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O MALE FACTUM: RECTILINEARITY AND KEPLER'S DISCOVERY OF THE ELLIPSE

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Introduction

In the *Mysterium cosmographicum* of 1596, the twenty-five-year-old Johannes Kepler rashly banished straight lines from "the pattern of the universe". Straight lines "scarcely admit of order", Kepler wrote, since their homogeneity does not allow the specification of privileged locations by which one can construct a spatially ordered cosmos. Hence, God Himself could have no use for lines — only centres and spheres — in laying out the structure of this "complete, thoroughly ordered, and most splendid universe".[1] Twenty-five years later, Kepler reissued his *Mysterium* with additional notes and revisions. To the passage repudiating lines, he appended a note remarkable for its exclamatory tone:

> [*O male factum*.] O, what a mistake! Are we to reject them from the universe? ... For why should we eliminate lines from the archetype of the universe, seeing that God represented lines in his own work, that is, the motions of the planets?[2]

In the years between editions, Kepler learned that God had use for straight lines, after all. As Kepler had discovered, they were necessary to lay out the elliptical orbits of the planets. Straight lines were essential parts of the universe: they were elements in God's transcendental archetype of creation. The second edition's note is opaque, however. It does not indicate how the motions of the planets demonstrate God's need for straight lines. Why, then, did Kepler come to lament the rashness of his youth?

The answer lies deep in the details of Kepler's discovery of elliptical orbits in 1605. Kepler struggled to find an empirically adequate description and physically plausible explanation of Mars's path through the heavens. He realized, however, that his originally spherical notion of direction was insufficient to reconcile descriptions and explanations of the planet's motion. Crucially inspired by the "magnetic philosophy" of William Gilbert, Kepler adopted an oriented conception of space, which finally allowed a plausible mechanism to be constructed for elliptical motion — the true path of the planet. Yet, this oriented space required the stipulation of straight lines. Without straight lines, even God could not construct the planets' elliptical orbits.

The dissolution of the Aristotelian heavenly spheres was crucially important in the development of modern physics and astronomy. Equally important, however, was the replacement of the spherical conceptions of space that formed the foundation of Aristotelian science in favour of a rectilinear spatial geometry. In terrestrial physics, for instance, the move toward a rectilinear conception of inertia marked an epoch in scientific development, but a move towards rectilinearity in the heavens

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was also a distinctive advance of the early modern period. Kepler's discovery of the elliptical orbit represented an important step in this regard, since it crucially depends on a rectilinear conception of direction. Studying the history of Kepler's reasoning demonstrates the close integration of the conceptual move with the theoretical development of the physical sciences. Conversely, recognizing the move toward rectilinearity reveals important details of Kepler's struggle to reconcile geometrical descriptions and physical explanations of the Martian orbit. The examination of the conceptual underpinnings of Kepler's thought augments our understanding of the development of his astronomy.

This paper, therefore, makes contact with several discussions in the literature surrounding early modern science in general and Kepler in particular, such as the move towards rectilinear conceptions of space, conceptual change in the physical sciences, and the development of astronomy. For example, it picks up the themes of Jammer's and Koyré's broad treatments of spatial concepts, but relates conceptual development to physical reasoning, rather than metaphysics.[3] It thus parallels DiSalle's examination of spatial concepts in the move from classical mechanics to relativity, but addresses the period before Newton.[4] Similarly, the discussion supplements Ruffner's suggestion that Kepler began treating the motion of comets rectilinearly sometime between 1602 and 1618 by showing exactly how that novel description reflected the emergence of a fundamentally rectilinear conception of space in Kepler's astronomy.[5] Several authors have discussed Kepler's attempts to reconcile geometrical hypotheses with physical causes, but they have not addressed the significance of a rectilinear conception of space in the actual process of physical confirmation of the elliptical orbit. One exception is Goldstein and Hon, who do relate an important conceptual move in Kepler to his physics, though their concern is the move away from material orbs, not toward rectilinear space.[6] Additionally, the close examination of Kepler's final steps to the ellipse below fills some of the interstices in the many reconstructions of Kepler's astronomy. In particular, this paper depends heavily on a long letter Kepler wrote to David Fabricius, eventually dated 11 October 1605 and used as material for the *Astronomia nova*, in which many passages from the letter appear almost verbatim.[7] The significance of this letter has been noted by several authors, but it has not been studied in extensive detail or completely translated.[8] An extended exegesis, however, is left for elsewhere.

Two Desiderata: Descriptions and Explanations

Throughout his extended attempt to discover the Martian orbit, Kepler sought two desiderata for his models of the planet's motion. First, an orbital model had accurately to describe phenomena. That is, he desired a way to calculate planetary positions that agreed with what was actually observed in the sky, in the future, present, or past. This included deriving a longitude and latitude of a planet for a given time. None of Kepler's predecessors or contemporaries would have disputed that this was a central goal of astronomy. However, Kepler was also interested in the physical reality of

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his hypotheses, including the planetary distances they predicted. Distances could be checked against observations, at least indirectly, so a planetary model could not be considered an accurate description of phenomena if it did not predict proper distances. Thus, Kepler required that an hypothesis agree with observations in three respects: longitude, latitude, and distance.

Second, Kepler required plausible physical explanations for his astronomical hypotheses. Kepler's Protestant Neoplatonism implied that, since man is created in God's image, human knowledge can approximate divine understanding and, via diligent effort, continually improve that approximation, just as straight lines can asymptotically approximate curves.[9] Therefore, God's design of the universe, including the causes of planetary motions, must be comprehensible, at least in the fullness of time. As a result, Kepler sought an astronomy "based upon causes".[10] He attempted to explain, as well as describe, the planetary orbits.

This is not to say, however, that Kepler presumed that a cause he proposes is *the* cause of the observed behaviour. He believed that it is always within God's power to bring about the phenomena in a different way. Still, Kepler thought, God's action is amenable to human grasp, so the cause must be something accessible to reason *in principle*. Consequently, hypotheses that do not admit reasonable explanations cannot be realized. So, when Kepler suggested a physical explanation for a hypothesis, he sought to demonstrate only that the hypothesis is *compatible* with human reason, and thus within the realm of possible truth. Kepler only required that a hypothesis is physically *plausible* — that it could be explained in a humanly comprehensible manner.[11] In general, Kepler's discussion of physical causes is in the hypothetical voice. He leaves aside the question of whether the proposed explanation really accounts for the phenomena.[12]

Still, Kepler sometimes blurs this fine distinction. A case in point is Kepler's talk of "planetary minds".[13] Often it seems that Kepler is really attributing spiritual minds to the planets. He then implies that the minds are responsible for moving the planets about. One might think, then, that the minds are to be taken as the causes of planetary motions. This is not Kepler's intended meaning. Instead, Kepler uses minds as stand-ins for physical mechanisms he does not understand. This is coherent if we remember that Kepler is only trying to test the plausibility of his hypotheses. Could an ellipse, Kepler asks, for example, be comprehensibly constructed (other than by mere stipulation)? The easiest way to answer this is to assume that the planet itself is rational. If the planetary mind has a method to 'measure' its position and 'deduce' an elliptical movement, then the resulting path can be rationally constructed. If this is the case, then the ellipse is a possible path for the planet. The unknown *real* cause of the motion will *a fortiori* be rationally comprehensible, as well.[14] Conversely, if a planetary mind *cannot* devise the path, the model itself becomes suspect.

Ultimately, an elliptical orbit would satisfy both of Kepler's desiderata. Showing that the ellipse accurately described the Martian longitudes, distances, and latitudes was a long, difficult project. This descriptive project, though, was completed early in 1605. The explanatory project, however, presented its own, separate difficulties,

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which were not resolved for several months. We now turn to that story as it developed in the summer of 1605.

Explanatory Problem: "Respecting the Sine"

In early 1605, in the midst of testing yet another of his many hypotheses, Kepler found that correct planetary distances were related to the versed sine of the eccentric anomaly. Given some point in the planet's eccentric orbit, Kepler could say how far Mars is from the Sun. The eccentric anomaly, meanwhile, could be derived by tabulation from the mean anomaly, the measure of time. In other words, Kepler could derive accurate distances for any given time. By 1605 Kepler had also guessed, almost by a process of elimination, that the Martian orbit was elliptical, though he had not yet conceived a geometrical model that could generate that ellipse. Now, the assumption of an elliptical orbit provided enough constraints to construct the orbit, since the geometrical properties of an ellipse allowed Kepler to relate the distance to longitude. The resulting elliptical model satisfied Kepler's first desideratum. It allowed the computation of observed longitudes "to the nail", and, while the values for distance and, consequently, latitude were "somewhat more lax", they were within observational error.[15] Subsequent observations might adjust his orbital parameters, but the ellipse was an accurate description of the true planetary orbit. Kepler had achieved empirical adequacy.

Still, Kepler remained unsatisfied. He still needed a plausible explanation of the ellipse. Since the *Mysterium*, Kepler had assumed that the motion of a planet could be explained by the effect of two forces. First, an *anima motrix* emanating from the Sun carried the planets about the zodiac. This "extrinsic force" is something similar to both light and magnetism (Kepler's description changes over time) whose force decreases in some relation to distance. By 1605, Kepler had settled on a magnetic action emanating from the Sun that decreases in proportion to distance from the Sun. Since, at the time, Kepler generally assumed that force is proportional to speed, he concluded that there was a direct proportion between distance and the planet's 'delay' in an arc of orbit. That is, the time a planet took to traverse equal (small) arcs of the orbit was proportional to the distance the (small) arc was from the Sun and, therefore, to the (small) area of the circular orbit swept out.[16]

A planet's general revolution around the Sun could be explained by this "magnetic" force. However, planets do not move at constant speed around the Sun. Instead, they speed up and slow down as they approach and recede from the Sun. Kepler attributed this change in speed to a second power, a *vis insita* inherent in the planet itself, which somehow regulates a planet's distance from the Sun.[17] Kepler assumed that the *vis insita* was something like a magnetic attraction and repulsion to the Sun, which was supposed to be some sort of magnetic monopole. Thus, if the planet could 'magnetically' move itself to the correct distance, the Sun's "extrinsic" power would carry the planet around at the proper speed.

When he constructed the elliptical orbit, Kepler still did not have a proper notion

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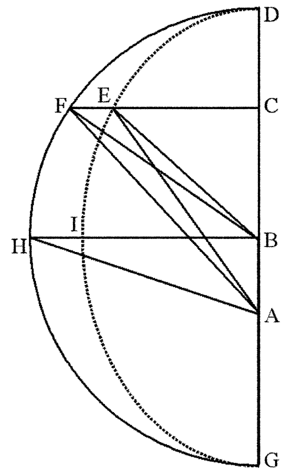

FIG. 1.

of the *vis insita*. Hence, he had set aside consideration of causes and worked directly from the geometrical properties of the ellipse. Once he returned to thinking about causes, however, he found that the ellipse could not easily be accounted for by his *anima motrix* and *vis insita*. The elliptical orbit required that the 'measure' of the planet's approach toward the Sun is the *sine* of the eccentric anomaly. As Kepler would later explain, the small "incursions" from the circumscribing eccentric orbit to the ellipse increase proportionally to the sine of the eccentric anomaly.[18] Thus, in Figure 1, when the eccentric anomaly is angle *DBF*, the planet will be found at *E*, such that the perpendicular approach *FE* is to the greatest approach *HI* as the perpendicular *FC* is to the (unit) radius *BD*. The perpendicular *FC* 'measures' the approach *FE*. In other words, the ellipse requires the *vis insita* to vary according to a magnitude that is always parallel to itself, the perpendicular to the apsidal line.

The perpendicular is also important to locate the planet in the ellipse. Once Kepler had discovered that planet's distance from the Sun varied as the versed sine of the eccentric anomaly, he assumed that the eccentric anomaly would actually measure the angle to the planet itself. Upon testing this assumption, however, he found that the orbit produced was not elliptical. Instead, the properties of the ellipse required the distance to be projected onto the perpendicular to the apsidal line dropped from the point on the eccentric circle corresponding to the eccentric anomaly (i.e., at *E* in Figure 1). In other words, the planet had to move toward and away from the Sun *as if* it were moving along the perpendicular. The "incursions" from the circumscribing eccentric are always directed along the perpendicular.

It was not obvious, however, how a magnetic *vis insita* could bring about such a movement of the planet:

> But there is also something else that I find wanting in this hypothesis: because striving to the point of insanity I am unable to produce the natural cause why Mars,

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> to which libration in the diameter is attributed with such great credibility (indeed, the thing was reducing so beautifully to magnetic virtues for us), should rather want to go in an ellipse or some path close to it. Nevertheless, I think magnetic virtues may not always respect the sine, but something somewhat different.[19]

If magnetic forces were responsible for the elliptical path, Kepler thinks, the planet should "librate in the diameter" of an epicycle that always coincides with a radius to the Sun. Yet this does not produce an ellipse,[20] which he knows to be the correct orbit. Kepler does not understand how "magnetic virtues" could be compatible with an elliptical orbit, since he does not see how the planet can both measure and find itself along a perpendicular to the apsidal line. He cannot fathom how the planet can be made to "respect the sine".[21]

In fact, the difficulty runs deeper than the incompatibility between orbital geometry and magnetic explanation. Even considering the planet as a *mind* (the limiting case of physical plausibility), it was not clear how it could find its way along the ellipse. Like most of his contemporaries and predecessors, Kepler adopted a *centred* or *spherical* concept of direction in order to describe astronomical phenomena. In this conceptual framework, directions are specified and interpreted in relation to a stipulated centre. Two spatial 'pointings' (of objects, motions, etc.) are described as being in the same direction if they bear the same relation to the spatial centre. In effect, this entails that a direction is conceived as a deflection from a radius to the centre. Two rays, for instance, forming similar angles with radii to the centre are described as "pointing in the same direction".

As an illustration, consider Copernicus's "third motion" of the Earth. This 'motion' is a direct consequence of a centred concept of direction. As the Earth orbits the Sun, its rotational axis stays roughly parallel to itself, always pointing toward the same region of the heavens near Polaris. Since the Earth's axis is inclined to the ecliptic, this entails that the axis is tilted toward the Sun at the summer solstice and tilted away from the Sun at the winter solstice. Copernicus describes this behaviour in relation to a stipulated centre: the centre of the Earth's orbit, a point near the Sun. Since the axis changes its relation to this point — sometimes tilting toward it, sometimes away — Copernicus asserts that the *direction* of the Earth's axis is different at different times of the year. In other words, the Earth's axis changes direction. It *moves*.[22] Copernicus labels this change of direction the Earth's "third motion". But this description of the phenomenon *as a change of direction* depends on the concept of direction used to specify and compare directions.

The situation for Kepler is precisely analogous to Copernicus's "third motion". In order to "respect the sine", the planet has to recognize a linear magnitude that is always pointed parallel to itself, perpendicular to the apsidal line. But Kepler, like Copernicus, conceives of direction in relation to a spatial centre, so this remaining parallel is actually conceived of as a continual change of direction — a "turning" — since the perpendicular to the apsidal line bears different relations to the spatial centre (in this case, the Sun) at each point in the orbit.[23] In other words, Kepler

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supposes that "respecting the sine" requires the planet to recognize a magnitude pointing in an infinity of different directions that constantly change at an unknown, varying rate. Kepler considers this task impossibly difficult, even for a planetary mind. Neither Kepler, nor his supposed planetary mind, can recognize the perpendicular to the apsidal line *as a single, unitary direction*.

Altogether, Kepler could conceive of no way, even if it has a mind, for a planet to move itself along an elliptical orbit. This meant, for Kepler, that the ellipse was irreconcilable with physical causes. If the planet, even conceived as a mind, cannot "respect the sine", no physical cause could either. Kepler does not see how an elliptical orbit, which he is convinced accurately describes the Martian orbit, could be *explained*. Yet, Kepler thought, if the orbit is inexplicable, it is *impossible*, despite its empirical accuracy. Without a plausible explanation, the ellipse would have to be rejected. He was driven "to the point of insanity" by the problem.

An Explanatory Mechanism: Magnetic Balance

At this point in his work on Mars, Kepler had become convinced that the planet's orbit was elliptical, but he still sought a plausible physical mechanism that could account for it. Kepler had long thought that something like magnetism was responsible for moving the planets, but he could not understand how magnetism might "respect the sine" of the anomaly, as required by the ellipse. Frustrated, Kepler apparently consulted the leading early-modern treatment of magnetism, William Gilbert's *De magnete*, published in 1600.[24]

Gilbert spurred Kepler's consideration of physical causes in two crucial respects. First, he corrected Kepler's method of representing magnetic action:

> Let us take the shape of the body of the planet to be the same as proposed above. I have said above that it is the same, whether the planet is considered as a globe or as a plane circle; now I say this as well, that it is the same whether it is considered as a plane circle or as a line. For it is certain from Gilbert the Englishman — and also in itself without his authority — that magnetic virtue extends in a straight line. [*Virtutem magneticam porrigi in rectum*.] So just as a globe is conceived to consist of infinite circular planes parallel to the eccentric, of all of which the disposition [*ratio*] is the same, so because of this rectilinear virtue the plane circle consists of an infinity of straight lines, of each of which the disposition is again the same. So the body of the planet can be considered as some straight line, since it does not obstruct any of the others, as I falsely maintained above.[25]

Earlier, Kepler had assumed that the planet's magnetic attraction was proportional to its bulk. Therefore, he related the 'strength' of magnetic action to volumes, areas, and circular angles. From Gilbert, however, Kepler has learned that magnetic virtue is fundamentally linear — it "extends in a right line". Thus, though magnetic action propagates spherically through space, the action itself always respects the magnetic

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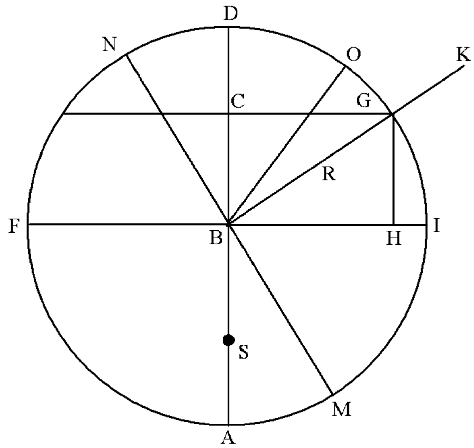

FIG. 2.

axis, the line extending through the magnet's poles. Consequently, Kepler need only consider its magnetic axis, a "right line", to study the inherent magnetic virtue of the planet.

As we have seen, Kepler supposed that the *vis insita* was a result of a magnetic interaction between the planet and the Sun. After Gilbert's correction, Kepler could now consider the influence of the Sun on two ends of a linear axis to represent that interaction. This representation, however, suggested the action of a magnetic 'balance', whose motion is determined by the different forces acting on the attracted and repelled ends of the axis:

> So [in Figure 2] let *AD* be the magnetic axis fleeing in *A*, approaching in *D*, representing one of the infinity of straight lines of virtue of the body of Mars. Now let *B* be the middle point of *AD*, with the Sun in *BI*. The reason why none of the said approaches or flights happens is because *A* and *D* are equal in action. Therefore, this is like equilibrium. See my *Optics*, chapter I.[26]

The action of the Sun on the magnetic axis of the planet is like the "equilibrium" found in balances. To understand the action of the *vis insita*, Kepler only needed to understand the action of a magnetic balance under the influence of a central magnetic power. Yet, Kepler had already considered the behaviour of balances under incident forces in his *Optics*, published in 1604. Thanks to Gilbert, Kepler realized he *already* had a mechanism that might explain how the *vis insita* "respects the sine".

The mechanism that leaped to Kepler's mind is found, in fact, in a passing remark in Chapter 1, Proposition 20, of the *Optics* where Kepler seeks to prove that "Light that has approached the surface of a denser medium obliquely, is refracted towards the perpendicular to the surface".[27] Throughout his proof, Kepler considers the action of light as a mechanical problem of impact or pressure. He compares light encountering a refracting surface to the percussive action of a "missile" striking a "panel" or to the

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continuous pressure exerted by a stream encountering an oar.[28] He then tries to determine how the panel or oar will move as a result of the impact. To do this, however, he draws the analogy between the panel or oar and the arms of a balance.

The proof of the proposition begins with a statical "protheorem" that establishes Kepler's "law of the balance".[29] Kepler considers a balance loaded unequally so that it comes to equilibrium inclined to the horizontal. According to Kepler, each arm of the balance can be found, depending on the loads, anywhere between (in Figure 3) *B* and *F*:

> About centre *A*, with radius *AC*, let the circle *AD* be described, and in it the perpendicular *BAF*. It is evident that neither of the weights at *C* and *D* can either descend lower than *F* or be raised higher than *B*.[30]

The total possible descent of either arm of the balance, then, is from *B* to *F*. Accordingly, claims Kepler, the arms will divide this total descent between themselves in a ratio equal to that between their respective loads:

> And since both are of this nature, that they tend to the bottom, and they mutually compete with each other, they divide up the descent *BF* between themselves in that ratio in which they themselves are. From *D* and *C* let the perpendiculars *DG*, *CH* be drawn. Now from what has been said, *BH*, the descent of the weight *C*, will be to *BG*, the descent of the weight *D*, as the weight *C* is to *D*.[31]

In other words, the vertical descents of the arms will be in the same proportion as their weights.[32] For example, if the weight on *D* is twice the weight on *C*, then the

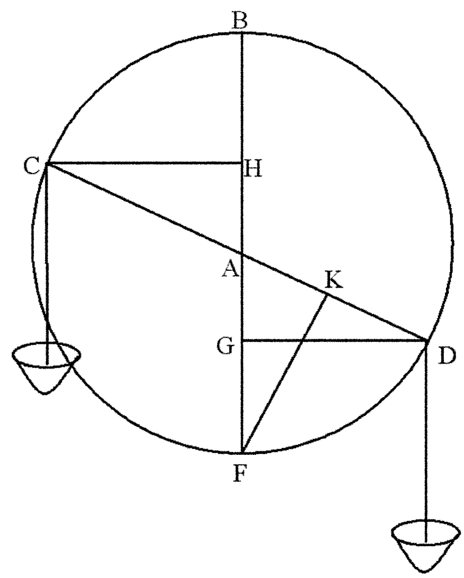

FIG. 3.

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vertical position of *D* (i.e., *G*) will be twice as far along *BF* as the vertical position of *C* (i.e., *H*). This implies that *BG* will be two-thirds of *BF*, while *BH* will be one-third of the distance.

Kepler goes on to show that his "law of the balance", applicable to equal-armed balances, is equivalent to the mechanical law of the lever, which dictates that a unequal-armed balance will equilibrate horizontally if the length of its arms are proportional to its loads:

> I say that this is the proportion of the unequal armed balance. For also, because *HAC*, *GAD* are equal, and *CA*, *AD* are equal, and *H*, *G* are right, *AH*, *AG* will also be equal, and therefore also the remainders of the equals *HB*, *GF*. Therefore, as *C* is to *D*, so is *FG* to *GB*. From *F* let a perpendicular be drawn to *CD*, and let this be *FK*. Therefore, since *CAH*, *FAK* are equal, and *CA*, *AF* are equal, and *H*, *K* are right, *CH*, *FK* will also be equal. Likewise also *AH*, *AK*. Consequently, the remainders of the equals *AB*, *AD*, *AF*, that is, *HB*, *GF*, and *KD*, are also equal. Therefore, as *C* is to *D*, so is *DK* to *KC*. And if the beam *CD*, thus loaded, be suspended from the support at *K*, it will be the ratio of the unequal armed balance, and *C*, *D* will weigh equally, as is demonstrated in mechanics.[33]

By a series of constructions and comparisons, Kepler shows how a point, *K*, can be found on the equal-armed balance such that, if the balance were hung from that support instead of its centre, the resulting unequal-armed balance would have arms *CK* and *KD* such that the ratio of their lengths would be equal to the ratio of the weights *C* and *D*. In this case, the resulting balance would "weigh equally" — horizontally — since it satisfies the classical law of the lever: weight *C* is to weight *D* as *KD* is to *CK*.

Kepler then applies this statical "protheorem" to the mechanical problem of impact or pressure by which he represents the action of light. First, he considers a body striking the panel or oar directly, along the perpendicular. In Figure 4:

> Let *AB* be a panel, *C* the centre, *ED* perpendicular through it; as, if a globe or missile were carried from *E* into the panel *AB*, it would drive it forward towards *D*; or as if *AB* were oars, of equal length on both sides, and *ED* were a river. For since *ECA*, *ECB* are right, the arms *AC*, *CB* are placed in equal balance, and meet the impact of the mobile body with an equal power.[34]

When the incident body strikes, the ends of the panel or the oars, *A* and *B*, will be driven forward by the impact. The distance each end is moved, however, depends on the 'power' with which it resists the action of the impact. To measure this power of resistance, Kepler compares the situation to a balance. He implies that ends of the panel or oars are akin to the arms of the balance, while the force of impact or pressure is similar to the force exerted by the balance's support. This case, therefore, is similar to a level balance, where each arm exerts equal resistance to the support. Carrying the analogy back to the case of impact, Kepler concludes that *A* and *B* have equal power of resistance, and thus are moved equally forward.

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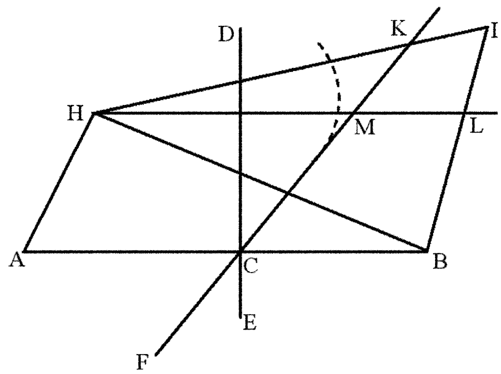

FIG. 4.

Next, Kepler considers the case of an oblique impact or pressure:

> Now let the oblique *FC* strike at *C* and let it be extended to *K*. And let the missile or the stream rush in from *F* to *AB*. Since the angle *ACF* is less than the angle *FCB*, the parts *AC*, *CB* will not be impelled with equal force, but the one that resists more will feel the blow more. And the one that faces at an obtuse angle resists more than that facing at an acute angle. Therefore, the exterior part *CB* will resist more.... Therefore, there is a greater impression of violent motion on *CB*. So when *AB* is moved in position, *B* advances more than *A*.[35]

An oblique impact is akin to an equal-armed balance loaded with unequal weights. In this case, Kepler argues, the ends of the balance resist the force of the support unequally. As a result, the balance will come to equilibrium tilted downward toward the heavier, more resistant, weight. In this position, the support makes an obtuse angle with the heavier weight. (The balance is imagined to hang from a support rather than rest on a fulcrum.) Again arguing by analogy, Kepler concludes that the end of the panel or oar making an obtuse angle with the path of the incident percussent will resist its action more, and therefore be moved farther, than the end or oar making an acute angle. Hence, the panel or oars will be moved from *AB* to *HI* by the force of the impact or pressure.

Though it is not relevant to his argument in the *Optics*, Kepler makes a crucial observation in relation to this argument. Considering specifically the action of a stream on an obliquely positioned oar, Kepler says that, since the end of the oar *B* resists the impact more than *A*, it will be moved further. However, he continues, if the oars are "artificially held back in this position *AB*", the "oar *AB* will at length be pushed forth to the shore", toward *B*.[36] In other words, if the oar is kept parallel to itself, "in this position *AB*", by some "artificial" faculty resisting rotation, then the effect of the stream's pressure will be to move the oar in the direction of *B*, parallel to its length.[37]

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In the *Optics*, Kepler did not try to deduce the distance the oar moves, since he was interested in the oar's deflection when it was not held "in position". The balance analogy, however, does provide a way to measure that motion, which Kepler adduces in his discussion of Mars's orbit (see Figure 2):

> So now let the Sun be in *BCG*. And let the circle *DG* with centre *B* and radius *BD* be drawn, and a [line] perpendicular to *DA* be drawn from *G*, the intersection of the circle with the line of the Sun. If therefore *GB* is the support and *AB*, *BD* the arms of a balance, as *DC* to *CA* will be the strength of angle *DBG* to the strength of *ABG*.[38]

Kepler considers the magnetic force of the Sun in the same way as he considered the action of light on a refracting surface — as a balance problem. The impinging solar force (like the light "missile") is likened to the force exerted by the support of a balance (the line to the Sun, *GB*), while its effect is determined by the "strengths" of resistance in the planet such that the more "resisting" end of the planet's magnetic axis will have a greater effect. The forces on this "balance" can be measured according to the "law of the balance".

In fact, the quantification of the planet's own approach toward and away from the Sun follows quite easily from the balance model:

> And so this flight is as much as *DC*, and the seeking as much as *AC*. Take from *AC* the equal of *DC*, which is *AS*. Therefore *SC* is the measure of the seeking, and *AD* the measure of the seeking at no angle. And as *AD* to *SC*, thus *BD* to *BC* or *GH*. Therefore the sine of the digression of the planet from apogee or perigee measures the speed of the approach.[39]

The planet's magnetic axis is supposed to be attracted to the Sun at end *D* and repulsed at end *A*. Thus, the strength of the magnetic attraction is assumed to act at *D*, while the repulsion acts at *A*. Assuming that the position of the balance is "like equilibrium", Kepler has shown (in the statical "protheorem") that there is a point *C*, found by constructing a perpendicular to *AD* from *G*, such that *DC* is to *CA* as the repulsive action is to the attractive action. That is, the "flight is as much as *DC*, and the seeking as much as *CA*". The difference between the seeking and flight, then, will be measured by the difference between these distances, which is *SC*. Thus, *SC* is to the net attractive force as *AD* is to the maximum attractive force possible ("the seeking at no angle"). Halving both these quantities (which preserves the proportions), we find that the net attraction is measured by *BC* or *GH*, the sine of angle *IBG*. Thus, the "sine of the digression of the planet from apogee or perigee"[40] measures the force of attraction or repulsion. Since Kepler still thinks force is proportional speed, the same sine also measures "the speed of the approach". The planet's attraction and repulsion can "respect the sine".[41] A *vis insita* acting in accord with this magnetic force can therefore move the planet toward and away from the Sun the distance necessary to produce the correct elliptical orbit.

At this point, then, Kepler has constructed a mechanism by which a magnetic action

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can be made to "respect the sine". All the mechanism requires is that the planet's magnetic axis be somehow kept parallel to itself and perpendicular to the apsidal line throughout the orbit. As we have seen, though, this is not a trivial requirement. Indeed, Kepler's inability to conceive this direction as a direction is what drove him "to insanity". It is not clear how even a planetary mind might recognize this direction as a direction and keep its axis "in position" along it.

Concept of Direction: An Oriented Space

At this point, we discover Gilbert's second crucial influence on Kepler's thinking. In *De magnete*, Gilbert tries to use magnetism to account causally for the motions of the Earth described by Copernicus. In particular, he addresses the very problem we have already noted is analogous to Kepler's own: how to explain Copernicus's "third motion" of the Earth — the change of direction of the Earth's axis so that it remains parallel to itself.[42]

To solve the problem, Gilbert introduces a new way of describing direction. He describes direction in relation to a rectilinear spatial orientation instead of a spatial centre. In this oriented space, directions are specified and interpreted as deflections from the privileged orientation that is everywhere parallel to itself. Parallel 'pointings' or rays, for instance, are described as having the *same* direction since they will all have the same deflection from the orientation.

Thus, for Gilbert, the direction in which the Earth's axis points does not change. He says that even as the Earth revolves around the Sun, its axis stays pointed in the *same* direction:

> For like as a loadstone … does by its native verticity, according to the magnetic laws, conform its poles to the poles of the common mother, — so, were the Earth to vary from her natural direction and from her position in the universe, or were her poles to be pulled toward the rising or the setting Sun, or other points whatsoever in the visible firmament (were that possible), they would recur again by a magnetic movement to north and south, and halt at the same points where now they stand.[43]

"North and south", the direction in which the axis points, is now described in such a way that it remains the same at all times. Even as the Earth moves "from her position in the universe" as it revolves around the Sun, the axis remains at the "same points". The direction of the Earth's axis, always parallel to itself, is no longer described as an infinity of different, constantly changing directions, it is *one* direction and always the same. In other words, Gilbert has introduced a novel concept of direction. As Gilbert would later write, Copernicus's "third motion" is "not a motion at all, but the direction of the Earth is stable".[44] The Earth's axis does not change direction. It *stays*.[45]

The new concept of direction, and the resulting description of a 'staying' instead of a motion, vastly simplifies Gilbert's explanatory project. To explain Copernicus's "third motion", one would have to appeal to an *active* cause — some power that

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brings about a change; in this case, the changing direction of the Earth's axis. The description of this cause, moreover, would have to account for various features of the change. The purported cause would have to explain, for example, the speed and sense of the axis's rotation. On the other hand, Gilbert's description of the phenomena as a non-motion allows him to explain it by appealing to a *static* cause — a cause that maintains a *stasis*. Gilbert does not need to say how his static cause operates. He does not need, for instance, to say how fast the axis would move under its influence. In other words, the cause required to explain the phenomena is simpler when the phenomena is described as a stasis rather than a motion. All Gilbert needs is a cause that *keeps* the axis pointed in the same direction.

In fact, Gilbert simply postulates that the spatial orientation he supposes in order to describe the behaviour of the Earth's axis has a physical instantiation. He claims there is a magnetic field that permeates the universe along its orientation. He calls this field the "law of the whole" or the "common mother".[46] Gilbert then argues that the Earth itself is a large spherical magnet (whose magnetic and rotational axes coincide). Hence, the Earth aligns its axis with the rectilinear orientation because of its innate magnetic power or "verticity". "By its native verticity", the Earth can "conform its poles to the poles of the common mother" just as a magnetized needle aligns itself with the Earth's magnetic field. The orientation itself, as the "law of the whole", recognizes and keeps the axis pointed in *a single, unitary direction*.

The important thing to note here, though, is that Gilbert's explanation requires an oriented concept of direction. Only once a spatial orientation has been presupposed does it become possible to describe the "law of the whole" as a real magnetic virtue that follows its structure. And it is on the basis of this virtue that Gilbert explains the fixity of the Earth's poles. By endowing the universe with a rectilinear orientation, Gilbert has made it possible to explain the Earth's "third motion".

Kepler realized that Gilbert's treatment of the "third motion" could simplify his own explanatory project. The problem he faces is similar to Gilbert's: how to explain why a planet's 'direction' remains parallel to itself throughout an orbit, in this case perpendicular to the apsidal line. Kepler had thought that this entailed that the planet's direction continually changes, a motion he could not readily explain. But Gilbert had shown that — so long as one assumes a rectilinear spatial orientation — the Earth's axis can be kept parallel to itself by a simple, static cause that merely keeps the axis pointed in the *same* direction. By introducing an oriented concept of direction, it becomes plausible to suppose that some faculty, be it magnetic or mental, can keep Mars's magnetic axis fixed "in position" perpendicular to the apsidal line. If this is the case, the planet's magnetic axis can measure the magnetic virtue proportional to the sine of the eccentric anomaly, and move itself toward or away from the Sun accordingly — exactly as required by the geometric construction of the ellipse. Adopting Gilbert's oriented concept of direction solves Kepler's explanatory problem.[47] Kepler finally has a plausible mechanism by which the planet can "respect the sine". The ellipse becomes a physically possible orbit.

In the *Astronomia nova*, Kepler explains this arrangement. He envisages a magnetic

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axis in the body of Mars, which is kept parallel either by a "retentive" force or an "animate faculty":[48]

> As before, let there be certain regions of the planetary body in which there is a magnetic force of direction along a line tending towards the Sun. However, contrary to the previous case, let it be an attribute, not of the nature of the body, but of an animate faculty of the sort that governs the body of the planet from within, that as it is swept along by the Sun, it keeps that magnetic axis always directed at the same fixed stars.... The result will be a battle between the animate faculty and the magnetic faculty, and the animate will win.... On the basis of these presuppositions, the planet's mind will be able to intuit and perceive the strength of the angle from the wrestling match between the animate faculty, which is designed to keep the magnetic axis in line, and the magnetic power of directing it towards the Sun.[49]

The magnetic axis tends to point toward the Sun. It is held in place, however, by the animate faculty, which counteracts the magnetic power. Yet, the magnetic power of direction is increased as the axis is more inclined to the Sun, so the animate faculty will have to "struggle" more vigorously to keep the axis in line. Thus, by sensing this "struggle", the planet can "intuit" the "strength" or force that measures the proper speed of its approach. In other words, the magnetic power of the planet becomes a measuring device by which the planet learns the proper action of its inherent *vis insita*:

> There was consequently a need for us to equip the mind with an animate faculty, as well as a magnetic one, and to contrive a battle between the two which would remind the mind of its duties, of which it could not have been reminded by the equality of either the times or the spaces traversed. So again we have asked nature to assist the mind.[50]

The natural, magnetic faculty assists the planetary mind to determine the sine of the eccentric anomaly (as opposed to the anomalies themselves — "the times and spaces traversed"). Once the mind knows this, it can (by measuring the apparent solar diameter) fulfil its "duty" to move itself to the proper distance to the Sun.

Kepler admits that this magnetic/animate mechanism might seem bizarre to some readers:

> Moreover, I do not know whether I have given sufficient proof to the philosophical reader of this perceptive cognition of the Sun and the fixed stars, which I myself so easily accept, and bestow upon the planet's mind.[51]

Thus, Kepler does not believe he has proven that Mars possesses a mind, or even a magnetic axis as he has described. This, though, is not really his concern. The aim all along has been merely to establish a plausible physical cause, however far-fetched:

> I will be satisfied if this magnetic example demonstrates the general possibility of the proposed mechanism. Concerning its details, however, I have doubts.[52]

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If Kepler can establish the "general possibility" of the mechanism, even by appealing to minds, he can cache the mechanism, and thus the elliptical orbit it produces, in the realm of the possible. In this, at least, he has succeeded.[53]

Conclusion: The Need for Linearity

Kepler, finally, has satisfied all his desiderata. He has constructed the true, "physical" path of the Martian orbit. It is causally explicable and agrees with observations:

> Furthermore, at the same time you see both that that most earnestly desired union is now finally complete.... Everything I sought has been accomplished; the causes of each eccentricity are given. You have an astronomy without hypotheses [*Astronomiam habes sine hypothesibus*]. Of course it seems that up to now it had been an hypothesis when I said that Mars's eccentric is a perfect ellipse. But this was previously concluded from physical causes; it is not therefore a hypothesis in my Commentaries. It is indeed in the calculation, but it is also a true supposition of the true path of the planets, giving the distances and the equations.[54]

When Kepler first proposed the ellipse, it was merely an hypothesis — a mathematical conceit. Still, it was a good description of the orbit, and accurate planetary positions and distances could be calculated. Now, however, plausible physical causes of the motion have been given, so the ellipse transcends mere empirical adequacy. It is the "true supposition of the true path", both descriptive and explicable. By reconciling description and explanation, Kepler has produced "astronomy without hypotheses".

Kepler could not have effected this "most earnestly desired union", though, without adopting an oriented concept of direction. The accurate description of the Martian orbit required geometric constructions that could not be described in reference to a single spatial centre. In particular, the perpendicular to the apsidal line is a direction that cannot be specified in relation to a presupposed centre. As a result, Kepler could not conceive how the planet might "respect the sine". The solution only came when Kepler realized that magnetic action could be described on the basis of a "right line" — a rectilinear orientation of the cosmos. By adopting a rectilinear orientation like Gilbert, Kepler could *assume* the direction in space he needed. He could then describe a plausible magnetic or animate faculty on the basis of that direction, and use this faculty to explain the ellipse. The presupposition of a rectilinear orientation allows the reconciliation of description and explanation. It allows the true discovery of the ellipse.

However, a rectilinear orientation was precisely the sort of thing that Kepler had banished from the universe in the first edition of the *Mysterium*. His long and trying struggle with the Martian orbit showed that the presupposition of a rectilinear orientation is necessary to establish and explain the true elliptical orbits of the planets. Thus, God himself could not have constructed this "complete, thoroughly ordered universe" without appeal to "right lines". Kepler was wrong to eschew lines in the *Mysterium*.

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Enlightened by his own "conquest of Mars", and his encounter with William Gilbert, Kepler candidly reported his error in the *Mysterium*'s second edition.

Kepler's adoption of an oriented concept of direction was always limited to part of his treatment of the *vis insita*. In general, Kepler's spatial framework remained centered, not oriented. He continued, for example, to privilege a geometric centre, embodied by the Sun. Indeed, the *anima motrix* moving the planets around their orbits is always, for Kepler, described on the basis of a spherical framework. It emanates radially from the centre and moves planets circularly about it. The ultimate effect of the *vis insita*, meanwhile, is always to move the planet along a radius to the Sun.[55] Nevertheless, Kepler's rectilinear representation of the animate faculty of the planet constituted an essential step in the emergence of inertial space in the early modern period. Eventually, Newton would combine a rectilinearly represented attractive force with a notion of inertia similarly rectified by Descartes. This synthesis laid the foundation for modern physical science. Yet this synthesis was made possible, in part, by Kepler's move toward a rectilinear concept of direction.

Acknowledgments

I am especially indebted to Paolo Palmieri and Nicholas Jardine for their translation assistance. Owen Gingerich and the anonymous referees for this journal were extraordinarily helpful in preparing the final version of the paper.

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1. Johannes Kepler, *Mysterium cosmographicum: The secret of the universe*, transl. by A. M. Duncan (New York, 1981), 95–97. Kepler does admit finite straightness as the "distinguishing features" — i.e., the geometrical boundaries — of solid bodies. His argument is that the universe as a whole is inherently ordered by God and, therefore, laid out spherically about a single centre.
2. *Ibid*., 102–3.
3. Max Jammer, *Concepts of space* (Cambridge, 1954); and Alexandre Koyré, *From the closed world to the infinite universe* (Baltimore, 1957).
4. Robert DiSalle, "Spacetime theory as physical geometry", *Erkenntnis*, xlii (1995), 317–37; and Robert DiSalle, *Understanding space-time* (Cambridge, 2006). See also John Earman, *World enough and space-time* (Cambridge, 1989).
5. J. A. Ruffner, "The curved and the straight: Cometary theory from Kepler to Hevelius", *Journal for the history of astronomy*, ii (1971), 178–94.
6. Bernard R. Goldstein and Giora Hon, "Kepler's move from *orbs* to *orbits*: Documenting a revolutionary scientific concept", *Perspectives on science*, xiii (2005), 74–111.
7. Johannes Kepler, *Johannes Kepler gesammelte Werke*, ed. by Walther von Dyck and Max Caspar (Munich, 1937– ; hereafter *KGW*), xv, 240–80.
8. See, for examples, J. L. E. Dreyer, *A history of astronomy from Thales to Kepler* (New York, 1953), 402; Alexandre Koyré, *The astronomical revolution: Copernicus, Kepler, Borelli*, transl. by R. E. W. Maddison (Ithaca, 1973), 259–61; Bruce Stephenson, *Kepler's physical astronomy* (Princeton, 1994), 107; James R. Voelkel, *The composition of Kepler's Astronomia nova* (Princeton, NJ, 2001), chap. 8; and Curtis Wilson, "Kepler's derivation of the elliptical path", *Isis*, lix (1968), 4–25, pp. 13–14.
9. Kepler, *Mysterium* (ref. 1), 93. Kepler is here referring to Nicholas of Cusa. See E. J. Aiton, "Celestial spheres and circles", *History of science*, xix (1981), 75–114, p. 91. For more regarding Kepler's

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epistemology and theology, see Peter Barker and Bernard R. Goldstein, "Theological foundations of Kepler's astronomy", *Osiris*, xvi (2001), 88–113; J. V. Field, *Kepler's geometrical cosmology* (Chicago, 1988); Nicholas Jardine, *The birth of history and philosophy of science: Kepler's A defence of Tycho against Ursus* (Cambridge, 1984), chap. 7; Job Kozhamthadam, *The discovery of Kepler's laws* (Notre Dame, 1994); David C. Lindberg, "The genesis of Kepler's theory of light: Light metaphysics from Plotinus to Kepler", *Osiris*, 2nd ser., ii (1986), 5–42; and Rhonda Martens, *Kepler's philosophy and the new astronomy* (Princeton, 2000).

10. The full title of the *Astronomia nova* is *New astronomy based upon causes or celestial physics*.
11. We say rather "plausible", rather than "possible" to emphasize the epistemic constraint on hypotheses. All hypotheses are "possible", but only those that admit reasonable explanation are "plausible". Note that Kepler's criteria for admissible hypothesis is not so different from Copernicus's reason for rejecting the Ptolemaic system. Copernicus argued that Ptolemy's use of an equant did not admit of explanation on the basis of accepted physical principles, and was therefore implausible.
12. Kepler wrote that all physical sciences include "a certain amount of conjecture". Johannes Kepler, *New astronomy*, transl. by William H. Donahue (Cambridge, 1992; hereafter *Astronomia nova*), 47.
13. E.g., in *ibid*., chap. 57; Johannes Kepler, *Epitome of Copernican astronomy & Harmonies of the world*, transl. by Charles Glenn Wallis (Amherst, 1995), 52ff.
14. Stephenson, *Kepler's physical astronomy* (ref. 8), 3. Kepler's method here is akin to Descartes's subsequent method of radical doubt in his *Meditations on first philosophy*. Just as Kepler assumes planetary minds as a limiting case of comprehensibility, Descartes assumes a deceiving Demon as a limiting case of incomprehensibility. See René Descartes, *Meditations on first philosophy*, transl. by John Cottingham (Cambridge, 1996), 12–15.
15. "Computaui inde aequationes Eccentri in situbus acronychiis, officium faciunt ad unguem, de distantiis quominus idem dicam fecit earum inquirendarum Methodus paulò laxior, quae semper me circa 100 particulas in dubio relinquit, etiam cum optimae sunt obseruationes. Nosti enim optimas obseruationes uno minuto peccare posse. At unum minutum vitiat distantiam immaniter, si Planeta propè ☉ vel ☍ ☉ fuerit. Hoc tamen certum habeas; quam proximè verum venire." Kepler, *KGW*, xv, 249–50.
16. Kepler, *Mysterium*, 62–65; Kepler, *Astronomia nova*, 372–5. By 1605, Kepler only possessed a preliminary version of the Area Law for which he is famous. It was not worked out in full generality until the *Epitome of Copernican astronomy*. See E. J. Aiton, "Kepler's second law of planetary motion", *Isis*, lx (1969), 75–90; E. J. Aiton, "Infinitesimals and the area law", in *Internationales Kepler-symposium, Weil der Stadt 1971*, ed. by Fritz Krafft, Karl Meyer and Bernhard Sticker (Hildesheim, 1973), 285–305; Peter Barker and Bernard R. Goldstein, "Distance and velocity in Kepler's astronomy", *Annals of science*, li (1994), 59–73; A. E. L. Davis, "The mathematics of the area law: Kepler's successful proof in *Epitome astronomiae Copernicanae* (1621)", *Archive for history of exact sciences*, lvii (2003), 355–93; and Stephenson, *Kepler's physical astronomy* (ref. 8), 161ff.
17. Kepler, *Astronomia nova*, 404ff.
18. See *ibid*., chap. 60.
19. "Sed et aliud est quod desidero in hac hypothesi: nempe quod ad insaniam usque contendens causam naturalem confingere non possum, cur Mars cui tanta cum probabilitate libratio in diametro tribuebatur (res enim nobis ad virtutes magneticas pulchrè admodum recidebat) potius velit ire Ellipsin vel ei proximam uiam. Fortasse tamen puto uirtutes magneticas non omnino respicere sinus sed aliud aliquid." Kepler, *KGW*, xv, 251. See the corresponding passage in *Astronomia nova*, chap. 58: Kepler, *Astronomia nova*, 576.
20. In fact, it generates the infamous *via buccosa*. The *via buccosa* is especially promising because the libration along the diameter of an epicycle is "measured" by the increase of the eccentric anomaly caused by the *anima motrix*. In other words, the "magnetic" *vis insita* moving the body toward and away from the Sun is straightforwardly related to the "magnetic" *anima motrix* and everything "reduces so beautifully to magnetic virtues". Declaring himself a "wretch [*miser*]",

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Kepler reports the failure of this model to Fabricius. See Kepler, *KGW*, xv, 249. See also Kepler, *Astronomia nova*, chap. 58.

21. Though it was initially troubling, Kepler eventually dispensed with this second problem by redefining astronomical terms. In the elliptical hypothesis, the eccentric angle does not measure the anomaly to the planet, but to an imaginary point on the circumscribing eccentric circle, while the planet is found on a perpendicular dropped from that point. At first, Kepler worried that this meant a descent along a radius to the Sun could not generate an ellipse, since he believed the *anima motrix* would carry the planet according to the eccentric anomaly, and the planet's radial descent according to the versed sine would place it on the *via buccosa*. To handle this particular difficulty, Kepler redefines his astronomical terms. He uses angular measures on the circumscribing eccentric to designate points on the ellipse. Thus, in Figure 1, arc *DE* is "named" by arc *DF*. The "eccentric anomaly", now really arc *DE*, is specified by angle *DBF*, where *F* is a point on the circle. From the properties of an ellipse, Kepler knows that area *DEA* is proportional to area *DFA*. This allows him to employ his provisional area law, since he knows that area *DFA* (not area *DEA*, as in the later formulation) is proportional to the mean anomaly. Kepler can say, therefore, that the *anima motrix*, which is governed by the area law, causes the forward motion of point *E*, not point *F*. So the *anima motrix* causes the increase of the eccentric anomaly as it has been redefined, and allows a descent along a radius to mimic the "incursion" along a perpendicular required by the ellipse. Without this remarkable redefinition, Kepler could not have dealt with the elliptical geometry he needed. Since this solution does not require the planet itself to recognize or measure the perpendicular directly, we set it aside, though the same problem arises again in relation to the *vis insita*, as seen below. Note, however, the crucial importance of expressing the action of the *anima motrix* by an *area* law, as opposed to the earlier distance law. Kepler, *KGW*, xv, 250–1. See the corresponding discussion in Kepler, *Astronomia nova*, chaps. 59–60, esp. p. 593. See also Stephenson, *Kepler's physical astronomy* (ref. 8), 126–30.
22. Specifically, the axis of the Earth rotates around the normal to the ecliptic, making roughly one revolution a year in the sense opposite to the Earth's annual orbit around the Sun. If the Earth were not to "move" this way, by contrast, the axis would always point in the "same direction" — which would require it to tilt toward or away from the Sun at all times.
23. "Omnino sapit magneticam vim Eccentricitas, vt est in meis Commentariis: ut si globus Martis haberet axem magneticum, vno polo Solis appetentem, altero fugientem, eoque axe porrigeretur in longitudines medias, tunc quamdiu versatur in descendente semicirculo, maximè in longitudine media, porrigit polum appetentem versus Solem, itaque semper ad Solem accedit, sed maxime in longitudine media, nihil in apsidibus." Kepler, *KGW*, xv, 251.
24. Kepler apparently acquired and read *De magnete* shortly after its appearance in 1600. He mentions it in some detail in his *Apologia pro Tychone contra Ursum*, which was composed shortly thereafter. Jardine, *The birth of history and philosophy of science* (ref. 9), 146.
25. " Sit nobis eadem figura coporis planetarii proposita quae supra. Dixi supra perinde esse, siue planeta consideretur vt globus, siue vt planum circuli; jam etiam hoc dico, perinde esse, siue vt planum circuli consideretur siue vt linea. Nam certam est ex Gilberto Anglo, et per se etiam sine eius authoritate, Virtutem magneticam porrigi in rectum. Quare vt globus fingitur constare ex infinitis circularibus planis, Eccentrico parallelis, quorum omnium eadem est ratio, ita circuli planum propter hanc virtutis rectitudinem, ex infinitis constat rectis, quarum rursum omnium eadem est ratio. Ergo planetae corpus ita considerari potest, vt quaelibet recta, cum nulla aliam impediat, vt supra falso confinxi." Kepler, *KGW*, xv, 253.
26. "Sit ergo *AD* axis magneticus fugiens in *A*, appropinquans in *D*, repraesentans vnam ex infinitis rectis virtuosis corporis Martii. Sit autem *B* punctum medium inter *AD*, Sole in *BI*, dictum appropinquationem vt fugam fieri nullam, causa est, quia *A* et *D* sunt in opere aequali. Ergo hoc est quasi aequipondium. Vide me *Optica* cap. I." *Ibid*., xv, 253–4.
27. Johannes Kepler, *Optics: Paralipomena to Witelo & optical part of astronomy*, transl. by William H. Donahue (Santa Fe, 2000; hereafter, *Optics*), 27.
28. In the *Optics,* Kepler defines "violent motion" or "impulse" as an attribute of light. Therefore, he

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conceives the action of light as something similar to the action of hard bodies colliding. Reflection, for example, is not merely a turning back of a light ray, but a "repercussion [*repercussus*]". See *ibid*., 26, 34. The reference to optics is also significant because light, for Kepler, is both a geometrical and a natural phenomenon. Thus, if the Sun's action could be compared to light, it might similarly be brought under mathematical description, and thus made comprehensible. For recent work on this issue, see Raz Chen-Morris, "Optics, imagination, and the construction of scientific observation in Kepler's new science", *The monist*, lxxxiv (2001), 453–86; Ofer Gal and Raz Chen-Morris, "The archaeology of the inverse square law: (1) Metaphysical images and mathematical practices", *History of science*, xliii (2005), 391–414; and Ofer Gal and Raz Chen-Morris, "Nature's drawing: Motion as mathematical order in Kepler and Galileo" (forthcoming).

29. Kepler, *Optics*, 33.
30. *Ibid*., 32.
31. *Ibid*.
32. In fact, this assertion is false.
33. Kepler, *Optics*, 32.
34. *Ibid*., 33.
35. *Ibid*., 33–34.
36. *Ibid*., 34.
37. This is apparently the source of the stream analogy that appears in chap. 57 of the *Astronomia nova*, although in that case, the oar is assumed to turn, rather than remain parallel.
38. "Sit iam Sol in *BGK*. Et centro *B* spacio *BD* circulus *DG* delineatur, et ex *G*, sectione circuli cum linea Solis perpendicularis in *DA* ducatur. Si igitur *GB* sit trutina, et *AB*, *BD* brachia librae, erit vt *DC* ad *CA* sic fortitudo anguli *DBG* ad fortitudinem *ABG*." Kepler, *KGW*, xv, 254.
39. "Itaque fuga hic tanta est, quanta *DC* appetentia tanta quanta *AC*. Aufer ab *AC* aequalem ipsi *DC*, quae sit *AS*. Ergo *SC* est hic modulus appetentiae, et *AD* mensura appententiae angulo nullo. Et vt *AD* ad *SC*, sic *BD* ad *BC* vel *GH*. Ergo sinus digressionis planetae ab apogaeo vel perigaeo metitur celeritatem accedendi." *Ibid*., xv, 253–4.
40. Earlier, Kepler uses "digression of the planet from apogee" to signify the mean anomaly. Here, he means the eccentric anomaly.
41. Note that this only applies to the instantaneous motion caused by the *vis insita*. Kepler had yet to realize that the accumulation of such motions, each proportional to the sine of the anomaly, would be proportional to the versed sine (i.e., cosine), as required by the ellipse. In yet another remarkable flash of intuition, Kepler would come to this conclusion shortly hereafter, but that is a subject for elsewhere. See Kepler, *KGW*, xv, 255.
42. Gilbert does not seek to explain *all* Copernicus's earthly motions. He only aims to explain the motions Copernicus ascribes to the Earth in and of itself — i.e., the first and third motions. Gilbert does not, on the other hand, have anything to say about Copernicus's second motion, the annual orbit of the Earth around the Sun. In fact, Gilbert never explicitly affirms that the Earth moves through the cosmos. Nevertheless, Gilbert accepts, without comment, Copernicus's conclusion that the Earth is not the centre of planetary orbits. Far more telling, though, is the very fact that Gilbert sees the need to explain the third motion at all. Recall that Copernicus introduces the third motion to account for the apparent stability of the Earth's axis, *given* that the Earth is orbiting the Sun. If one assumes, conversely, that the Earth does not orbit, and remains in place, presumably one would also assume that its axis would remain in place, obviating any need to explain the appearance of stability. Hence, the very fact that Gilbert sees it necessary to explain the fixity of the Earth's axis implies that he accepts Copernicus's second motion. For further support of this position, see Gad Freudenthal, "Theory of matter and cosmology in William Gilbert's *De magnete*", *Isis*, lxxiv (1983), 22–37, pp. 33ff.
43. William Gilbert, *De magnete*, transl. by P. Fleury Mottelay (New York, 1958), 180.
44. "Tertius his motus a Copernico inductus, non est motus omnino, sed telluris est directio stabilis, dum

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in circulo mango fertur, dum unam partem coeli constanter respicit." William Gilbert, *De mundo nostro sublunari philosophia nova* (Amsterdam, 1651), 165.

45. This discussion ignores the very slow precession of the Earth's equinoxes, attributed by Copernicus to a small difference between the second and third motions and by Gilbert to a slow "wobble" of the "common mother" and, therefore, the Earth's axis.
46. Gilbert, *De magnete* (ref. 43), 66, 180.
47. Kepler explicitly credits Gilbert with the notion that magnetic axes remain parallel to themselves in the *Astronomia nova*, chap. 57, and in the *Epitome*, 4.3.1. Kepler, *Astronomia nova*, 550–1; Kepler, *Epitome of copernican astronomy & Harmonies of the world* (ref. 13), 95. In fact, Kepler was not alone in seeing the importance of Gilbert's thesis. There is a parallel discussion of Gilbert in Galileo Galilei, *Dialogue concerning the two chief world systems*, transl. by Stillman Drake (Berkeley, 1967), 345–55, 410. Galileo endorses Gilbert's conclusion that the "third motion" is not a motion, but fails to comprehend the conceptual shift underlying that conclusion. This confusion leads to difficulties surrounding Galileo's argument for the motion of the Earth based on the motion of sunspots. See Owen Gingerich, "The Galileo sunspot controversy: Proof and persuasion", *Journal for the history of astronomy*, xxxiv (2003), 77–78.
48. Kepler gives both explanations. See Kepler, *Astronomia nova*, 553.
49. *Ibid*., 567.
50. *Ibid*., 569.
51. *Ibid*., 570.
52. *Ibid*., 559. Kepler was disappointed to find, for instance, that the Earth's axis is roughly *parallel* to its apsidal line, not perpendicular, as needed in the case of Mars, and that its direction slowly changes. Nevertheless, Kepler decided to "relinquish" these objections, emphasizing, meanwhile, the *conceptual* importance of Gilbert's treatment of the third motion over its factual content. ("Nam in meis Commentariis relicta fuit haec obiectio: si planetae per directionem axis in easdem mundi plagas virtute magnetica eccentricitates conficiunt, Terra idem faciet. At Terrae axis is solum directus est qui porrigitur à ♋ in ♑ .... Illam vero obiectionem de Telluris axe in apsidum lineam inconstanter tamen porrecto superis discutiendam relinquamus." Kepler, *KGW*, xv, 254–6.) Kepler also ignored the very important point that the anomaly in question here is the *eccentric* anomaly, which violates his rejection of physical references to empty points, such as the eccentric centre. Thus, the anomaly should be the *equated* anomaly, which is measured to the body of the Sun. In the *Astronomia nova*, Kepler dismisses this difference as insignificant. Kepler, *Astronomia nova*, 558; Stephenson, *Kepler's physical astronomy* (ref. 8), 115–16. Later, in the *Epitome of Copernican astronomy*, Kepler introduces another libration of the magnetic axis precisely equal to the optical equation — the difference of the two anomalies. Thus, the magnetic axis "measures" the eccentric anomaly even though it is physically affected according to the equated anomaly. See Kepler, *Epitome of Copernican astronomy & Harmonies of the world* (ref. 13), 99–106; and Stephenson, *Kepler's physical astronomy* (ref. 8), 146–72.
53. In fact, Kepler emphasized the importance of the magnetic balance for planetary orbits throughout his career. He repeats the full explanation, including his "law of the balance", in the *Epitome*, Book V. Kepler, *Epitome of Copernican astronomy & Harmonies of the world* (ref. 13), 128–33. One of the goddesses atop the Temple of Urania on Kepler's frontispiece of the *Rudolphine Tables* (1627) holds an unequally weighted balance with the Sun at its fulcrum, which may also be a reference to this mechanism. The goddess is flanked by images of Magnetica, holding a compass and lodestone, and Geometria, displaying the elliptical orbit with its perpendiculars to the apsidal line. Though he has previously claimed otherwise, Owen Gingerich has told me he concurs in this speculation. See Owen Gingerich, "Johannes Kepler and the *Rudolphine Tables*" in *The great Copernicus chase* (Cambridge, 1992), 123–31.

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