas the subordinate relation R is an *object* one must be acquainted with in order to understand the complex in question. As Tully has it: “in [...] [the] ‘understanding complex’ there is to be found only one relating relation, namely what ‘U’ stands for. The embedded relation, *similarity*, is only a term of the U relation, just as A , B and $R(x,y)$ are. Russell thus retains the ‘brick’ interpretation [namely the 1912 account] of these relations”.¹¹⁹ The form of the complex, likewise, is an object-constituent of the understanding complex (representing the way in which the constituents of the corresponding complex are put together). Russell provides the following diagram in order to illustrate the structure of the understanding complex [see TK: 118]:



This is the diagram symbolising the fact that S understands that A and B are similar. Russell explains the diagram by saying that in it “one relation goes from S to the four objects; one relation goes from $R(x,y)$ to $similarity$, and another to A and B , while one relation goes from $similarity$ to A and B ” [TK: 118]. So, the subject S has a relation of acquaintance with all the constituents of the complex ‘A

¹¹⁹ Tully [1988: 310].

is similar to B' , namely with A , B , and the relation 'similarity'. Moreover, the subject must be acquainted with the logical form of a dual complex xRy , or $R(x, y)$.

There seems to be a tension in Russell's discussion of the multiple relation of understanding in the 1913 manuscript. On the one hand, as we have just seen, Russell claims that the subordinate relation (the relation other than judging) is a mere object. On the other hand, he surreptitiously seems to grant that the subordinate relation works as relation in the understanding complex, arguing that, in the diagram above, 'a relation goes from similarity to A and B '. What this means, I take it, is that the relation of similarity between A and B is *instantiated*. I fail to see what the arrow's going from similarity to A and B in Russell's diagram can mean if not that there is a relation of similarity between A and B . But if this is granted, it would immediately resurrect the problems of the 1910 account. As Candlish observes: "Only [...] [the] relative complexity [of the diagram represented above] disguises the fact that A is similar to B still appears in it as a unified whole, now attached to the form as well as to the judging mind".¹²⁰

Of course, if the subordinate relation occurs in the understanding complex as relating relation, as Russell's explanation of the diagram suggests, then no complex would simply *correspond* to a judging/understanding complex; the 'corresponding' complex, in that case, would be a unified part of the understanding one. The truth-maker would be an integral part of the truth-bearer. In no way, therefore, does the notion of logical form in the 1913 account seem to improve the multiple relation theory and in particular solve the problems the theory suffered from under the 1910-1912 accounts. In fact, the 1913 account,

¹²⁰ Candlish [1996: 123].

despite the introduction of logical form, still retains a twofold reading of the subordinate relation, seen both as *object* and as *relation*.

[7.4] Conclusion

We have been discussing Russell's development of thought on judgement from *The Principles of Mathematics* (1903) to *Theory of Knowledge* (1913). We have seen that his early account of truth and judgement was embedded in a metaphysical framework in which truth was considered an unanalysable property of propositions, of bits of reality. From this it followed a conception of judgement – analysed in section [5.3] – as a dual relation between a mind and a proposition, and of truth or falsehood of judgements as acquaintance with true or false propositions.

Initially put forward as a tentative hypothesis in 1906, and then presented and argued for in the 1910 paper 'On the Nature of Truth and Falsehood', the multiple relation theory of judgement was seen as the only theory able to dispense with objective truths and falsehoods and to give a clear account of truth, now seen as a property of judgements given by their correspondence to facts. In 1910, we saw in section [6.1], Russell claimed that the embedded relation in a judgement complex is one of the objects of it, but granted, at the same time, that it sometime has a *sense* or *direction*. What is left without explanation is how an object can have a sense at all, namely how it can proceed from an object to another, without being a relating relation.

The 1912 modification of the multiple relation theory (discussed in section [6.2]) tried to deal with this problem, withdrawing the status of providing a sense

or direction from the subordinate relation, now seen as a mere object, namely as a *brick* in the judgement structure. But this modification makes it difficult for Russell to specify what constitutes the unity of the judgement and differentiates it from an array of simultaneous instances of dual relation of acquaintance, where objects are not combined with one another. Secondly, this account leaves Russell without an explanation of the different role played, within a judgement complex, by its objects and its subordinate relation (the constituents of the judgement), a distinction that he wants to preserve.

In chapter [7] we argued that the 1913 theory presented in *Theory of Knowledge*, with the introduction of the notion of logical form, tries to overcome the difficulties of both the previous theories. The notion of logical form, however, is unclear and does not improve on the early accounts. Overall, the 1913 account is defective (as were the previous two ones), in its treatment of the subordinate relation; Russell considers it, as he did in 1912, as an *object*; this was due to his willingness to avoid the unwelcome consequence of identifying judging complex and judged one. On the other hand he seems to grant, as he did in 1910, that the subordinate relation has a different status from the other constituents of the judgement complex. But he does not explain what this different status is, if not that of being a relating relation. This problem thus makes Russell's 1913 account no better off than his early attempts to devise a multiple relation theory of judgement.

Part Three

The *Tractatus* on Language and Logic

Chapter Eight

Wittgenstein's Criticism of Russell's Theory of Judgement

[8.1] Introduction

The multiple relational account of judgement and understanding presented in the *Theory of Knowledge* manuscript is likely to be the 'crucial part' of Russell's work Wittgenstein criticised, leaving Russell 'paralysed',¹²¹ and thus forcing him to abandon his work on the book. We have some evidence of Wittgenstein's criticism of Russell's theory, to be found especially in the 1913 *Notes on Logic*, the *Tractatus*, and private correspondence. As has been noted, however, "the clues that are tantalizingly scattered through the surviving documents have been thought to stop just short of telling us quite what Wittgenstein's objection was".¹²² As a consequence, a number of interpretations as to what Wittgenstein's objection to Russell exactly amounted to have been advanced, often in opposition to each other. In this chapter I intend to identify the *core* of Wittgenstein's objection by looking at some of the interpretations recently (and less recently) advanced, showing that they are less in opposition to each other than is generally thought. The element of Wittgenstein's criticism that I will identify as the core of his objection is in fact discussed in (almost) all interpretations of the Russell/Wittgenstein controversy.

Russell's *official* position, in all the variants of the multiple relation theory, is that there is no unity in what is judged; this is due to the fact that the subordinate relation in a judgement complex is not considered as a relating relation, but as an

¹²¹ See [NB: 122].

¹²² Potter [2009: 118].

object, a term. A judgement is a multiple relation between different objects, a judging mind and the constituents of the judgement, *object-terms* and an *object-relation*. I believe that Wittgenstein's criticism of Russell's theory of judgement is primarily directed against this feature of the multiple relation theory, its treatment of the elements (the terms) of a judgement as essentially on a par; Wittgenstein, in other words, opposed the idea that there is no unity in what is judged.

Wittgenstein's objection to Russell's theory of judgement is easy to identify. It appears in two slightly different instances in the *Notes on Logic* and once in the *Tractatus*. In the *Notes* Wittgenstein writes:

The proper theory of judgement must make it impossible to judge nonsense [NL: 95].

And:

Every right theory of judgement must make it impossible for me to judge that this table penholders the book. Russell's theory does not satisfy this requirement [NL: 103].

In the *Tractatus* Wittgenstein writes, clearly reminiscently of what he had written in the *Notes*:

The correct explanation of the form of the proposition, 'A makes the judgement *p*', must show that it is impossible for a judgement to be a piece of nonsense. (Russell's theory does not satisfy this requirement) [TLP: 5.5422].

According to Wittgenstein, then, Russell's theory of judgement is faulty because it allows a judgement to be nonsensical, namely it allows for the possibility of nonsensical judgements. How is this related to Russell's treatment of the subordinate relation of a judgement complex as being on a par with its other constituents? In order to understand the nature of Wittgenstein's criticism we will

consider some of the interpretations that have been advanced. I concentrate, in particular, on Griffin's interpretation,¹²³ on Hanks',¹²⁴ and on Potter's.¹²⁵

[8.2] Types

Nicholas Griffin holds that Wittgenstein's criticism "came in two instalments".¹²⁶ The first instalment is the one Russell discusses in his letter to Ottoline Morrell of 21 May 1913, where he says that Wittgenstein came to see him "with a refutation of the theory of judgement which I used to hold. He was right, but I think the correction required is not very serious. I shall have to make up my mind within a week, as I shall soon reach judgement".¹²⁷ This letter was written just after Russell completed the chapter 'On the Acquaintance Involved in our Knowledge of Relations', namely the first of the chapters of *Theory of Knowledge* that Russell left unpublished. Griffin argues that the point of Wittgenstein's objection was that on Russell's multiple relation theory of judgement there is no difference in logical type between the objects of judgement, because they are considered to be on a par. If this is the point of Wittgenstein's objection, then it was essentially the one Wittgenstein made in January, when he sent Russell a letter in which he wrote:

[E]very theory of types must be rendered superfluous by a proper theory of symbolism: For instance if I analyse the proposition Socrates is mortal into Socrates, mortality and $(\exists x, y) \varepsilon_1(x, y)$ I want a theory of types to tell me that 'mortality is Socrates' is nonsensical, because if I treat 'mortality' as a proper name (as I did) there is nothing to prevent me to make the substitution the wrong way round. *But* if I analyse [it] (as I now do) into Socrates and $(\exists x). x$ is mortal or generally into x and $(\exists x) \phi(x)$ it becomes impossible to substitute the wrong way round because the two symbols are now of a different *kind* themselves [NB: 122].

¹²³ Griffin [1985].

¹²⁴ Hanks [2007].

¹²⁵ Potter [2009: 118-131].

¹²⁶ Griffin [1985: 226-227].

¹²⁷ Quoted in Griffin [1985: 227].

Wittgenstein suggests that treating a property as if it could be symbolised by a proper name (which in turn means treating the predicate as if it were a term, an object-term) makes it possible nonsensical combinations of words. Although not directly linked to Russell's theory of judgement, or indeed to *any* theory of judgement, it is easy to apply these considerations to the analysis of judgement, claiming that if in a judgement the subordinate relation (or predicate) is treated as if it were an object term, then this would allow the possibility of nonsensical judgements; in other words, if one analyses *S*'s judgement that Socrates is mortal as a complex made up by *S*, Socrates and Mortality, then the possibility of *S*'s judging that Mortality is Socrates is not ruled out.

Griffin is right in stressing the relevance of the January letter for understanding the nature of Wittgenstein's objection to Russell; what Wittgenstein says in the letter nicely connects with the passages from the *Notes on Logic* and the *Tractatus* where he criticises Russell's theory of judgement for not ruling out the possibility of nonsense judgements. This, however, cannot be the whole of the story; after all, the January letter (if at all directed against Russell's multiple relation theory) cannot be directed against the account of judgement (and understanding) in *Theory of Knowledge*, the work on which Russell started in May, knowing for a long time about Wittgenstein's criticism. The theory of judgement presented there – as we saw – is a modification of his previous accounts of judgement; it has been suggested that such modification was intended to meet Wittgenstein's objection, sketched above.¹²⁸

¹²⁸ Griffin [1985: 228-229] and Potter [2009: 123].

In what sense can the 1913 account be said to be a response to Wittgenstein's objection? As we saw in the previous chapter, Russell, in 1913, includes the logical form of a complex as an element in the judgement complex. Griffin argues that "[t]here is no mention in *Theory of Knowledge* of logical form until parts of the manuscript written after Wittgenstein's 20 May criticisms. This might suggest, that the doctrine of logical form was added to the theory in response to Wittgenstein".¹²⁹ Why should the introduction of logical form in the analysis of judgement make it invulnerable to Wittgenstein's criticism? Griffin argues as follows:

It would not be surprising if Russell had thought that the inclusion of logical form would protect his theory against the objection that he treated both the terms and the relating relation of the object-'complex' as on the same level. For the logical form in the 1913 theory is responsible for marking the different roles that the two types of constituent play in the object-'complex'.¹³⁰

Now, Russell is explicit that the notion of logical form has to do with the way the constituents of a complex are put together. "[W]hen all the constituents of a complex have been enumerated, there remains something that may be called the 'form' of the complex, which is the way in which the constituents are combined in the complex" [TK: 98]. And this, in turn, seems indeed related to the different logical role played by the elements of a complex. "[I]n order to understand 'A and B are similar' we must know what is supposed to be done with A and B and similarity, i.e., what it is for two terms to have a relation; that is, we must understand the form of the complex [...]" [TK: 116]. Therefore, for Griffin, the introduction of logical form would enable Russell to meet Wittgenstein's criticism

¹²⁹ Griffin [1985: 228].

¹³⁰ Griffin [1985: 229].

that he previously treated both particulars and the relating relation as all *terms* in the complex.

Besides, Griffin claims that Russell modifies his treatment of the subordinate relation in those chapters of *Theory of Knowledge* written after Wittgenstein's visit in May. Russell in fact says that "subject and predicate obviously differ *logically*, and not merely as two particulars differ" [TK: 90], recognising that particulars and universals are given in logically different ways. It thus seems that Russell realised the difficulties involved in his *brick* account of universals (account fully endorsed in 1912) and that, under Wittgenstein's criticism, amended the theory accordingly. This correction, according to Griffin, "can be seen as a sustained attempt to preserve the distinct role of the relating relation in the object-'complex' within the overall framework of the multiple relation theory".¹³¹ Besides, Griffin seems to grant that this change was *enough* to block Wittgenstein's objection.

The reason why Griffin thinks so is that he believes the fatal criticism Wittgenstein moved to Russell's theory is another one, namely the one contained in a letter sent to Russell in June, where Wittgenstein writes:

I can now express my objection to your theory of judgement exactly: I believe it is obvious that, from the proposition 'A judges that (say) *a* is in a relation R to *b*', if correctly analysed, the proposition ' $aRb \vee \sim aRb$ ' must follow directly *without the use of any other premiss*. This condition is not fulfilled by your theory [NB: 122].

This, as has been pointed out, amounts to claiming that a judgement cannot be a piece of nonsense. ' $aRb \vee \sim aRb$ ' being a tautology, it follows from any proposition, provided it is significant, and thus provided it is not nonsense. So, in what sense is

¹³¹ Griffin [1985: 231].

this criticism different from the one considered above? The key passage of Wittgenstein's objection is identified by Griffin in the requirement that the tautology should follow 'without any other premises', and this is precisely what Russell's theory, according to Griffin, cannot guarantee. Russell could not grant that ' $aRb \vee \sim aRb$ ' follows from the judgement that aRb unless one makes the further stipulations that a and b are individuals and that R is a relation. Griffin claims that Wittgenstein would not accept these stipulations because "to make them would require further judgements. We are trying to analyse what is supposed to be the simplest type of elementary judgement. But to do so would seem to involve us in yet further judgements".¹³² Such judgements, moreover, are judgements about the type classification of the constituents of judgement (are judgements about the types of a , b , and R), and so, it seems, they cannot ground simpler, lower-level, judgements such as that a is in a relation R to b . Griffin concludes by saying that "[t]his shot is certainly powerful enough to be fatal, and I see no way Russell could have avoided it".¹³³

I have two observations about Griffin's interpretation. The first regards his contention that Russell is forced to introduce further premises (namely that a and b are individuals and that R is a relation) in order to rule out the possibility that aRb is nonsense. Griffin thinks that Wittgenstein's early criticism of Russell (the one expressed in the January letter) is based on the idea that on Russell's account particulars and universals are on the same level; in turn, their being on a par is responsible for the arising of nonsense ('Mortality is Socrates'). At the same time,

¹³² Griffin [1985: 242]. Those further judgements are called by Griffin 'type assigning judgements'. See Griffin [1991: 550].

¹³³ Griffin [1991: 552].

we have seen that Griffin thinks that the introduction of the logical form of the complex enables Russell to block Wittgenstein's objection, because the logical form, for Griffin, is 'responsible for marking the different logical role of the constituents of the object-complex'. But if so, why does Russell need to grant further judgements in order to guarantee that the judgement that aRb is significant? That is, why does he have to resort to type-assigning judgements? Since S 's judgement that aRb has to be analysed as $J[S, a, b, R, xRy]$, the presence of the logical form would reflect the adequacy of type of the constituents of the judgement, without any need for further judgements about the logical type of the constituents in question. Thus, the 'first instalment' of Wittgenstein's criticism seems in conflict with the 'second instalment', the one Griffin thinks to be fatal for Russell's theory.

My second observation concerns the evidence we have for identifying the nature of Wittgenstein's objection to Russell. We have plenty of remarks by Russell about Wittgenstein's criticism that sank his theory of judgement. Nowhere does he mention that this concerns type assigning judgements. All the evidence we have points unquestionably towards the idea that Wittgenstein criticised Russell's theory on the grounds that it treated the subordinate relation as if it were a *term*, and that *this* is responsible for the possibility of nonsensical judgements. In a passage from the 1918 *The Philosophy of Logical Atomism*, where he discusses the multiple relation theory, he says the theory is "a little unduly simple, because [...] [it] treat[s] the object verb as if one could put it as just an object like the terms, as if one could put 'loves' on a level with Desdemona and Cassio as a term for the relation 'believe'" [PLA: 199]. Further evidence comes from a set of lectures

Russell delivered at Harvard in 1914, therefore only one year after the abandonment of his theory; he presents his 'old theory of judgement' as one according to which the fact that S judges xRy is analysed as "a relation between S , x , R and y , before commenting, in what is plainly an allusion to Wittgenstein's criticism, that 'if R was a thing you could substitute another thing (z) for it, and if you do, the j[udgement] is meaningless'".¹³⁴

If my observations are correct, and the evidence we have of Wittgenstein's criticism not misleading, then this suggests that Wittgenstein's criticism did not come in two instalments, but is what Griffin thinks is the first instalment of the criticism (the one about the difference between particulars and universals), making it, *pace* Griffin, the really fatal one for Russell's theory of judgement.

[8.3] Unity

A recent interpretation of Wittgenstein's criticism of Russell's theory of judgement is presented by Peter Hanks. Hanks' account is put forward in direct opposition to Griffin's. Hanks' fundamental contention is that Wittgenstein's objection was not at all about type restrictions on judgements; instead, "the real problem that Wittgenstein raised was essentially [...] the problem of the unity of the proposition".¹³⁵ According to Hanks, Wittgenstein's objection about nonsense raises an "internal problem for the multiple relation theory. It is directed at the core of Russell's theory of judgement, the idea that judgement is a multiple relation between a subject and several terms".¹³⁶

¹³⁴ Potter [2009: 123].

¹³⁵ Hanks [2007: 122].

¹³⁶ Hanks [2007: 137].

Hanks argues that Wittgenstein's complaint towards Russell's theory is that a judgement must involve *unity*, the unity of what is judged (the unity of the judged proposition), whereas the multiple relation theory of judgement requires that the objects of the judgement, the constituents of the judgement other than the judging mind, are all *terms*; on the multiple relation theory, no unity can be found in what is judged. Hanks takes the following passage from the *Notes on Logic* to suggest this interpretation:

[W]e overlook that ordinary language would not contain the whole propositions if it did not need them: However, e.g., 'not-*p*' may be explained, there must always be a meaning given to the question 'what is denied?' [NL: 101].

This seems to have been of some importance to Wittgenstein, for he repeated and enlarged this point (with a reference to the analysis of judgement) in another excerpt from the *Notes*:

When we say A judges that etc., then we have to mention a whole proposition which A judges. It will not do either to mention only its constituents, or its constituents and form, but not in the proper order. This shows that a proposition itself must occur in the statement that it is judged; however, for instance, 'not-*p*' may be explained, the question what is negated must have a meaning [NL: 94].

In both passages Wittgenstein claims that in the analysis of judgement the *whole* judged proposition must occur; it will not do – Wittgenstein contends – to mention only its constituents or its constituents and form¹³⁷ but not in the proper order. Hanks' reading of Wittgenstein's objection is that the notion of *nonsense*, deployed by Wittgenstein in his objection, should not be read as something deriving from type-conflations, but as the dissolution of *propositional unity*. Nonsense, in other

¹³⁷ Hanks mistakenly interprets Wittgenstein's use of the notion of *form* as if it were the same as Russell's. In fact, it is not; by the 'form' of a proposition Wittgenstein meant its predicate (or relational) expression, whereas Russell meant the *logical form* of the proposition (or complex).

words, is not the violation of type-ordering, but, more simply, the absence of the possibility of truth. As Hanks says:

When Wittgenstein says that any correct theory of judgement must show that it is impossible to judge nonsense, by 'nonsense' he does not mean something that violates type restrictions. Rather, he means something that is not capable of being true or false. And the collection of *a*, *b*, and *R*, considered as a disunified collection, is not something that can be true or false.¹³⁸

The idea is therefore that, for instance, the complex '*aRb*', in order to be judged, must have a unity, must have a propositional articulation, a *sense*; its separate constituents, considered as autonomous entities (as does the multiple relation theory), have no such articulation, are just a blend of terms, an array of things; the combination of distinct autonomous elements, *a*, *R*, *b*, (though allegedly unified by the judging relation), is *nonsense*; it is not, according to Wittgenstein, something that can be judged to be either true or false.

Hanks, as said, opposes Griffin's interpretation that Wittgenstein's criticism of Russell had to do with type distinctions. He claims:

Only a proposition can be judged to be true – a collection of items, *even if they are of the right number and variety of types*, is not the sort of thing that can be true or false and hence not the sort of thing that can be judged.¹³⁹

And:

Wittgenstein's objection is not really about types or type restrictions. Even if the terms of a judgement meet all the requirements necessary for making up a possible fact, as long as those terms are disunified and separate they are not something that can be judged.¹⁴⁰

This, however, seems questionable. Hanks' interpretation is consistent with Griffin's interpretation of the first instalment of Wittgenstein's criticism, and,

¹³⁸ Hanks [2007: 138].

¹³⁹ Hanks [2007: *ibid.*].

¹⁴⁰ Hanks [2007: 138].

moreover, it is also consistent with a reading of Wittgenstein's objection as being about type-distinctions. There cannot be propositional unity if the items of a judgement (or proposition) are not of the *right* logical type: propositional unity implies the appropriateness of type of propositional constituents. But part of Wittgenstein's reasoning seems to suggest that appropriateness of types presupposes propositional unity. It is hard to see what *a disunified collection of terms of the right variety of types*, to use Hanks' expression, amounts to. If terms are unified into a proposition, into a judgeable whole, then they are of the right type. Conversely, if terms are disconnected from each other, disunified, then they are not of the right variety of types. There seems to be no way of identifying *difference* in logical type outside the context of a proposition, and so no room for the idea that items in *isolation* belong to different logical types. Propositional unity and appropriateness of type go along with each other. If this is right then, *pace* Hanks, there is no real distinction between nonsense as the absence of propositional unity and nonsense as violation of type-restrictions.

The nonsensicality of 'this table penholders the book' (Wittgenstein's own example in the *Notes*) can be seen as arising either from the lack of propositional unity (as Hanks would favour), for it is simply an array of items without logical articulation, or from the violation of type-restrictions, namely from the fact that those items (table, penholder, book) belong to the same logical type (this would be a Griffin style reading). But the two readings are at bottom the same. It is true that "the collection of *a*, *R*, and *b*, considered as a disunified collection, is not something that can be true or false".¹⁴¹ But this is so because, in order for it to be a

¹⁴¹ Hanks [2007: 138].

disunified collection, its items must violate the type restrictions required for having a unified complex capable of making sense. But this is *exactly* Griffin's point in his exegesis of Wittgenstein's 13 January letter. If one treats the property in the judgement that 'Socrates is mortal', as if it were a *term*, namely as symbolised by the proper name 'Mortality', one can get the nonsensical judgement 'Mortality is Socrates'. This, again, can be seen as nonsense either because there is no propositional articulation in it, because it is a 'disunified collection of terms' (to use Hank's favourite expression), or because in it all the items belong to the same logical type (they are all individuals, symbolised by proper names); but one should not think that these two are different examples of nonsense; the two amount to the same thing.

I now briefly summarise the results we have reached so far. Griffin's interpretation of Wittgenstein's objection mistakenly splits it in two 'instalments'. Wittgenstein's objection is basically a single (and a simple) one. On Russell's theory of judgement no unity can be found in what is judged, for the items with which the judgement is concerned do not respect type restrictions. Russell in fact claims that both *individuals* and *universals*, in the analysis of judgement, have to be considered *bricks* in the judgement's structure, and not its *cement* (this entails that, grammatically, they both can be symbolised by proper names). Wittgenstein objected to this analysis, claiming that a different role for the subordinate relation must be recognised, on pain of allowing nonsensical judgements. Griffin is thus right in his analysis of the first instalment of Wittgenstein's criticism, but wrong in claiming that that was just a part of Wittgenstein's objection, the second part of which has to do with type-assigning judgements.

Hanks, on the other hand, correctly identifies Wittgenstein's objection as being about the problem of the unity of the proposition, but mistakenly thinks this to be incompatible with interpreting Wittgenstein's point as being about type restrictions. The two points are different ways of looking at the same problem.

[8.4] Forms

I want now to turn to the interpretation of Wittgenstein's objection advanced by Michael Potter. At bottom, Potter's interpretation is the same as the one we have offered while analysing Griffin's and Hanks' interpretations. Russell's theory, Potter argues, does not satisfy the requirement that it should be impossible to judge that this table penholders the book, "because in it all the components of what is judged, including the verb, occurred as terms. Russell therefore did not have the resources to distinguish, in what is judged, between terms (such as 'similarity') which derive from verbs and those (such as 'penholders') which do not".¹⁴²

Wittgenstein's objection is therefore, for Potter, about the difference in role a theory of judgement must recognise between the terms of the judgement, its object-terms and its subordinate relation. They cannot be treated as if they were on a par, for this would entail the disappearing of their logical difference and therefore the disappearing of *unity* and indeed of *sense*. The following remarks aptly summarise Potter's reading of Wittgenstein's criticism of Russell's theory of judgement:

¹⁴² Potter [2009: 122].

The nub of Wittgenstein's complaint against Russell's analysis [...] was that in it the verb of the proposition believed did not occur as a verb.¹⁴³

And:

If I say 'A judges that p ', I do not, of course, myself judge that p , but I do, in the course of saying it, have to express what it is that A is judging (namely that p). If I am to do that, I must use the verb of p as a verb. Russell's theory, in any of its variants, forces the verb of p to occur as a term in the judging relation, and in such a position it cannot function as a verb.¹⁴⁴

Now, the account offered by Potter is therefore not dissimilar from the 'first instalment' of Wittgenstein's criticism identified by Griffin (with its insistence on the peculiar status of the subordinate relation) or from Hanks' reading according to which Wittgenstein's point was about propositional unity and sense.

As well as Griffin, Potter claims that Russell's theory of judgement in *Theory of Knowledge* is a *response* to Wittgenstein's criticism. In particular, the fact that the main element of novelty in that account is the notion of logical form suggests that the inclusion of logical form in the judgement complex was thought by Russell to meet Wittgenstein's objection. But since, instead, Russell gave up writing the book, something must have gone wrong with such a response. We saw that Wittgenstein criticised Russell's notion of logical form in the *Notes on Logic* [see NL: 100-101]; that notion is in fact, as we noted when discussing *Theory of Knowledge*, highly problematic and unclear.¹⁴⁵

What Wittgenstein must have found wrong in Russell's account is the idea that the logical form of a complex (or of a proposition) can be understood as a logical *object*, and that, furthermore, we can have acquaintance with it [see TK: 97,

¹⁴³ Potter [2009: 218].

¹⁴⁴ Potter [2009: 130].

¹⁴⁵ See section [7.2].

101]. This contrasts with Wittgenstein's early idea that "logic must turn out to be a *totally* different kind than any other science" [NB: 120], namely with his idea that logic does not have a subject matter, or, to put it differently, that there are not objects which constitute the realm of logic. Besides, for Wittgenstein, one cannot have a relation of acquaintance with forms (for forms are not things),¹⁴⁶ and thus one cannot have a relation of acquaintance with logical forms.¹⁴⁷ Logical forms are not objects. But this, however, is the role Russell grants for them in *Theory of Knowledge*, to be objects of acquaintance, which enter into a judgement complex as terms, as constituents, in the complex.

This is the reason why Russell could not avoid allowing nonsensical judgements, and this is the reason why the notion of logical form, insofar as it is understood in that way, as an *object*, is totally unable to block Wittgenstein's objection about nonsense. As long as the logical form of a complex is thought to be a *term* in the judgement structure, then, contrary to Russell's intentions (as they are presented by Griffin), it cannot be "responsible for marking the different roles that the two type of constituent play in the object-'complex'",¹⁴⁸ so cannot play a role in marking the difference between the relation (the *verb*) and the particular components of a judgement. Their being all objects (*bricks* in the judgement structure) makes them be on the same level, and thus makes any logical articulation impossible; the result is therefore nonsense.

¹⁴⁶ "[W]e can [...] not say that a relation which in certain cases holds between things holds sometimes between forms and things. This goes against Russell's theory of judgement" [NL: 105].

¹⁴⁷ In the *Tractatus* Wittgenstein will reaffirm that there is no such thing as acquaintance with logical forms, namely something like Russellian 'logical experience'; see [TLP: 5.552].

¹⁴⁸ Griffin [1985: 229].

In conclusion, Wittgenstein's objection about the role of the subordinate relation, its occurring as a term in the judgement structure (and thus its occurring in a way that prevents the proposition judged to express a sense), cannot be overcome by the introduction of the logical form of the proposition, as long as this is itself understood as an *object*, itself understood as a *brick*, a *constituent*, of the judgement complex.

[8.5] Russell's Theory of Judgment and Wittgenstein's Objection

In this final section I want to briefly identify those passages in Russell's presentations of the multiple relation theory where one can find the treatment of the subordinate relation Wittgenstein criticised, namely the treatment of the *verb* of the complex as a *term*, that is, as an *object*.

On Russell's 1910 account, the subordinate relation in a judgement complex is an *object*, not providing the unity of what is judged. If there were unity in what is judged, then this would make a judgement create the complex which is supposed to be judged, and would thus make false judgements impossible. But, as we saw in section [6.1], Russell wants to grant that the subordinate relation is somehow different from other judgemental components. Unlike them, it may have a *sense* or *direction*. But if the relation is treated as an *object*, Russell cannot say, in order to explain the difference between asymmetrical complexes like 'A loves B' and 'B loves A', that in one complex the relation *proceeds* from A to B, while in the other it *proceeds* from B to A.

Russell, in 1912, admittedly revised this account in order to cope with this problem. But the modified account presented in *The Problems of Philosophy*, is in no

better a predicament. In it the subordinate relation is explicitly treated as a term in the structure. But if Wittgenstein's objection is precisely directed against this treatment of the subordinate relation, he could not have accepted an account according to which the subordinate relation "as it occurs in the act of believing, is one of the objects – it is a brick in the structure, not the cement. The cement is the relation 'believing'" [PP: 74]. This clearly represents the target of Wittgenstein's complaint. On this account particulars and the subordinate relations are clearly on the same level, they are both objects (bricks in the structure).

In *Theory of Knowledge*, as highlighted by Griffin, Russell recognises that subject and predicate differ *logically* [see TK: 80]. However, the overall account of judgement there pushes in the opposite direction, namely in the direction of considering all the constituents of the judged complex as on a par. In fact, in *Theory of Knowledge*, the only relating relation in a judgement, that is to say, the only relation working as *verb*, is the relation of judging, not the subordinate relation, which is treated, once again, as a component of the judgement complex. As we have seen, the introduction of the notion of logical form cannot be of any help here, as Russell must have recognised, for it is understood as being another term among terms, another brick in the structure; this does not solve the problems posed by Russell's treatment of the subordinate relation.¹⁴⁹

Wittgenstein's objection, by Russell's own account, left him 'paralysed'. An example of Russell's paralysis is to be found, in my opinion, in the lectures on the philosophy of logical atomism, delivered in 1918. We have already partially

¹⁴⁹ See Pears [1989: 179].

quoted a relevant passage in this context. Russell says there that the analysis of judgement must face two impossibilities:

The first is the impossibility of treating the proposition believed as an independent entity, entering as a unit into the occurrence of the belief, and the other is the impossibility of putting the subordinate verb on a level with its terms as an object term in belief. That is a point in which I think that the theory of judgement which I set forth once in print some years ago was a little unduly simple, because I did then treat the object verb as if one could put it as just an object like the terms, as if one could put 'loves' on a level with Desdemona and Cassio as a term for the relation 'believe' [PLA: 199].

Here, instead of advancing a *positive* view on judgement, Russell states two impossible approaches to the analysis of judgement. Russell perhaps thought the first approach not to be available to him because of his willingness to reject the conception of propositions as unities – as objective mind-independent entities (conception he embraced in *The Principles of Mathematics*). At the same time, he was fully convinced that his previous – opposite – account of judgement, the multiple relation theory, was an impossible approach to the matter of judgement either: the second impossibility Russell mentions, namely the 'impossibility of putting the subordinate verb on a level with other terms as an object in the judgement' is, if my interpretation is correct, precisely the core of Wittgenstein's objection to Russell's theory of judgement.

The story we have sketched here has some advantages; it describes Wittgenstein's objection as being directed against *every* variant of the multiple relation theory Russell developed in the years 1910-1913, and so it explains why Russell ended up abandoning the theory altogether, instead of working on some further modification of it. Secondly, it matches the evidence we have from Russell's own recollections of Wittgenstein's objection.

Chapter Nine

Sense and Propositional Unity in the *Notes on Logic*

[9.1] 'A Correct Theory of Propositions'

We analysed Wittgenstein's criticism of Russell's multiple relation theory of judgement, and concluded that – on Wittgenstein's view – Russell's account was mistaken in assimilating the role of the subordinate relation to that of the other components of the judgement complex. In so doing Russell's account was prone to the charge of allowing nonsensical judgements, because all the components of the subordinate complex are terms. But, for Wittgenstein, only what makes sense (is true or false) can be judged, and for something to make sense, it has to have articulation, unity, and thus must contain something that works differently from how its other components work; it must have something functioning as *verb*, a *form*.¹⁵⁰

According to Wittgenstein, in order to successfully develop a theory of judgement or belief, it is preliminarily necessary to give a correct account of representation, of the nature and articulation of a proposition; as he puts this thought in the *Notes on Logic*:

The epistemological questions concerning the nature of judgement and belief cannot be solved without a correct apprehension of the form of the proposition [NL: 106].

The same idea is expressed in a letter to Russell, where he argues that his objection to Russell's theory of judgement "can only be removed by a correct theory of propositions" [NB: 122]. In the *Notes on Logic* Wittgenstein aims at providing just

¹⁵⁰ As Wittgenstein puts it in the *Notes on Logic*: "[W]hat is true must already contain the verb" [NL: 100].

that, a correct theory of representation, and thus of the nature of a proposition. In this sense his account of symbolism in the *Notes on Logic* is a direct development of his critical insights about Russell's theory of judgement.

Thomas Ricketts, in an influential paper on Wittgenstein's *Tractatus*,¹⁵¹ gives pivotal importance to the notion of a 'correct theory of propositions' played in Wittgenstein's philosophy. He sees in Wittgenstein's January 1913 letter to Russell – where he presents his early sketchy theory of propositional articulation – the seeds of the mature theory of propositions presented in the *Tractatus*. For Ricketts the January letter gives evidence that

Wittgenstein breaks with the Russellian view of language by ceasing to treat unary and relational predicates as names of ontological atoms combined by a copula with names of individuals. This [...] is the root of the conception of sentences as pictures.¹⁵²

I think this claim of Ricketts' is right. In the previous chapter, besides, we saw that Wittgenstein's January letter to Russell plays a role in Wittgenstein's criticism of Russell's theory of judgement. But if the account of sense developed in the *Notes* is Wittgenstein's way of implementing his criticism of Russell's theory of judgement (and thus his insights in the January letter), as I will show, then one can say that the seeds of the mature picture theory can be traced back to Wittgenstein's critique of the multiple relation theory of judgement. In this chapter we will elucidate these connections by analysing the account of the nature of the proposition given by Wittgenstein in the *Notes on Logic*.

¹⁵¹ Ricketts [1996].

¹⁵² Ricketts [1996: 70].

[9.2] Names and Forms

To understand a proposition, for Wittgenstein, means to understand its sense. As Wittgenstein says: "Every proposition is essentially true-false: to understand it, we must know both what must be the case if it is true, and what must be the case if it is false. Thus a proposition has two *poles*, corresponding to the case of its truth and the case of its falsehood. We call this the *sense* of a proposition" [NL: 98-99]. The sense of a proposition, on Wittgenstein's account, is independent of its truth and falsity (independent of what he calls its 'meaning'). But the sense of a proposition is indissolubly related to the possibility, for the proposition, to be either true or false (to have a meaning). "I understand the proposition 'aRb' when I know that either the fact that aRb or the fact that not aRb corresponds to it" [NL: 104]. This is Wittgenstein's idea of the bipolarity of the proposition, an idea that, as we will see in later chapters, is crucial for an appraisal of the picture theory and, indeed, for Wittgenstein's understanding of the nature of logic as a whole.

The notion of sense so introduced brings us to Wittgenstein's thought that understanding the sense of a proposition depends on understanding its indefinables, for "[a] proposition must be understood when *all* its indefinables are understood" [NL: 99]; the indefinables in a proposition are "of two sorts: names, and forms" [NL: 96]. Now, what is involved in understanding the indefinables of a proposition (and thus its sense)? Wittgenstein says: "If we know the meaning of 'a' and 'b', and if we know what ' xRy ' means for all x 's and y 's, then we also understand 'aRb'" [NL: 104]. This passage is perhaps not immediately and completely clear; but it is clear that Wittgenstein implies, in it, that there is a

difference in the way the components of a proposition symbolise [see also NL: 99].

The difference between the symbolising roles of different propositional components is discussed in the *Notes on Logic* in the following passage:

[T]he form of a proposition symbolizes in the following way: Let us consider symbols of the form 'xRy'; to these correspond primarily pairs of objects, of which one has the name 'x', the other the name 'y'. The x's and y's stand in various relations to each other, among others the relation R holds between some, but not between others. I now determine the sense of 'xRy' by laying down: when the facts behave in regard to 'xRy' so that the meaning of 'x' stands in the relation R to the meaning of 'y', then I say that they [the facts] are of 'like sense' with the proposition 'xRy': otherwise, 'of opposite sense'; I correlate the facts to the symbol 'xRy' *by* thus dividing them into those of like sense and those of opposite sense [NL: 104].

In this passage Wittgenstein claims that the form of a proposition presupposes that the names occurring in it have meaning (symbolise). Thus an understanding of the notion of the symbolising role of names seems to be preliminarily required before we can address the question of how forms symbolise.¹⁵³ Wittgenstein does not discuss the notion of meaning of a name in the *Notes on Logic*, but it is safe to assume that he saw the relation between a name and its object as being quite unproblematic. A name simply means an object, by referring to it. As Ricketts puts it, in the *Notes*, "Wittgenstein says nothing about names and designation, following Russell in treating names as unproblematic labels for objects".¹⁵⁴

But, as Wittgenstein remarks, the form of a proposition symbolises differently: the form of a proposition is not a label for a thing; instead it *discriminates* between *facts*. The proposition itself is for Wittgenstein a fact (precisely by virtue of its having a form):

Propositions [which are symbols having reference to facts] are themselves facts [NL: 97].

¹⁵³ In fact, as Wittgenstein writes in the *Notes Dictated to Moore*: "The type of a symbol of a relation is partly fixed by [the] type of [a] symbol of [a] thing, since the symbol of [the] latter type must occur in it" [NM: 111].

¹⁵⁴ Ricketts [1996: 74].

In 'aRb' it is not the complex that symbolises but the fact that the symbol 'a' stands in a certain relation to the symbol 'b' [NL: 96].

So the symbolising role of the form of a proposition is not the complex propositional symbol, but the *fact* that the names in it are arranged (stand to one another) in a certain way, that is, according to a certain *form*. In the symbol 'aRb', what symbolises, besides the names 'a' and 'b', is the fact that those two names stand in a certain relation with one another, the relation of *R*-left-flanking and *R*-right-flanking one another. This very fact symbolises another fact in the world, and thus have a symbolic role.

In the passage from the *Notes* quoted above the expression 'xRy' is an expression for the *form* of a proposition; the form 'xRy' discriminates between facts whose two constituents are related by a *R*-relation from those whose constituent things are not so related. The former are called by Wittgenstein facts of *like sense* with that particular form, the latter are facts of *opposite sense*. To use an example: let's suppose that the form in question is the form 'x is-smaller-than y'. This form discriminates between those facts one of whose two constituents (the one that has the *x*-position) is smaller than the other (the one that has the *y*-position) from those facts in which the constituent that has the *x*-position is not smaller than the one that has the *y*-position. The former are of like sense with the form, the latter of opposite sense. Facts of like sense are thus facts that verify the proposition; facts of opposite sense are facts that falsify it.¹⁵⁵

¹⁵⁵ This account seems problematic, however; it seems implausible to assume that *all* the facts can be categorised as being of either *like* or *opposite* sense with propositions. The fact that Stirling is smaller than Edinburgh, for example, seems simply *irrelevant* to the sense of the proposition 'Stirling is north of Edinburgh' and so it seems neither of like nor of opposite sense with that proposition.

For Wittgenstein, both the relation between a name and its meaning (object) and the relation between a propositional form and the form of the fact it represents are *conventional*.¹⁵⁶ It is a matter of convention that the name ‘*a*’ names the object *a*; likewise, the fact that ‘*a*’ *R*-left-flanks ‘*b*’ in the propositional complex ‘*aRb*’ might be used to symbolise, for instance, the fact that the object *a* is smaller than the object *b* is arbitrary, because it could have been used to symbolise another fact, namely, for example, the fact that *a* is older than *b*. As Wittgenstein observes, “that this inkpot is on this table may express that I sit in this chair” [NL: 97].¹⁵⁷ So there is some degree of arbitrariness involved in representation; and such arbitrariness applies to both names and forms; the form of a proposition represents by *convention* [see NL: 102]. An important characteristic of Wittgenstein’s account of propositional sense in the *Notes on Logic* is thus that it does not require *identity* of form between propositions and represented facts. A particular relation between linguistic items in a proposition can represent that a *different* relation holds between objects in reality.

[9.3] Wittgenstein, Russell, Frege

At the heart of Wittgenstein’s conception of propositional articulateness in the *Notes on Logic* is therefore a distinction between the symbolising role of forms and names. The basic idea is that names stand for things, objects, whereas forms do not

¹⁵⁶ As Wittgenstein has it in the *Notes Dictated to Moore*: “The relation [between a proposition and reality] depends upon the conventions by which we have given meanings to our simples (names and relations)” [NM: 112].

¹⁵⁷ See Ricketts [1996: 71]. More generally, thus, Wittgenstein’s idea of representation can be summarised by saying “that a certain thing is the case in the symbol says that a certain thing is the case in the world” [NL: 96]. Wittgenstein seems careful here not to imply that the fact that a certain thing is the case in the symbol says that the very *same* thing is the case in the world. See also Potter [2009: 113].

stand for objects at all. In the proposition ' aRb ', ' R ' is not a name. "Symbols are not what they seem to be. In ' aRb ' ' R ' looks like a substantive but is not one. What symbolises in ' aRb ' is that R occurs between a and b " [NL: 98]. What symbolises in ' aRb ' is not the symbol R , but the fact that a relation occurs between the symbols a and b – the relation of R -left/right flanking. That relation symbolises by discriminating between objects that stand to one another in the relation R and objects that do not so stand. In particular, owing to the two names occurring in the proposition, the proposition picks out the objects a and b . If those objects are related by the relation R , then the fact ' aRb ' is of like sense with the form of the proposition, and it therefore agrees with the sense of the proposition, that is to say, makes the proposition true.

This conception of a division of propositional constituents into two different categories – names and forms – marks a contrast with Russell's conception of judgement, and this is why Wittgenstein's account can be said to be a development of his critique of the multiple relation theory. As Ricketts says, in contrasting Wittgenstein's position with Russell's:

The Russellian view [...] assimilates the correlation of relational predicates to relations to the use of proper names to label individuals. On Wittgenstein's alternative view, forms of sentences symbolize via a general rule setting forth when sentences of that form agree and disagree with the facts. The general rule depends on a structural similarity between sentences and the facts that verify them if they are true.¹⁵⁸

This is evident as regards Russell's multiple relational analysis of judgement; according to it, in the judgement that aRb , the only relating relation, the only relation actually working as *verb*, is the judging relation itself; all the elements of the complex aRb , the individuals and the relation, figure in the judging complex as

¹⁵⁸ Ricketts [1996: 72].

terms; they are all *objects* (items that can be represented, in language, by *names*) in the judgement structure. But this, as we saw in the last chapter, is essentially the target of Wittgenstein's criticism. "When we say A judges that etc., then we have to mention a whole proposition which A judges" [NL: 94]. Thus the unity that is peculiar to a proposition (due to the interplay of names and a form) must be respected if a judgement is to make sense. By treating the components of the judged complex as if they were on a par – as if they were all objects – Russell failed to provide a satisfactory account of judgement (and thus of representation) – allowing the possibility of nonsensical judgements.

The account proposed in the *Notes on Logic* brings Wittgenstein close to Frege, who envisaged a similar difference between components of thoughts and sentences (saturated and unsaturated ones). It is highly significant that the January letter to Russell partly quoted in section [8.2] was written by Wittgenstein shortly after a visit to Frege.¹⁵⁹ In the letter Wittgenstein rejects a Russellian analysis of propositional complexity as given by *terms* related by a copula, on the assumption that properties and relations are copulae themselves, namely function as incomplete entities saturated by terms (what can be symbolised by proper names).

I have changed my views on 'atomic' complexes: I now think that qualities, relations (like love) etc. are all copulae! [NB: 121].

This idea is clearly reminiscent of Frege's treatment of sentential complexity, and is the origin of Wittgenstein's distinction between forms and names above discussed. As Potter says, in discussing the parallel between Wittgenstein and Frege:

¹⁵⁹ See [NB: 121-122].

Frege [...] had conceived of the expressions for concepts and relations as what he called *unsaturated*, i.e. as containing argument places which have to be completed with names of objects in order to form a complete sentence. What corresponds in Wittgenstein's conception to a saturated expression is what he in 1913 calls a 'name'; what corresponds to an unsaturated expression is what he calls a 'form'. [...] The two kinds of components of a proposition are fundamentally different, as Frege's two kinds – saturated and unsaturated expressions – had been.¹⁶⁰

We can thus see that the problems related to the multiple relation theory of judgement, and its treatment of judgemental components as all on a par, pulled Wittgenstein from a Russellian conception of propositional complexity (which he adopted in the early stages of his career) to a Fregean view according to which in a proposition two distinct categories of components – with different symbolising functions – must be recognised.

[9.4] The Picturing Analogy

In the *Tractatus* propositions are conceived of as *pictures*. But in the *Notes on Logic*, apart from one instance,¹⁶¹ the word 'picture' does not figure at all. Nonetheless, I argue,¹⁶² Wittgenstein's conception of sense, there, requires that a proposition and the fact it represents share some *symmetry*, some structural *similarity*. Wittgenstein writes:

Every proposition which says something indefinable about one thing is a subject-predicate proposition; every proposition which says something indefinable about two things expresses a dual relation between these things, and so on. Thus every proposition which contains only one name and one indefinable form is a subject-predicate proposition, and so on [NL: 107].

Every proposition about one object is a proposition where only one name (and one unary form) is present. Likewise, every proposition that says something about two

¹⁶⁰ Potter [2009: 113].

¹⁶¹ [NL: 106].

¹⁶² Following Potter [2009: 224-227].

things (that represents two things as arranged in a certain way) is a proposition containing two names (and one relational form). In order for a proposition to represent a fact, therefore, the proposition itself must share a structural similarity with the fact in question.¹⁶³ We can see here the emergence of a conception that is crucial to the picture theory – the idea that a proposition and its represented fact must share the same logical multiplicity [see TLP: 4.04].

However, the picturing analogy itself, as it is conceived in the *Tractatus*, according to which a proposition is a picture of reality because it shares with it an *identical* mode of combination of elements, is not present in the *Notes on Logic* or the *Notes Dictated to Moore*, and with good reason. Wittgenstein's account of sense there does not require *identity* of form between a proposition and what the proposition represents. As we saw Wittgenstein arguing, in fact, both the relation between objects and names, and the relation between the form of the represented fact and the propositional form are conventional.

Wittgenstein's picturing analogy emerged as a refinement of his early conception of sense and propositional articulation, refinement that we will consider in details in the next chapter. However, one can say that the seeds of that conception are present in the *Notes'* account of how a proposition has sense. In order for a proposition to say what it does, it has to have a structural similarity with the represented situation.

¹⁶³ In the *Notes Dictated to Moore* Wittgenstein seems to realise that his conception of propositional sense allows him to establish a *general* correspondence between propositions and reality. See [NM: 112].

Chapter Ten

Sense and Propositional Unity in the *Tractatus*

[10.1] Introduction

The account of propositional complexity in the *Notes on Logic* rests on the division of propositional constituents into two different logical categories, names and forms. Names are labels for objects. Forms are, on the other hand, the relational element (the copula) that holds names together in propositions. Forms are then, on Wittgenstein's early account, the source of the unity and sense of the proposition. Names combine with forms to create propositions. The crucial point, then, is that both elements must be present in a proposition. "Propositions cannot consist of names alone" [NL: 96]. This view, I argued, derives from Wittgenstein's criticism of Russell's theory of judgement, and it amounts to Wittgenstein's alternative account of propositional unity. Another characteristic of Wittgenstein's early account of propositional complexity and sense is that a proposition's having the form it does (and thus the sense it does) determines a structural similarity – or correspondence – between the proposition and what it represents; on this account there is thus harmony between language and reality.

In this chapter we will analyse to what extent and in what form these two features of Wittgenstein's early account of propositional complexity survive and are developed into the mature account presented in the *Tractatus*, that is, the picture theory. The best way to address these two questions is perhaps to reverse their order. Only by seeing how Wittgenstein implements and deepens, in the *Tractatus*, the idea that the dimension of language somehow mirrors that of reality,

and that therefore the structure of linguistic propositions reflects (depicts) the structure of facts, can one appreciate how Wittgenstein conceives of propositional constituents to be and how they contribute to the sense the proposition expresses. In the next sections I will discuss Wittgenstein's general theory of representation – the picture theory – and then show how such an understanding of the theory should guide our interpretation and identification of propositional constituents.

[10.2] Propositions as Pictures

The idea we have seen in the *Notes on Logic*, that a proposition and its corresponding fact have some structural similarity, is of course the basic idea behind the picture theory presented in the *Tractatus*. A proposition is there conceived of as a *picture* of a fact (or state of affairs), as a linguistic *representative* fact. This may well be seen as a deepening of the conception held in the *Notes on Logic*: there, as seen, it is the form of a proposition that determines a structural similarity with the fact it is about. In the *Tractatus*, the structural similarity between propositions and states of affairs is understood in terms of the former being pictures of the latter. For Wittgenstein, a picture is a “likeness” of what it pictures [TLP: 4.012], between the two there is an “inner similarity” [TLP: 4.0141]; thus “something [must be] identical in a picture and what it depicts, to enable the one to be a picture of the other at all” [TLP: 2.161]. A fact is thus a picture of another because it shares with it a formal – structural – similarity or identity. In this section I will explore and discuss this idea.

In the *Tractatus*, the discussion of the pictorial – representative – nature of propositions is preceded by a discussion of the nature of a picture (*Bild*) in general.

Incidentally, that means that the picture theory is not only a theory of language meaning; it should be conceived of as a general theory of representation.¹⁶⁴ The representational nature of a proposition, its pictorial nature, is in turn grounded in the proposition's sharing something with the fact depicted. Wittgenstein puts this point in the following terms:

The fact that the elements of a picture are related to one another in a determinate way represents that things are related to one another in the same way.

Let us call this connexion of its elements the structure of the picture, and let us call the possibility of this structure the pictorial form of the picture [TLP: 2.15].

Wittgenstein says here that a picture represents by virtue of its elements' being arranged in the same way as elements in reality are arranged, by sharing with what it represents the same *structure*. Now, a picture is a fact, and every fact has a structure, namely a particular way in which its elements are arranged. But that of course does not mean that every fact is a picture. A necessary condition for something to be a picture is that its elements must be arranged in the same way as the elements of another fact. I clarify this by saying that a picture has a *pictorial structure*.

The notion of (pictorial) form is, as 2.15 shows, defined by means of the notion of structure. In general terms, "[f]orm is the possibility of structure" [TLP: 2.033]. Pictorial form is therefore "the possibility of the kind of structure that counts as a picture",¹⁶⁵ or, according to my terminology, the possibility of pictorial structure: "Pictorial form is the possibility that things are related in the same way as the elements of the picture" [TLP: 2.151]. So every fact, just by virtue of being a

¹⁶⁴ As observed by Kenny [1973: 44].

¹⁶⁵ Pears [1987: 130].

fact, has a structure and thus realises a form. Only a picture, in representing another fact, realises a pictorial form, and has thus a pictorial structure.

Now, how is pictorial structure achieved? The structure of a fact, as said, is the particular arrangement of the elements of the fact. Pictorial structure, on the other hand, is the structure of a picture, namely of a fact that represents things as being arranged in the way in which the elements of the picture are arranged. This requires a relationship between the elements of the picture and the elements of what is pictured. This relationship Wittgenstein calls 'pictorial relationship'.

The pictorial relationship consists of the correlations of the picture's elements with things [TLP: 2.1514].

As Wittgenstein explains, the correlations between the elements of a picture and the objects of the fact to be pictured make the former be representatives of the latter. But then, picture and pictured fact have to have the same number of constituents, in order for the elements of the picture to have a one-to-one correspondence to the objects of the fact, as TLP 2.1514 claims. This is what Wittgenstein actually says later in the *Tractatus* with regard to propositions: "In a proposition there must be exactly as many distinguishable parts as in the situation that it represents. [...] The two must possess the same logical (mathematical) multiplicity" [TLP: 4.04].

It is important to note that the notion of pictorial relationship cannot be severed from the notion of pictorial form. In fact, the mere obtaining of such a relationship, the mere correspondence between the elements of a picture and things in the world, does not make a fact into a picture. What must be granted is that the structure of the picture, the arrangement of its elements, should be a

possible arrangement for the elements of the fact to be represented; but this is to say that in order for a picture to be a picture of a fact, they must also share the same *pictorial form*. “A picture can depict any reality whose form it has” [TLP: 2.171]. Conversely a picture cannot represent a reality whose form it does not have.

The three fundamental (and intertwined) notions of Wittgenstein’s conception of a picture in the *Tractatus* are thus those of (pictorial) structure, pictorial relationship, and pictorial form. In order to be a picture, then, a fact must have a pictorial structure, related by a pictorial relationship to the structure of the fact to be represented, and must share with it the same pictorial form. To sum up then:

- (1) What depicts must be a *fact*, and therefore it must have a *structure*. Such structure (the combination of its elements), besides, must be a *pictorial structure*, must show how things are arranged in reality [TLP: 2.14-2.15].
- (2) The two facts must be related by a *pictorial relationship*, by a one-to-one correspondence between the elements of the picture and things in reality. The two facts must thus have the same logical multiplicity [TLP: 2.1513-2.1514].
- (3) The possible combinations of the elements of the picture must be possible combinations for the elements of the fact that is to be depicted (represented). That is to say, both facts must present the same possibility of structure (the same *pictorial form*) [TLP: 2.15-2.151, 2.16-2.17].¹⁶⁶

¹⁶⁶ The three conditions I have identified as the conditions in order for a fact to be a picture are *roughly* the same as what Morris identifies as being the general commitments of Wittgenstein’s general theory of representation. See Morris [2008: 122].

The picture theory of meaning presented in the *Tractatus* is the application to propositions of the general theory of representation discussed above. The conditions we have identified for something to be a picture are applied by Wittgenstein, in the *Tractatus*, to linguistic sentences as well, and this is the straightforward (literal) sense in which sentences are, on Wittgenstein's theory, pictures (or models) of reality [see TLP: 4.01].

Wittgenstein begins his discussion of the notion of a proposition – and its pictorial nature – by saying that a proposition is the expression, perceptible by the senses, of a thought [see TLP: 3.1], where a thought is itself “a logical picture of facts” [TLP: 3]. A thought is itself a fact, and its constituents correspond to the constituents of the proposition expressing it. As Wittgenstein put it, in a letter to Russell in which he clarified his position in the *Tractatus*:

[A] Gedanke [Thought] is a Tatsache [Fact]: [...] [and] it must have constituents which correspond to the words of Language [NB: 130].

A proposition expresses a thought by projecting a possible situation (what is represented) into the ‘propositional sign’. “I call the sign with which we express a thought a propositional sign. – And a proposition is a propositional sign in its projective relation to the world” [TLP: 3.12]. In turn, this means that “[i]n a proposition a thought can be expressed in such a way that elements of the propositional sign correspond to the objects of the thought” [TLP: 3.2].

A propositional sign is conceived by Wittgenstein as the *material* element through which the sense of a proposition is expressed. Wittgenstein insists on the idea that a propositional sign is a fact. This is its crucial characteristic (and not, for instance, its being composed of words). As Wittgenstein puts it:

What constitutes a propositional sign is that in it its elements (the words) stand in a determinate relation to one another.

A propositional sign is a fact [TLP: 3.14].

The essence of a propositional sign is very clearly seen if we imagine one composed of spatial objects (such as tables, chairs, and books) instead of written signs [TLP: 3.1431].

A propositional sign is not in itself a picture. It is a fact, and not every fact is a picture, because – as we said – not every fact has a pictorial structure. A fact (a propositional sign) acquires a *pictorial structure* when it *represents* another fact, when the arrangement of its elements is the same as the arrangement of the elements in the pictured situation. This means that a propositional sign expresses a sense when its structure becomes pictorial. But this amounts to the ‘projection of a possible situation’ into the propositional sign, and thus amounts to the notion of a thought, for “the method of projection is to think the sense of the proposition” [TLP: 3.11]. As Wittgenstein puts it, later in the *Tractatus*, “[a] propositional sign applied and thought out, is a thought” [TLP: 3.5], but, equivalently, a propositional sign applied and thought out expresses a sense, or in short, *is* a proposition, for it has a pictorial structure which puts it in a projective (representative) relation to the world [see TLP: 3.12]. So, since the propositional sign in its representative relation to the world is a proposition with a sense, and since a propositional sign in such a relation to the world is a propositional sign applied and thought out (hence a thought), then “[a] thought is a proposition with a sense” [TLP: 4].

Besides being a fact, a proposition (a picture) in order to represent, must also fulfil conditions 2 and 3 discussed above. So it must share with the represented fact a pictorial relationship and the same pictorial form. The obtaining

of a pictorial relationship between a proposition and what it depicts is explained by Wittgenstein through the idea that the elements of the proposition (names) correspond to, and are representative of, the elements of the situation to be pictured [see TLP: 3.22]. In a proposition “there must be exactly as many distinguishable parts as in the situation that it represents” [TLP: 4.04]. This claim is then extended to incorporate the combination of pictorial elements (and thus condition 3). “One name stands for one thing, another for another thing, and they are combined with one another. In this way the whole group presents – like a *tableau vivant* – a state of affairs” [TLP: 4.0311]. Thus, the fact that a picture, or proposition, has a pictorial structure entails that it has the same number of constituents as what it represents, and that it realises the possibility of having the same structure as the represented situation (form).

To sum up: in the *Notes on Logic* Wittgenstein envisaged that propositions have sense, and thus have a structural similarity with the relevant situation they represent. In the *Tractatus* this idea is developed in the claim that having sense amounts to propositional elements’ being arranged as the elements of the situation to be represented are arranged. Thus the structures of what represents and the structure of what is represented are *identical*. In the next section I analyse how the *Tractatus*’ account of identity of structure between propositions and facts can help understand Wittgenstein’s conception of the nature of propositional constituents.

[10.3] Propositional Constituents, Identity of Structure and Isomorphism

In the *Notes on Logic* the idea that the structure of a proposition corresponds to the structure of a fact is grounded in the distinction between names and forms. Since

the notion of correspondence – or structural similarity – between a proposition and a fact is in the *Tractatus* deepened into the conception of a proposition's being a picture of a fact, one might expect the division of propositional constituents to be maintained there. In fact, when looking at the *Tractatus'* account of propositional complexity, one is struck by the contrast that that account bears with the view held in the *Notes*. In overt contradiction to the *Notes'* claim that propositions cannot consist of names alone, in the *Tractatus* Wittgenstein says that a (elementary) “proposition consists of names. It is a nexus, a concatenation, of names” [TLP: 4.22]; it “consist[s] in names in immediate combination” [TLP: 4.221]. Wittgenstein says here that the components of a proposition are no longer “forms and constituents” [NL: 96], that is, forms and names, but names *alone*. Names are in immediate combination in propositions, directly related to one another; the function of copula, that the form of a proposition was meant to have in the *Notes*, is then taken over by names themselves.

On the surface, then, the view of propositions the *Tractatus* presents is at odds with that of the *Notes*. Where the latter saw it necessary to introduce two distinct propositional constituents to guarantee propositional unity, the *Tractatus* argues for a – as it were – ‘monistic’ view of propositional articulation; propositions are made up by names in combination. Forms – in the *Notes on Logic's* understanding of the term – disappear from the scene. But this conclusion may be premature. Thomas Ricketts, for example, argues that the difference between the two accounts is “more terminological than substantive. The Tractarian conception

of sentences as picture is more a natural deepening than a revision of the [...] [Notes'] conception of sentences and representation".¹⁶⁷

If my reading of Wittgenstein's account of sense in the *Notes on Logic* is right, then its distinction between names and forms implies a rejection of Russell's view of judgement. This difference between the ways in which names and forms symbolise is the outcome of Wittgenstein's criticism of Russell's theory of judgement, one of whose main feature was treating the subordinate relation in a judgement (the relation other than judging) as a term, namely as on a par with the particulars involved in the judgement. This conception, in Russell, was not tied up to a theory of language. But Wittgenstein draws this connection (arguing that the problems with Russell's theory of judgement could only be overcome by a correct theory of symbolism). What in Russell's theory of complexes is a term, is, in Wittgenstein's early theory of proposition, a name. Unlike Russell, who treated relations as being terms too, on Wittgenstein's account in the *Notes* relations are not objects, cannot be symbolised by names: "There is no thing which is the form of a proposition, and no name which is the name of a form" [NL: 105]. Ricketts summarises this difference between Russell's account and Wittgenstein's nicely:

Russell takes relations to be a type of thing – they are constituents of facts, objects of acquaintance [...]; they may themselves have properties and be the relata of still further relations. All this is what the reality of relations comes to for him. So conceived, Wittgenstein rejects the reality of relations, Russell's most cherished ontological thesis. Relations are not things, are not entities; relations cannot be labelled or designated. Unlike 'a' and 'b', 'R' is not a symbol in 'aRb'. Instead, roughly put, the holding of a relation over objects is symbolised by the holding of a relation over names of those objects.¹⁶⁸

¹⁶⁷ Ricketts [1996: 73]. Likewise, in [2002: 236], Ricketts argues that Wittgenstein's conception of sense in the *Notes on Logic* "is Wittgenstein's overpowering insight of 1913 that sets him on the path to the philosophy of the *Tractatus*".

¹⁶⁸ Ricketts [1996: 72].

This, in turn, coheres with the passage from the *Notes* where Wittgenstein says: “‘Not: ‘the complex sign aRb ’ says that a stands in the relation R to b ; but *that* ‘ a ’ stands in a certain relation to ‘ b ;’ says *that* aRb ” [NL: 106].¹⁶⁹ The symbolising fact in ‘ aRb ’ is that *two* names are related by a relation (itself conventionally indicated by the sign ‘ R ’);¹⁷⁰ that represents another fact (with the same structure), the fact that two things (the bearers of the names ‘ a ’ and ‘ b ’) are related by a relation. So, according to this account, things are symbolised by names, whereas relations between objects are symbolised by relations between names. This I call a ‘nominalistic’ interpretation of propositional complexity, according to which relations are *not* represented by names in propositions.

Ricketts maintains that this account is overall maintained in the *Tractatus*, despite Wittgenstein, there, does not mention forms as components of propositions. For Ricketts, in the *Tractatus*, “relations are not among the [...] objects [...]. Wittgenstein rejects Russell’s view of relations as ontological atoms that have the role of joining other ontological atoms together into complexes. Nor is there any ontological glue linguistically symbolized by the copula that binds individuals and relations into atomic facts”.¹⁷¹ And he goes on to say:

Nothing in a sentence goes proxy for or names a relation. [...] Rather, the atomic sentences in which names are representatives (*vertreten*) of objects represent (*darstellen*) those objects as related in a particular way. There is no *vertreten* of relations, but only the *darstellen* in atomic sentences of the holding of relations, the modelling of objects as combined in a particular way. Thus, the role played in [...] [the *Notes on Logic*] by Wittgenstein’s distinction between the way that names symbolize and the way forms symbolize is taken up in the *Tractatus* by the distinction between *vertreten* and *darstellen*.¹⁷²

¹⁶⁹ See also [TLP: 3.1432].

¹⁷⁰ See Potter [2009: 113-114].

¹⁷¹ Ricketts [1996: 75-76].

¹⁷² Ricketts [1996: 76].

Thus names combine with one another in an elementary proposition [see TLP: 4.221], and this represents a state of affairs, where objects are combined with one another [see TLP: 2.01]. Wittgenstein is clear that in elementary propositions nothing more than names in immediate combination is present. At the same time this applies to state of affairs, where objects are the only constituents. This emerges clearly from a letter to Ogden, the first translator of the *Tractatus*, where Wittgenstein, in discussing the correct translation of 2.03, writes:

Here instead of 'hang one on another' it should be 'hang one in another' as the links of a chain do! The meaning is *that there isn't anything third* that connects the links but that the links themselves make connexion with one another [LO: 23].

Objects can combine with one another, as names can, without the intervention of any other entity (ontological or propositional glue) because they have in themselves their own possibilities of combination into states of affairs and propositions (what characterises their form). But that means that there is not, on the *Tractatus'* view, a rule by means of which a name acquires meaning, and, subsequently to that, a rule that fixes whether propositions are true or false (as it was in the *Notes*); those rules are interdependent and must be given at the same time: Names can thus be given meaning only within the context of a proposition.

As Ricketts explains:

In [...] [the *Notes on Logic*], Wittgenstein says nothing about names and designation, following Russell in treating names as unproblematic labels for objects. Just here the *Tractatus* improves on the treatment of [...] [the *Notes on Logic*]. Rules of designation and rules of agreement presuppose each other in the following fashion. Rules of agreement presuppose the possibility of correlating names with objects: that one name ENVY-leftflanks another says that *the bearer* of the first name envies *the bearer* of the second. A less obvious presupposition runs the other direction. It is only within sentences after the erection of rules of agreement that names symbolize, designate, or mean objects. There is no giving of *names*, no dubbing, apart from the erection of these rules.¹⁷³

¹⁷³ Ricketts [1996: 74-75].

Ricketts thus argues that there is continuity in Wittgenstein's account of propositional complexity from the *Notes on Logic* to the *Tractatus*. In the *Notes* Wittgenstein distinguishes two different propositional components with different symbolising roles, and thus distinguishes what Ricketts calls *rules of designation* for names and *rules of agreement* for forms. Rules of designation are prior; after the establishment of them, by means of which objects are given names, the form of the proposition in which those names occur symbolises by putting the names in a relation with each other, and thus the proposition is true – if the objects named by those names stand to one another in such a relation – and is false otherwise. This is the rule of agreement of the form of a proposition, which determines the sense the proposition has.

In the *Tractatus*, Ricketts argues, rules of designation and rules of agreement are inter-dependent. Names can only be given meaning within the context of a proposition [see TLP: 3.3], so the meaning of names presupposes their combination into propositions, and this, in turn, represents a state of affairs, where objects are combined with one another. So, as Ricketts has it, the *Vertreten* of names in propositions presupposes the *Darstellen* of a state of affairs by a proposition, and thus the correctness or incorrectness – truth or falsity – of the proposition, and vice versa. Rules of designation are now defined as the possibility for names to stand for (*Vertreten*) objects, where rules of agreement for forms are now given by propositions' representing (*Darstellen*) – or picturing – (correctly or incorrectly) states of affairs. An important feature of this account is that relations are not represented by names in propositions, and that, therefore, relations are not objects. In this sense Wittgenstein would have remained faithful

to his early conviction that there is no such *thing* as the form of a proposition, and no name for a propositional form.

Ricketts' account seems promising, for it shows a coherent development in Wittgenstein's conception of sense from the early criticism of Russell's theory of judgement, via his early alternative account of sense (the *Notes on Logic*), to the mature account presented in the *Tractatus* (the picture theory). In particular, that account is able to explain in what respect the picture theory of propositions in the *Tractatus* arises out of the account of propositional sense in the *Notes* and the criticism of Russell; the role forms had in the *Notes* eventually develops into the role of a proposition's picturing (*Darstellen*) a state of affairs by presenting its elements in immediate combination. Consistently throughout the early phase of his career, then, so Ricketts' account goes, Wittgenstein thought that relations are not represented in propositions by names.

The account just presented, however, seems to present a difficulty as an interpretation of Wittgenstein's position in the *Tractatus*. We saw that according to Ricketts relations between objects (that is to say, states of affairs) are represented (*Darstellt*) by relations among names, that is to say, by propositions. This in turn is what makes propositions agree or disagree with facts, namely be true or false. However, when discussing the representative nature of pictures (of which propositions are instances), as we saw, Wittgenstein says:

The fact that the elements of a picture are related to one another in a determinate way represents that things are related in the same way.

Let us call this connexion of its elements the structure of a picture, and let us call the possibility of this structure the pictorial form of the picture [TLP: 2.15].

As said, Wittgenstein in the *Tractatus* defines the notion of form as the possibility of structure; pictorial form is thus the possibility of pictorial structure:

Pictorial form is the possibility that things are related to one another in the same way as the elements of the picture [TLP: 2.151].

Wittgenstein in the above passages clearly claims that there is an *identity* of structure between pictures and what they picture; this is what makes the structure of a picture a pictorial structure. Since propositions are pictures, one should conclude that, in a proposition (at least in an elementary one), the arrangement of its elements (names), its structure, is pictorial by virtue of being the *same* as the arrangement of the elements in the pictured state of affairs.

Ricketts' account does not seem to be able to handle this requirement. Ricketts argues – consistently with Wittgenstein – that the arrangement of elements in a proposition (the relation between its names) is representative of a state of affairs (representative of the arrangement of its objects). But the way in which names are arranged (related) in a proposition may be (and generally is) very *different* from the way things may be arranged in reality. Ricketts claims that relations between objects are not represented by names in propositions, but are represented by relations between propositional elements: in explaining this point, Ricketts writes:

Can we think of elementary sentences on the model of 'a envies b' as I have been urging? 4.22 states that an elementary sentence consists of names. I have insisted that 'a envies b' has only two names. Is my interpretation consistent with 4.22? Here we must remember that sentences are facts: the sentence in question is 'a's ENVY-leftflanking 'b'. This fact is a chaining together of elements in a way entirely analogous to the way in which the state of affairs of a's envying b is a combination of objects.¹⁷⁴

¹⁷⁴ Ricketts [1996: 76].

Rickett's interpretation is coherent with Wittgenstein's account in the *Notes on Logic*, where the sense of a true proposition determines a formal *similarity* between its structure and the fact that verifies it (a proposition's having as components two names and a form might be made true by a fact having – likewise – two objects related by a relation). But the *Tractatus* account requires more than that. As we saw, Wittgenstein in the *Tractatus* claims that the arrangement of the elements in a proposition (the relation between them) has to be the *same arrangement* of the elements in the state of affairs represented. And surely the relation between the names 'a' and 'b' in the proposition 'a envies b', a relation defined by Ricketts as exemplified by the fact that 'a' envy-leftflanks 'b', is *not* the same relation linking together the objects *a* and *b* in the state of affairs 'a envies b'. In the latter the relation is that of 'envying', while in the former the relation is that of 'envy-leftflanking'.

According to Ricketts' account, therefore, a picture (a proposition) may represent a state of affairs even though its elements (names) are *not* related in the *same* way as the elements of the corresponding state of affairs, running thus blatantly contrary to Wittgenstein's requirement of identity of structure between picture and pictured state of affairs. Whereas Wittgenstein, at 2.15, claims that the arrangement of elements in a picture represents an arrangement of things in reality by showing that very arrangement realised, Ricketts claims that identity of structure is *not* necessary for a picture to represent a state of affairs.

Ricketts' conclusion is due, as we saw, to his 'nominalistic' interpretation according to which relations among objects in states of affairs are not represented (in the sense of *Vertreten*) by names in propositions, but by propositions

themselves (namely by the arrangement of their propositional elements – names). This makes it difficult to account for the claim that the combination of names in propositions is the *same* as that between elements of states of affairs (objects). Ricketts's solution is that the claim that the structure of a proposition (picture) has to be *identical* to that of the pictured state of affairs must be dropped. He says:

[S]trict identity between the ways that pictorial elements can combine into pictures and objects into states of affairs is not necessary for a picture to present a state of affairs.¹⁷⁵

Ricketts is not alone in holding this. Irving Copi, who – perhaps first – argued for the view that in the *Tractatus* properties and relations are not objects (and therefore that they are not *named* in propositions) reaches a similar outcome. Relations among names in propositions need not be the same as the relations holding among named objects. As he has it:

[N]ot all relations among pictured objects are represented by placing the pictorial representations of those objects in identically the same relation that is to be represented. [...] Although in pictorial representation relations are represented by relations, they are often represented by relations other than themselves.¹⁷⁶

Now, Wittgenstein's requirement that there has to be identity of structure (and therefore identity of ways of combination of elements between pictures and pictured states of affairs) may indeed seem puzzling. How can relations between words in propositions be the same as the relations holding among objects in reality? This made David Pears ask: when Wittgenstein claims that objects are combined in the same way as are the elements of a picture (and a proposition), “can he really mean in the *same way*”?¹⁷⁷

¹⁷⁵ Ricketts [1996: 74].

¹⁷⁶ Copi [1958:157].

¹⁷⁷ Pears [1977: 194].

For Ricketts, as said, the answer is 'no'. Ricketts' reason is that the idea of identity of structure (and therefore of identity of relations between picture and pictured state of affairs) is due to a faulty translation of 2.15. Pears' and McGuinness' translation, which renders the German "[...] stellt vor, dass sich die Sachen so zu einander verhalten" as "represents that the things are related in the same way" is for Ricketts "philosophically tendentious for being misleadingly definite".¹⁷⁸ This does not really seem a fair point. Even Ricketts' favoured alternative translation – due to Ogden – which reads "represents that things are so combined with one another" seems to imply *identity* of ways of combination between pictorial elements and elements of the state of affairs. The 'so' there cannot really suggest anything else than 'in the same way'.

According to David Pears Wittgenstein argues that the combination of elements in a picture – insofar as 'picture' is understood as a material likeness, such as a photograph or a drawing (having a material or natural similarity with what it represents) – is identical to the combination of the objects in the corresponding state of affairs; but that strong requirement, on the other hand, does not hold for propositions. Pears argues that the criterion of sameness of way of combination between elements of a picture and of the pictured state of affairs is in the *Tractatus* "sufficiently relaxed to fit propositions".¹⁷⁹ Pears is right in saying that the requirement of identity of combination, presented by Wittgenstein at 2.15 regarding pictures, is not *explicitly* applied to propositions. There are no passages in the *Tractatus* where Wittgenstein claims that the relations between the elements of a proposition are the same as the relations among the objects of the

¹⁷⁸ Ricketts [1996: 98].

¹⁷⁹ Pears [1977: 194].

corresponding state of affairs. However, there are no passages that (at least overtly) contradict that requirement either.

Ricketts' position is that, instead of identity of structure between pictures and what they picture, we should look for a *formal similarity*, namely a relation of *isomorphism* between structures. As he says:

Modeling does not require that the pictorial elements and the represented objects share the very same possibilities of combination. It only requires a formal 'isomorphism' between the possible configurations of pictorial elements into pictures and of objects into facts.¹⁸⁰

The notion of isomorphism implies that elements in the picture are univocally associated to elements in the situation to be represented, and that relations among those elements univocally correspond to relations among the elements of the situation to be pictured. Thus, even granting that elements in pictures differ from the elements in the situations to be pictured, and that relations among those elements are not identical to the relations among objects in situations, it is however possible to univocally associate relations among pictorial/propositional elements to relations among objects in reality, and thus to represent situations in reality.

Picture and pictured situation will, on this account, have a *formal*, not *substantial*, identity, due to their having the same logical multiplicity and to the possibility of univocally correlate relations among objects to relations among pictorial elements. As soon as one knows how to associate which pictorial element with which object, and which relation among those elements with which relation among objects in reality, then one is able to read off the situation from the picture. On this line of interpretation, then, Wittgenstein's picture theory of representation

¹⁸⁰ Ricketts [1996: 78].

does not require that the structure of the picture is *identical* to the structure of the situation to be represented. It only requires that the two are isomorphic with one another.

[10.4] Realism and Identity of Structure

I presented the isomorphism account as arising from the ‘nominalistic’ interpretation (held by Ricketts and Copi) of the Tractarian conception of propositional constituents. This account relaxes the requirement of identity of structure between pictures and states of affairs into a requirement the two structures are *isomorphic*, without undermining the other crucial tenets of the picture theory (i.e. that a picture is a fact and that picture and pictured fact have the same logical multiplicity). The isomorphism account, however, is *absent* from the *Tractatus*. There is no explicit statement, there, that representation relies on the idea that a picture is isomorphic with the structure of the state of affairs it represents. On the contrary, as we saw, Wittgenstein claims that a structure is pictorial when it displays the *same* arrangement of elements as the state of affairs to be represented. In this section I argue that – by resisting the nominalistic interpretation – it is possible to maintain the requirement that representation relies on the arrangement of elements in propositions being *identical* to the arrangement of elements in reality, as the *Tractatus* indeed claims.

Resisting the nominalistic account frees one from thinking that names, in propositions, are only names of particulars. Names might also be names of properties and relations. This account – which one might call, in opposition to the ‘nominalist’ interpretation, ‘realist’ – may be briefly sketched as follows. In a

relational proposition ' aRb ' we have three names, namely three symbolising elements, ' a ', ' b ' and the relation between them, which we symbolise by means of putting the sign ' R ' between ' a ' and ' b '; so two are names of individuals whereas the third is a relational name. Given the parallelism between the dimension of the representation and of what is represented, this implies that in the represented state of affairs we have three objects, a and b united by the *R-from-to* relation (which is itself an object). This is explained by Stenius as follows:

Since what stands for an element of the fact described by sentence ' aRb ' is the ' R '-from-to relation and not the letter ' R ', we may say that the *symbols* (the symbolizing elements) occurring in the sentence ' aRb ' are ' a ', ' b ' and the ' R '-from-to relation. Using the word 'symbol' in such a way we may state that the symbols in a sentence are not always linguistic *objects* like words or letters but may also be '*universals*' like the ' R '-from-to relation in [...] [aRb].¹⁸¹

This interpretation has as its originator Frank Ramsey, who, in interpreting Wittgenstein's account of the articulation of ' aRb ', writes that "the name of R is not the word ' R ', but the relation we make between ' a ' and ' b ' by writing ' aRb '".¹⁸² Both the nominalist and the realist account of propositional articulation provide an account of Wittgenstein's claim that propositions consist of names in immediate combination. According to the nominalist account the notion of a name is to be understood *narrowly* as comprising only names of particulars and not of predicates and relations. According to the realist interpretation, on the other hand, names also include relational and predicate names.

The nominalistic interpretation rejects Wittgenstein's claim that a picture represents objects as being arranged *in the same way* as its pictorial elements are arranged, and weakens it into the requirement that the two structures (that of the

¹⁸¹ Stenius [1981: 116].

¹⁸² Ramsey [1954: 145].

picture and that of the represented state affairs) are isomorphic, but not identical. If we adopt a realist stance, however, we are not forced into this predicament. By assuming that the relational expression in a proposition is itself a symbol for an object, and that therefore the relational component of a state of affairs is itself an object, one can maintain that the structure of a proposition is identical to the structure of the pictured state of affairs. The proposition '*aRb*' is an immediate combination of names, '*a*', '*b*', and the '*R* left/right-flanking' relation, names which represent (in the sense of *Vertreten*) the objects *a*, *b*, and an arbitrary dual relation. The state of affairs in question is itself a combination of those objects. All these names are conventional. The names '*a*' and '*b*' could have meant something different than the objects *a* and *b*, and the '*R* left/right-flanking' relation could have meant a different dual relation than the one that it does.

However, as long as those 'arbitrary stipulations' are in play, then the sense of the proposition is fixed, and the combination of those names is indeed *identical* to the combination of those objects in the possible situation. At this point, one may also note that Wittgenstein, when discussing the combination of elements in a picture/proposition, does not use the word 'relation' (*Beziehung*), but other expressions instead.¹⁸³ So, on Wittgenstein's view, the *combination* of pictorial elements in pictures (or names in propositions) is identical to the combination, or arrangement, of elements in the pictured state of affairs. But the *relation* between *names* (for example the relational element in the proposition '*aRb*') need not be, and in fact is not, identical to the relation between particular elements in states of affairs.

¹⁸³ I owe this point to Peter Sullivan. This difference is sometime lost in the Pears and McGuinness translation, while it is maintained in the Ogden translation.

If this is correct then, a realist interpreter can endorse the isomorphism account without dropping the requirement that the combination of propositional elements is *identical* to the combination of elements of the represented state of affairs, that is, without holding the isomorphism account as a way of *relaxing* the identity requirement (as does the nominalist). The identity of possibility of combination for the representing and the represented fact can naturally be fleshed out in terms of isomorphism. An isomorphic representation can still be said to share something *identical* with what it represents. The identity lies precisely in the fact that – although elements and relations may be different in the represented and the representing fact (for names are conventional) – the combination of entities and relations in the representing fact is identical to the combination of entities and relations in the represented one. Such a representation would still be isomorphic, but it would have, nonetheless, something *identical* in common with what it represents, that is, the manner of combination of its elements, its logical structure.¹⁸⁴ In this respect then, the realist account is superior to the nominalistic one, because it is able to vindicate – while the latter is not – Wittgenstein’s requirement of identity of structure (combination of elements) between pictures and what they picture.

It can be argued, however, that both the nominalistic and the realistic accounts are mistaken as accounts of what Wittgenstein thought propositional constituents to be. Both in fact presuppose that Wittgenstein was following Russell (and Frege) in holding that there is an ultimate difference between particulars and universals (difference that in Frege takes the form of a difference between objects

¹⁸⁴ In *this* sense, I believe, the isomorphism account is endorsed by Stenius [1981: 113].

and functions of different level), and thus a difference between names of particulars and predicate/relational expressions. Where they differ is in interpreting the *Tractatus*' notion of a 'name'; for the realist, this includes predicate and relational names, for the nominalist, Tractarian names are only names of particulars.

The question whether names are only names of particulars or also predicate or relational names actually amounts to the question of what form propositions have. Wittgenstein, however, in the *Tractatus*, is *agnostic* about the composition of elementary propositions. For Wittgenstein the question of what elementary propositions there are cannot be answered *a priori*, and can be settled only once the analysis of propositions into their simple components has been carried out. It is thus "the application of logic" (truth-functional analysis) that decides what form elementary propositions have, but "[w]hat belongs to its application, logic cannot anticipate" [see TLP: 5.557]. If so then, the very debate over what kinds of names Wittgenstein thinks Tractarian names to be (which also amounts to what kind of entities Wittgenstein includes among objects - only particulars, or also universals) is misguided, for Wittgenstein did not provide an explicit position on this.¹⁸⁵ Sometimes, though, Wittgenstein uses *examples* of elementary propositions, and then he uses either Frege's notation or Russell's. An example of the former occurs at 4.24, where Wittgenstein calls an elementary proposition a "function of the names, in the form ' fx ', ' $\varphi(x, y)$ ', etc." An example of the latter can be found at the already discussed 3.1432 (a passage which directly derives from the *Notes*) where Wittgenstein writes a proposition in the form ' aRb '. But those formulations have to

¹⁸⁵ See Johnston [2008].

be taken as elucidatory examples of elementary propositions, not as suggesting any particular interpretation of the composition (articulation) of them.¹⁸⁶

[10.5] From the *Notes on Logic* to the *Tractatus*

Ricketts' interpretation of the *Tractatus*' conception of propositional complexity provides a natural and smooth reading of Wittgenstein's development of thought from his criticism of Russell's theory of judgement to the account of sentential complexity in the *Tractatus*. Wittgenstein saw a flaw in Russell's treatment of relations in his multiple relational account of judgement, its conceiving of a (subordinate) relation as a term. The account of propositional complexity in the *Notes* relies on the different symbolising role of names and forms, both seen as indispensable constituents of propositions. Now, Ricketts' account builds precisely on *this* aspect to stress that the *Tractatus* does not substantially revise the *Notes on Logic* account of representation and sentential complexity.

We saw, on the other hand, that Ricketts cannot explain Wittgenstein's claim that the structure of propositions is identical to the structure of states of affairs, and has therefore to resort to the isomorphism account to modify and relax that claim. So this story is, despite its initial plausibility, mistaken. I think that a different account, able to guarantee a general compatibility (through difference) between the account in the *Notes* and the *Tractatus*, can be drawn. Wittgenstein's early notion of indefinable components of propositions (names and forms) is, in the *Tractatus*, transformed into the general notion of a symbolising (in the sense of

¹⁸⁶ In particular, as we will see in section [11.3], Wittgenstein's use of Frege's notation does not imply a commitment to a Fregean distinction between different kinds of propositional entities.

Vertreten) entity, a *name*. The notion of a name, which in the *Notes* is understood in a narrow sense as a name of a particular, undergoes a significant transformation in the *Tractatus*, and, moreover, retains the function that the notion of a *form* had in the *Notes*. Ricketts is therefore mistaken when he claims that “the role played in [...] [The *Notes on Logic*] by Wittgenstein’s distinction between the way that names symbolize and the way that forms symbolize is taken up in the *Tractatus* by the distinction between *vertreten* and *darstellen*”.¹⁸⁷

The difference between the symbolising roles of names and forms in the *Notes* is in the *Tractatus* accommodated by a broader and more general understanding of *Vertreten*, where the contextual understanding that the *Notes on Logic* envisaged for forms is extended to all sentential expressions, to all names. Every sub-propositional expression, according to the *Tractatus*, has to be contextually understood, that is: its understanding cannot be divorced from the understanding of its possibility of combination with other expressions in propositions. A name, now understood as an ‘expression’, is in fact said by Wittgenstein to presuppose “the forms of all the propositions in which it can occur” [TLP: 3.311]. This means that understanding every name, every expression, presupposes, for the *Tractatus*, understanding every proposition in which the expression in question may occur. This is the reason why Wittgenstein claims that “[a]ll variables can be constructed as propositional variables (even the variable name)” [TLP: 3.314].¹⁸⁸ But this of course means that for Wittgenstein the understanding of a name cannot be divorced from the understanding of its

¹⁸⁷ Ricketts [1996: 76].

¹⁸⁸ See Stenius [1976: 80-81], and, for a more recent and developed account, Johnston [2007: 242-243].

possibilities of combination to form propositions. This characteristic, that Wittgenstein in the *Notes on Logic* attributed to forms in order to account for propositional unity and thus in order to overcome the shortcomings of Russell's theory of judgement, in the *Tractatus* he extends to all names.

The continuity in Wittgenstein's thought on representation between the *Notes on Logic* and the *Tractatus* should thus be sought not in an alleged consistent treatment of the notion of a name, but in the generalisation of the *Notes'* notion of the form of a proposition, of a propositional component that presupposes its possibility of combination with other propositional elements. The mutual dependence of names and propositions (or, better, of terms and judgements) is totally absent from Russell's conception of judgement, which seeks to explain the nature of judgement and understanding by solely relying on the notion of acquaintance with the objects involved. But acquaintance, in that a simple and immediate dual relation, cannot ground the notion of understanding, of grasping the role played by propositional constituents in propositions, as it is conceived of by Wittgenstein. To be acquainted with an object (logical or not) does not require and does not presuppose, for Russell, the understanding of its possibilities of combination with others.¹⁸⁹ For this reason acquaintance, the basic ingredient of Russell's theory of judgement, was not for Wittgenstein a notion by means of which one could account for propositional articulateness, unity, and therefore sense.

¹⁸⁹ In line with this, Pears [2006: 13] characterises Russell's theory as "an attempt to base sense on acquaintance with objects, interpreted extensionally, and so without any knowledge of their possibilities of combination with one another".

Chapter Eleven

Language and Logic in the *Tractatus*

[11.1] Language, Truth and Logic: Pictures and Truth-functions

In chapters [8] and [9] we discussed how Wittgenstein's objection to Russell's theory of judgement – and his conviction that it could only be removed by a correct theory of symbolism – led him to the account of sense and propositional articulation set forth in the *Notes on Logic*, which is, if my interpretation is correct, delineated in direct opposition to Russell's account of judgement. We then analysed in chapter [10] how that account – according to which there is a structural similarity between propositions and facts, is transformed and elaborated in the *Tractatus*, where the idea of a harmony between language and world is grounded in the idea that propositions are pictures of states of affairs.

In the *Tractatus* Wittgenstein also argues that being a picture is a condition for being true or false.

A proposition can be true or false only in virtue of being a picture of reality [TLP: 4.06].

And in the *Notebooks* he writes:

We can say straight away: Instead of: This proposition has such and such a sense: this proposition represents such and such a situation.

It portrays it logically.

Only in this way can *the proposition* be true or false: it can only agree or disagree with reality by being *a picture* of a situation [NB: 2.10.14].

In the *Notebooks* entry for 3.10.14 (repeated almost literally in the *Tractatus* at 4.032), Wittgenstein makes the relation between pictures and truth/falsity depend on the fact that pictures are essentially *logically articulated*. “The proposition is a

picture of a situation only in *so far* as it is logically articulated". Thus "a simple – non-articulated – sign can be neither true nor false" [NB: 3.10.14]. A picture's capability of being true or false depends thus on its having a *structure*, by means of which it represents (depicts) a state of affairs in reality. Something can thus be true or false only when it respects conditions 1 – 3 discussed in section [10.2]: being a fact (1), sharing with what is represented a pictorial relationship (2) and a pictorial form (3). A picture is therefore a *representational* (or *pictorial*) structured fact, that is, a fact representing elements in reality to be combined in the same way as its elements are combined. Its being a representation (a picture) of reality makes therefore the proposition *intrinsically* true or false; if things in reality are combined as it shows them to be, then the proposition is true, and otherwise false. What emerges is therefore a conception of truth and falsity as *agreement* and *disagreement* with reality.

A picture agrees with reality or fails to agree; it is correct or incorrect, true or false [TLP: 2.21].

And:

The agreement or disagreement of [...] [a picture's] sense with reality constitutes its truth or falsity [TLP: 2.222].

The idea of agreeing and disagreeing with reality amounts to the idea that when a picture is correct/true, then what it represents exists, and fails to exist when the picture is incorrect/false. As Wittgenstein has it:

If an elementary proposition is true, the state of affairs exists: if an elementary proposition is false, the state of affairs does not exist [TLP: 4.25].

The pictorial account of propositional articulation discussed above, as is well recognised, directly applies only to *elementary* propositions, that is, to those propositions which are the result of a process of logical analysis, of decomposition of ordinary propositions into their constituent elements. Wittgenstein's account of complex - molecular - propositions (among which, importantly, propositions of logic are to be found) relies on the idea that all complex propositions are truth-functions of elementary ones, namely propositions whose truth-value is determined by the truth-value of their constituent propositions; examples of such propositions are those obtained by the application of the familiar truth-functional connectives such as ' \sim ' (negation), '&' (conjunction), ' \vee ' (disjunction), and so forth. As TLP 5 reads: "A proposition is a truth-function of elementary propositions".

On the one hand, thus, we have elementary propositions, which directly picture or model states of affairs, and on the other we have molecular propositions, truth-functions of elementary propositions, in which the latter, with their pictorial role, occur as truth-arguments. While an elementary proposition asserts the existence of a state of affairs [see TLP: 4.21], by representing it, molecular propositions are expressions of "agreement and disagreement with truth-possibilities of elementary propositions" [TLP: 4.4]; that is to say, they are expressions of agreement and disagreement with the truth-conditions of elementary propositions [see TLP: 4.41], agreement and disagreement which can be expressed in a truth-table. The propositions of logic, tautologies and contradictions, are a subset of the set of all molecular propositions; the former express agreement with all the truth-possibilities of elementary propositions, and

the latter express disagreement with all the truth-possibilities of elementary propositions.

Now, the picture of language we seem to be presented with is thus *bipartite*. Elementary propositions are pictures of states of affairs, and molecular propositions truth-functions of them. Two, therefore, seem to be the principles on which the *Tractatus* bases its account of language: the principle of pictorial representation *and* the principle of truth-functionality; this reading of Wittgenstein's analysis of language and logic in the *Tractatus*, which I label the 'bipartite reading', seems quite natural. However, it also seems problematic under many respects. Brian McGuinness expresses worries about it in the following terms:

[I]n the first part of the *Tractatus* [...] we seem to be told that the essence of a proposition is to be a picture, while in the later parts we are told that its essence is to be a truth-function [...]. [A] [...] serious difficulty is that the two accounts seem to be quite separate things, and, if this is so, cannot both be adequate accounts of what it is to be a proposition.¹⁹⁰

More recently, Michael Morris argued:

There is a risk of understanding Wittgenstein's account of language [...] as falling into two completely unconnected parts: one which is appropriate to the conception of elementary sentences as models, the other which concerns the construction of other sentences out of elementary sentences. For the account of elementary sentences as models, the crucial notion of sentential form seems to be concerned with the ways in which names can be arranged to form a sentence [...]. By contrast, the notion of sentential form which seems to be relevant to the account of logic seems to be concerned with the ways in which whole sentences can combine with each other.¹⁹¹

In fact, throughout the *Tractatus*, Wittgenstein speaks of propositions both as 'pictures' and as 'truth-functions', not always carefully distinguishing propositions which are directly pictures (elementary ones) from those that are

¹⁹⁰ McGuinness [2002a: 65-66].

¹⁹¹ Morris [2008: 234-235].

truth-functions of elementary propositions (molecular ones). He seems to suggest that the nature of a proposition is *both* its representational character and its truth-functional structure.

The bipartite reading also seems to entail that an account of logical interconnectedness can be reached only when a prior understanding of the pictorial character of (the elementary bits of) language is achieved; this is so because the principle of truth-functionality seems to entail the view that logical articulation is the result of the application of truth-functional connectives to elementary propositions, whose articulation, in turn, is describable in terms of pictorial representation. Against this perspective, Peter Winch said:

The question of how propositions are related to their negations is a special case of the question of what relations hold between a proposition and other propositions – the relations which enable us to infer one proposition from others. It would be tempting to say that this is the question: What is logic? But this might misleadingly suggest that the nature of logic has only become an issue at *this* stage [...].¹⁹²

According to Winch, it is a deep misunderstanding of Wittgenstein's position to argue that an understanding of logic depends on a prior, and independent, understanding of the pictorial character of language.

These two criticisms point towards two related shortcomings of the bipartite reading; in the first place, as McGuinness notes, such an interpretation makes the relation between elementary propositions and molecular ones mysterious (i.e. it does not explain it in any way). Secondly, Winch points out that that reading gives a wrong explanation of the role of logic, and of its relation to language's ability to express sense, encouraging the idea that the latter is prior to the former. In both cases, the bipartite reading does not seem able to explain the

¹⁹² Winch [1969: 3].

deep interconnections between elementary propositions and molecular ones, and, more generally, between language and logic.

[11.2] The Internal Unity of Language and Logic

In this section I argue that the bipartite reading amounts to a misunderstanding of Wittgenstein's conception of language and logic in the *Tractatus*. The fact that there is some stringent (internal) relation between propositional (and logical) articulation and a proposition's ability to express the sense that it does, is something that Wittgenstein acknowledges very early in his philosophical career.

In 1912, when Wittgenstein was still studying under Russell, and when the two were mainly discussing the nature of logic and the status of logical propositions, Wittgenstein wrote to Russell:

I believe that our problems can be traced down to the *atomic* propositions. This you will see if you try to explain precisely in what way the Copula in such a proposition has meaning.

I cannot explain it and I think that as soon as an exact answer to this question is given the problem of 'v' and of the apparent variable will be brought *very* near to their solution if not solved. I now think about 'Socrates is human' [...] [NB: 121].

Wittgenstein is here saying that a solution to the problem of explaining the nature of logical interconnectedness (the problem of 'v') will be achieved by a correct explanation of the role of the copula in an elementary (or atomic) proposition. The suggestion thus amounts to the idea that an explanation of logical articulation will be given by an explanation of propositional articulation, at the atomic level, thus of the articulation of elementary propositions.

In the wartime *Notebooks* we find a similar attitude towards the relation between the pictorial nature of propositions and the nature of logical articulation.

As regards the latter, Wittgenstein writes:

The problems of negation, of disjunction, of true and false, are only reflections of the one great problem in the variously placed great and small mirrors of philosophy [NB: 6.3.15].

The single great problem is identified by Wittgenstein as that of giving an explanation of the nature of the proposition: “My *whole* task consists in explaining the nature of the proposition” [NB: 22.1.15]. This passage clearly suggests that Wittgenstein thought that the problems relative to logical articulation (‘the problems of negation, of disjunction’ etc.) are derivative from the problem of giving an account of the nature of the proposition, and that, therefore, an explanation of the nature of the proposition, of how a proposition expresses its sense, will also bring a solution to the former problems.

This testifies that Wittgenstein felt there cannot be any dualism between an account of the nature of the proposition and an account of logical interconnectedness. These are simply two aspects of the same problem, which are to be given the same solution. It is not that the solution of the problem of logical articulation has to wait until an explanation to the nature of elementary sense is given, as the bipartite reading suggests. Wittgenstein is here implying that there are not two different problems, but only one. Marie McGinn puts this thought in the following words:

Wittgenstein is convinced that we shall see everything clearly – the nature and the status of the propositions of logic, negation, disjunction, inference, truth and falsity – when we see this one thing clearly: the nature of the proposition. It is not that we shall be able to *deduce*, say, the status of the propositions of logic, or the nature of negation, from the nature of the proposition [...]. It is rather that coming to see the nature of the proposition clearly *is, at the*

very same time, coming to see negation and the status of the propositions of logic clearly: we have here, not a number of separate problems, but one great problem. If the problem is to be solved, then it must be solved all at once and in its entirety. The idea of the single great problem is that once the nature of a proposition has become clear, then everything will be clear: the nature and status of the propositions of logic, the nature of negation, inference, and so on.¹⁹³

In the same vein, Peter Winch argues as follows:

[I]t is vital to our understanding of Wittgenstein to see that the nature of logic is already being inquired into in Wittgenstein's treatment of the puzzle about the relation between propositions and facts. This point can perhaps be expressed in the form of another problem: what is the relation between a proposition's ability to state a fact and its ability to stand in logical relations to other propositions? Now Wittgenstein thought [...] that there must *be* such a relation; that it is not merely a contingent matter that a proposition can combine these two functions; that unless propositions had logical relations with each other they would not state facts (i.e. would not be propositions) and unless they stated facts, they would not have logical relations with other propositions.¹⁹⁴

Basically the same point is made by Elizabeth Anscombe, when she argues that the pictorial character of propositions is not to be divorced from truth-functional interconnectedness.

Indeed, we should not regard Wittgenstein's theory of the proposition as a *synthesis* of a picture theory and the theory of truth-functions; his picture theory and theory of truth-functions are one and the same.¹⁹⁵

How does Wittgenstein develop the idea that an account of the nature of the proposition must be, by itself, also an account able to explain the nature of logical articulateness, and therefore that the picture theory also amounts to a truth-functional account of the nature of the proposition? In a nutshell, Wittgenstein thought that logical constants are already present in the articulation of an elementary proposition, that is, of a proposition which is a picture of reality. This is the reason behind his thought that the nature of logic is grounded in, and

¹⁹³ McGinn [2006: 15-16].

¹⁹⁴ Winch [1969: 3-4].

¹⁹⁵ Anscombe [1959: 81]. The point is directly made against von Wright [1955: 533], who claims that "Wittgenstein's *Tractatus* may be called a synthesis of the theory of truth-functions and the idea that language is a picture of reality".

even exhausted by, the pictorial character of language. We have statements of this position in the *Notebooks*:

All logical constants are already contained in the elementary proposition [NB: 5.11.14].

And in the *Tractatus*:

An elementary proposition really contains all logical operations in itself. For '*fa*' says the same thing as

$$'(\exists x) . fx . x = a'$$

Wherever there is compositeness, argument and function are present, and where these are present, we have already all the logical constants [TLP: 5.47].¹⁹⁶

Although the basic idea of these passages seems clear enough (logical articulateness is already given as soon as elementary – pictorial – compositeness is given), it is by no means easy to derive a coherent and full account of the relation between language and logic out of it.

The task is thus that of making sense of the view embedded in the passages quoted above, the idea that the nature of logic is to be clarified by a correct account of the nature of an elementary proposition (its pictorial character). The task is thus, at bottom, that of explaining how Wittgenstein accounted for the internal unity of language and logic.

[11.3] Propositional Complexity and the Functional Model

Wittgenstein's claim, at 5.47, that compositeness entails function and argument seems to encourage the thought that it is the functional structure of an elementary

¹⁹⁶ See also [TLP: 5.515], where Wittgenstein, after saying that it is evident that only propositions can be combined with one another by symbols for logical constants such as '*v*', and '*.*', says that the reason is that "the symbol in '*p*' and '*q*' itself presupposes '*v*', '*~*', etc."

proposition that entails the presence, in it, of all logical constants. In this section I explore the viability of an analysis of propositional structure in terms of the functional model, within the framework of the picture theory discussed in the previous chapter, and its significance for an understanding of the nature of logic.

Besides 5.47, there are other remarks in the *Tractatus* explicitly suggesting that propositional articulation is thought of by Wittgenstein as function-argument articulation. Some examples are:

Like Frege and Russell I construe a proposition as a function of the expressions contained in it [TLP: 3.318].

I write elementary propositions as functions of names, so that they have the form ' fx ', ' $\phi(x, y)$ ', etc.

Or I indicate them by the letters ' p ', ' q ', ' r ' [TLP: 4.24].

The idea that propositional articulation must be understood along the lines proposed by the functional model is clearly reminiscent of Frege's analysis, and Frege is in fact explicitly mentioned by Wittgenstein at 3.318, quoted above. A reference to Frege also occurs in the *Prototractatus* predecessor of 4.24, which reads as follows:

Generally in what follows I indicate elementary propositions by the letters p, q, r, s, t , or else (like Frege) I write them as functions of their objects in the form ' $\phi(x)$ ', ' $\psi(x, y)$ ', etc. [PT: 4.2212].

This suggests that looking at Frege's conception of propositional articulation might clarify Wittgenstein's position in the *Tractatus*.

In his *Begriffsschrift*¹⁹⁷ Frege analysed what he called 'assertible/judgeable' or 'conceptual' content, that is, the content of a judgement, or of a proposition, as

¹⁹⁷ Frege's mature position on function and argument articulation is however expressed in the essay 'Function and Concept' and in the early sections of *The Basic Laws of Arithmetic*.

being analysable into function and argument, where the function is the *constant* part and the argument the *replaceable* one. The conceptual content as a whole is therefore seen as a *combination* of argument and function. As Frege has it:

If, in an expression [...], a simple or complex symbol occurs in one or more places and we imagine it as replaceable by another (but the same one each time) at all or some of these places, then we call the part of the expression that shows itself invariant a function and the replaceable part its argument [CN: § 9].

The functional model, Frege argues, is applicable to any possible object of judgement (in Wittgenstein's terminology, to any proposition). In *Begriffsschrift* Frege represents the expression of a (indeterminate) function of the argument A by putting A in brackets following a Greek (capital) letter, which – together with an empty place in parentheses – is the symbol for the function: ' $\Phi (A)$ '. A function of two arguments A and B is expressed by the two letters in brackets following a (Greek capital) letter – together with two empty places in parentheses, namely ' $\Psi (A, B)$ ', (which says something different from ' $\Psi (B, A)$ '). Now, this analysis seems very similar to Wittgenstein's characterisation of an elementary proposition, as it is sketched at 4.24: there we have expressions (elementary propositions) such as ' $f(x)$ ' and ' $\phi(x, y)$ ', which have a functional structure, the former presenting a function of one argument ' x ', and the second a function of two arguments ' x ' and ' y '.

For Frege, however, the function and argument articulation is meant to capture a logical and ontological difference between *functions* and *objects*, difference that is reproduced in grammar by the distinction between names (*Eigennamen*) and predicate or relational expressions. Predicate and relational expressions are – as well as the functions which they stand for – *incomplete*, or

unsaturated, whereas proper names, by standing for saturated objects are themselves *complete*, or *saturated*.¹⁹⁸ As Stenius remarks:

The main point in Frege's arguments seems to be the introduction of a distinction between [...] two main categories of entities, namely complete entities or objects, and unsaturated entities or functions. Among the complete entities belong numbers and truth-values since both are meanings of complete expressions. [...] Again, both 'functions' and 'concepts' are counted as belonging among the unsaturated entities, since their signs are unsaturated.¹⁹⁹

Although we saw that Wittgenstein's analysis of propositional articulation and unity in the *Notes on Logic* relied on a Fregean distinction between different categories of propositional constituents, *names* and *forms* (a distinction which parallels Frege's distinction between objects and functions), by the time of the *Tractatus* such an analysis was abandoned. There are not two different categories of indefinables in the *Tractatus* (forms and names), but only names. Likewise, the constituents of states of affairs are objects (and not forms *and* objects). Of course, since Wittgenstein thought that propositional constituents are names, they cannot be names in the Fregean sense, they cannot be complete or saturated entities. As I argued in section [10.5], the conception of a name, in the *Tractatus*, is broader than that of the *Notes* (and Frege's); in this respect, names in the *Tractatus* retain some degree of the unsaturatedness that Frege reserved to predicate-relational expressions and the young Wittgenstein, in the *Notes*, reserved to forms. Every

¹⁹⁸ As Frege remarks: "Objects stand opposed to functions. Accordingly [...] objects [are] everything that is not a function, for example, numbers, truth-values, and the courses-of-values [...]. The names of objects - the proper names - therefore carry no argument places; they are saturated, like the objects themselves" [BLA: 37].

¹⁹⁹ Stenius [1976: 76]. Instead of function we might want to speak of relation, but the point does not change. As Potter puts it: "[For Frege] [t]he relation possesses an unsaturatedness analogous to that of the relation symbol" [2009: 115], and conversely, "whatever refers to something unsaturated is itself unsaturated" [2009: 117]. On the other hand, Proops [2000: 70] says, in my opinion quite erroneously, that "[f]or Frege a relational expression is not unsaturated in the same sense as its reference, for a relational expression is an *object*".

name, every expression, the *Tractatus* claims, presupposes the forms of all those propositions in which it may enter as a constituent.

This brief description of the functional model, however, seems to engender the following interpretive problem: Wittgenstein, unlike Frege, retains the function-argument model, as a way to analyse propositional complexity, *without* endorsing the corresponding distinction of logical (and ontological) entities into unsaturated functions and saturated objects. It might therefore seem that the functional model proposed by Frege is at odds with Wittgenstein's conception of propositional articulation. The problem, in other words, is that the idea of propositions as functions of their expressions – the idea embedded in the functional model endorsed by Wittgenstein at 3.318 and 4.24, seems thus not to be in accord with the conception – proposed at 4.22 – of a proposition as an immediate concatenation of names.

So how can the functional model cohere with Wittgenstein's conception of propositional articulation? Elizabeth Anscombe remarks that "if the elementary proposition consists of names in immediate connection – if it is just a concatenation of names – then it is not *reproduced*, even if it can faithfully be represented by a formula consisting of some letters for names and some letters for functions".²⁰⁰ And she goes on; "If for the moment we may give 'a-b-c-d' as an elementary proposition, then 'a-b-c-()' and a-(')-(')-d' would be two different functions; which might be represented as 'fx', ' $\phi(x,y)$ ' respectively; and the representations of 'a-b-c-d' as a value of these two functions would be 'fd',

²⁰⁰ Anscombe [1959: 100].

' $\phi(b,c)$ '".²⁰¹ Now, according to Anscombe we should not think that, for Wittgenstein, the function-argument analysis of a proposition reveals the composition (structure) of it; this is the reason why she claims that the functional model does not *reproduce* the structure of an elementary proposition.

By writing an elementary proposition as ' $f(x)$ ', therefore, Wittgenstein is not suggesting that that proposition has *two* constituents (the function and the argument). Thus, Anscombe remarks, "' $f(b)$ ' symbolizes an elementary proposition, but not necessarily one in whose sense (the atomic fact) *only* two objects occur".²⁰² By adopting the functional model one can thus *represent* elementary propositions, without, by that, *reproduce* their structure. The function-argument analysis in the *Tractatus*, therefore, is not meant to reveal any fundamental distinction between different propositional constituents (functions and objects).²⁰³

But now a question naturally arises. What is the significance of Wittgenstein's adoption of the functional model as a way of representing elementary propositions? My answer is that the representation of propositions according to the functional model is, in Wittgenstein, simply a way of insisting on the idea that a proposition is *articulate, complex*, and therefore provided with a structure. In fact, only something composite can be analysed into function and

²⁰¹ Anscombe [1959: 101].

²⁰² Anscombe [1959: 102].

²⁰³ McGinn [2006: 193-197] does not seem to notice this. For her, unless we know the structure of elementary propositions, it is impossible to know whether they may be analysed according to the functional model. As she puts it: "Wittgenstein [...] acknowledged that the logical form of elementary propositions is not something that can be anticipated a priori. This clearly raises a question: how does he know that ' fx ' and ' $\phi(x,y)$ ' represent the forms of elementary propositions", and concludes that "in the absence of any examples of elementary propositions and their constituents, [...] [Wittgenstein] is not in a position to introduce variables to represent them" [2006: 195, 197].

argument. This explains why Wittgenstein says that “wherever there is compositeness, argument and functions are present” [TLP: 5.47]. Function and argument, therefore, do not characterise, for Wittgenstein, a *particular* kind of compositeness, nor do they characterise a particular logical difference between propositional constituents; what they do characterise is (elementary) propositional compositeness (structure) *itself*.²⁰⁴

This brings us back to our previous discussion of Wittgenstein’s conception of a proposition as a picture, in which the pictorial nature of a proposition was explained in terms of its having a pictorial *structure*, that is, in its having representative elements arranged, or combined, in the same way as the elements of the state of affairs to be pictured or represented. Since the notion of a logical constant is, as 5.47 says, tied up to the notion of function and argument, and, as we have seen in this section, the functional model amounts to the notion of propositional compositeness, then the notion of logical constant (and therefore Wittgenstein’s explanation of the logical propositions) must be explained by resorting to his conception of propositional compositeness, that is, to his idea of the pictorial nature of a proposition.

²⁰⁴ To this one might reply by arguing that the function-argument analysis does not necessarily characterise *propositional compositeness*, because it may as well characterise *non-propositional compositeness* – for instance the compositeness of an expression such as ‘the capital of Scotland’. This is in fact what Frege thought; in *Begriffsschrift* he says that the functional analysis can be applied to expressions “whose content need not be assertible” [CN: § 9], that is to say, need not be a possible object of assertion or judgement (or, in Wittgensteinian terminology, need not be a proposition). Wittgenstein’s adoption of Russell’s theory of descriptions [see TLP: 4.0031, NB: 129], however, allowed him to rule out *non-propositional* (or *sub-propositional*) compositeness.

Chapter Twelve

The Logic of the *Tractatus*

[12.1] Bipolarity, *ab*-functions and Logic

We saw in section [11.2] that Wittgenstein often hints at the idea that an understanding of the nature of the proposition will imply a grasp of the nature of logical articulation (logical constants) and of logic as a whole, that the latter will be clarified by a correct account of the former. This idea runs contrary to the reading according to which the *Tractatus* provides two separate and independent explanations of the nature of the proposition, the one labelled 'bipartite reading'. What we need, and what Wittgenstein saw himself as providing, is a *unified* account of the nature of propositional and logical articulateness, which is, at bottom, a unified account of the nature of language and logic. Our task now is thus to answer the question: why are all logical constants already implied in an elementary proposition? That is: why are all logical constants implied by propositional compositeness (which, as seen, can be represented by the function-argument articulation)? From this we will then move on to consider what this idea of logical constants' being implied in elementary propositions implies for an understanding of the nature of logic.

My contention is that the answer to the first question lies in Wittgenstein's notion of bipolarity, thus in the relation between sense and truth. The idea that the analysis of the notion of bipolarity – introduced in the *Notes on Logic* – contains the answer to the puzzle represented by Wittgenstein's claim that logical constants are

present in an elementary proposition has been long ago noticed,²⁰⁵ but never – it seems to me – fully discussed or very persuasively explained. Let us recall how Wittgenstein explains the sense of an elementary proposition in terms of bipolarity in the *Notes on Logic*:

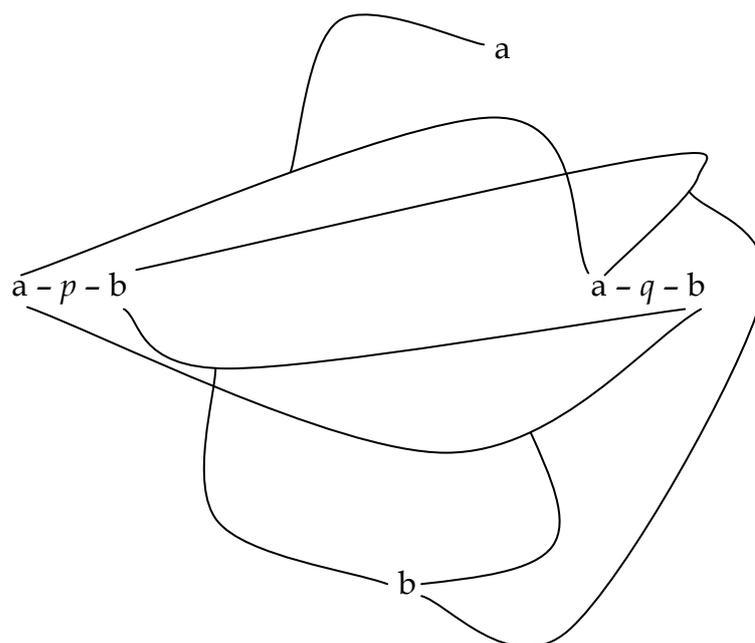
Every proposition is essentially true-false: to understand it, we must know both what must be the case if it is true, and what must be the case if it is false. Thus a proposition has two *poles*, corresponding to the case of its truth and the case of its falsehood. We call this the *sense* of a proposition [NL: 98-99].

Thus, the sense of a proposition is the possibility of its being true or false, its having truth-poles, which Wittgenstein, in the *Notes on Logic*, chose to indicate by the letters *a* and *b*.²⁰⁶ So the proposition '*p*' together with its truth poles is rendered by Wittgenstein as '*a-p-b*'.

Poles are essential in Wittgenstein's account of molecular propositions in the *Notes on Logic*. Molecular propositions are called by Wittgenstein '*ab*-functions' [see NL: 102-103]. The molecular proposition 'not *p*', for instance, is rendered in Wittgenstein's *ab*-notation as '*b-a-p-b-a*'. What this means is the following: the proposition '*p*' together with its true-false poles is, as seen, '*a-p-b*'. What negation does is reversing the truth-conditions of the proposition, thus turning the *a*-pole into a *b*-pole and the *b*-pole into an *a*-pole, and thus getting as a result the formula '*b-a-p-b-a*'. A different example, involving two elementary propositions, is the proposition '*p* and *q*'. In order to express this proposition in his *ab*-notation, Wittgenstein uses (not in the *Notes on Logic*, but in correspondence with Russell, in the *Moore Notes*, and also in the *Tractatus*) truth-diagrams, of his own devising, like the following:

²⁰⁵ For example by McGuinness [2002b: 109-110].

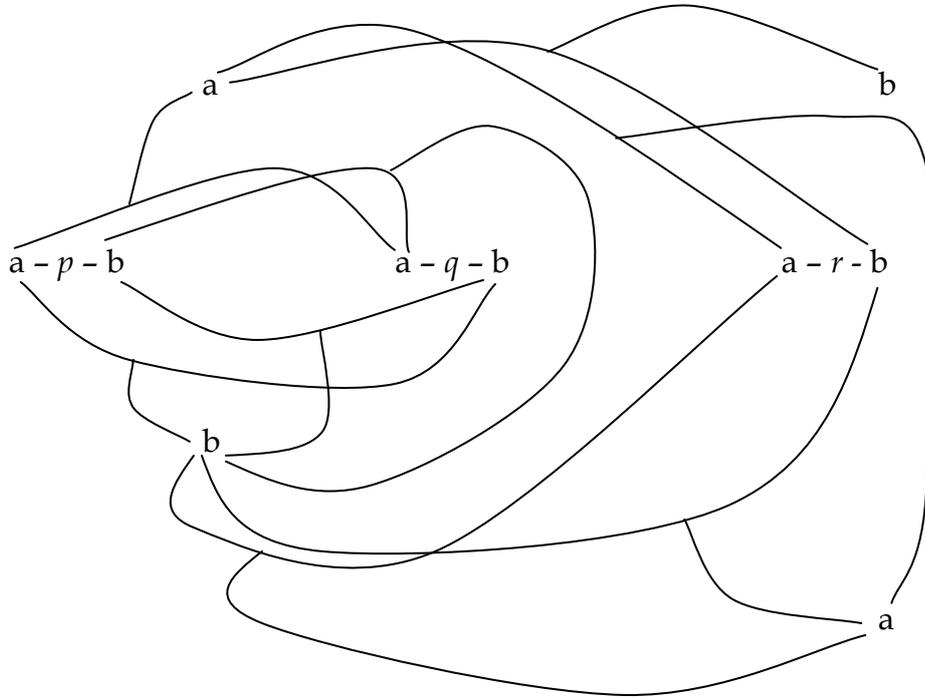
²⁰⁶ He sometimes used W and F instead. See Potter [2009: 173-174] on this.



A conjunctive proposition is only true when both its conjuncts are true, and false in all other cases. This is exactly what the diagram shows. The outer *a*-pole (the true-pole) is only connected with the connection between the *a*-pole of '*p*' and the *a*-pole of '*q*' (that is to say, with the possibility of '*p*' and '*q*' being both true), and the outer *b*-pole (the false-pole) is connected with all other connections of the poles of '*p*' and '*q*'. *ab*-functions, then, as well as elementary propositions, have *ab*-poles (are essentially true/false) and thus are bipolar in the same way in which elementary propositions are. This is made clear by truth-diagrams, which show how a molecular proposition gets its *ab*-poles by correlations with the *ab*-poles of the original elementary propositions.

Of course, as Wittgenstein reminds us: "As the *ab* functions of atomic propositions are bi-polar propositions again we can perform *ab* operations on them. We shall, by doing so, correlate two new outside poles via the old outside poles to the poles of the atomic propositions" [NL: 94]. That is: *ab*-functions can be

iterated. This is again made visible by the truth-diagram method. A proposition such as $'(p \ \& \ q) \supset r'$ is in fact represented diagrammatically as follows:



First the ab -poles of $'p'$ and $'q'$ get connected to form the ab -poles of $'p \ \& \ q'$. Then the ab -poles of that proposition are in turn connected to the ab -poles of the elementary proposition $'r'$ to determine the ab -poles of the molecular proposition $'(p \ \& \ q) \supset r'$.

We can see from this that, on Wittgenstein's account, the link between elementary and molecular propositions is provided by the notion of bipolarity, with its embedded discrimination between facts; an elementary proposition is true or false (has ab -poles) because its form discriminates between two classes of facts, of like and opposite sense. ab -functions (molecular propositions) simply exploit the discriminations made by the (forms of) elementary propositions occurring as truth-arguments in them. A proposition's having ab -poles, truth-conditions, is thus

everything that is needed in order to account for the relations between propositions, because *ab*-functions simply operate on elementary propositions' *ab*-poles to generate propositions with *ab*-poles, with truth-conditions. As Wittgenstein remarks, "The *ab*-functions use the discriminations of facts, which their arguments bring forth, in order to generate new discriminations" [NL: 105].

From this we are also in a position to see another important consequence of Wittgenstein's account of molecular propositions, namely the fact that nothing is introduced by *ab*-functions that was not already provided by the elementary propositions occurring as arguments in them. Molecular propositions, in fact, do not add anything to the elementary propositions out of which they are constructed, for they only imply re-arrangements of the elementary proposition's *ab*-poles. This point is expressed by Wittgenstein as follows:

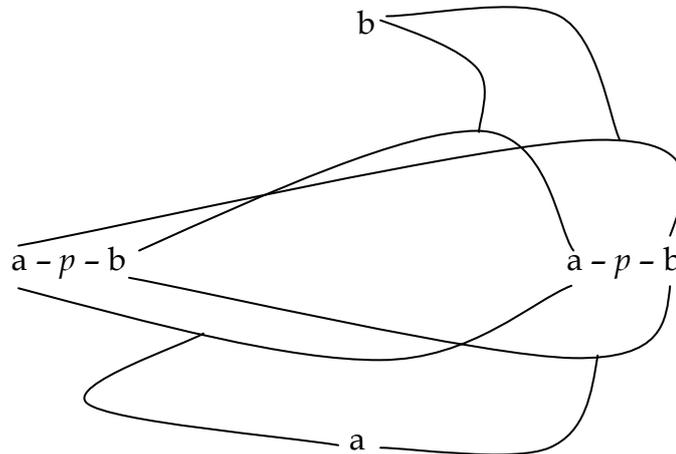
Molecular propositions contain nothing beyond what is contained in their atoms; they add no material information above that contained in their atoms.

All that is essential about molecular functions is their T-F schema (i.e. the statement of the cases when they are true and the cases when they are false) [NL: 98].

And this is, *in nuce*, the core of Wittgenstein's *Grundgedanke* in the *Tractatus*, which we will be discussing in section [12.3].

We have seen how Wittgenstein accounts for molecular propositions in his *ab*-notation. But we have said nothing about logical propositions. In the *Notes on Logic* there is no direct use of truth-diagrams to show whether a proposition belongs to logic or not. At the time of the composition of the *Notes*, Wittgenstein was still unclear about the status of the propositions of logic, wavering between considering them "generalizations of tautologies" [see NB: 128] (admittedly under Russell's influence) and taking them to be tautologies *tout court*. However, in a

letter sent to Russell from Norway in 1913, he provides an example of truth-diagram for the proposition ' $p \equiv p'$ ', that is to say, the proposition ' p if and only if p' '.²⁰⁷ The example is the following:



Now, this proposition is a biconditional, and a biconditional is true when either both its left-hand and right hand are true, or when they are both false. In the diagram we can in fact see that the outer a -pole (that is the true-pole) is connected with the connection of a -poles of the atomic proposition and with the connection of b -poles of the same proposition. The b -pole, on the other hand, is connected with the connection between the a -pole of the first occurrence of ' p ' and the b -pole of the second occurrence of ' p ', and with the connection with the b -pole of the first occurrence of ' p ' and the a -pole of the second occurrence of ' p '. These two connections, however, are *impossible*. In fact the same proposition, ' p ', cannot be at the same time both true and false. So the outer b -pole is connected with an impossible link, the possibility of ' p 's being simultaneously true and false. The outer pole, therefore does not indicate a genuine possibility, and is to be ignored; all we are left with is the a -pole, that is, the case of the proposition's being true.

²⁰⁷ See [NB: 129].

This proposition is hence always true; it is a kind of proposition that Wittgenstein in the *Notes Dictated to Moore* and in the *Tractatus* will call a *tautology*. Such a proposition has only an *a*-pole for whatever assignment of truth-values to the elementary propositions occurring in it (for whatever connection of *a*-poles and *b*-poles of the elementary propositions).

On Wittgenstein's account, therefore, propositions of logic are arrived at by means of the *same* procedure used to obtain molecular propositions out of elementary ones. Propositions of logic are *ab*-functions of elementary propositions like any other molecular proposition. But if so then, we can say that, despite there being no general statement, in the *Notes on Logic*, for the conclusion that all logical constants are already implied in an elementary proposition, and therefore for the view that logical complexity is already contained in the nature of the proposition, Wittgenstein's discussion of the intertwined notions of sense, bipolarity and *ab*-functions seems to provide an account of language and logic that implements that general idea. Elementary propositions already ensure the possibility of logical complexity, because the latter simply consists in a rearrangement of the *ab*-poles of elementary propositions; *ab*-poles, besides, are given by, and even *coincide* with, a proposition's having sense. As Wittgenstein put this crisply in the *Notebooks*:

The logical constants of the proposition are the conditions of its truth [NB: 7.12.14].

[12.2] Truth-operations

In this section I intend to show that the conception of the relation between (elementary) sense and logical complexity discussed above is overall maintained

in the *Tractatus*, in spite of the numerous alterations Wittgenstein makes of his early account of the sense of propositions and of *ab*-functions.

We have already discussed how Wittgenstein refines his account of sense in the *Tractatus*. The notion of propositional sense – and its underlying distinction between constituents and forms – is dropped, and the sense of a proposition is explained in terms of a proposition's being a *picture* of reality, sharing logical form with the depicted situation. Besides, *ab*-functions disappear from the scene. In the *Tractatus* a molecular proposition is no longer considered an *ab*-function, but a *truth*-function of elementary propositions [see TLP: 5].

The *Notes on Logic's* account of the sense of a proposition in terms of its being 'essentially true-false', however, is overall maintained in the *Tractatus*, where Wittgenstein claims that "[t]he sense of a proposition is its agreement and disagreement with possibilities of existence and non-existence of states of affairs" [TLP: 4.2]. A proposition's being a picture of a possible situation makes it intrinsically true or false. A proposition '*p*', for instance, expresses agreement with the possibility of existence of the state of affairs that *p*, and disagreement with the possibility of non-existence of the same state of affairs. It is then true if the depicted state of affairs exists (is a – positive – fact), and false if it does not exist [see TLP: 4.25]. *ab*-poles thus becomes, in the *Tractatus*, truth-possibilities, or truth-conditions. "Truth-possibilities of elementary propositions mean possibilities of existence and non-existence of states of affairs" [TLP: 4.3]. In the context of the discussion of truth-possibilities Wittgenstein introduces the familiar truth-tabular notation. As he says, truth-possibilities can be represented by schemata like the following:

<i>p</i>	<i>q</i>	<i>r</i>		<i>p</i>	<i>q</i>		<i>p</i>
T	T	T		T	T		T
F	T	T		F	T		F
T	F	T		T	F		
T	T	F		F	F		
F	F	T					
F	T	F					
T	F	F					
F	F	F					

We can thus see that in the truth-tabular notation, the proposition '*p*' is assigned T-F poles, that is, truth-possibilities, where "'T' means 'true' [and] 'F' means false" [TLP: 4.31]. The bipolarity of propositions, despite the changes in its notational expression, is thus reaffirmed.²⁰⁸

We said that the *Tractatus* conceives of molecular propositions as truth-functions of elementary propositions. "Truth-functions of elementary propositions are results of operations with elementary propositions as bases. (These operations I call truth-operations)" [TLP: 5.234]. Logical constants are in the *Tractatus* taken to be operations; "Negation, logical addition, logical multiplication, etc. etc. are operations" [TLP: 5.2341], and an operation is conceived as "what has to be done to the one proposition in order to make the other out of it" [TLP: 5.23]. What is new in the *Tractatus* with respect to the relation between elementary and molecular propositions, then, is precisely the notion of a truth-operation. This we have to investigate in some detail.

When discussing truth-operations, Wittgenstein insists that "[o]perations and functions must not be confused with each other" [TLP: 5.25], because "[t]ruth-

²⁰⁸ I thus agree with Proops [2000: 2] that "[a]lthough bi-polarity is not explicitly mentioned in the *Tractatus*, Wittgenstein is unlikely to have changed his mind about it".

functions are not material functions” [TLP: 5.44]. The reason for this is that “[a] function cannot be its own argument, whereas an operation can take one of its own results as its base” [TLP: 5.251]. Besides – apparently unlike a function – “no statement is made by an operation, but only by its result, and this depends on the bases of the operation” [TLP: 5.25].

It has been argued, however, that Wittgenstein stresses a difference that is not, by the *Tractatus*' own lights, really justified. As Hylton – following Max Black – says, it appears as if “everything, or almost everything, that Wittgenstein says about operations could with equal correctness be said about functions”.²⁰⁹ For example, the claim that no statement is made by an operation seems hardly helpful in distinguishing operations and functions, for no statement is made by a function either; the function x^2 , for instance, does not assert anything. Likewise, the fact that functions cannot be their own arguments does not seem to distinguish them from operations, for surely operations themselves cannot be operated on; (it makes no sense to negate ‘what has to be done to a proposition in order to get another proposition out of it’); what can be operated on is the *result* of an operation, but, likewise, the result of the saturation of a function with an argument can be, at least in some cases, be the argument of that very function.

According to Hylton, when Wittgenstein opposes operations and functions he does not have in mind Frege’s (mathematical) notion of a function, but, instead, Russell’s and Whitehead’s notion of a *propositional function*, as that notion is

²⁰⁹ Hylton [2005: 140]. See also Black [1964: 259-260].

discussed in *Principia Mathematica*.²¹⁰ A propositional function is, in *Principia*, “something which contains a variable x , and expresses a proposition as soon as a value is assigned to x ” [PM: 38]. So, for example, “ x is a man” or “Socrates is φ ” are propositional functions. In fact, Russell and Whitehead discuss logical constants – sentential connectives – precisely in terms of propositional functions (or of ‘aggregations of propositions’). “An aggregation of propositions [...] into a single proposition more complex than its constituents, is a function with *propositions as arguments*” [PM: 6], and – arguably – propositions as values. The resulting proposition (the value of the function) will thus be a molecular proposition. Russell identifies four cases of ‘aggregations of propositions’ which have ‘fundamental importance’; the “Contradictory Function”, the “Logical Sum” (also called “Disjunctive Function”), the “Logical Product” (or “Conjunctive Function”) and the “Implicative Function” [PM: 6-7, see also 93-94], which correspond to the sentential connectives of ‘negation’, ‘disjunction’, ‘conjunction’, and the ‘material conditional’. Thus, what Wittgenstein conceives of as operations [TLP: 5.2341], Russell conceives of in terms of propositional functions.

Interpreting Wittgenstein’s notion of a function as corresponding to Russell’s and Whitehead’s notion of a propositional function is helpful in making sense of the opposition he draws between functions and operations. One of Wittgenstein’s reasons for distinguishing functions and operations is, as we saw, that “no statement is made by an operation, but only by its result, and this depends on the bases of the operation” [TLP: 5.25]. This claim seems baffling

²¹⁰ Hylton [2005: 141 ff.], and Landini [2007: 127]. Perhaps significantly, in *Principia* Russell and Whitehead write that “when the word ‘function’ is used in the sequel, ‘propositional function’ is always meant” [PM: 39].

because no statement is made by a function either, and so it seems powerless in justifying the distinction between functions and operations. However, in *Principia Mathematica*, Russell and Whitehead discuss (and admit) precisely the notion of asserting a propositional function. As they put it:

When we assert something containing a real variable, we cannot strictly be said to assert a *proposition*, for we only obtain a definite proposition by assigning a value to the variable, and then our assertion only applies to one definite case [...]. When what we assert contains a real variable, we are asserting a wholly undetermined one of all the propositions that result from giving various values to the variable. It will be convenient to speak of such assertions as *asserting a propositional function* [PM: 18, see also 92-93].

In this passage Russell and Whitehead speak of asserting a propositional function, in terms of asserting any value of the propositional function in question. Wittgenstein opposes to this conception of logical constants his conception of a logical operation, in which the notion of assertion finds no place; it does not make sense to speak of asserting the operation of, say, negation, that is, asserting what is to be done to one proposition in order to get another (of opposite sense) out of it.

If Wittgenstein contrasted his notion of an operation with Russell's notion of a propositional function, the possibility seems open of assuming that he might have endorsed an account according to which logical constants (and thus logical operations) express (mathematical) functions, in roughly the way Frege thought.²¹¹ On a Fregean account a proposition is the result of the combination of an unsaturated entity (a functional expression) with a saturated entity, or more than one. Frege's account, however, entails that this very conception of sentential complexity can be extended, with the introduction of some particular functions (truth-functions) to account for logical relations between propositions; thus, the

²¹¹ This is argued by Landini [2007: 128], who claims that "Wittgenstein's point is simply that, unlike *Principia*, he adopts the mathematical notion of a function as primitive. That is, his notion of an operation is just the mathematical notion of a function".

relation that a propositional constituent (a name) has to the proposition of which it is a constituent is in principle *no* different from the relation a proposition has to the more complex proposition of which it is a constituent. A proposition, on this account, occurs in another in *no* different way from how a sub-propositional constituent occurs in a proposition. Both are arguments of some functions.

This conception, however, cannot be attributed to Wittgenstein. In the *Tractatus* he claims that “[i]t is self-evident that \vee , \supset , etc., are not relations in the sense in which right and left are relations” [TLP: 5.42]. And, as Sullivan has observed, this remark is meant to be a criticism of Frege’s conception:

Frege is the more obvious target, since he did straightforwardly maintain that the truth-functional constants were first-level relational expressions, that is, incomplete expressions with gaps for singular terms. In *Grundgesetze* all complete expressions, simple and complex singular terms and propositions, are of the same logical category. Since the logical category of all other expressions is fixed relatively to complete expressions, there is no logical difference between an expression that takes two terms to make a proposition and an expression that takes two propositions to make a bigger proposition.²¹²

Right and left are for Wittgenstein *functions* in Frege’s sense, and he is claiming that logical operations are *not* like that. In the *Notes Dictated to Moore* Wittgenstein remarked that the relations between propositions are not of the same kind as the relations between names and propositions.

There are *internal* relations between one proposition and another; but a proposition cannot have to another *the* internal relation which a name has to the proposition of which it is a constituent, and which ought to be meant by stating it ‘occurs’ in it. In this sense one proposition can’t ‘occur’ in another [NM: 116].

There seems to be a clear denial, here, of Frege’s overall account, with its assimilation of logical constants (and thus of logical operations) to functions.

²¹² Sullivan [2000: 178].

As we saw, the *Tractatus* characterises an operation as “what has to be done to one proposition in order to make the other out of it” [TLP: 5.23]. This seems to go against the idea that an operation is either a *propositional function* in Russell’s sense, or a *function* in Frege’s sense. In fact, Wittgenstein’s characterisation seems to suggest that an operation, as Hylton writes, is “less like an entity, that might be a constituent of a more complex entity, than it is something that we do”,²¹³ that is to say, some kind of *procedure* or *process*.

In fact, Wittgenstein discusses operations in terms of *rules* for constructing propositions. At 5.512 Wittgenstein writes:

‘ $\sim p$ ’ is true, if ‘ p ’ is false. Therefore, in the proposition ‘ $\sim p$ ’, when it is true, ‘ p ’ is a false proposition. How then can the stroke ‘ \sim ’ make it agree with reality?

But in ‘ $\sim p$ ’ it is not ‘ \sim ’ that negates; it is rather what is common to all the signs of this notation that negate p .

That is to say the common rule that governs the construction of ‘ $\sim p$ ’, ‘ $\sim\sim p$ ’, ‘ $\sim p \vee \sim p$ ’, ‘ $\sim p \cdot \sim p$ ’, etc. etc. (ad inf.). And this common factor mirrors negation.

And he goes on:

We might say that what is common to all symbols that affirm both p and q is the proposition ‘ $p.q$ ’; and that what is common to all symbols that affirm either p or q is the proposition ‘ $p \vee q$ ’. [...] [TLP: 5.513].

Although perhaps not very clearly, Wittgenstein is here saying that it is not the expression of an operation that performs the operation in question. In ‘ $\sim p$ ’, it is not the symbol ‘ \sim ’ that negates, as well as in ‘ $p.q$ ’, it is not the ‘.’ that conjuncts. The negative factor in ‘ $\sim p$ ’ is represented, Wittgenstein says, by “‘what is common to all the signs of this notation that negate p ’”. That common element is a rule that governs the construction of propositions in which ‘ p ’ is negated; and that common rule is the operation of negation. In ‘ $p.q$ ’, on the other hand, the conjunctive element is what is common to all propositions that affirm both ‘ p ’ and ‘ q ’, and

²¹³ Hylton [2005: 152].

what is common to all those propositions is that of having been constructed according to the same *rule*. This rule, in turn, is the operation of conjunction.

The existence of these rules, of procedures for constructing propositions according to logical operations, seems for Wittgenstein to depend on the establishment of a (suitable) notation. As soon as we have a notation (a language) then we have the rules for logical operations.

Once a notation has been established, there will be in it a rule governing the construction of all propositions that negate p , a rule governing the construction of all propositions that affirm p , and a rule governing the construction of all propositions that affirm p or q ; and so on [...] [TLP: 5.514].

Logical operations are thus already in place in the establishment of a notation, of a language. They are not something over and above the workings of propositions of language. They are rules for the construction of propositions (the results of operations) given other simpler propositions (the bases of the operations).²¹⁴ This will bring us directly to the Grundgedanke of the *Tractatus*, which we will be discussing in the next section. Before turning to that, however, I want to draw some conclusive considerations about Wittgenstein's conception of logical operations.

We saw above that Wittgenstein discusses the nature of logical constants, in the *Tractatus* and the *Notes on Logic*, especially in opposition to Russell's conception of them. The logical notation (the TF-notation and the truth-tabular

²¹⁴ My discussion of Wittgenstein's conception of logical complexity does not address quantification. In the *Tractatus*, however, Wittgenstein gives a truth-functional account of quantifiers, and thus general propositions are not a counterexample to the thesis that logical complexity is given by the application of truth-operations. Recent discussions of Wittgenstein's treatment of general propositions can be found in McGinn [2006: 234-240] and Morris [2008: 215-225].

method) in the *Tractatus* is meant to express these insights on the nature of logical constants. Proops puts the point as follows:

Wittgenstein regards [his notation] as helping to forestall certain philosophical misunderstandings that he takes to be invited by Russell and Whitehead's notation. For example, he sees Russell's notation as encouraging the view that logical connectives signify by having reference. It is at very least much harder to get this impression from [...] [Wittgenstein's notation].²¹⁵

Now, of course Wittgenstein's notation discourages the thought that logical connectives have meaning by having reference, but that cannot be taken as a reason for the claim that logical connectives do not refer. After all, as Sullivan remarks:

The TF-notation of the *Tractatus* realizes all of Wittgenstein's early claims about the logical constants. But then it was designed for just that purpose. It can help clarify the claims, but cannot do much to motivate them.²¹⁶

I think that Wittgenstein's reasons for adopting his T-F notation have to do with his conception of *sense*. The sense of an elementary proposition, we know, is the situation it logically portrays. An elementary proposition has two truth-possibilities, corresponding to the case of the existence and the case of the non-existence of the state of affairs it represents. Molecular propositions have sense (truth-conditions) by agreeing and disagreeing with the truth-possibilities of elementary propositions. As 4.4 reads: "A proposition is an expression of agreement and disagreement with truth-possibilities of elementary propositions". And later: "The expression of agreement and disagreement with the truth-possibilities of elementary propositions expresses the truth-conditions of a proposition" [TLP: 4.431]. The proposition '*p.q*', for instance, is rendered in

²¹⁵ Proops [2000: 16].

²¹⁶ Sullivan [2000: 181].

Wittgenstein's T-F notation as '(TFFF) (p, q)'. This proposition has sense – truth-conditions – by agreeing with some of the truth-possibilities of the elementary propositions, and disagreeing with others. In particular, such a proposition agrees with the possibility in which ' p ' and ' q ' are both true and disagrees with all others. The proposition ' $\sim(p \supset \sim q)$ ' has the same truth-conditions as ' $p.q$ '; both can thus be represented by the expression '(TFFF) (p, q)'. If sense amounts to truth-conditions, and thus if in order to understand a proposition one needs to know what would be the case if the proposition is true and what would be the case if the proposition is false, then the two propositions have the same sense and are thus, at bottom, the same proposition. Wittgenstein's notation has thus a perspicuity that Russell's lacks, for Russell's notation invites the thought that ' $p.q$ ' and ' $\sim(p \supset \sim q)$ ' are different propositions, for they are values of different propositional functions.²¹⁷

So, we can see that it is not Wittgenstein's T-F notation that is meant to provide *justification* for Wittgenstein's conception of logical constants. It is Wittgenstein's conception of *sense* that grounds both his choice of notation and that motivates the Grundgedanke of the *Tractatus*, to which we will now turn our attention.

[12.3] The Grundgedanke of the *Tractatus*

In the *Tractatus* Wittgenstein writes:

²¹⁷ As Hylton [2005: 144] argues: “[O]n Russell’s account a proposition which is obtained by application of the propositional function *disjunction* to two propositions p and q is a disjunctive proposition – it contains a constituent corresponding to disjunction. It must thus be a different proposition from that which we obtain if we first apply to each of p and q the propositional function corresponding to negation, then take the resulting propositions as arguments to the propositional function corresponding to conjunction, and then take the resulting proposition as argument to the propositional function corresponding to negation. In short: for Russell ' $p \vee q$ ' must represent a different proposition from that represented by ' $\sim(\sim p \cdot \sim q)$ '. But this is precisely the result that Wittgenstein wants to avoid”.

My fundamental idea is that the 'logical constants' are not representatives; that there can be no representatives of the *logic* of facts [TLP: 4.0312].

Wittgenstein's claim is that logical constants do not represent [*nicht vertreten*], that is to say, do not refer to objects, and thus, at bottom, are not names.²¹⁸ Logical constants are not representatives in the sense that they are not involved in a proposition's representational activity. This is what makes them entirely different from names, which, on the contrary, are integral part of propositions' pictorial role. In fact, the very possibility of propositions, as Wittgenstein says in the same section of the *Tractatus*, "is based on the principle that objects have signs as their representatives".

As we saw, Wittgenstein claims that there is *nothing* represented by logical connectives such as 'or', 'and', and so forth. They are not names of entities, such as Fregean functions or Russell's propositional functions. They are simply expressions of the agreement and disagreement with the possibilities of truth of elementary propositions; thus they can be entirely dispensed with, as Wittgenstein's notation shows; the proposition ' $p.q$ ', whose formulation in Russellian notation might encourage the idea that there is *something* corresponding to the dot, can be rewritten as '(TFFF) (p, q)', where *nothing* corresponds to the logical constant. Again, the notation makes it clear that the molecular proposition

²¹⁸ I take these formulations to be equivalent. To be a name *is* to refer to an object. McDonough [1986: 38] questions this, by arguing that "the deep meaning of the fundamental idea is that though there are signs which contribute to the representational content of the proposition, there are other signs, the logical ones, which have a completely different role. This is very different from simply saying that Wittgenstein's fundamental idea is that the logical constants do not refer to an object". McDonough, I think, is right in stressing that the role of logical constants is to be understood in connection with a proposition's representational content. However, as said, I do not believe that holding that logical constants are not part of a proposition's representational content means something *very* (or indeed *any*) different from claiming that they do not stand for objects. In fact, as Wittgenstein remarks, "only in the nexus of a proposition does a name have meaning" [TLP: 3.3]. Signs can be representatives – can stand for objects – only as part of a proposition's expressing the sense it does, only in so far as they contribute to its representational content.

in question does not contain anything over and above what is contained in the elementary propositions of which it is a truth-function.

In the idea, expressed in the *Grundgedanke*, that logical constants are not representatives, and are therefore not involved in the characterisation of a proposition's representational content, we can thus also find the justification for Wittgenstein's claim that "an operation is not the mark of a form" [TLP: 5.241], that an operation does not necessarily characterise the sense of a proposition obtained as the result of its application. "The occurrence of an operation does not characterise the sense of a proposition" [TLP: 5.25]. So the sense of a proposition where a logical constant 'occurs' is not necessarily determined by the occurring logical constant. The latter only expresses that an operation has been applied to the sense of the elementary propositions that are its bases. So, no proposition has an *intrinsically* negative, or conjunctive, or disjunctive form, because no proposition has, as part of its sense, elements corresponding to negation, conjunction, or disjunction. As Wittgenstein says, as regards negation, "the occurrence of negation in a proposition is not enough to characterize its sense ($\sim\sim p = p$)" [TLP: 4.0621]. ' $\sim\sim p$ ' is equivalent to ' p ', in the same way in which ' $p \vee q$ ' is equivalent to ' $\sim(\sim p.\sim q)$ '. They have the same sense, that is, the same truth-conditions. As McGuinness observes:

The truth-functions do not *introduce* new discriminations of facts, new material functions, but merely operate with the discriminations already introduced when the components of propositions were given a meaning. [...] [N]othing is introduced by the logical constants, which is not present in the atomic proposition.²¹⁹

²¹⁹ McGuinness [2002b: 113].

Wittgenstein's idea that logical constants are not representatives, that, as he wrote to Russell as early as 1912 – “there are NO *logical* constants” [NB: 120], in the sense that there are no elements (logical objects) corresponding to them, together with his idea that logical constants express operations (rules for the construction of propositions), plays thus a crucial role in Wittgenstein's conception that nothing more than the sense of elementary propositions is needed in order to account for truth-functional articulation, and hence for logical complexity also. Once the pictorial character of a proposition is understood, then everything is given in order to understand how propositions can enter into logical relations with one another. Those relations are already implied by propositions' own nature, by propositions' determining their sense.²²⁰

This, to be sure, is not to say that given an understanding of the sense of a proposition (the nature of a proposition) we are in a position to *deduce* the nature of logical relations among propositions, as if the latter were something over and above the former. There might be such a risk, I think, in describing Wittgenstein's conception of a molecular proposition as a “logical construction out of an elementary proposition”.²²¹ A construction out of elementary propositions might well be something over and above the elementary propositions themselves. But the crucial point Wittgenstein is concerned to stress is that logical constants are *contained* in elementary propositions, that logical operations do not introduce any new material information. Logical relations between propositions are given by propositions' expressing the sense they do. This is the reason why logic, for

²²⁰ Wittgenstein's *Grundgedanke*, in fact, also explains why “[i]t immediately strikes one as probable that the introduction of elementary propositions provides the basis for understanding all other kinds of proposition” [TLP: 4.411].

²²¹ McGinn [2006: 231].

Wittgenstein, is given at the level of elementary propositions, that is to say, is given as soon as propositions saying something about reality are given, and this is therefore also the reason why he says that “[a]n elementary proposition really contains all logical operations in itself” [TLP: 5.47].

[12.4] Truth and Logical Truth

We have seen how Wittgenstein’s notion of the sense of a proposition justifies his claim that an elementary proposition contains all logical constants in itself. In the *Tractatus* Wittgenstein contends that the sense of a proposition amounts to its being a picture, and that this is what makes it true or false. Besides, in the *Tractatus* Wittgenstein abandons the *ab*-notation (in favour of the T-F one) and represents the truth-conditions of molecular propositions by means of truth-tables. The new notation makes it easy to see how one can get molecular propositions out of elementary ones. Operations operate on propositions’ T-F poles to get other propositions with T-F poles as a result. Besides, the new notation makes it evident in what sense propositions of logic are products of the general procedure of truth-functionally constructing molecular propositions out of simpler ones. This Wittgenstein discusses as follows:

Among the possible groups of truth-conditions there are two extreme cases.

In one of these cases the proposition is true for all the truth-possibilities of the elementary propositions. We say that the truth-conditions are *tautological*.

In the second case the proposition is false for all the truth-possibilities: the truth-conditions are *contradictory*.

In the first case we call the proposition a tautology; in the second, a contradiction [TLP: 4.46].

Propositions of logic are tautologies (and contradictions), and these are, as Wittgenstein has it, the two extreme cases of the truth-functional scheme. This is

easily seen. At 5.101, for example, Wittgenstein provides a schema of the truth-functional construction of propositions out of two elementary propositions. He writes:

The truth-functions of a given number of elementary propositions can always be set out in a schema of the following kind:

(TTTT) (p, q) Tautology (If p then p , and if q then q .) $(p \supset p \cdot q \supset q)$
 (FTTT) (p, q) In words: Not both p and q . $(\sim(p \cdot q))$
 (TFTT) (p, q) " " : If q then p . $(q \supset p)$
 (TTFT) (p, q) " " : If p then q . $(p \supset q)$
 (TTTF) (p, q) " " : p or q . $(p \vee q)$
 (FFTT) (p, q) " " : Not q . $(\sim q)$
 (FTFT) (p, q) " " : Not p . $(\sim p)$
 (FTTF) (p, q) " " : p or q , but not both. $(p \cdot \sim q \vee \sim p \cdot q)$
 (TFFT) (p, q) " " : If p then p , and if q then q . $(p \equiv q)$
 (TFTF) (p, q) " " : p
 (TTFF) (p, q) " " : q
 (FFFT) (p, q) " " : Neither p nor q . $(\sim p \cdot \sim q \text{ or } p \mid q)$
 (FFTF) (p, q) " " : p and not q . $(p \cdot \sim q)$
 (FTFF) (p, q) " " : q and not p . $(q \cdot \sim p)$
 (TFFF) (p, q) " " : q and p . $(q \cdot p)$
 (FFFF) (p, q) Contradiction (p and not p , and q and not q .) $(p \cdot \sim p \cdot q \cdot \sim q)$ [...] [TLP: 5.101].

Tautologies and contradictions are molecular propositions in the very *same* sense in which other propositions are. As well as other molecular propositions, they are the result of operations on elementary propositions, which deliver truth-functions as result. They are peculiar truth-functions, though, because, unlike other truth-functions, they are either true for all truth-possibilities of elementary propositions (tautologies) or false for all truth-possibilities of them (contradictions). In this sense, they are the extreme cases – or the “limiting cases” [TLP: 4.466] – of the process of the truth-functional construction of propositions.

Wittgenstein’s conception of the propositions of logic makes it clear why they are in some important respect *contiguous* with other molecular propositions (as well as other truth-functions, they have truth-functional articulation); in this sense tautologies and contradictions “are part of the symbolism, just as ‘0’ is part

of the symbolism of arithmetic" [TLP: 4.4611], that is; "in constructing a truth-table that yields a tautology one is following the same rules as one would follow in constructing any type of truth-table".²²² This is the reason why Wittgenstein claims that "[t]autologies and contradictions are not [...] nonsensical" [TLP: *ibid.*]. By operating on the truth-possibilities of propositions, their T-F poles, it is possible to arrive at propositions that, strictly speaking, do not have T-F poles, that are not true for some truth-possibilities of elementary propositions and false for others, but are either true for all truth-possibilities of elementary propositions or false for all of them.

The notion of bipolarity, we have observed, implies that propositions are essentially true or false, that they have poles corresponding to the case of their truth and the case of their falsity, truth-possibilities. Operations, by means of which molecular propositions are obtained, operate on propositions' T-F poles, and deliver propositions with T-F poles as a result. Some of these propositions, the propositions of logic, only have either a T-pole or an F-pole. Thus Wittgenstein's notion of bipolarity justifies the fact that the propositions of logic are *propositions*, that they are truth-functions. Logical truth is thus, on this account, conceived of as a by-product of contingent truth. Propositions, by their having T-F poles, and therefore sense, agree and disagree with some possibilities of existence of states of affairs; their truth-conditions "determine the range [...] [they] leave[...] open to the facts" [TLP: 4.463]. In a tautology (or a contradiction), the senses of its constituent propositions are brought into *equilibrium* with one another, so that they "cancel one another" [TLP: 4.462]; the resulting proposition thus *lacks* sense, and

²²² Mounce [1981: 43].

does not have, properly speaking, truth-conditions [TLP: 4.461]. Propositions of logic, then, in spite of their not being nonsensical (*unsinnig*), are senseless (*sinnlos*). But since having sense amounts to saying something, a tautology's (or contradiction's) being senseless amounts to its saying nothing. "Propositions show what they say: tautologies and contradictions show that they say nothing" [TLP: *ibid.*]. In turn, this also implies that a tautology is not a picture, that is, does not stand in a representational relation to reality.

We have discussed, so far, the reason why Wittgenstein thinks that propositions of logic are truth-functions, and are, therefore, molecular propositions like any other truth-function, despite their being senseless propositions. But this is, of course, only half of the story. Wittgenstein also claims, in fact, that the propositions of logic have a *peculiar* status among other propositions, and so that they are *not*, in some important respect, like other truth-functions. As he puts it:

The correct explanation of the propositions of logic must assign to them a unique status among all propositions [TLP: 6.112].

Wittgenstein's claim about the peculiarity of the propositions of logic is of course on a par with his early conviction that logic is of "a *totally* different kind than any other science" [NB: 120]. Logic has a unique status among other sciences precisely because its propositions differ from other propositions. Wittgenstein discusses this difference as follows:

It is the peculiar mark of logical propositions that one can recognize that they are true from the symbol alone, and this fact contains in itself the whole philosophy of logic. And so too

it is a very important fact that the truth or falsity of non-logical propositions *cannot* be recognized from the propositions alone [TLP: 6.113].²²³

The characteristic mark of propositions of logic – Wittgenstein argues – is that they can be recognised as true by mere investigation of their articulation, by the ‘symbol alone’. However, it is a requirement of Wittgenstein’s conception of a proposition that “in order to tell whether [it] [...] is true or false we must compare it with reality” [TLP: 2.223], we must consider whether the proposition agrees or disagrees with reality [see TLP: 2.21]. In other words, a proposition’s being true (or false) means that it is correspondence-true (or false), namely true (or false) by virtue of corresponding (or failing to correspond) to reality. But this exactly amounts to the claim that a proposition – unlike a tautology – cannot be recognised as true (or false) from the symbol alone. “It is impossible to tell from the picture alone whether it is true or false” [TLP: 2.224].

The idea that a proposition (a picture) can be true or false only by virtue of agreeing or disagreeing with reality seems to entail that Wittgenstein must deny that propositions of logic can, after all, be true. This is argued for by Ian Proops, according to whom “tautologies cannot be truths – at least not in the strict sense of the word”.²²⁴ There is actually some force in Proops’ ascription of this thesis to Wittgenstein. In the *Moore Notes* (where we find the earliest characterisation of the propositions of logic in terms of tautologies), Wittgenstein explicitly claims that “logical propositions are neither true nor false” [NM: 109]. For Proops, when Wittgenstein in the *Tractatus* says that tautologies are true, “he intends this only in

²²³ Wittgenstein sometimes speaks of the propositions of logic as comprising *both* tautologies and contradictions, and sometimes, like at 6.113, as being tautologies only. I will follow his more consistent usage of considering only tautologies to be the propositions of logic.

²²⁴ Proops [2000: 3].

a honorary sense".²²⁵ Actually, the thesis that propositions of logic are neither true nor false can be deduced from other commitments of Wittgenstein's in the *Tractatus*, which we have previously discussed. In fact, Wittgenstein claims that "a proposition can be true or false only in virtue of being a picture of reality" [TLP: 4.06]. But, since "[t]autologies and contradictions are not pictures of reality" [TLP: 4.462], it follows that tautologies and contradictions cannot be either true or false. However, there are numerous passages in the *Tractatus* where Wittgenstein commits himself to the claim that tautologies are true (and contradictions false). A tautology is said to be "unconditionally true" [TLP: 4.461], a tautology's truth "is certain" [TLP: 4.464], and, as we saw, it can be recognised 'from the symbol alone'.

Wittgenstein seems to be committed to contradictory claims. On the one hand, if truth is correspondence-truth, dependent on the relation between a proposition and reality, then he should claim that tautologies are not true, for their truth does not depend on the existence and non-existence of states of affairs. On the other hand, the truth-functional construction of propositions out of elementary ones inevitably leads to propositions with tautological (and contradictory) truth-conditions, namely propositions that are *true* for any truth-possibility of elementary propositions.

The solution to this problem, I think, lies in realising that there are *two* notions of truth in play here. Contingent truth – correspondence-truth – applies to non-logical propositions, to pictures, whereas logical truth to tautologies. This difference, and the relation between the two notions of truth, is discussed by Levine as follows:

²²⁵ Proops [2000: 4].

[B]y holding that the bearers of correspondence truth-values are sentences which are 'pictures of reality', or are sentences with sense, or are sentences that represent possible situations, Wittgenstein has to deny that 'p or not-p' [...] is a 'picture of reality' or represents a possible situation or has sense. [...] Given his consistent application of [...] [the] correspondence requirement [...] Wittgenstein is forced to deny that the notion of truth and falsity that applies to sentences of logic and their negations is the same notion of truth and falsity – a correspondence notion of truth and falsity – which applies to every other sentences.²²⁶

By 'correspondence requirement' Levine means the idea, that Russell came to adopt after 1910, that the truth or falsehood of a belief or judgement – or in Wittgenstein's conception – of a proposition, is not an intrinsic feature of it, but depends on something external to it. The correspondence requisite is embedded into Wittgenstein's notion of the sense of a proposition as bipolarity. On the one hand, the bipolarity of a proposition amounts to its being true or false, its having T-F poles; in a deeper sense, though, bipolarity entails that a proposition is *contingently* true or false; it has to have the possibility of truth and the possibility of falsity. And this is fully consistent with Wittgenstein's claim that the truth (or falsity) of a proposition cannot be discovered from an analysis of the proposition itself. A proposition has to be compared with reality in order to be discovered to be true or false, and it is true only if it agrees (or corresponds) to reality, if its truth-conditions are met by the world, and false otherwise.

But this account cannot hold good for the propositions of logic. As Levine remarks, a proposition such as '*p* or not *p*' cannot be said to be made true by something external to the proposition, by its agreement with reality. That proposition is in fact true regardless of the way the world is, and the world cannot meet (or fail to meet) its truth-conditions because a tautology does not have any. If so, then, tautologies are not propositions that represent reality.

²²⁶ Levine [1996: 161].

Tautologies and contradictions [...] do not represent any possible situations. For the former admit *all* possible situations, and latter *none*.

In a tautology the conditions of agreement with the world – the representational relations – cancel one another, so that it does not stand in any representational relation to reality [TLP: 4.462].

Now, the propositions of logic still have, in a loose sense, a connection with the world. The elementary propositions of which they are truth-functions must be bipolar propositions, pictures; tautologies “presuppose that names have meaning and elementary propositions sense” [TLP: 6.124].²²⁷ However, the truth-conditions of the elementary propositions that occur in a tautology ‘cancel one another’, so that the resulting proposition does not have truth-conditions, and does not represent reality.

Propositions of logic, therefore, are not pictures – they are senseless – because they do not represent any possible situation and so say nothing. This Wittgenstein discusses in terms of tautology’s lacking *content*, and having no *subject-matter*.

The propositions of logic [...] have no ‘subject-matter’ [TLP: 6.124].

All theories that make a proposition of logic appear to have content are false [TLP: 6.111].

Wittgenstein’s account of logical truth thus begins with an account of empirical truth. The twofold sense of bipolarity we have mentioned above explains why tautologies are at the same time contiguous with other molecular propositions, and radically different from them. They are contiguous with other truth-functions because they are themselves truth-functions. They are different from them because their truth is not due to their truth-conditions’ being met, and can thus be

²²⁷ As Wittgenstein writes in the *Notebooks*: “In the tautology the elementary proposition does, of course, still portray” [NB: 12.11.14].

recognised by a mere inspection of their articulation, unlike other propositions, whose truth (or falsity) depends on the way the world is, on the world's meeting (or failing to meet) their truth-conditions.

I approvingly quoted Levine saying that Wittgenstein's endorsement of the correspondence requirement (integral part of his notion of bipolarity) led him to deny that propositions of logic are true in the same sense in which contingent propositions are true. The adoption of the correspondence requirement thus led Wittgenstein to formulate a clear distinction between *empirical* truth and *logical* truth. We observed in section [6.1] that the correspondence requirement was originally formulated by Russell in response to his early (unsuccessful) account of truth and falsehood, the *intuitional* theory of truth. In the *Principles of Mathematics*, such account of truth is applied to *all* propositions; we do not find, there, a clear distinction between contingent truth and logical (or necessary) truth. Logic, therefore, is not presented there as a science of a totally different kind than any other else. For Russell logic is contiguous with other sciences; its difference from them only consists in the fact that it is more general. "Symbolic logic - he observes - is distinguished from the various special branches of mathematics mainly by its generality" [PoM: 11].

In his post-1910 philosophy, however, Russell insists that the truth and falsity of judgements depends on something external to the judgement itself; *his* way of implementing the correspondence requirement is to say that the truth or falsehood of a judgement depends on the presence or absence of a truth-maker in reality. It is worth noting, I believe, that after Russell came to adopt the correspondence requirement, he also came close to a conception of logic as

tautologous. In *The Philosophy of Logical Atomism*, a work where the correspondence requirement plays a large part in Russell's conception of truth, Russell affirms that "[e]verything that is a proposition of logic has got to be in some sense or other like a tautology. [...] [Propositions of logic] have a certain peculiar quality which marks them out from other propositions and enables us to know them a priori" [PLA: 211].

For Wittgenstein, what marks propositions of logic out of other propositions – as we saw – is precisely the fact that they can be known as true from the symbol alone (from their internal articulation), namely without the need of comparing them with reality. Russell, however, seems still quite unclear about this, for he does not actually ground the idea that logic is tautologous on the idea that propositions of logic are not 'correspondence-true', and in the same passage from *The Philosophy of Logical Atomism* quoted above, he admits that he is unable to tell what the characteristic that distinguishes propositions of logic from other propositions exactly is.

[12.5] Entailment

In this section we will discuss Wittgenstein's conception of entailment, or logical consequence. Sections 5.11-5.121 of the *Tractatus* give the following concise account of entailment:

If all the truth-grounds that are common to a number of propositions are at the same time truth-grounds of a certain proposition, then we say that the truth of that proposition follows from the truth of the others [TLP: 5.11].

In particular, the truth of a proposition ' p ' follows from the truth of another proposition ' q ' if all the truth-grounds of the latter are truth-grounds of the former [TLP: 5.12].

The truth-grounds of the one are contained in those of the other: p follows from q [TLP: 5.121].

This truth-functional account of logical consequence occurs within Wittgenstein's general discussion of the nature of molecular propositions. The truth-grounds of a proposition – Wittgenstein tells us – “are those truth-possibilities of [...] [a proposition's] truth-arguments that make it true” [TLP: 5.101]. Truth-grounds are thus the circumstances in which a proposition is true, they are represented by the T's in a truth-table. Wittgenstein's account of logical consequence can therefore be reformulated as follows: if all the circumstances in which a number of propositions are true are also circumstances in which a certain proposition is true, then the truth of the latter proposition follows from – or is entailed by – the truth of the former propositions.

The following truth-table provides a very simple example of the truth of one proposition's being entailed by the truth of others. In particular the example shows that the proposition ' $\sim p$ ' is entailed by – or follows from – the two propositions ' $p \supset q$ ' and ' $\sim q$ ':

p	q		$p \supset q$	$\sim q$	$\sim p$
T	T		T	F	F
F	T		T	F	T
T	F		F	T	F
F	F		T	T	T

We first list the truth-possibilities of the elementary propositions ' p ' and ' q ' in the first two columns of the truth-table. We then determine the truth-conditions of the three propositions of our example, ' $p \supset q$ ', ' $\sim q$ ', and ' $\sim p$ '. We can see that the

proposition ' $\sim p$ ' follows from the propositions ' $p \supset q$ ' and ' $\sim q$ ', because in all the circumstances in which the latter propositions are both true, the proposition ' $\sim p$ ' is also true. Thus the truth-grounds that are common to the propositions ' $p \supset q$ ', ' $\sim q$ ' are also truth-grounds of the proposition ' $\sim p$ '.²²⁸ This, for Wittgenstein, also amounts to the idea that a proposition's being entailed by others can be explained in terms of the sense of that proposition's being contained in the sense of the other propositions. "If p follows from q , the sense of ' p ' is contained in the sense of ' q '" [TLP: 5.122].

At 5.13-5.131 Wittgenstein builds on the notion of entailment:

When the truth of one proposition follows from the truth of others, we can see this from the structure of the propositions [TLP: 5.13].

If the truth of one proposition follows from the truth of others, this finds expression in relations in which the forms of the propositions stand to one another: nor is it necessary for us to set up these relations between them, by combining them with one another in a single proposition; on the contrary, the relations are internal, and their existence is an immediate result of the existence of the propositions [TLP: 5.131].²²⁹

Wittgenstein says here that entailment is a *structural* relation between propositions. The structure of propositions shows if the truth of one follows from the truth of others. A structural relation is called by Wittgenstein an *internal* relation [TLP: 4.122]. Two propositions have an internal relation with one another simply by virtue of being the propositions they are.²³⁰ One can see that one

²²⁸ Wittgenstein will later put the same point more simply as follows; "In the T F notation we can [...] say that if the Ts of one proposition include those of another then the second follows from the first" [WLC: 57].

²²⁹ I think that the expressions 'structure of a proposition' and 'form of a proposition' in 5.13 and 5.131 respectively, are used by Wittgenstein as synonymous.

²³⁰ Proops [2002: 288] correctly remarks that the holding of an internal relation between propositions "consists in nothing more than the propositions' *being the very propositions that they are*". For an account of the notion of entailment as an internal relation see Proops [2000: 99-101].

proposition follows from another (or more than one) by simply understanding the sense of the propositions in question, their truth-conditions.

Wittgenstein goes on discussing the notion of entailment by claiming that, in order to see if propositions are in such a relation with one another, one need not combine them into a single proposition. This means, I take it, that one need not combine them into a single proposition and check whether the proposition is a *tautology*. Proops discusses this point of Wittgenstein's conception of entailment as follows:

[I]n order to appreciate that q follows from p , there is no need to first show that the proposition 'if p then q ' can be known to hold on the basis of logic alone. This is so because we do not need to think of the validity of the inference as owed to the logical truth of the corresponding conditional [...]. Rather, the validity of the inference is owed merely to an internal relation between the forms of the premise and the conclusion.²³¹

Although Proops puts his point in terms of inference, it straightforwardly applies to entailment. In order to see whether one proposition follows from another, it is not necessary to put them together into a single proposition and consider whether the latter is a logical proposition. Although this *is* a way of finding out if the relation of entailment holds between two propositions,²³² it is *not* necessary. The relation of entailment's being an internal relation between propositions, one need only analyse the structure of the propositions (their senses) to see whether one is entailed by the other. The existence of tautological conditionals, Wittgenstein is implying, is not the *justification* for the fact that the propositions figuring as their antecedents entail the propositions that figure as their consequents. This fact is due to the existence of an internal relation between propositions.

²³¹ Proops [2002: 294].

²³² Wittgenstein in fact says that "[i]f, for example, two propositions ' p ' and ' q ' in the combination ' $p \supset q$ ' yield a tautology, then it is clear that q follows from p " [TLP: 6.1221].

Wittgenstein's discussion of *entailment* in the *Tractatus* is intermingled with considerations about the nature of *inference*. In the *Tractatus* he seems to base the latter on the former, in the sense that he thinks the *justification* for an inference from a proposition to another to lie in the presence of a relation of entailment between the two propositions. As he puts it:

If p follows from q , I can make an inference from q to p , deduce p from q .
The nature of the inference can be gathered only from the two propositions.
They themselves are the only possible justification of the inference.
'Laws of inference', which are supposed to justify inferences, as in the works of Frege and Russell, have no sense, and would be superfluous [TLP: 5.132].

This conception of the relation between inference and entailment also emerges in some excerpts from a series of lectures Wittgenstein gave in Cambridge in 1930-1932. There he argues that:

Inference is the transition from one proposition to another, a transition which we justify by saying that e.g. q follows from p . This relation is entirely determined when the two propositions are given [WLC: 56].

Inference is justified by an internal relation which we see; the only justification of the transition is our looking at the two terms and seeing the internal relation between them [WLC: 57].

Why does Wittgenstein speak of the internal relation of entailment in terms of its being the *justification* for an *inference*? And what does Wittgenstein mean by saying that "one elementary proposition cannot be deduced from another" [TLP: 5.134]? Does he mean that one cannot *infer* an elementary proposition from another? Of course, this seems wrong. One *can* infer an elementary proposition from another, mistakenly taking, for instance, the former to follow from the latter. But this is a *psychological* notion of inference that is of no concern to Wittgenstein. Wittgenstein would say that such an inference is not *justified*. And he says that the

notion of *entailment* provides the relevant justification for an inference. So, when Wittgenstein is discussing inference in this context, he is assuming the notion of a (deductively) justified inference, therefore of an inference that is based on the existence of the internal relation of entailment between two propositions.

The holding of a relation of entailment between two propositions, Wittgenstein argues, is the *only* justification for inferring one from the other. And this only depends on the structures of the propositions, on the sense they express. Understanding the propositions ' p ', ' q ', and ' $p \supset q$ ', for Wittgenstein, is enough to understand that ' q ' follows from the other two propositions, and thus to infer it from them. The deduction of ' q ' from ' p ' and ' $p \supset q$ ' rests solely on the structures of the propositions, which are in the internal relation of entailment with one another. The deduction, Wittgenstein argues, is justified only by this, and not by the existence of a rule of inference such as 'from ' p ', and ' $p \supset q$ ', infer ' q ', that is somehow meant to license it. Deductions, or inferences, need not be justified by something (a rule of inference) *external* to the propositions themselves. The propositions involved in the inference, the premises and the conclusion of the inference, for Wittgenstein, provide the only justification for it.²³³

We have been discussing Wittgenstein's idea that all logical constants/operations are already present in an elementary proposition. The reason is that propositions have sense and thus have poles corresponding to the case of their truth and the case of their falsehood. This is everything that is needed

²³³ Wittgenstein made essentially the same point in the *Notes on Logic*: "Logical inferences can, it is true, be made in accordance with Frege's or Russell's laws of deduction, but this cannot justify the inference; and therefore they are not primitive propositions of logic. If p follows from q , it can also be inferred from q , and the 'manner of deduction' is indifferent" [NL: 100]. I will return to the notion of inference in section [13.4].

in order to give an account of logical operations. Logical operations operate on propositions' T-F poles, to deliver molecular propositions and, thus, tautologies and contradictions as well. The possibility of logical propositions rests on T-F articulation, and hence, at bottom, on linguistic sense. Therefore, as soon as elementary propositions are given, logic as a whole is given, for it depends on operations on propositions' sense and structure.

The discussion of Wittgenstein's conception of entailment, however, provides us with a different reason for the claim that logic is given as soon as elementary propositions (with T-F poles) are given. Traditionally, logic has been conceived of as the science of valid inference, the science that studies valid argument forms. In this sense, logic "is concerned with the laws whereby we justifiably move from one judgement or assertion of truth to another".²³⁴ For Wittgenstein these laws are nothing over and above propositions of language and their truth-functional articulation. The justification for inferring one proposition from others is afforded by the existence of some internal relations holding between them, notably the internal relation of entailment. And this relation, we have noted, merely depends on the propositions' in question being the very propositions they are (in this sense it is an *internal* relation). Propositions, by having sense and T-F poles, stand in logical (internal) relations with one another. In particular, the internal relation of entailment, which holds among some propositions, affords the only possible justification for inference. Thus, logic, as the science of valid inference, is as well given as soon as propositions are given.

²³⁴ McGinn [2006: 54].

Chapter Thirteen

Frege and Wittgenstein: The Nature of Logic

[13.1] Introduction

In part three of this thesis we analysed the development of Wittgenstein's conception of the nature of the proposition, and the relation between his conception of language and his philosophy of logic. We have discussed and explored Wittgenstein's idea that logical operations are contained in elementary propositions, and that, therefore, there are no logical constants, representative on their own; this grounds Wittgenstein's conception of logical truth as a by-product of contingent truth. Logical consequence or entailment, furthermore, in being an internal relation between propositions, only depends on the very nature of propositions, on their representative structures. These are all corollaries of the general conception that logical articulation is grounded in the way in which propositions express the sense they do, and are thus true or false.

Our discussion was thus aimed at clarifying Wittgenstein's basic intuition about the nature of logic, the idea that the nature of logic has to be explained by giving an account of linguistic significance, which is to begin by providing an account of the structure and articulation of the simplest kind of propositions, namely elementary ones. This vindicates Wittgenstein's early claim that questions about the nature of logical constants and logical propositions "can be traced down to the *atomic* propositions" [NB: 121]. As McGinn comments, this is justified by an understanding of the nature of elementary propositions as providing everything that is needed in order to account for the nature of logic.

Once the essence of the elementary proposition is made perspicuous, we shall, [...] [Wittgenstein] believes, already have everything we need to understand the relation between elementary propositions and the propositions of logic, the nature of negation and inference, and so on. It is not [...] that we shall be able to deduce, say, the status of the propositions of logic from the nature of an elementary proposition. It is rather that in coming to see clearly into the workings of an elementary proposition we are coming to see clearly into the essence of representation, and this is all that is needed to make all of the problems disappear.²³⁵

In this last chapter of my thesis, I bring together these views of Wittgenstein's with Frege's views on logic. When discussing Frege's views on the nature of logic in the first part of this thesis, I highlighted some internal tensions Frege's conception of logic is prone to. In particular, we saw that Frege thinks logic is the most general science, *descriptive* of the most general laws of truth; at the same time, however, especially in his early thought, he also retains the idea that logic is – in contrast to more specific sciences – *formal*, for it provides the framework within which particular sciences can be expressed. Besides, Frege thinks that logic is not only *descriptive*, but also *normative* for thought (insofar as it aims at truth), and grounds the normativity of logic on its being the most general science. Finally, we saw that there is a further strand in Frege's view according to which logic is also *constitutive* of thought, in the sense that accordance with the laws of logic is a pre-requisite for thought as such; thus accepting the laws of logic – Frege often seems to hold – is a necessary condition for being a thinker at all.

Although it is surely true that Wittgenstein's ideas on the nature of logic in the *Tractatus* are an integral part of a tradition which “was begun by Frege and was continued later by, among others, Russell”,²³⁶ it is also true that Wittgenstein rejects some of the tenets of Frege's conception of logic discussed in the first part of this thesis and summarised above. In contrasting Wittgenstein's views and

²³⁵ McGinn [2006: 83].

²³⁶ Hintikka, Hintikka [1986: 87].

Frege's, my aim is to show that Wittgenstein's conception of the nature of logic, delineated in previous chapters, is free from the internal tensions that affect Frege's thinking on the matter.

[13.2] The Generality of Logic

We discussed in chapter [1] that, according to Frege, the aim of logic is to discover and establish laws of truth. In this respect, logic is contiguous with other sciences. However, whereas particular sciences aim at discovering truths with regard to their peculiar subject-matter (e.g. physics aims at discovering truths about mass and motion, chemistry about the composition of elements, and their reciprocal reactions, and so forth), logic aims at discovering *the* laws of truth, which hold for any science whatsoever. As Frege says:

[W]e do not demand of [...] [logic] that it should go into what is peculiar to each branch of knowledge and its subject-matter. On the contrary, the task we assign to logic is only that of saying what holds with the utmost generality for all thinking, whatever its subject-matter [L2: 128].²³⁷

According to this aspect of Frege's view of logic, which we discussed in section [2.3] and at the beginning of section [3.1], logic is the *most general* science. Laws of logic, for Frege, assert what is the case, no less than laws of physics do; but, unlike the latter, the former are "the most general laws" [BLA: 12]. Therefore, according to this strand in Frege's conception, logic *is* a science, whose general laws of truth encompass those of particular sciences. Since logic is a science, it is legitimate to ask what its subject-matter is. For Frege, the answer is that the subject-matter of

²³⁷ Russell, as we saw, shared a similar perspective on the generality of logic. See [PoM: xi], [TK: 97-98].

logic is *everything*. Laws of logic, for Frege, express substantive generality, for they are universally quantified propositions.²³⁸

According to Frege's 'universalist' conception, then, it is primarily generality that marks logic from other sciences, and thus the propositions of logic are different from other propositions because they are more general. On this view, there is no distinction in *kind* between logical and non-logical propositions; the distinction is a distinction in *degree* of generality. As Tascheck writes:

[W]hat distinguishes logic from other sciences is the fact that its laws are in no significant way restricted with respect to subject matter. The laws of logic are laws that hold of all reality, all of 'what is'. There is no subject matter – there is, so to speak, no truth – that is not subsumed under the laws of logic; and it is in this sense that they count, for Frege, as the most general laws of truth.²³⁹

On this conception the contiguity of logical and non-logical propositions is explained with the idea that both have *content*. However, the content of the propositions of logic is maximally general, unlike that of non-logical propositions.

We have discussed in section [12.4] Wittgenstein's conception of the nature of the propositions of logic. For Wittgenstein, there is a distinction in *kind* between logical propositions and non-logical ones; Logic is of "a *totally* different kind than any other science" [NB: 120]. And this in turns implies a rejection of the idea that logical propositions have *content* (however general that content is conceived to be), as non-logical ones do.

All theories that make a proposition of logic appear to have content are false [TLP: 6.111].

²³⁸ I speak of laws of logic, for Frege, to be 'propositions', but, on his account, laws of logic are 'thoughts'. This, however, does not matter much in the present context.

²³⁹ Tascheck [2008: 381].

Propositions of logic differ from non-logical propositions precisely because the former – in contrast with the latter – do not have content, do not stand in a representational relation with reality. They are ‘sinnloss’ combinations of signs, because they exploit the representational nature of their constituent propositions by arranging them into propositions which do not have a representational function. This is what distinguishes them from other (contingent) propositions; on the other hand generality is *not*, for Wittgenstein, the mark of the logical as opposed to the non-logical.

The mark of a logical proposition is *not* general validity.

To be general means no more than to be accidentally valid for all things. An ungeneralized proposition can be tautological just as well as a generalized one [TLP: 6.1231].

The general validity of logic might be called essential, in contrast with the accidental general validity of such propositions as ‘All men are mortal’ [6.1232].²⁴⁰

The fact that a proposition is general does not imply that it is logical, and that, therefore, it has no representational (descriptive) content. In fact, “[w]e can describe the world completely by means of fully generalized propositions” [TLP: 5.526]. Fully generalised propositions, if they are not of a tautological form, “are not logical propositions, and this explains the feeling that, even if they were true, their truth could only be the result of a fortunate accident” [TLP: 6.1232].

Against the idea that propositions of logic have general content and maximally general validity – a view we saw to be part of Frege’s conception of logic – Wittgenstein argues that they have *essential* validity, independent of how things are in the world, and solely dependent on the internal articulation of the

²⁴⁰ Wittgenstein arrived at this conclusion gradually. As we mentioned in section [12.1], early in his career (ca. 1913) he was still under Russell’s influence in thinking generality as being, somehow, implicated in logical propositions.

propositions themselves, namely on their symbolic nature. Whereas any genuine proposition must be 'compared with reality' in order to know whether it is true or false, propositions of logic can be recognised as true solely by inspection of their logical articulation. By means of their having constituent propositions combined they way they are, propositions of logic are severed from reality, and do not stand in any representational relation to the world. The representational content of the elementary propositions is, in a tautology, brought into *equilibrium*, annihilated as it were, by its structural articulation; As we saw: "Tautology and contradiction are the limiting cases – indeed the disintegration – of the combination of signs" [TLP: 4.466]. Because of their being the 'disintegration' of the combination of signs, propositions of logic are senseless and say nothing. They do not have content. Contrary to Frege (and Russell), Wittgenstein argues that logic has thus no subject-matter, it does not describe features of reality, however general. But, by virtue of their constituent propositions (pictures of reality) being arranged in a particular way, so as to cancel one's another sense, propositions of logic are *necessarily* true.

Wittgenstein's opposition to the idea that logic is the most general science also amounts to an opposition to the idea that logic is a *descriptive* science, a contention of Frege's conception of logic. We saw in chapter [2] that on Frege's account the ideas that logic is *descriptive* and *general* are interconnected: logic is descriptive of the most general laws of truth. Our analysis has shown that on Wittgenstein's view logic is *not* the most general descriptive science.

The contrast between Wittgenstein and Frege is, however, deeper than just that; not only is logic not a general descriptive science: for Wittgenstein logic is *no* science at all:

Logic is not a body of doctrine, but a mirror-image of the world [TLP: 6.13].

Logic, for Wittgenstein, far from being a science with its own subject-matter, constitutes the framework of language (and thought), and the world. It defines their boundaries and limits, by showing the formal (structural) properties and relations of language (the essence of representation) and of the world [see TLP: 6.12]. Logic, for Wittgenstein, “pervades the world: the limits of the world are also its limits” [TLP: 5.61]. But this, to be sure, also means that logic pervades language. For Wittgenstein, in fact, logic is embedded into the very nature of language and thought, it forms its underlying structure (what we might call its *grammar* or *syntax*). The principle of truth-functionality, with its T-F articulation, reveals such structure, reveals how sentential complexity (and therefore logical complexity also) can be generated. Knowing how it is possible to obtain molecular propositions out of elementary ones, that is, knowing how propositions of language acquire their T-F poles, and therefore have *sense*, puts one in a position to understand the nature of logic, for logic underlies (and is revealed by) the representative nature of propositions of language, their possibility of being true or false. Logical operations in fact, as we said at length in previous chapters, operate on propositions’ T-F poles, poles that amount to propositions’ sense. A proposition’s having sense is thus all that is needed to carry out truth-operations, and thus to obtain molecular (and logical) propositions out of elementary ones.

We have here, then, another affirmation of the *leitmotif* of this dissertation, the idea that, on Wittgenstein’s conception, the nature of logic is made clear by a correct account of linguistic representation, by a correct account of the nature of the proposition; this, again, has as its corollary the idea that logical complexity is

contained in (forms part of) linguistic complexity, and that elementary propositions have in themselves the possibility of all logical operations (and thus the possibility of all the propositions of logic). To use McGinn's words:

Logic is given as soon as a language in which we express judgements about the world is given; it is, in some sense, already complete or entire when we have a language that we use to say how things are. Frege's and Russell's treatment of logic as a body of doctrine, Wittgenstein believes, fails to make clear that by acquiring a language in which we express thoughts that are true or false, we have already grasped the whole of logic.²⁴¹

For Wittgenstein logic, if the one discussed above is the correct perspective, is thus not a science (it does not matter how general it is conceived it to be), but – rather – if I am allowed to use a rather metaphorical expression, the *expressiveness* of language (of any language), its capability and possibility of making sense, of expressing thoughts that are true or false. As soon as we have propositions with sense, propositions with pictorial, representative, nature, the whole of logic is given. The conception of logic that emerges from the *Tractatus* is thus in direct opposition to any conception that sees logic as a (general) science. Logic, for Wittgenstein, cannot be a science in the sense of being a particular set of true propositions; logic is *not*, in this sense, the totality of tautologies, but the truth-functional (and bipolar) structure of language, which contains the possibility of all logical (necessarily true) propositions.²⁴²

This, I think, contains the *decisive* point about Wittgenstein's conception of the nature of logic, which he himself alludes to at 6.124:

²⁴¹ McGinn [2006: 69].

²⁴² I therefore disagree with Proops [2000: 3] who interprets Wittgenstein as holding that “[l]ogic is to be viewed [...] as a body of sentences, which, because they convey no information [...], are to be regarded as expressing no thoughts”. Proops' attribution to Wittgenstein of the idea that logic is 'a body of sentences' fails to give justice to Wittgenstein's idea that logic as a whole is given as soon as elementary propositions are given.

We have said that some things are arbitrary in the symbols that we use and that some things are not. In logic it is only the latter that express: but that means that logic is not a field in which *we* express what we wish with the help of signs, but rather one in which the nature of the natural and inevitable signs speaks for itself [TLP: 6.124].

Wittgenstein is here urging – I believe – that logic is not a field (however general) that can be *described* by means of language; logic is instead the symbolic, representative, nature of language. Logic is contained in, and given by, the expressiveness of language, the *sense* of linguistic propositions, their possibility of expressing thoughts that describe reality and are thus true or false.

If my reading is correct, it may thus sound strange to find Wittgenstein writing, in the very same section of the *Tractatus* quoted above, that propositions of logic describe something nonetheless. He there says that logical propositions “describe [*beschreiben*] the scaffolding of the world, or, rather they represent [*stellen*] it” (my emphasis). I think Wittgenstein is simply being loose in his use of terminology here. Of course, by the *Tractatus*’ own lights, the ‘scaffolding of the world’, the set of the world’s formal and structural properties, cannot be represented by propositions; instead, it is reflected by propositions’ having the formal properties they have, which gives them the possibility of representing reality. This is something Wittgenstein had already been clear about at the time of the *Notes Dictated to Moore*: There he says:

Logical so-called propositions *show* [the] logical properties of language and therefore of [the] Universe, but *say* nothing.

It is impossible to *say* what these properties are, because in order to do so, you would need a language, which hadn’t got the properties in question, and it is impossible that this should be a *proper* language [NM: 108].

The propositions of logic, far from describing, or representing anything, *show* the formal properties of language and the world, and they do so precisely by virtue of their combining sensible propositions into propositions that say nothing.

The fact that the propositions of logic are tautologies *shows* the formal – logical – properties of language and the world [TLP: 6.12].

[13.3] Frege and Wittgenstein on the Formality of Logic

Wittgenstein's rejection of the idea that propositions of logic are the most general descriptive laws of truth is motivated by the thought that what concerns logic is essential – as opposed to general – truth; and essential, or necessary, truth is due to logical propositions' being the limiting case of truth-functional combination of elementary propositions. This explains – as we saw – both the contiguity of logical propositions with non-logical ones, and the difference between them, which is something that, on the other hand, poses difficulty for Frege's and Russell's conception of the nature of logical propositions.

In chapter [3], however, we highlighted a strand in Frege's reflection on logic that sits uneasily with the idea that the distinction between the logical and the non-logical is a question of generality. This strand emerges in Frege's discussion of the *formal* as opposed to the *material* part of a language, which suggests that logic is merely formal, and thus detached from any material content. According to this strand, the formulae of Begriffsschrift are empty schemata [see ACN: 97] which are to be supplanted by contentful propositions in order to produce a perspicuous language in which logical relations between propositions (the content) are made fully evident. As Frege says, a Begriffsschrift "must have simple modes of expression for the logical relations which, limited to the

necessary, can be easily and surely mastered. These forms must be suitable for combining most intimately with a content" [S]CN: 88]. From this it emerges that a Begriffsschrift, in Frege's conception, is the logico-syntactic skeleton of language, which can be applied to different fields, just by supplanting it with particular propositions (the non-logical vocabulary, the content). In § 1 of the *Begriffsschrift*, as we saw, Frege says that he uses two kinds of symbols, those with a completely *fixed sense*, and those which can *signify various things*. This difference between signs reflects the difference between the logico-syntactical part of the language and its material part. Frege explains the link between these two parts of a language by saying that in his Begriffsschrift he puts together the existing symbols of mathematics with new symbols he introduces, in order "to form a single formula language" [ACN: 93]. And he goes on saying that "the existing symbols [of mathematics] correspond to the word-stems of [ordinary] language; while the symbols I add to them are comparable to the suffixes and [deductive] formwords [*Formwörter*] that logically interrelate the contents embedded in the stems" [ACN: *ibid.*].

The symbols with a completely fixed meaning that Frege adds to his Begriffsschrift, with the exception of the judgement and horizontal stroke (which do not concern us here) include symbols for truth-functional connectives, in particular, the symbols for conditionality [CN: § 5] and negation [CN: § 7], the only two used in Frege's Begriffsschrift, as all the others are obtainable from – and definable in terms of – the former. Now, this means that Frege's Begriffsschrift, in its being the logical structure of language, contains the resources for obtaining all the propositions of logic – tautologies and contradictions – in line with what

Wittgenstein claims at 6.124. Just by combining the content of actual propositions with the syntactical elements of Frege's *Begriffsschrift* – the logical connectives, one can obtain all tautologies and contradictions.

If one looks at Frege's *Begriffsschrift* in this way, then one can see more than a mere analogy with Wittgenstein's conception of logic in the *Tractatus*. Logical interconnectedness is explained in the *Begriffsschrift* as the result of the combination of the contents of propositions (the vocabulary) with the formal part of the notational language, that providing its syntax (truth-functionality). But from this it seems to emerge a conception of logic at odds with the idea that logic is contentful and general. The *Begriffsschrift* is not a *science* in its own right; it lacks the vocabulary that would make it able to have a subject-matter; this is *not* to say that its subject-matter is the most general there is, namely that is *everything*. It has none, for it simply provides the logico-syntactical resources for making a language *able* to have a subject-matter, to speak about something.

The tension in Frege's conception between the view of logic according to which logic has *substantive* generality and the view on which logic is *formal* is thus resolved by Wittgenstein by dropping altogether the idea that logic is a science, whose propositions are contentful and general, and embracing the view that logic is formal. Logic is formal because it does not have a content of its own; truth-operations, by means of which logical complexity is generated, do not add any material information that is not already involved in elementary propositions' representational activity.

In section [3.2] we saw that there is evidence for the conclusion that Frege rejected, at least at one point in his career, the idea that logic is formal. Frege

thought that logic cannot be completely formal, for if it were, “it would be without content. Just as the concept point belongs to geometry, so logic too, has its own concepts and relations; and it is only in virtue of this that it can have a content. Toward what is thus proper to it, its relation is not at all formal. No science is completely formal; but even gravitational mechanics is formal to a certain degree, in so far as optical and chemical properties are all the same to it. [...] To logic, for example, there belong the following: negation, identity, subsumption, subordination of concepts”.²⁴³ As we saw, this means that for Frege, as MacFarlane explains it, logic “cannot abstract from all semantic content: it must attend, at least, to the semantic contents of the logical expressions, which on Frege’s view function semantically just like non-logical expressions”.²⁴⁴ So, on Frege’s view, logic cannot be entirely formal because logic has *its own* concepts, because there are concepts – like those referred to by connectives and quantifiers – which are specifically logical.

Our discussion – in sections [12.2] and [12.3] – of Wittgenstein’s conception of logical constants as *operations* for constructing propositions, and thus as not being representatives on their own, enables us see that Wittgenstein has none of the reasons Frege has for rejecting the idea that logic is formal. For Wittgenstein, logical expressions do *not* have semantic content, in the sense that they do not refer to objects (or concepts); we saw that this is the essence of the *Grundgedanke* of the *Tractatus*; logical constants are not representatives, for there is nothing they refer to; they are signs for truth-operations, and truth-operations are already implied in a proposition’s expressing the sense it does, in its being either true or

²⁴³ See MacFarlane [2002: 29].

²⁴⁴ MacFarlane [2002: 29-30].

false; they are already contained in elementary propositions. At the same time, unlike Kant, who thought – in line with the ‘schematic’ conception of logic – that logical expressions, while not having semantic content, are indications of the *form* of a judgement (or of a proposition), for Wittgenstein – as we saw in section [12.2] – “an operation is not the mark of a form” [TLP: 5.241], because it is not by itself always enough to characterise the sense of a proposition.

[13.4] Frege and Wittgenstein on the Normative Status of Logic

In chapter [2] we discussed Frege’s views on the twofold – *descriptive* and *normative* – status of logic. This difference follows from the status Frege retains for logic, its being a *science*. For Frege logic is a descriptive as well as a normative science. In holding that logic has a normative status, Frege opposes the psychologistic view of logic according to which logical laws describe how thinking takes place; in contrast, Frege thinks that logic establishes norms for guiding the intellect in the attainment of truth. For Frege the normative status of logic (prescribing norms for thinking is so far as thinking is concerned with truth) stems from, and is grounded on, its descriptive status (being the science of truth).

[L]aws of truth [...] must provide the norm for holding something to be true. And these will be the laws of logic proper [L2: 146].

From the laws of truth there follow prescriptions about asserting, thinking, judging, inferring [T: 351].

We can now also consider how Wittgenstein’s conception of logic fits in with Frege’s repeated conviction that logic is to provide norms for thought insofar as it aims at truth, that is, that logic has an intrinsically *normative* status. I believe

there is no room, on Wittgenstein's account, for the idea that logic is normative for thought in Frege's sense. As Picardi has observed:

The most thorough rejection of the conception of logic as a normative science is to be found in Wittgenstein's *Tractatus Logico-Philosophicus*. Wittgenstein rejected the entire phraseology of laws of thought and laws of inference which had still been used by Russell and Frege. By connecting logic and language in the closest possible way, he made it virtually impossible to formulate the question as to the normative vs. ideal status of logical propositions [...]. On the one hand, the laws of logic are downgraded from laws of thought to tautologies; on the other, logic is built into the very fabric of language and thought. There is no stepping outside the limits of our language and hence, the limits of our logic.²⁴⁵

For Frege logic has a normative status because its laws are laws of truth; and from the laws of truth there follow prescriptions on how to think, or judge, or infer, insofar as one is to do so truly. Logical laws have therefore some kind of priority – and authority – over other laws; laws of logic are laws to which laws pertaining to other sciences must conform; they are the most general laws of truth, and are thus laws of truth as such. On the 'universalist' conception of logic – as we saw – logic is in fact a science alongside others. It differs from particular sciences because it is more general. So while laws of specific sciences tell us how to think truly on matters pertaining to their fields, logic establishes the laws of truth, which hold in any field whatsoever. Any law, for Frege, has normative force, but because logic is the most general science, it has a wider scope of normative force than any other science. Laws of logic – says Frege – “have a special title to the name ‘laws of thought’ only if we mean to assert that they are the most general laws” [BLA: 12].

We already discussed Wittgenstein's opposition to any conception of logic that takes logic to be a descriptive and completely general science. If this framework is rejected, the idea of the normativity of logic – at least in the terms in which Frege conceives of it (as deriving from its being the most general science) –

²⁴⁵ Picardi [1997: 170-171].

must also be abandoned. Logic, for Wittgenstein, is not a general science that has authority over others. Logic, for Wittgenstein, is *constitutive* of language because it is a pre-condition for a proposition's expressing its sense, which it does by sharing logical form with reality, and thus acquiring the possibility of being true or false, the possibility of representing reality correctly or incorrectly.

In this sense, then:

[L]ogic is not something outside language that is authoritative for thought insofar as we think truly. Rather, logic is internal to representation and thought as such.²⁴⁶

Logic, for Wittgenstein, is internal to language because a proposition's expressing the sense it does – its saying something – makes it intrinsically true or false, makes it occupying a place in logical space. And as soon as everything that is essential to a proposition's representational capability is in place, then logic as a whole is given, for any further logical construction is obtainable by means of logical operations, which operate on propositions' sense, or truth-conditions. If this is a correct reading of Wittgenstein's conception of logic, then, on his account the idea that logic is normative for thought – as Frege held – evaporates. Logic is *internal* to thought and language, insofar as logic is internal to the possibility of expressing a sense and thus being true or false. The idea that is in play in Frege, that logic has a special normative status due to its being the most general science, cannot find any room on Wittgenstein's conception.

But this, one might argue, means to dismiss the question of the normativity of logic too quickly; after all, Wittgenstein agrees that logic is the “theory of forms and of *inference*” [TLP: 6.1224, my emphasis], and we saw in section [12.5] that he

²⁴⁶ McGinn [2006: 100].

explicitly discusses the logical (internal) relation of entailment in terms of its providing the justification for an inference. In some sense, then, logic seems implicated in justifying – licensing – inferences, and if this is the case, then logic retains some kind of normativity on Wittgenstein’s perspective also. At the already quoted 5.132, Wittgenstein writes:

If p follows from q , I can make an inference from q to p , deduce p from q .
The nature of the inference can be gathered only from the two propositions.
They themselves are the only possible justification of the inference.
‘Laws of inference’, which are supposed to justify inferences, as in the works of Frege and Russell, have no sense, and would be superfluous [TLP: 5.132].

As already observed, Wittgenstein here says that an inference from one proposition to another need not be justified (or licensed) by a ‘law of inference’, something *external* to – and admittedly, on Frege’s perspective, *more general* than – the two propositions themselves. The inference is on the other hand fully justified by the two propositions alone, by their being in an *internal* relation with one another, the internal relation of entailment. So it is the relation of logical consequence (or entailment) holding between two propositions that grounds one’s inferring one proposition from another; but this is in no way different from saying that the inference is justified by the internal structure of the propositions, thus by the propositions alone.

At 6.1264 Wittgenstein seems to make a different point:

Every proposition of logic is a *modus ponens* represented in signs.

Modus ponens is a rule (or law) of inference. By means of *modus ponens* one can move from two propositions to another. It is exemplified in the inference from ‘ p ’ and ‘ $p \supset q$ ’ to ‘ q ’. What does it mean to say that every proposition of logic is a

modus ponens represented in signs? Does it mean that every proposition of logic can be conceived of as a law of inference? If this were the case, then Wittgenstein could straightforwardly claim that logical propositions have a normative status that other propositions lack. Logic would be intrinsically normative because its propositions are laws of inference, and thus justify and license inferences.

It would be misleading, however, to think that, on Wittgenstein's perspective, it is a proposition's being a tautology that licenses or justifies one's inference, for example that it is the fact that the proposition $(p \ \& \ (p \supset q)) \supset q$ is a tautology that licenses one's inferring ' q ' from ' p ' and ' $p \supset q$ '. As Wittgenstein says, "we can actually do without logical propositions" [TLP: 6.122]. The inference is only justified by the internal relation of entailment that holds between the propositions in question, and thus by the propositions themselves. 'They themselves are the only possible justification of the inference'.²⁴⁷ So laws of logic (logical propositions), for Wittgenstein, do not have a normative function *qua* laws of logic. For Frege, on the other hand, this is indeed the case; laws of logic are normative because they are the most general laws of truth, and for this reason they have authority for thinking (regardless of its object) insofar as it aims at truth. But if this conception of logic is rejected, particularly the idea that laws of logic have a special title to be called normative *because* they are the most general laws, then the normativity of logic, in the sense in which Frege conceived of it, disappears.

²⁴⁷ Now, it can be questioned whether we have two different and competing views here, namely the view that the internal relations between propositions justify or license inferences *and* the view that the propositions of logic do. For Wittgenstein these amount to the same thing. As he puts it in the *Notes Dictated to Moore*. "E.g., take ϕa , $\phi a \supset \psi a$, ψa . By merely looking at these three, I can see that 3 follows from 1 and 2; i.e. I can see what is called the truth of a logical proposition, namely of [the] proposition ϕa . $\phi a \supset \psi a : \supset : \psi a$. But this is *not* a proposition; but by seeing that it is a tautology I can see what I already saw by looking at the three propositions: the difference is that I *now* see THAT it is a tautology [NM: 108-109].

[13.5] Wittgenstein and the Constitutivity of Logic: The Impossibility of Illogical Thought

In chapter [4] we discussed Frege's stance on the question whether logic is constitutive of thought, and we highlighted, in the context of our discussion of logical aliens, a potential tension in Frege's account of the nature of logic, between the 'normative' strand and the 'constitutive' strand in his thought on the matter. This tension is easily formulated; the gist of Frege's conception of logic as normative is that the laws of logic teach how one *ought* to think, and not merely how one *does* think; this forms the core of Frege's anti-psychologistic view of logic. Logic establishes laws for the use of thought insofar as one wants to think (judge, infer) truly. But Frege also retained a different role for logic, not only that of providing norms for thought, but also that of being *constitutive* of thought as such, that is, of being internal to thought. On this second - 'constitutive' - strand, logic is involved in the very possibility of judging and inferring, in the very possibility of thought. If this is right, then thought is - at least partly - constituted by the acceptance of the laws of logic; logic - as Frege says - governs "everything thinkable"; we cannot reject the laws of logic if we do not want to renounce to thought altogether, and fall into "complete confusion" [FA: 21].

But if the constitutive role logic has for thought is to be taken seriously, the normative role of logic seems, on Frege's perspective, to be lost. What sense can be made of the idea that logic states how we ought to think, and not how we do think, if logic is already implicated in the possibility of thought, if logic is internal to thinking? As Cerbone wrote: "[T]he availability of *logical* notions such as contradiction and entailment serves as a condition on the possibility of judging

[...]. But if this is so, then logic cannot be seen as bearing a normative relation to our judgements: it does not prescribe how we *ought* to proceed in our making judgements; it instead forms a background for our making judgments at all".²⁴⁸

At the beginning of this section we said that this is a *potential* tension in Frege's thought, because – as discussed in section [4.3] – it is not clear how deep his commitment to the 'constitutivity thesis' is. In the passage from the Introduction to *The Basic Laws of Arithmetic* where Frege discusses the logical aliens' scenario, he seems to refrain from fully endorsing it; he thus seems to argue that logical aliens – beings that reject the laws of logic – constitute a genuine possibility; if this is true then 'illogical thought' – which they exemplify – will count as a form of thought, one that, however, will never reach the truth; by not accepting the laws of logic (by not being sensitive to logic's normative dimension), logical aliens' judgements and inferences are always wrong. This line of thought seems to be more consistent with Frege's idea that acceptance of the laws of logic (recognition of their normative status in that laws of truth) enables one to think truly, and thus with the 'normative' strand in Frege's conception of logic.

In the *Tractatus*, on the other hand, the idea that logic is constitutive of thought, as already anticipated in section [4.3], is endorsed and repeatedly affirmed. The clearest statements of the 'constitutivity thesis' – are the following:

Thought can never be of anything illogical, since, if it were, we should have to think illogically [TLP: 3.03].

It used to be said that God could create anything except what would be contrary to the laws of logic. The truth is that we could not *say* what an 'illogical' world would look like [TLP: 3.031].

²⁴⁸ Cerbone [2000: 298].

What makes logic *a priori* is the *impossibility* of illogical thought [TLP: 5.4731].

Wittgenstein seems to say here that logic is involved in the very possibility of thought, and that, therefore, there cannot be 'illogical thought'. We cannot say what an illogical world would look like, because we could not *think* about it. In the *Tractatus* we find Wittgenstein applying the 'constitutivity thesis' to language also, arguing thus against the possibility of a language that contradicts logic:

It is as impossible to represent in language anything that 'contradicts logic' as it is in geometry to represent by its co-ordinates a figure that contradicts the laws of space, or to give the co-ordinates of a point that does not exist [TLP: 3.032].

The same idea is also expressed clearly in the early *Notes Dictated to Moore*:

[It is] impossible to construct [an] illogical language [NM: 108].

From these passages it thus emerges that Wittgenstein is deeply committed to the idea that logic is constitutive of the possibility of thought and language; illogical thought and language are impossible.

The sections of the *Tractatus* where Wittgenstein gives voice to the 'constitutivity thesis' are for the most part comments on section 3, which provides the following definition of the notion of a thought.

A logical picture of facts is a thought.

And section 4 develops this by stating:

A thought is a proposition with a sense.

Thus Wittgenstein's endorsement of the 'constitutivity thesis' with regard to thought and language is justified by the fact that for him, the notion of a thought is

already tied up to logical adequacy; a thought is – as seen in section [10.2] – a *logical picture*, or a proposition with a *sense*. In the *Tractatus*, then, as has been said, “there is no extra-logical notion of a thought”,²⁴⁹ and there is no extra-logical notion of a proposition either. A thought, then, insofar as it is a logical picture, insofar as it is a proposition with a sense, cannot be of anything illogical, because “[t]he possibility of all imagery, of all pictorial modes of expression, is contained in the logic of depiction [TLP: 4.015].

The logic of depiction implies that all pictures are “constructed according to a common logical pattern” [TLP: 4.014], that is, are all ‘logical pictures’. The logic of depiction – says Wittgenstein – is expressed by the existence of a rule of translation, or of projection [see TLP: 4.0141], by means of which a state of affairs can be represented in language, that is to say, through a proposition that is either true or false. The rule of projection amounts to the conditions for a fact to be a picture – discussed in section [10.2]. By projecting a possible situation into a propositional sign, a fact is given a pictorial structure, and becomes true or false, depending on whether it depicts (represents) reality correctly or incorrectly.

As McGinn puts it:

Logic, the essence of representation as such, constitutes everything that is essential to the rules of projection whereby we determine the place of a proposition in logical space; it is what is common to any system of representation within which we express propositions that can be compared to reality for truth and falsity.²⁵⁰

When we have propositions (thoughts) that represent reality, and are thus true or false, we already have logic, because logic (the system of rules of projection) is everything that is needed to make representation as such possible. Logic can thus

²⁴⁹ Cerbone [2000: 294].

²⁵⁰ McGinn [2006: 100].

be said to be the necessary condition for the possibility of representation; thus there cannot be something like a 'logical experience', as Russell thought, because logic is prior to – a condition of – any experience that something is so [see TLP: 5.552]. Logic is what makes the representation that something *is so* (and thus true or false), possible. As Wittgenstein puts this in the *Notebooks*; "In order for a proposition to be true, it must first and foremost be *capable* of truth, and that is all that concerns logic" [NB: 29.10.14]. Propositions and thoughts, by their having sense, by their saying that something is so, presuppose a system of rules of projection (or translation) between language and reality, and are thus already in logical order, are 'logical pictures'.

We have also seen how Wittgenstein extends this account of logic to molecular propositions. Logical constants do not introduce any new element which is not already involved in the representational function of elementary propositions. Truth-functions are bipolar propositions in exactly the same way in which elementary propositions are. The whole of the logic of depiction is thus given at the level of elementary propositions, and this is the reason why all logical constants, or operations, are already contained in an elementary proposition.

In the *Tractatus* Wittgenstein claims that there actually is only *one* logical constant, which is "what *all* propositions, by their very nature, had in common with one another"; but that – Wittgenstein says – "is the general propositional form" [TLP 5.47], the "essence of the proposition" [TLP: 5.471]. The general form of the proposition, to use Wittgenstein's own words, is "a description of the propositions of *any* sign-language *whatsoever*" such that "every possible sense can be expressed by a symbol satisfying the description. [...] The general form of a

proposition is: This is how things stand" [TLP: 4.5]. The general form of the proposition, the *essence* of the proposition, is thus its pictorial character, its capability of expressing sense – representing reality – and thus of being true or false. As McGinn writes:

The general form of a proposition expresses the essence of a proposition, that is, what all propositions that represent states of affairs have in common. It is given as soon as a language in which we express judgements about the world is given. In acquiring language we have already grasped the general form of a proposition, that is, we have already grasped the whole of logic, the essence of representation as such.²⁵¹

Insofar as logic is everything that is needed to represent reality – to say anything about the world – then logic is a “mirror-image of the world” [TLP: 6.13]; logic is everything that is needed in order for reality to be represented by language; language and reality share the same logical structure, for logical form is indeed the “form of reality” [TLP: 2.18]. Therefore to give the essence of a proposition – to give an account of its pictorial (representative) character – “means to give the essence of all description, and thus the essence of the world” [TLP: 5.4711].

If the one developed above is a correct reading of Wittgenstein’s view of logic, then we have thus reached an explanation of Wittgenstein’s conception of the *unity* of language and logic, of why the problems related to individual logical constants (the problem of logical interconnectedness), as Wittgenstein repeatedly states, are only reflections of the “one great problem” [NB: 6.3.15], the problem of giving the essence of the proposition, and thus the essence of the pictorial character of language and thought.

²⁵¹ McGinn [2006: 240].

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