Frege, Gottlob (1848–1925)

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Friedrich Ludwig Gottlob Frege was born on November 8, 1848, in Wismar, Grand-Duchy of Mecklenburg-Schwerin, Germany. Frege’s father, Karl Alexander Frege, studied Protestant theology and founded a private girls’ high school in Wismar. In 1846 he married one of the teachers of the school, Auguste Bialloblotzky, who was descended from an old Polish aristocratic family, who left Poland for Germany in the seventeenth century as a result of religious persecution. After her husband’s death in 1855, Auguste directed the school.

Frege attended die Große Stadtschule zu Wismar. In April 1869 he matriculated at the University of Jena. Between 1871 and 1873 Frege lived in Göttingen, where he received his PhD with the dissertation “On a Geometrical Representation of Imaginary Figures in a Plane.” The supervisor of Frege’s dissertation, Ernst Schering, was a former student of Carl Friedrich Gauss and editor of Gauss’s papers. Immediately after that, Frege started working on his Habilitation (second dissertation) in Jena. In 1874 he wrote and defended his Habilitation, titled “Methods of Calculation Based on an Extension of the Concept of Quantity,” and became *Privatdozent* for mathematics with the right to teach in the University of Jena.

Frege received decisive support in his career from Ernst Abbe, who was one of his professors in Jena. In the spring of 1874, however, Abbe left university teaching in order to manage the Carl Zeiss Corporation in Jena, which applied the newest achievements in mathematics and physics to the production of, among other things, the best microscopes of the time. Abbe’s retirement from his teaching post partly explains the speed in which Frege was promoted in the University of Jena—he was at least partly to replace Abbe as an university teacher. In 1879, after publishing his *Begriffsschrift*, Frege became an extraordinary professor. In 1896 he was appointed Ordinary Honorary Professor of the Carl-Zeiss Foundation, which had been set up by Abbe ten years earlier. In the first years after his Habilitation, Frege’s income was considerably below the average in Germany. Starting from 1886, however, he regularly received an anonymous donation from the Carl-Zeiss Foundation, which helped him to reach the middle-class standard.

In 1887 Frege married Margarete Liesberg (1856–1904), who came from a little town near Wismar. They had no children. In 1908 Frege received the guardianship of two children, Alfred

(1903–1944) and Toni (1905–1990) Fuchs. Alfred grew up with Frege, while his sister lived in a vicarage (*Pfarerrhaus*) near Jena. Between August 1921 and August 1922, Frege adopted Alfred officially, so that the boy became his legal successor. After 1902 Frege had a housekeeper, Meta Arndt (1879–1942), also from Mecklenburg.

When he retired in 1918, Frege sold his house in Jena and moved to Bad Kleinen (the anonymous financial donation from Wittgenstein that Frege received at the beginning of 1918 was also helpful in this), some 25 kilometers south of Wismar. Toward the end of his days, Frege started building a house in Pastow, near Rostock. Frege died on July 26, 1925, in Bad Kleinen before the house was finished. When it was ready, his son Alfred moved in as well as Frege's former housekeeper Meta Arndt, who lived there until her death in 1942.

Beside his teaching duties, Frege was a member of the Deutsche Akademie der Naturforscher (Leopoldina), Jenaische Gesellschaft für Medizin und Naturwissenschaft, and the Deutsche Mathematiker Vereinigung, founded by his professor in Jena, Hermann Schäfer. From 1899 to 1900 Frege was deputy co-treasurer at the latter society. In 1919 Frege joined the conservative Deutsche Philosophische Gesellschaft.

Frege's work as a university teacher was judged differently over time. In the beginning it was praised, in his later years it was sharply criticized. The negative tendency grew after the worst crisis in his life in the mid-1900s: in 1902 he saw that his logic was inaccurate, in 1904 his wife died, and in 1905 Ernst Abbe died. In the same year, Frege had serious nervous symptoms for which he had to undergo a cure. As a result, his lecturing suffered.

One of the effects of these developments was that the authorities of the University of Jena grew increasingly critical of Frege. The trustees of the University declined suggestions to officially celebrate, first, Frege's 60th birthday, then his 75th anniversary. The declared reason was that Frege was a bad teacher having a very small number of students. Many of his lectures were cancelled because no students came at all. In truth, Frege's lectures were difficult to follow, but, obviously, good in their logical quality (2004), which is also confirmed by the fact that Frege's fellow professors (Otto Liebmann and Rudolf Eucken, among them) sent their sons to study with him. Frege's most prominent student was Rudolf Carnap, who between 1910 and 1914 attended three courses of his lectures.

Politically, Frege was an admirer of Bismarck and his national-liberal party. His ideal was a strong Germany, an iron regency, a strong army, and a powerful fleet to guarantee security so as to develop the economic, technical, and cultural power of Germany as well as to keep social democracy, the most dangerous enemy at home, under control. In 1918 Frege composed the draft "Vorschläge für ein Wahlgesetz" (Suggestions for an Electoral Law) and sent its type-copies to some members of the local assembly, unfortunately, to no avail. It proposed a new mode of elections, which would take into account every single vote; in this way, all political streams were to be represented in the elected body. Being already a terminally ill man who was also facing financial ruin, in 1924 Frege wrote down a political notebook ([1924] 1994) in which he expressed sympathy with Adolf Hitler and a radical form of anti-Semitism.

As a young scholar, Frege evidenced the flourishing of mathematics of his time, which gave birth to even new calculi. The problem was that nobody knew whether they were true or just games. Frege saw it as his task to put mathematics on a sound foundation, which would make it real science that achieves truths. (Later, in *Remarks on the Foundations of Mathematics* (1956), Wittgenstein would attack exactly this point: mathematics does not need foundations—let mathematicians freely develop their calculi!)

To this purpose, in his first book, *Begriffsschrift* (*Conceptual Notation*, 1879), Frege advanced an ideal—or perfect—language of the "pure thought" that is better adjusted to serve science and mathematics. (As a matter of fact, Frege built up this program on Leibniz's idea, which he was acquainted with through the mediation of Adolf Trendelenburg.) Ordinary language hasn't the resources to do this. This was also Frege's new logic: "a formula language, modelled upon that of arithmetic," as the subtitle of Frege's book has it.

The objective of Frege's language—logic was a kind of ideography—was to make it graphically perfect in order to show how we think. Frege believed that "the spatial relations of written symbols on a two-dimensional writing surface can be employed in far more diverse ways to express inner relationships than the mere following and preceding in one-dimensional time, and this facilitates the apprehension of that to which we wish to direct our attention" ([1882b] 1979: 87). The new language shows how "pure thinking" works in a perspicuous way, yielding it a *Übersichtlichkeit der Darstellung* (perspicuity of presentation—a term, later widely used by Wittgenstein). The proofs and the inference chains in this language are to be completely formalized and the appeal to intuition to be cut down completely.

Importantly enough, this was a program for a real, but perfect, *language*. Similarly to the ordinary language, it is intrinsically connected with thinking's content (in this connection, in

Conceptual Notation, Frege introduced the term "judgeable content"). It is also inextricably connected with the Being, understood as an absolute singularity. Frege's ideal language is not purely formal, neither is it just a calculus. It is a project for *lingua characterica* not for *calculus ratiotinator* (Heijenoort 1967).

Apparently, exactly this complex task, to create a perfect language, modeled upon the language of arithmetic (not on algebra, as it was by George Boole) brought Frege to discover the universal quantifier. This new technique made the expression of generality an easy task for the first time. It conferred on logic more expressive power and so made both the Aristotelian syllogistic and George Bool's "algebra of logic" obsolete. Following this track, Frege also replaced the conventional subject-predicate logic with function-argument logic. In *Conceptual Notation* he also developed a propositional logic and built up a formal *system* for the first time.

Frege's new logic was adopted first by Peano and then by Bertrand Russell in *The Principles of Mathematics* (1903). However, it became generally acknowledged only after Alfred North Whitehead and Russell's *Principia mathematica* (1910–1913) was published.

Conceptual Notation was badly received. Some of the reviewers complained that the baroque symbolic made it difficult to understand. Frege's acquaintance from his Göttingen time, Carl Stumpf, advised him to publish his ideas in a nonformal style. Frege followed this tip and wrote his next book, *Grundlagen der Arithmetik* (*The Foundations of Arithmetic*, 1884) without technicalities. In it he applied his new logic to the theory of natural numbers. Directing attention to arithmetic was not accidental. Mathematical analysis rapidly developed in these years and the role of arithmetic as a model of human knowledge increased. This development was especially prominent in Göttingen, which was dominated by Gauss's tradition.

In order to meet this objective, Frege defined the primitive ideas of arithmetic in terms of logic. Numbers are nothing but concepts that can be explored with the resources of conceptual notation. Furthermore, Frege built up the elementary arithmetic as an axiom system. In this way he articulated his form of logicism, which consisted in reducing elementary arithmetic to logic. Later Russell advanced a similar program in *The Principles of Mathematics* (1903) developed independently from Frege. In contrast to Frege, however, Russell tried to reduce the whole of mathematics to logic, while Frege maintained the Kantian view that geometry is based on synthetic a priori truths and so is not reducible to logic.

While working on the book, Frege gradually realized that he was developing an interdisciplinary study that explores problems of philosophy and mathematic together. Some interpreters also consider *The Foundations of Arithmetic* as pioneering in the philosophy of mathematics—despite the fact that Kant and Jacob Friedrich Fries also produced works in this realm. *The Foundations* is also famous for the devastating criticism of the mainstream theory of numbers, especially that of the empiricists John Locke and John Stuart Mill. Moreover, the interdisciplinary approach induced Frege to converge philosophy of mathematics with philosophy of language. Questions of the form "what is the number 7?" are to be answered in the form "what is the sense of the sentences in which 7 occurs?"

Another stance Frege developed in the mid-1880s was his radical form of anti-psychologism. It held that logic investigates pure thinking and not the psychology of thinking. This position of Frege was highly influential, affecting, among others, the work of Edmund Husserl. It, however, didn't start with Frege but with his philosophy professor in Göttingen, Hermann Lotze (Gabriel 2002). What was new with Frege was that he connected the anti-psychologism stance with the uncertainty of the ordinary language. The two aberrations are to be jointly eliminated in the logic of pure thought achieved via ideal (perfect) language.

At the beginning of the 1890s Frege made further refinements in his logic. Above all, he replaced the basic function-argument distinction with the concept-object distinction. Object is anything that has a meaning as a singular term (proper name). In contrast to Russell, who was guided by epistemology, Frege's objective was not to construct language with building blocks that come from experience. This makes it clear why, in his logic, proper names are not to be simple. In Frege's logic, both concrete entities, such as chairs, and abstract entities, such as numbers, shapes, proofs, directions, and classes (Frege adopted Cantor's naive theory of classes) are objects. It follows that numbers already exist in the "third realm" and wait to be discovered like the objects of any other science do. It also follows that numbers are not simply mechanically deducible from the axioms of arithmetic. In Kant's sense, they are both analytic and synthetic (1884: § 88).

Concepts, as well as relations and functions, are radically different from objects. (Similarly, first-order concepts are radically different from second-order concepts.) They are unsaturated expressions that became saturated when they receive a fixed object as an argument. Singular terms are saturated expressions as well.

Besides having meaning, singular terms also have sense. Moreover, one and the same object can correspond to different proper names that have discrete senses. It follows that an identity assertion, for example, that "Phosphorus is Hesperus," can be informative—it informs us that two proper names with different senses, "Hesperus" and "Phosphorus," have the same meaning, the planet we call today Venus.

The sense of sentences, in contrast to proper names, is the thought that can be defined as the thing that can be true or false. We communicate with other persons via transporting to our collocutors the same—identical—thought. The decisive point here is that language is guided by living persons who determine, in acts of will, what is true and what is false. They make this by way of their judgments, which are expressed in assertions (*Behauptung*) made with assertoric force. In the last resort, only "whoever understands a proposition uttered with assertoric force adds to it his recognition [*Anerkennung*] of the truth" (1980: 79). This is a speech-act theory of truth, or a recognition theory of truth, with a clear connection to the redundancy theory of truth. Recognized, or admitted ([1892] 1984: 165n), is the true-*value* of the assertion. The ethical "coloring" in this term, value, is not accidental. It goes together with Frege's insistence that one is to "seriously take the sentence to be true or false" (162). Exactly this "seriousness" connects language with reality. Sentences in a novel are not true or false. Since they are part of a fiction, they are not immediately connected with reality: they are part of a *game* to produce "aesthetic delight," not information.

Frege's conception also makes it clear why he opposed the aggregative conception of judgment. Judgment is not a complex of ideas, as maintained by Johann Friedrich Herbart, Boole, and Mill. Judging is a process of *organically* connecting the parts of the concept. Thinking is not just an association of ideas, as Hume maintained. The connection between concept and object is not chemical but biological—it is organic.

Thoughts are articulated in sentences, not in words. This was Frege's famous "context principle": "[do not] ask for the meaning of a word in isolation, but only in the context of a proposition" (1884: x). We do not also think in mental images. Mental images are not constitutive for the meaning of words but the *role* the words play by determining the truth-conditions of the sentences in which they occur. That is what makes the context principle unavoidable. Thoughts are something objective but immaterial—such as the planetary axial tilt or the equator. Later Frege maintained that thoughts pertain to the "third realm" of being, the first realm being the material world and the second the world of imagination ([1918] 1984: 363).

This position, consequently followed by Frege, made logic and the logically informed philosophy of language the leading philosophical discipline. *This* was the turn in philosophy made by Frege: prima philosophia is neither epistemology, which was the leading discipline in the modern philosophy, nor ontology, as it was for Aristotle and Aquinas. There is, however, an alternative interpretation that suggests Frege was an epistemologist since his objective was the truth of mathematics. He also strived to make mathematics an exact *science*.

Between 1891 and 1903 Frege worked on his magnum opus, *The Basic Laws of Arithmetic*. It closely followed the project of *The Foundations* for deriving elementary arithmetic from logic, despite the fact that it had fewer axioms and more inference rules. The second volume of the book explored the theory of cardinal numbers and started analyzing the theory of real numbers but he didn't finish it. Apparently, Frege planned a third volume which was never written. The reason for this was that when the second volume was in press, Frege received a letter from Russell from June 16, 1902, that informed him about the paradox of classes. This meant that the naive set theory, on which Frege's logicism was based, leads to mistakes. It followed the worst crisis in Frege's life. Significantly, nobody (neither Peano, nor Couturat, nor Whitehead, for example) was as pessimistic in their assessment of the consequences of the paradox as Frege was. Apparently, this followed from his main objective—to achieve ultimate *truth* in mathematics.

In the years between 1904 and 1917, Frege principally stopped creatively writing and concentrated mainly on polemical works. Notorious is a squabble between Frege and his colleague, and neighbor, the Jena mathematician Johannes Thomae ([1906] 1984). More acknowledgements came from Cambridge (England). In 1912 Russell invited Frege to lecture at Cambridge, in 1911 and 1912 Wittgenstein visited him in Germany and had long discussions with him. In 1912 Russell's former student, Philip Jourdain, published a paper on Frege and in 1915 translated into English and published in *The Monist* the "Introduction" to the *Basic Laws.* Between 1913 and 1920 Frege and Wittgenstein exchanged letters and in 1919 Wittgenstein sent him a manuscript of his *Tractatus*.

Apparently, these clear signs of recognition motivated Frege to restart work in logic preparing a book that was to expose his logical-philosophical-mathematical system. Earlier, Frege made three unsuccessful attempts to display his ideas on logic: “Logic” (1897), “Introduction to Logic” (1906), and “Logic in Mathematics” (1914), all of them published posthumously in 1979. The result was his unfinished “Logical Investigations” (1918–1923), the first three chapters of which were published as articles in a philosophy journal (1984: 351–407). There are some new ideas in them (perhaps, formulated in connection with his dialogue with Wittgenstein) and also some changes in his style.

Around 1923, however, Frege became convinced that it was a grave mistake to try to found his philosophy of arithmetic on (naive) set theory. To unify mathematics, analysis and theory of numbers included, his philosophy must be based on geometry and intuition. In other words, Frege returned to the old Kantian view that mathematics is a synthetic a priori science based on intuition ([1924/1925] 1979).

According to Michael Dummett, of all philosophers, perhaps of all theorists of any kind, Frege pursued the most extraordinarily single-minded course (1981: 6). Today, however, we know that Frege was not a solitary scholar at all—he was deeply embedded in the mathematics and the philosophy of his time and place. His former teacher in Wismar and friend in Jena, Leo Sachse, was Herbart’s follower, which makes it probable that Frege adopted his understanding of numbers as concepts and also his interpretation of the idea of existence from Herbart. Without any doubt, however, philosophically, Frege learned most from his professor in Göttingen, Hermann Lotze. This is especially well documented in Frege’s “17 Key Sentences to Logic” ([1882a] 1979), which are nothing but a conspectus of Lotze’s “great” *Logic* (1874). The anti-psychologism, the strict discrimination between genesis (psychology) and value, the project for reducing mathematics to logic, and much more were already formulated by Lotze.

Frege’s new logic remained unnoticed and unappreciated for a long period of time. His theory of quantification was first adopted by Peano. Russell discovered it via Peano and used it first in his *The Principles of Mathematics* and ultimately with Whitehead in *Principia mathematica*. Only then Frege’s ideas from *Conceptual Notation* became mainstream logic. Very helpful for his reception was Russell’s appendix on Frege published in *The Principles* (1903: 501–522).

Frege’s influence in philosophy came much later. It started with the publishing of Wittgenstein’s *Tractatus* in which the “Preface” declared that he was profoundly “indebted to Frege’s great works.” Through the *Tractatus*, Frege’s ideas also became leading in the Vienna

Circle—despite the fact that his philosophy of language was often treated there as philosophy of science. Frege's real influence on analytic philosophy took place only after the Second World War. In *Philosophical Investigations*, Wittgenstein transformed Frege's insistence that not the mental images but the role of the words determine their meaning into the statement that the meaning of the word is its use in the language (1953: § 43). The Oxford ordinary language philosopher J. L. Austin translated Frege's *Foundations* into English and advanced his speech-act theory following Frege's idea of "assertoric force." Gilbert Ryle developed himself as a crypto-Fregean, criticizing the "Fido"-Fido theory of meaning, namely, that the meaning of sentences can be only contextually determined. The real turn in Frege's influence on analytic philosophy, however, came after Dummett, who started as a Fregean in this Oxonian milieu, published his first book on Frege (1973). In Dummett's interpretation, Frege was the sole founding father of analytic philosophy, which can be defined in terms of Frege's philosophy of language.

It only remains to specify what kind of analytic philosopher Frege was. Since the analytic philosophy of G. E. Moore and Russell started as radically anti-Hegelian, according to Dummett (1967: 225), Frege, too, but much earlier, instigated a realist revolt against Hegelian idealism. Dummett explains the fact that Frege never attacked German idealism, but massively criticized J. S. Mill's empiricism, with the words that he simply passed it by. New investigations, however, suggest a different story (Milkov 2015). As a young scholar Frege, together with Ernst Abbe, took part in Karl Snell's Sunday circle in Jena. The group was influenced by Schelling and the German romanticists, and had contacts with the Kuno Fischer group, which was influenced by Kant and Hegel. These contacts can be clearly discerned in Frege's logic. Despite the fact that it consequently pursued exact results in logic, and was thus antipodal to Hegel's "fluent" dialectics, Frege's logic also decisively fought the formalist, mechanicist understanding of logic and opposed it with logic with contents. He also understood language as an activity of living persons, defended the organicist treatment of the glue that connects concept and object, and much more. These were stances typical of German idealism.

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