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Frege and German Philosophical Idealism

1. Frege and the German Idealists

The received view has it that analytic philosophy emerged in reaction to the German idealists, above all Hegel, and their British epigones (the British neo-Hegelians). According to Bertrand Russell, German idealism failed to achieve solid results in philosophy. The distinguished later analytic philosopher, Michael Dummett, saw Gottlob Frege as a key figure in the concerted effort to throw off idealism: "In the history of philosophy Frege would have to be classified as a member of the realist revolt against Hegelian idealism" (Dummett [4], 225).

This paper establishes that while Frege too sought "solid" results in Russell's sense, and on that count qualifies as analytic philosopher, he nonetheless took a radically different view of idealism than did Russell. Frege never spoke against Hegel, Schelling, or Fichte.[1] What's more, like the German idealists, his sworn enemy was empiricism as paradigmatically exemplified, for Frege, by John Stuart Mill (Bertrand Russell's godfather). By contrast with empiricism, which he regarded as "shallow", Frege discussed "the basis of arithmetic [that] lies deeper, it seems, than that of any of the empirical sciences" (Frege [8], §14).[2]

Beyond targeting empiricism and evincing other sympathies with German idealist thinking, Frege actually integrated elements of German philosophical idealism into his logic. When one takes into account the scholarly milieu in which Frege pursued his formative studies, the readiness with which he did this is not difficult to explain. Frege served his philosophical apprenticeship in an academic environment dominated by German idealists. He attended the University of Jena in the 1870s, where the faculty was organized into three informal clubs: One was led by the mathematician Karl Snell; the philosopher Kuno Fischer headed a second group; and the zoologist and popular philosopher Ernst Haeckel oversaw

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[1] Dummett's explanation: "Frege barely troubled to attack Idealism at all; he simply passed it by" (Dummett [4], 225) is not convincing.

[2] This claim is clearly opposed to the message of the manifesto of the Vienna Circle, one of the most important documents of the early analytic philosophy: "There are no depths in science" (Hahn [26], 15).

the third club. Frege belonged to Snell's "Sunday Circle" which met until 1880. Through the mediation of philosophy professor Karl Fortlage, however, this group, "influenced by Schelling and the German Romantics", maintained close contact with Kuno Fischer's group, in which the thought of Kant and Hegel predominated (Kreiser [32], 13).[3] Add to this that Snell was the teacher and intellectual guide of Frege's professor and mentor Ernst Abbe (Snell's son-in-law) and it should come as no surprise that Frege attached high importance to his participation in the "Sunday Circle".

The first Anglophone scholar to point out what Frege's thought owes to nineteenth-century Germany philosophy was Hans Sluga.[4] Sluga argued that Frege followed the philosophical-logical tradition originating with Leibniz and Kant, and which Trendelenburg and Lotze developed significantly just prior to and into Frege's time. Sluga has identified this current of philosophical thought as the tradition of "classical German philosophy." About the same time, a philosophical historian writing in German, Gottfried Gabriel, did much to bring this tradition to light, casting Frege as neo-Kantian (Gabriel [21]).

Advancing beyond Sluga and Gabriel, the present paper reveals that through the mediation of Trendelenburg and especially of Lotze many elements of German idealism found their way into Frege's logic and philosophy. Indeed, albeit clearly intending to transform the philosophy of the time, Trendelenburg and Lotze, while often critical of Hegel, were not anti-Hegelian. Rather, their objective was to reform German idealism.

Under Hegel's influence Trendelenburg, senior to Lotze by fifteen years, began to explore thinking as a process. As a result, he changed the very architecture of the received philosophical logic. As V. Peckhaus explains, "the traditional [Aristotelian] core, the theory of inferences, with syllogistics at its center, was pushed into the background. The new core was the theory of judgment" (Peckhaus [40], 16). Lotze, for his part, was an openly eclectic philosopher who while renouncing Leibniz, undertook to systematize the most pregnant thoughts of Kant, Hegel, Fichte and Schelling (Milkov [38]).

What needs to be borne in mind here is that Trendelenburg and Lotze influenced Frege along fundamentally different lines. Trendelenburg

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[3] See also Sluga [46], 321.

[4] In Germany, this connection was well-known. Bruno Bauch, for example, discussed it in [1], 62. See also Goedeke [22], 116. Scholz [43] too connected Frege's ideas with those of Leibniz and Kant.

looms as a defining influence in Frege's effort to formulate an artificial language that can better express our thoughts, something Frege first presented in the *Conceptual Notation* (1879, see also Frege [17]). Lotze's impact becomes evident mainly immediately after that, when Frege was to make philosophical sense of his *lingua characterica*. As we shall see through the course of this discussion, German idealism exerted a formative influence on Frege both early and late in his philosophical development.

That said, the received view is nonetheless accurate in that there were many respects in which Frege and the German idealists were antipodal. Above all, Frege stressed discursive exactness as opposed to "dialectical transition" from one "characterization or formula" into another (Hegel [27], § 81). Moreover, he did not hesitate to marry mathematics and philosophy: Frege was convinced that philosophy could thereby make mathematics more precise. The classical German idealists, by contrast, as well as such distinguished successors as Trendelenburg and Lotze, counted themselves "pure philosophers" and so refused on principle to incorporate logical or mathematical formulas in their theoretical explorations.

All of this led Frege scholars erroneously to conclude that Frege was a philosophical logician who was radically anti-Hegelian. As has already been adumbrated, Frege borrowed many elements of the philosophy of the German idealists. In what might otherwise strike one as paradoxical, these elements proved instrumental, we shall see, to Frege in achieving rigor and exactness in logic.

2. Anti-Mechanicism, Pro-Organicism

We have noted that while the classical Aristotelian logic started with concepts, went on to treat judgments (propositions), and ended with inferences, Frege's post-Trendelenburgian logic commenced with judgments (Frege [7], 94) – and he had good reasons for this.

What Frege most opposed is the so-called "aggregative" conception of judgments. The mainstream logicians of his time conceived of judgments as complexes of concepts.[5] They "found it difficult to distinguish between a combination of terms which constitutes a judgment and one which constitutes merely a complex concept" (Sluga [44], 483). Frege directed his

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[5] Russell was one among them, especially before August 1900 (cf. Milkov [37]). Beaney ([2], 203) also underlines that Frege's function–argument analysis is quite different from the "decompositional conception of analysis embraced by Moore and Russell". Cf. also Hylton [30].

criticism against these "mechanical logicians," most notably George Boole, who considered thinking a process of pure calculation. Frege found that Boolean logic "represents only part of our thinking; [but] the whole can never be carried out by a machine or be replaced by a purely mechanical activity" (Frege [5], 35).

This opposition to mechanistic philosophical logic has a long tradition in German philosophy. It originated with Leibniz who held that in their analytic predilections, Descartes and Locke went too far. As a corrective, Leibniz reintroduced ideas of Aristotelian metaphysics. The German idealists employed analogous argumentation, as did Hermann Lotze.

Against the mechanistic logicians, Frege advanced a kind of "logical organicism." This found expression in the fact that, similarly to the German idealists, he regularly used biological metaphors in his logic. The following enumeration of Frege's recourse to such metaphors over the years makes manifest his unswerving commitment to logical organicism:

i. In "Boole's Logical Calculus and the Conceptual Notation" (1880/81) Frege maintained that the starting point of his logic is the event of judging. Judging is a process that organically connects the parts of the concept. We can discriminate the elements of the concept as discrete individuals only after the concept is already constructed (Frege [5], 17, 19).

ii. In *Foundations of Arithmetic* (1884) Frege held that arithmetical definitions contain in themselves all ordinal numbers, similar to how seeds contain the trees, and not as beams are contained in a house (Frege [8], § 88).

iii. In *Basic Laws of Arithmetic* (1893) Frege compared arithmetic with a tree "that unfolds in a variety of methods and axioms, while the roots go in the depth" (Frege [12], xiii).

iv. Four years later (in 1897) Frege declares that our "thought is not an association of ideas – no more than an automaton ... is a living being" (Frege [13], 145).

v. And seventeen years beyond that we find him asserting that science "must endeavor to make the circle of improvable primitive truths as small as possible for the whole of mathematics is contained in these

primitive truths as in a kernel [*Keim*].[6] Our only concern is to generate the whole of mathematics from this kernel" (Frege [16], 204–205).

3. Frege's Two Types of Analysis

To grasp the role of Frege's logical organism one needs to recognize that he employed two concepts of analysis. First, following Kant, Frege regarded knowledge in arithmetic as analytic in the sense that we derive it, *deduce* it, from definitions and general laws by logical means (Frege [8], § 3): We shall refer to this as "anlysis$_1$". Frege determined, however, that Kantian derivation (deduction) is also *synthetic*, in the sense that it creates something new, and he drew attention to this point in *Foundations*: "The conclusion we draw from it [the definition] extended our knowledge, and ought therefore, on Kant's view, to be regarded as synthetic; and yet they can be proved by purely logical means, and are thus analytic" (Frege [8], § 88).

Frege's position, we should note, stands clearly opposed to the view, widely accepted (for example, by Hume and Kant) prior to the ascendancy of post-Kantian German idealism, that analytic judgments are epistemologically sterile. Also Wittgenstein, lacking Frege's background in idealism, would call analytic judgments "tautological". Hegel, on the other hand, sharply criticized Aristotle's sterile logic, pleading for logic of content–sort that Frege was to advance too (cf. section 7).

The second type of analysis (call it "analysis$_2$") is *decompositional* in nature. The difference between analysis$_1$ and analysis$_2$ becomes patent when one revisits the previous example of the plant. A plant, to which Frege relates the particular numbers, is synthesized (or analysed$_1$) in a process of photo*synthesis*. We may decompose it, subject it to analysis$_2$, only afterwards, anatomizing it in order to determine, for instance, its composition. The living, existing plant, however, cannot as such undergo analysis$_2$.

While Frege's analysis$_2$ correlates with the scientific analysis, say, in chemistry, his analysis$_1$ is close to the growth and self-maintenance (synthesis) that distinguish biological organisms. Regrettably, many Fregeans

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[6] Apparently for "analytic" reasons, Frege's "*Keim*" was often translated as "kernel", instead of as "germ".

uncritically adopted the received judgment that the master was an "analytic philosopher" pure and simple. As a consequence they did much to play down the pivotal difference between the two alternative senses of "analysis" with which Frege operated, when they didn't simply ignore it.

An additional factor that militates against properly understanding Frege on this score is that he was, without question, also an adept at analysis$_2$. He maintained, for example, that logicians have the task of isolating what is logical from psychology and language, and doing so in the same way that scientists undertake to isolate the elements of chemistry. There is more to Frege's position, however, since "even when we have completely isolated what is logical in some form or phrase from the vernacular or in some combination of words, our task is still not complete. What we obtain," observes Frege, "will generally turn out to be complex; we have to analyse this, for here as elsewhere we only attain full insight by pressing forwards until we arrive at what is absolutely simple" (Frege [9], 6).

In short, the method of decompositional analysis plays a formative role in Frege's philosophical logic. This sort of analysis became especially prominent in his thought five years after he published his *Conceptual Notation*, in particular, when he discovered that numbers are objects and when concepts began to serve a defining role in his logic (Weiner [51], 70). These developments notwithstanding, analysis$_1$ prevailed over analysis$_2$ in Frege's thinking. It is of more than passing interest that this tendency finds a parallel, as we shall see, in the prime role of quantification theory over propositional calculus in his logic (see section 8, below).

4. "Saturatedness": Chemical or Biological Metaphor?

The standard account has it that Frege borrowed from chemistry the metaphor "saturated/unsaturated" which he employs to characterize the relation between function and argument (Potter [41], 13). On this matter, as with those noted in the preceding section, commentators have simply presumed that Frege was "analytic philosopher" – whatever they take this notion precisely to mean. Is this view correct?

Before probing the meaning of "saturated [*gesättigt*]" as Frege utilized it, one should be aware that the metaphor in question derives not

from chemistry but from biology: the German term "*satt*" ("full up") applies to organisms when their striving or desire to eat is satisfied. Be this as it may, scholars invoke this biological term across a wide range of non-biological disciplines: from epidemiology and demography to economics (a market can be "saturated") and mathematics. In chemistry "saturation" is understood as "the point at which a solution of a substance can dissolve no more of that substance"[7] – which is to say it means the achieving of a final stable state by one mass individual as a result of a dynamic process.

Frege however conceived the notion of being *gesättigt* along completely different lines. He held that in logic "the argument does not belong with a function, but goes together with the function to make up a complete whole [*vollständiges Ganzes*]" (Frege [10], 140). In other words, Frege's concept and object are *two* individuals that fuse in order to build up the "organic unity" of a thought – like two cells that merge in order to constitute a germ:[8] one of them "*ungesättigt*", the other one capable to make the first one "*satt*". Frege held that we invariably subvert such a thought, once formulated, by subjecting it to analysis$_1$. Similarly, he maintained that we do not construct a concept by subsuming the subject under the predicate. Rather, concept's different elements (its "characteristics [*Merkmale*]") grow together (*wachsen zusammen*) – are synthesized – so as to form ("build") it.

In an unpublished review of Arthur Schoenflies' book *The Logical Paradoxes of Set-theory*, Frege noted: "The unsaturatedness of one of the components is necessary, since otherwise the parts do not hold together. Of course two complete wholes can stand in a relation to one another; but then this relation is a third element – and one that is doubly unsaturated" (Frege [15], 177). In light of this position it comes as no surprise that when he read Wittgenstein's *Tractatus* Frege questioned: "What is the thing that binds [the objects in a state of affairs]" (Frege [20], 53)? To Frege's way of thinking such a connection should be a kind of concrescence (*Zusammenwachsen*) of the two (or more) objects into one.[9]

This judgment reflects Frege's view that one of the purposes of the function/argument and concept/object distinction is to replace the idea

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[7] "Saturation": http://en.wikipedia.org/wiki/Saturation_(chemistry)

[8] As already mentioned in section 2 (v), Frege's term "*Keim*" was often translated as "kernel" instead of as "germ" (see footnote 6).

[9] It is interesting to compare Frege's solution of this problem with that of Wittgenstein who was much more pronounced defender of analysis$_2$. According to Wittgenstein, the elements of the state of affairs hold together because of their topology alone: no fusion, as well as no mortar that connects them together is needed (cf. Milkov [36]).

that the content of a sentence is composed of constituents that are merely subsumed in one complex. Indeed, the Booleans had "assumed concepts to be pre-existent and ready-made and judgments to be composed from them by aggregation" (Sluga [47], 85). Frege, on the other hand, taught that concepts and propositions are to be synthesized, *created* (cf. section 2, (i)). This idea of "creation" simply does not obtain when one takes "saturation" in the chemical sense: no creation occurs in chemical saturations.

5. Life

Beyond Frege's recourse to the "saturation" metaphor, his organicism appears in a number of other forms. Perhaps the most significant instance stands at the very center of his philosophical logic, namely in the idea that thinking – the subject-matter of logic – is to be understood as embedded in human life.

When we are awake, we cognitively react to the events and situations of the external world, thus making judgments. Moreover, in judgments we advance (*fortschreiten*), asserts Frege, from a thought to its truth-value. This is the case since when we judge in *real life*, we are, as Frege puts it, "serious". In other words, in real life we know that matters have irreducibly practical import and that ultimately our survival is at stake when it comes to judging matters correctly. In contrast, when we *play*, we are not, in Frege's view, serious – we are not preoccupied with real life.[10] In play, our sentences accordingly have no truth-value. What they betoken is simply the exercise of our free will. In such cases what we produce is merely a series of *obiter dicta*, not propositions.

We can see now why judgment played a central role in Frege's logic: he argued in "On Sense and Meaning" that judgment "is something quite peculiar and incomparable" (Frege [11], 165). Judgment is such because it is the event (*das Fortschreiten*) that intrinsically connects logic to life.

Frege went on to claim that when we make judgments, we *strive* for truth. This striving is the "motor" that connects logic with the world: "It is the striving for truth that", as he put it, "drives us always to advance from the sense to the thing meant" (Frege [11], 163).

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[10] Cf. with Friedrich Schiller's famous saying "Life is serious, art is cheerful" from the prologue to *Wallensteins Lager*. That Frege was well acquainted with Schiller's works is clear from his reference to *Don Carlos* (Frege [13], 130). Frege also often opposed "poetry" to "truth" (Frege [12], xxi) with a hint to the subtitle of Goethe's autobiography *Aus meinem Leben. Dichtung und Wahrheit.*

It is evident from the foregoing that, like Hegel's logic and that of Lotze, Frege's logic is markedly anthropological in character. He argued that logic is embedded in human life and as such is a logic of human beings of flesh and blood, not a logic of robots or other machines.

6. Logical Voluntarism

Besides championing organicism in his philosophy of logic, Frege asserted that to make a judgment is to make a choice between opposing values, between truth and falsehood. The judgment *acknowledges* the truth of the content. "We grasp", he declares, "the content of a truth *before* we recognize it as true, but we grasp not only this; we grasp the opposite as well. When asking a question we are *poised* [*schwanken wir*] between opposite sentences" (Frege [9], 7; emphasis added) until we decide, in an act of will, its truth-value.

Frege's terms "before" and "poised" show that judgments are processes.[11] More precisely, a judgment is an act of inquiring whether a thought be true or false. It is based on our intuition – on the feeling that our thoughts constitute either a correct or a mistaken assessment of reality. Frege maintained, moreover, that the process of "accepting one of [the truth-values] and rejecting the other is one act" (Frege [14], 185): an act of exploration, one followed by an act of decision. Frege's activist view of judgment proved of fundamental importance in his logic, and gives us leave to speak of a Fregean "logical voluntarism". This logical voluntarism found expression in Frege's claim that assertoric force is one of the constitutive elements of our articulation of a judgment[12] – a position that reveals another facet of Frege's debt to German idealism, particularly to the philosophy of J. G. Fichte. Wittgenstein, who in contrast to Frege had, as we've noted, no idealist background, promptly rejected this thinking as Frege gave expression to it in his logical symbolism: "Frege's 'judgment stroke' '⊢' ", declared Wittgenstein, "is logically quite meaningless" (Wittgenstein [52], 4.442).

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[11] This point supports Paul Linke's statement that "Frege actually brought psychology into, meaning right inside, his new foundations for logic" (Linke [33], 67). His severe criticism of "psychologism" can be explained with the fact that "he confused the bad psychology which was prospering at the time with psychology in general" (Linke [33], 69).

[12] This conception was developed further by J. L. Austin, the first translator of Frege's *Grundlagen* into English, in the concept of "illocutionary force".

Significantly, Frege was convinced that this voluntarism does not contradict the objectivity of logic that he defended. His conviction on this head appears to derive from the fact that his Conceptual Notation was a language (*lingua characterica*)[13] and so intrinsically connected with Being, understood as an absolute singularity.[14] From this it follows, on Frege's view, that when two persons judge a situation "seriously", in his sense, they make the same judgment. In other words, the "seriousness" secures the objectivity of judgments – a position that Michael Dummett saw fit to label as "realist."

At the same time, however, it is also the case that thinking is possible, according to Frege, only because it originates with the activity of the human will. This, of course, is an anti-realist position, one that exhibits affinities with Kant's epistemology. Indeed, according to Kant himself the knowing person *constructs* his knowledge. This point goes some way toward explaining why Frege didn't see himself as an anti-Kantian logician, and also why among the host of commentators on Frege the majority of those who have a thorough grounding in Kant and in the Kantian tradition (Scholz [43], Sluga [45], Gabriel [21], Carl [3]) have not hesitated to identify Frege as a Kantian philosopher of logic.

7. Logic of Content

Beyond its other debts to German idealist thinking, Frege's logic also paralleled Hegel's project for a logic of content that opposed the formal logic of the Kantian type. Frege articulated his logic of content along two lines:

(i) In *Conceptual Notation* (1879) he sought to establish a *lingua characterica*, not just a *calculus ratiotinator*. This program undertook to present the thinking process in a transparent way, one that would yield a "perspicuity of presentation" (*Übersichtlichkeit der Darstellung*) of it (Frege [6], 88).[15]

Frege aimed to replace inconvenient, ordinary language that develops spontaneously and that manifests many defects, as measured against his new language. He was convinced that his new language would prove to

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[13] We are going to speak more about this in section 7 below.

[14] Many worlds are typical for Russell and Carnap, not for Frege (cf. Milkov [39]).

[15] My translation from German – N. M. This term was often used by the later Wittgenstein.

be a vehicle in which our thought finds its true articulation. Arguably, this project had its roots in *philosophia teutonica*[16] that followed the "Protestant principle which put the world of mind into its own home, so that it contemplates, knows and feels what otherwise lies beyond it, in its own terms" (Hegel [28], 826–7) and doesn't investigate it from outside, through quasi-objective principles. This impelled Frege to investigate thinking according to its own laws, and not with the help of formalist schemes.[17]

In order to present our thinking in its true form, Frege employed the relation of logical signs in two dimensions, something that resulted in his complex conceptual notation. As Frege saw it, "the spatial relations of written symbols on a two-dimensional writing surface can be employed in far more diverse ways to express inner relationships than the mere following and proceeding in one-dimensional time, and this facilitates the apprehension of that to which we wish to direct our attention" (Frege [6], 87) (cf. Milkov [35]).

This original approach to logic made feasible the allegedly impossible marriage between this formal discipline and autonomous philosophical exploration. With its help Frege circumvented logicians like Boole, Graßmann, Jevons and Schröder, all of whom failed to connect logical forms with philosophical content (Frege [6], 88). He also left behind the traditional German philosophers, including those who were well versed in mathematics and logic, like Hermann Lotze and Edmund Husserl, who strictly adhered to the dogma not to incorporate what are merely formal tools as substantive components of philosophical development.

(ii) The second line along which Frege developed his logic of content, beginning in *Conceptual Notation*, was that of logical semantics, which he treated as "judgeable content". In the early 1890s he further developed formal semantics by introducing the idea of the sense of a proposition. This innovation anchored Frege's logic in the world and thus categorically differentiated it from the constructs of the formal logicians. Indeed, his Conceptual Notation was not only logic but also a language that is intrinsically connected with Being[18] understood as an absolute singularity, and also with life.

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[16] Already R. M. Martin saw Frege's logic as following the "German traditions" opposing the English tradition in logic as presented by J. S. Mill and his friends (cf. Martin [34], 8).

[17] This point found expression in the fact that while Kant investigated the "pure reason", Frege's logic explored "pure thinking".

[18] Cf. footnote 14.

8. Intensional Logic

Frege's logic is intensional in that in it a *function* ranges over every *argument* that falls under it, and a *concept* defines every *object* that falls under it: "The concept has a power of collecting together far superior to the unifying power of synthetic apperception" (Frege [8], § 48). And this is not because the objects are spatial or temporal parts of concepts. It follows, rather, from the circumstance that the objects obey the "laws" of concepts. The essential point to note here is that the dependence relation is not immediate or intuitive – it is not realized because of inclusion in the volume of the whole. Rather, it is abstract: it is dependence "from a distance".

The intensional stance that distinguishes Frege's logic also governs the relation between propositional and predicate calculus. As van Heijenoort has noted, "in Frege's system the propositional calculus subsists embedded in quantification theory. . . . In that system the quantifiers binding individual variables range over all objects" (Heijenoort [29], 325).

A similar line of thought had earlier appeared in the German idealists, according to whom the Idea (the "Absolute") determines the characteristics and behavior of all individuals that fall under it with necessity of a law. One way to appreciate the singular nature of this conception is to compare it to the idealist aspect of Spinoza's thinking, which pictures "individuals as mere accidents of substance" (Inwood [31], 304). The fact that German idealist thought-determinations substantiate Frege's new logic[19] should hardly be surprising given that some of his contemporaries who lacked his comprehensive background in German idealism, Carl Stumpf, for example, also elaborated a radically anti-psychologist and anti-Millian philosophy of arithmetic, albeit one which was based on mereological logic (Stumpf [50]).[20]

Significantly, the absolute primacy that Frege's logic of quantifiers accorded the *function* and the *concept* introduced a new emphasis on the role of individuals in logic – indeed, the power of the general term makes it possible to fix the parameters of the particular that falls under it with

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[19] Of course, we would not deny that main inspiration of Frege's logic were Cauchy and Weierstrass, not Hegel or Schelling (cf. Grattan-Guinness [23]).

[20] Interestingly enough, between 1871 and 1874 Stumpf and Frege (both born in 1848) were at the University of Göttingen – Frege as doctoral student and Stumpf as "Privatdozent".

signal exactness. It was in this way that Frege foregrounded the problematic of reference, which was to become defining topic of twentieth-century philosophy of language.

Frege's intensional first-order logic has been subjected to considerable criticism on this point. A prominent recent commentator, Barry Smith, dismisses it as "fantology" (Smith [48]). While no one would deny that first-order logic has great expressive and inferential power, it lacks the resources to treat objects of the real world, such as universals, types, processes, and, we might add, mereological entities. Smith objects that its "universe of discourse consists of particular items" only (Smith [49], 110). Frege's first-order logic is of use only in mathematics, the objects of which are not situated in space and time. For object of the real world, on the other hand, one must have recourse to an alternative to predicate logic, namely a logic of terms.

Over the years, Frege accorded increasing significance to the role of intensions in logic. In "Function and Concept" he replaced the naïve function–argument logic of *Conceptual Notation* with logic of "course-of-value" (*Wertsverlauf*) of concepts. As the very name of this logic suggests, in contrast to the positions of the parts that constitute mereological wholes, the position of an individual (or argument) in a course of value is intrinsically indeterminate: it is "floating" – a factor that Frege symbolized with a curve – until it is identified. In his epistemology Frege spoke about "course-of-images" (*Vorstellungsverlauf*) (Frege [6], 83, 87) that can be determined only through his Conceptual Notation. Apparently, the indeterminateness of the general (the "absolute") was Frege's leading theoretical stance.

Russell readily embraced Frege's new logic, particularly in his idea of "denoting phrases."[21] He regarded the latter as a pivotal innovation, this because denoting phrases can indicate infinite collections of individuals with the help of singular (intensive) concepts; and they can do this precisely because their denotation is undetermined. Russell was convinced that this "discovery" resolves the paradox of infinity.[22] It was not long, however, before Russell detected another paradox: the paradox of classes. Apparently, Frege's logic simply led Russell to relocate paradox from one domain, the realm of infinity, to another, the realm of classes that range over infinite number of individuals (Milkov [37], 54). In other words, the

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[21] Russell first learned Frege's logic of quantification via Peano in August 1900.

[22] To this we would add that Russell was sensitive to the problem of infinity because of his Hegelian past: infinity was, according to Hegel, a prime paradox.

paradox of classes was a consequence of embracing class-concepts that range over infinite numbers of individuals.

Over the last hundred years or so it has turned out that the most effective way to address the "paradox of classes" is to accept multitudes that do not fall under a class-concept that ranges over them.[23] In other words, the paradoxes disappear when we abandon those ideas in Frege's logic that were supported by the philosophy of the German Idealism.

9. Epilogue

The foregoing history provides a considerably richer context than that of the received view for understanding of how Bertrand Russell advanced the project for a new, "analytic" philosophy as a philosophy that can make verifiable progress. Analytic inquiry of the sort he championed achieves its theoretical results step by step, such that if a particular philosophical theory proves false, some components of the theory may nonetheless be preserved – just as elements of refuted theories of science can be preserved.

What this essay has principally striven to make clear, however, is that the philosophical thought had made important progress along these lines long before Russell. The philosophy of the German idealism had introduced, decades earlier, new ideas that survived the demise of the grand theories that originally framed them. Later, succeeding generations thinkers who subscribed to a quite different methodology and who styled themselves as "analytic" philosophers were to reintroduce these originally idealist notions in totally different programs of inquiry. This explains much of the difficulty in recognizing these notions as they figure in leading currents of contemporary philosophical discourse. To make these origins and their philosophically pregnant implications explicit is the task of the historian of philosophy.

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[23] See Gödel [24], 135.

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