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THE FORMAL THEORY OF EVERYTHING: EXPLORATION OF HUSSERL'S THEORY OF MANIFOLDS

„Kant verstehen, heißt über ihn hinausgehen."
Wilhelm Windelband[1]

1. Opening

Husserl's theory of manifolds was developed for the first time in a very short form in the *Prolegomena* to his *Logical Investigations*, §§ 69–70 (pp. 248–53), then repeatedly discussed in *Ideas I*, §§ 71–2 (pp. 148–53), in *Formal and Transcendental Logic*, §§ 51–4 (pp. 142–54), and finally in the *Crisis*, § 9 (pp. 20–60). Husserl never lost sight of it: it was his *idée fixe*. He discussed this theme over forty years, expressing the same, in principle, ideas on it in different terms and versions. His discussions of it, however, were always cut short and inconclusive, so that he never developed his theory of manifolds in detail. Apparently, the reason for this was that he had not a clear idea about it; "it seems to serve [only] as a regulative ideal for future philosophical-mathematical work." (Smith 2002, p. 106)

Not only that Husserl himself failed to conclusively elaborate his theory of manifolds. It also remained a neglected theme in Husserl's studies. Only in the last years it was discussed in a number of essays (see Scanlon 1991, Hill 2000, Smith 2002, Gauthier 2004). This can be partially explained with the fact that Husserl's most instructive writings on this theme were published only recently. Here I mean *Logik und allgemeine Wissenschaftstheorie*, Chapter 2, first published in 1996, and *Einleitung in die Logik und Erkenntnistheorie*, §§ 18–19, available since 1984.

Our task in this paper will be to fill this gap in Husserl studies. The importance of this task results from the fact that the theory of manifolds plays a central role in Husserl's phenomenology. In fact, phenomenology and theory of manifolds are two alternative branches of a single discipline: the science of essences (*Wesenslehre*). The difference between them is that while phenomenology is the content science of essences, the theory of manifolds is the formal one (see Husserl 1913, pp. 149 f.). Consequence of this divergence is that whereas phenomenology is descriptive, the theory of manifolds is not. As we are going to see in the lines to come, the latter supplies primitive law-essences (*Wesensgesetze*), such like axioms, and is so deductive.

Main claim in this essay is that Husserl's theory of manifolds is twofold. It is (1) a theory of theories, advanced in the good German tradition of *Wissenschaftslehre*, launched first by Bernard Bolzano. The latter, in turn, is an offspring of the ancient tradition of *mathesis universalis*, explored in the modernity by Descartes and Leibniz. Further, (2) the theory of manifolds is also a formal theory of everything. Its objective is to provide a systematic account of how the things in the world (its parts) hang together in wholes of quite different kind. It is to be noted that Husserl explicitly speaks about (1) and only implicitly about (2). In this paper we shall also show that the theory of everything is intrinsically connected with the theory of theories.

We shall accomplish the tasks we took on in this investigation in two steps. The first step, made in Part One, will be to find out what Husserl's theory of manifolds is. In Part Two we shall try to develop this theory further, following its authentic spirit. In this sense we shall paraphrase the words of Wilhelm Windelband articulated in the motto to this essay thus: "To understand Husserl means to go beyond him."

I.

2. Levels of Husserl's Logic

(a) Pure and Transcendental Logic. Husserl's logic has two sides: (i) It turns towards what is subjective; (ii) it turns towards the objective order of the ideal objects (concepts) including. (i) is developed by the transcendental logic; (ii) by what he called *pure logic*. Husserl insisted that (ii) has primacy over (i). What pertains to the purely logical is something ideal and has nothing to do with the subjects. Nevertheless, (ii) finds by him a necessary completion in (i).

(b) Three Kinds of Logical Atoms. Husserl's logic claims that there are three kinds of logical atoms: *expression*, *meaning* and *object*. Logic analyses not only forms of expression (sentences) but also forms of meaning (propositions and thoughts) and forms of objects (states of affairs). The basic forms of meaning are: concepts, propositions, truths, connectives, etc.; the basic forms of objects are: object, state of affairs, unity, plurality, number, relation, connection, etc.

(c) Three Levels of Logic. Husserl accepts that there are three levels (*Stufen*) of pure logic:

• Traditional Aristotelian apophantic logic of truth.

• The logic of second level deals with objects of indeterminate, general kind. We find here the theory of cardinal numbers, the theory of ordinals, set theory, etc. This logic treats "forms of judgments, and forms of their constituents, forms of deduction, forms of demonstration, sets and relationships between sets, combinations, orders, quantities, objects in general, etc." (Hill 2000a, p. 168)

• Abstracting further, we reach a third level of logic, the level of the *theory of possible theories,* or the science of theory-forms. "This is a science which tracks down and investigates the legitimate relations between the essential types of possible theories (or realms). All real theories are particularization of relevant theory-forms." (Husserl 1900, p. 251)

The theory of theories is a completely new discipline. It is a such in both historical sense, in the sense that nobody spoke about it before Husserl, as well as in a theoretical sense, in the sense that it is a new level of logic. Indeed, whereas on the second level we deal with forms of propositions, forms of demonstrations, etc., on this level logic deals with forms of systems of propositions.

This is the level of pure forms in which we can modify the real theoretical systems *ad libitum*. This means that we can construct an infinite number of forms of possible disciplines. "Any individual theory is a particular instance of the theory form corresponding to it." (Hill 2000a, p. 170)

(d) Apophantic Logic and Formal Ontology. Pure logic can be also considered as consisting of just two layers. The lower level, the level of apophantic logic, investigates what can be said a priori. Its main categories are the proposition (predication) and the state of affairs. It is a logic of truth; it deals with forms of propositions and states of affairs.

The higher level of logic treats absolutely determined formal object-structures: sets, numbers, quantity, ordinals, multitudes, etc. (Husserl 1906/7, p. 78) This layer of logic is called *formal ontology*; it is a priori science of objects as such.

What is the relation between apophantic logic and formal ontology? On the one side, Husserl's formal logic depends on formal ontology. In the same time, however, formal ontology follows the laws of apophantic logic. The point is that formal ontology follows principles which are those of apophantic logic. Indeed, "we cannot think without thinking" (Husserl 1906/07, p. 94). This is the case since the stem of the pure logic is the apophantic logic. It is true that formal ontology is its highest level; nevertheless it is based on the stem of pure logic – the apophantic logic (p. 77).

3. Husserl's Theory of Pure Forms

(a) Disciplinal Forms. Husserl claimed further that different realms of investigation define different forms: physical form, mathematical form, logical form, ontological form, phenomenological form. "A rich ontology, then, will distinguish different types of form: in the do-

mains of linguistic, conceptual, and mathematical, as well as physical and mental entities." (Smith 2002, p. 104) In his *Logical Investigations* Husserl studied all these different forms. This means that the book examines rather different disciplines: pure logic, speech act theory (act, sense, reference), ontology (universals, parts/whole, ideal meanings), phenomenology (intentionality, structure of consciousness), epistemology (evidence, intuition).

(b) Objectival Forms. Different objects of cognition have also different forms. In this sense Husserl discriminated between:

- propositional form;
- forms of states of affairs;
- number forms (*Zahlenformen*);
- form of entailment (*Schlusselform*);
- categorical form;
- axiomatic form;
- theory form;
- disciplinal form;
- manifold form.

(c) Pure Forms. Husserl, however, did not only studied these disciplinal and objectival forms. He also investigated the pure form. In particular, Husserl's idea of a pure form was a product of his efforts to abstract the ideal forms in mathematics, logic, ontology, and phenomenology.[2] It is to be investigated by the discipline *mathesis universalis*. This is "a philosophical theory of the types of form that shape or situate entities of various types or categories". (p. 105) As we are going to see in the next section, Husserl connected it with his theory of manifolds.

4. The Idea of Husserl's Theory of Manifolds

Indeed, Husserl used to call his theory of theories also theory of manifolds. Some authors define his manifolds as "pure forms of possible theories which, like moulds, remain totally undetermined as to their content, but to which the thought must necessarily conform in order to be thought and known in a theoretical manner." (Hill 2000a, p. 169)

Husserl started to examine the manifolds at the beginning of the 1890s. In his first works "On the Concept of Number" (1887) and *Philosophy of Arithmetic* (1891), however, he rarely employed the term "manifold". Instead, he spoke about "quantity", "plurality", "totality", "aggregate", "collection", "set", "multiplicity [*Menge*]".

Now, multiplicity (*Menge*) was the concept Georg Cantor used to denote what we today call "set". In this connection it is instructive to remind the reader that Husserl taught at the same German University (at Halle a.d. Saale) in which Cantor was a professor. More than this: Cantor was a member of the committee that approved Husserl's Habilitation Dissertation "On the Concept of Number" in 1887. In the same time we must point to the fact that except for their Platonism, Husserl's manifolds differ significantly from Cantor's multiplicity; the latter pertains to the second level of Husserl's pure logic (as described in § 3, (c)), whereas the former to the third level.

But what exactly did the term *manifolds* mean? Husserl was explicit that he borrowed his concept of manifolds from the him contemporary geometry; in particular, from the *n*-dimensional manifolds as set up in the works of H. Grassmann, W. R. Hamilton, Sophus Lie and Georg Cantor. (Husserl 1900, p. 252) In this sense he called the theory of manifolds "a fine flower of modern mathematics" (p. 250). This means that this new discipline, the theory of manifolds, was not only Husserl's vision. It turn reality in the last years of the nineteenth-century mathematics. Husserl's dream was to extrapolate this discipline to the whole categorial realm of human knowledge.

This claim of Husserl was interpreted in three different ways. (a) His manifolds were seen as being close to Riemann's theory of varieties. (b) Most often Husserl's concept of manifold was explained referring to the manifold of three dimensions in Euclidean geometry. By the way, this interpretation can be straightforwardly supported with citations from Husserl 1917/18, p. 265. (c) Yvon Gauthier, by contrast, believes that on this point Husserl followed the general theory of forms or polynomials of Leopold Kroneker's work *Foundations of an Arithmetical Theory of Algebraic Quantities* (see Gauthier 2004, p. 122).

Despite this dissension in interpretation, we can easily outline unambiguous examples of the manifolds. One such an example: The conventional mathematician speaks of space. Instead, we can simply accept that space is the just mentioned Euclidean manifold of three dimensions, so that every object in space is subject to the laws of this particular manifold. (Husserl 1917/18, p. 265) In this way we avoid to talk in terms of space altogether. We speak of axiomatic forms instead, from which we can make inferences as to particular objects. Every object of such a manifold is subject to the laws of the manifold. At a next level of abstraction, we do not speak even of three dimensions but of *n* dimensions instead which have different particular manifolds. Not only do we not speak of space; we do not speak even of geometry. (p. 267).

In the light of the analysis made so far, it turns out that in the theory of manifolds the numbers are only number forms (*Zahlenformen*). Further, what is conventionally called arithmetic and algebra turns out to be nothing but hypothetical theories of particular manifolds. (p. 271) The signs 0, 1, etc., as well as +, x, =, were introduced in them only in order to make evident particular formal analogies.

It is time now to say something more about the pure manifolds. Husserl defines them as "endlessly opened sets of thought-of objectnesses [*Gegenständlichkeit*] which are defined trough axiomatic forms" (p. 274). This means that the objects in a pure manifold are absolutely undetermined.[3] Further, Husserl calls mathematical manifolds *definite manifolds* (Husserl 1913, p. 152).[4] These are systems of axioms, defined in purely analytic way, and so are completely and unambiguously determined – there are no places for contingencies in them.

Let us now chart the scheme of different types of manifolds in Husserl's theory of manifolds:

- Euclidean manifolds;
- other particular manifolds;
- pure manifolds;
- definite (mathematical) manifolds.

5. The Essence of Husserl's Theory of Manifolds

In an earlier exposition of his logic (from 1906/07), Husserl defined the theory of manifolds as the third, higher level of logic.[5] Later it was also called axiomatic mathematics. However – and we already have mentioned this ,– Husserl's theory of manifolds was also called *mathesis universalis*.

Conventionally, mathematics is understood as treating numbers and magnitudes. Instead, Husserl's *mathesis universalis* claims that "what is important in mathematics is to be found out not in its objects, but in the type of its method" (Husserl 1906/07, p. 80). Pure mathematics produces "calculation truths", of any kind. In geometry, for example, we calculate with constructions (*Gebilden*). But "we can [also] calculate with concepts and propositions, exactly as with lines, powers or surfaces" (p. 81).

In other words, instead of numbers, powers, energies, light-beams, Husserl claims that it is better to think of letters and of rules of calculating. If we accept this, then the problems in mathematics will be resolved in the higher possible completeness and generality. (Actually, we can consider the numbers, letters, etc. only as chips (*Spielmarken*) with the help of which we play the game of calculating.) In this way we shall forget that we have to compute with numbers. What matters here is the "tissue of entailments" (*Gewebe von Schlüssen*).

This is the realm of pure logic which can be also called super-mathematics (*Übermathematik*),[6] or mathematics of higher degree. It can be seen as nothing but an interconnection (*Verkettung*) of entailments. As just mentioned, this is not a mathematics of numbers or magnitudes. Rather, it is a mathematics (logic) of an indefinitely general realm of thinking. The only thing that is determined in it is the form. Contemporary mathematics calls such undefined realms *manifolds*; the particular theoretical systems in mathematics are its consequences.

We can construct manifolds, through contingent definitions, after which we can mathematically deduce theoretical systems which follow from them. Such a construction is a product of the creative mathematical imagination. (pp. 86–7)

A pure manifold is a class of objects; it is a construction of purely logical concept of possible objectnesses (*Gegenständlichkeiten*). The latter can be characterized through the forms of propositions which are valid for it. Actually, this is not a manifold of objects but of things which are thought of as objects. To put it in other words: A pure manifold consists of objects-senses, or of substrata-senses which are suitable to function in a system of judgments as substrata of predications (Husserl 1929, p. 148) Or: In the theory of manifolds we operate with pure logical, principal concepts (*Grundbegriffe*).

These characteristics of the theory of manifolds allow some authors to call it "a formal theory of everything" and to see it as nothing but "a philosophical theory of the types of form that shape or situate entities of various types or categories". (Smith 2002, p. 105)

6. Additional Notes on Husserl's Theory of Theories. Disciplinal Form

Husserl's theory of theories is part of his more comprehensive philosophy of science. The latter distinguishes between normative and practical disciplines, on the one hand, and theoretical sciences, on the other. Theoretical sciences are nomological disciplines and are analytical. They are the *formal mathesis*. As already noted in § 3, Husserl's declared aim is to advance "a new and higher form of formal mathesis – a science of the possible forms of theories – *mathesis universalis*". (Husserl 1917/18, p. 257) Husserls's *mathesis universalis* addresses the disciplinal form (another name for theory-form). Its aim is to gain a general concept of many formal, mutually independent axioms. (p. 272)

We can analyze this way many existing theories, for example, we can look for the pure form of the Euclidean geometry, in particular, for the proofs in it. The result of such kind of analysis is a number of axioms from which the whole theory can be deduced.[7] (In the real Euclidean geometry not all axioms are made explicit.)

There are different possible relations between disciplinal forms. Usually, several theoretical disciplines have one group of axioms as a form. In other situations, a disciplinal form can be a part of another disciplinal form. Another case: the system of axioms of one disciplinal form can be a formal restriction of another one. Etc. (pp. 262–3)

A disciplinal form can be widened up. We can explore different ways of its expansion. We can also modify its axioms. In both cases we simply play with the forms of the possible theories (with the disciplinal forms). The only condition by these experiments is not to change the system of this particular discipline. At the end, we contemplate the infinity of possible disciplinal forms in one vision. (p. 268) We, more precisely, try to see the regularities which rule in the contexts and modifications (*Zusammenhänge und Abwandlungen*) of the system. The further our theoretical illumination reaches, the perfect our deductive work in this particular theory is. Indeed, with the enlargement of the disciplinal form grows the power of the mathematical (high logical) thinking.

When an expert confronts such a possible theory, she can assess its applicability in her discipline. In other words: Husserl's new science constructs a priori forms of possible theories and possible sentences, and we can use these theory-forms when we discuss actual theoretical contents (Husserl 1906/07, p. 89). Here are meant, however, only the deductive disciplines. We can thus define Husserl's *mathesis universalis* as a science about the possible forms of the deductive disciplines.

7. The Connection Between the Theory of Manifolds and the Theory of Theories

The connection between the pure theory of manifolds and the theory of possible theories is that the first determines the second and this in such a way that Husserl considered them identical. The reason for this is that Husserl saw the manifold of a given deductive theory as the ontological form of the highly complex state of affairs presented by it. (Smith 2002, p. 110) As a matter of fact, the formal structure of any elementary state of affairs [*Rab*] can be seen as a simple type of manifold in Husserl's sense. (p. 115). On the other hand, any manifold can be seen as the form of a possible world.

This all means that Husserl's theory of manifolds, or his theory of everything, is nothing but formal ontology. This interpretation can be supported with Husserl's definition of the manifolds as “compossible totalities of objects in general” (Husserl 1936, § 9 (f), p. 45). It also conforms Husserl’s claim, discussed in § 2, (d), that formal ontology is the higher level of logic.

In order to make this conception more convincing, Husserl’s formal ontology posits complex forms of states of affairs which mirror the logical connectives and quantifiers. Husserl claims further that there are conjunctive, disjunctive, negative and hypothetical states of affairs.[8] He, for example, recognizes connectional states of affairs like [*Rab* & *Qcd*]. “This exceedingly complex state of affairs is, as it were, the “world” of the theory *T*, or rather that part of the actual world characterized by *T*.”[9] (Smith 2002, p. 118)

II.

In Part Two of this essay we shall advance our own conception of theory of manifolds which goes, in many points, beyond Husserl’s conception, despite the fact that we follow in this effort its true spirit. We shall make this in concert with the motto of this paper as paraphrased at the end of § 1.

8. Husserl’s Project and His Contemporaries

Different aspects of Husserl’s theory of manifolds were developed by some contemporary to him exact philosophers, in the first place, by Bertrand Russell and Ludwig Wittgenstein. We shall pass them in review here with the hope that this will help us to discover new perspectives in Husserl’s theory of manifolds. We shall see, at that, that while Russell developed something like a Husserlian theory of theories, Wittgenstein advanced a theory of manifolds which prima facie was quite different from that of Husserl. In truth, however, it disclosed important new aspects of it.

(a) Husserl and Russell. In his book *Our Knowledge of the External World* (1914) Russell developed – independently from Husserl – the idea of theory of theories in most clear form. In particular, he connected the theory of theories with the new symbolic logic. The latter “suggest[s to science] fruitful hypotheses which otherwise could hardly be thought of”. (Russell 1914, p. 51) Scientists must decide later which theory fits the facts they now know and which do not fit them.

This makes out the difference between the new and the old logic. Whereas the old logic is normative, the new (symbolic) logic is liberal: it assumes that there are many possible solutions to the problem under consideration. Its task is not to criticize such solutions but, quite the reverse, to promote them. It has the effect of "providing an infinite number of possible hypotheses to be applied in the analysis of any complex fact". (p. 68) In more concrete terms, Russell claimed that the new scientific philosophy, enriched by the ideas of the new symbolic logic, can supply to physics new hypotheses. Later we can chose the hypothesis which is most appropriate to the empirical data now available.

In this way (symbolic) logic supplies the method of research in philosophy. At the centre of this conception lie two claims of Russell: (i) The proper subject of philosophy is philosophical logic. (ii) The philosophical logician advances hypotheses build up on the analogy of symbolic logic in regions of science which are still not susceptible to systematic scientific analysis. This is a true marriage between philosophy and science.

The difference between Husserl and Russell on this point was that while Russell believed that the theory of theories is suggested by the new symbolic (mathematical) logic, Husserl claimed that his theory of manifolds was a result ("a flower") of mathematics. That difference, however, is scarcely a big one since to Russell logic is so good as identical with "pure" mathematics: the former is the essence of the latter.

(b) Husserl and Wittgenstein. The concept of manifold (multiplicity) played a central role also in Wittgenstein's *Tractatus*. At least prima facie, however, Wittgenstein used this concept in quite different sense from that of Husserl. In particular, he claimed that in a proposition there are exactly as many distinguishable parts as in the situation it represents. To Wittgenstein this means that they have the same logical (mathematical) manifold. The very picturing of reality is possible only because of this common manifold.

Wittgenstein was convinced that this way of connecting matter and mind (language) is much more promising that the old idealistic conception of "spatial spectacles": he termed so (in Wittgenstein 1922, 4.0412) Kant's claim that we see the matter through the "spectacles" of our aesthetic intuition. Wittgenstein set out that the two elements, fact and proposition, touch one another; this touch is realized in the manifold which is common to the two formations; it is something of an intersection of these two. It is the element on which the two quasi "hinge". (see Milkov 2001a, p. 408) Wittgenstein saw the manifold – the common element between matter and mind (language) – as an *indefinable*.[10] Being such, it, "of course, cannot itself be the subject of depiction. One cannot get away from it when depicting." (Wittgenstein 1922, 4.041) We can only show it.

In his works between 1929 and 1933 Wittgenstein continued to use the concept of manifold, with an important addendum, though. Now he connected it with the conception of copying actions. Similarly to pictures, the action we now make, following an exemplary action, has the same manifold as the action which we copy. (see Wittgenstein 1979, p. 112)

So far so good. The question now is what is the relatedness, and what the difference, between Wittgenstein's conception of manifolds and that of Husserl. In an effort to answer this question we can remind ourselves that Max Black has called the analogy between Husserl's pure theory of forms and Wittgenstein's conception of perfect, perspicuous language "striking". (Black 1964, p. 137) What is even more striking, however, is that the two conceptions were based on the idea of manifold. How this?

This is the case because, as we already have noted in § 7, Husserl's conception of manifolds can be seen as a kind of formal ontology. In this formal ontology, every state of affairs has its own manifold. The same with the complex states of affairs, including such macro-complexes as those presented in science. Further, states of affairs, of all kinds, can be seen as possible worlds. This is exactly how the things were conceived in the formal ontology of Wittgenstein's *Tractatus*.

In another place (Milkov 2001a) we have reconstructed Wittgenstein's Tractarian ontology as claiming that states of affairs are combinative compositions out of many aspects (forms) of objects and of states of affairs. In the world of pure possibility, there are many such compositions: in Husserl's idiom, we can interpret the forms of states of affairs (the manifolds) as "compossible totalities" out of forms of objects. (pp. 405 f.) Further, exactly like in Husserl's formal ontology, the scientific theories in the Tractarian ontology state possible worlds. The facts of the real world can make these possible worlds true or false.[11]

Finally, Husserl embraced a "robust *formal realism* holding that these types of form [the pure manifolds] are part of the world. Forms are abstract or ideal entities, along with numbers, universals, concepts, etc." (Smith 2002, p. 120) This means that forms do not reside in another world but are rather features of particulars in this world – they are something of *universalia in rebus*. The same with Wittgenstein. As David Pears have correctly noticed, the Tractarian ontology is approximately Aristotelian. In it "the forms revealed by logic are embedded in one and only one world of facts" (Pears 1987, i, p. 23).

9. Pure Manifolds, Really Pure Forms

Now we are going to develop a theory of pure manifolds, adding some new elements to Husserl's theory of manifolds. This is a logic of fourth level advanced in an effort to develop further his scheme of more and more abstract logics as described in § 4, (c).

In particular, we shall eschew Husserl's claim that "pure forms" are only of mathematical or formal-logical type. In contrast, we shall accept that they can be of quite different types. The difference with Husserl's pure forms is that our pure forms are not analytic a priori but synthetic a priori. We can call them *really pure forms*, and the discipline that investigates them *theory of pure manifolds*.[12]

Such really pure forms present (express) all courses of values (their ups and downs) of the elements in this particular manifold of elements. They exhibit an order of development, of priority and dependence, etc. The elements themselves can be of quite different kind: they are not only mathematical ones but also colours, shapes, etc., and further: pieces of music, types of animals, etc. The manifolds of such systems supply coordinate system for discriminating of their elements.

The task of this variant of the theory of manifolds is to describe something of a conceptual scheme,[13] with this important addition that it is not really a scheme of *concepts*, i.e. it is not "conceptual" proper. It is merely an ontological-logical scheme of objects; or merely of object-like formations (called by Husserl *objectnesses*). Nevertheless it is similar to a conceptual scheme in that it lays down the epistemological and logical conditions of thinking and perceiving in its terms. Every item of a scheme of this kind has a unique place on the cognitive map set out with its help. This means that such an item is identifiable as an unique thing in it.

In the lines bellow we shall give two examples of our theory of pure manifolds.

(a) Logical Chromathology (Logic of Colours). Programmatic for developing such a theory of pure manifolds are Wittgenstein's *Remarks on Colour*. (Wittgenstein 1977) In a word, they advance a systematic study of the colour circle. Such a study is worth doing since the relations between colours constitute a system which makes possible all discourses about them. The listing of the relations between the colours describes the manifolds of colours. Its family resemblance with the mathematical manifolds explains why Wittgenstein has called the discipline that he was engaged with in *Remarks on Colour* "geometry of colour", or "mathematics of colour". Indeed, colours have their manifold, exactly like mathematical objects, or the objects of geometry have their manifolds. The task of the mathematics of colour is not to "record anything that lies in the unknown nature of colour, nor do they record phenomenological laws, but rather they state or describe the structural relations within the system of colour conceptions that is defined by means of the colour circle". (McGinn 1991, p. 444)

The trouble here arises from our tendency to think of colour in terms of two-dimensional, monochrome patches of determinate colour. From here we make the conclusion that there must be a single, absolutely precise chromatic description of the world. This, however, is an illusion. In fact, the objects have a variety of textures and different degree of transparency, opacity and cloudiness. Our colour concepts interrelate with concepts like transparency and reflection, which require the notion of three-dimensionality or depth. Besides, the colour of an object is affected by its surroundings.

(b) Logical Biology. It (described e.g. in Milkov 2002) studies the forms of animals and other biological objects, both as species and as individuals. In particular, it "pass transformations of animal shapes in review" and describes them (Wittgenstein 1956, III, § 13)

This is a formal, strict and a priori, descriptive discipline.[14] Its possibility is based on the fact that biological individuals can be seen as mosaic of forms which actually makes out their manifold. Not only this. Every individual biological specie has its idiosyncratic manifold.

The study of biological manifolds is made through a comparison of different forms of biological individuals (in the same way in which we compare different colours), tracking down different analogies between them. An illuminating example of this method is given by Wittgenstein: "But might it not be that plants had been described in full detail, and then for the first time someone realized the analogies in their structure, analogies which had never been seen before? And so that he establishes a new order among the descriptions." (Wittgenstein 1980, § 950) Wittgenstein apparently hoped that this kind of analysis will help us to chart the manifold(s) of plants in a new way.

We can call the theory of really pure forms (or the theory of pure manifolds) a theory of natural forms, which is to be distinguished from the theory of natural kinds of Saul Kripke and Hilary Putnam. We call them "natural *forms*" because they are a priori. Here we speak of "*natural* forms" since these, similar to colours, are given to us and so are synthetic. They are neither invented, nor deduced by us. Rather, they pertain to the world "as we find it". They are among those a priori elements of human knowledge, through the rearrangement of which we receive new knowledge.[15]

We can see the natural forms as shapes. Metaphysics, on this understanding, investigates the transformations of such shapes. This study is a type of morphology; not a morphology of Goethe's type, however. The later is complementary in the sense that it strives to arrive at new knowledge, to make discoveries. This is not the case with the theory of natural forms which is analytic. Its aim is only to analyze a priori forms. Making this, however, it achieves a synthesis – by way of *drawing their map*. Still, it adds nothing to the forms themselves and so is not speculative.

In a sense, this discipline is a successor of the *philosophia teutonica* (the German mainstream philosophy of the 17–19th century) which has as its objective to see something as something, to discover *a* as *b*; it so follows the *principle of concept*. In the same time, however, our theory of pure manifolds is not speculative – and the *philosophia teutonica* was speculative *par excellence* – but analytic. Because of this, we can see it as an intersection point between analytic and continental philosophy.

10. Theory of Manifolds in the Humanities

We have already noted that Husserl accepted that his theory of manifolds is valid only in the theoretical disciplines. He claimed that when we try to formulate a pure form in the humanities, we reach "nothing more than the empty general truth that there is an infinite number of propositions connected in objective ways which are compatible with one another in that they do not contradict each other analytically."[16] (Hill 2002a, p. 171) Now we shall mention by way of two illustration (no more!) that this is not the case with the theory of pure manifolds.

(a) Logical Geography. Its aim is to see new aspects in geographic world; for instance, to see the Appenine peninsula as a boot, or France as a hexagon. These are most elementary examples but the method they illustrate can be quite helpful when we try to orient ourselves in a new geographical environment.

(*b) Logical History*. Similarly, the task of logical history is to make *conceptual shifts*: to discover new correlations between the facts already known – more precisely, between their forms. On the basis of these new logical determinations, we can advance new historical theories.

What is common to logical geography and logical history is that the conceptual shifts they make produce a priori models of the posteriori facts in the subject under scrutiny – and exactly these models are cases of manifolds in the humanities. Of course, their cognitive value is not as high as that achieved by systematic scientific pursuit. Nevertheless, they can help to gain new aspects of the geographic, resp. historical facts, which can bring important fruits in this very scientific discourse.

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NOTES

[1] Windelband 1924, i, p. iv.

[2] In the same way in which Cantor claimed that we can obtain the concept of set by abstracting the elements both from properties, as well as from the order in which they are given. On the similarities between Husserl and Cantor see Hill 2000b.

[3] Cf. with Wittgenstein's Tractarian objects which are also absolutely undefined. On Wittgenstein and Husserl see § 8 (b).

[4] See on them Scanlon 1991.

[5] We have already seen, however, that the third level of logic is preserved for the theory of theories. This is not a contradiction since we have previously declared (at the beginning of § 4) that Husserl used to call his theory of theories theory of manifolds. On the connection between theory of theories and theory of manifolds see § 7.

[6] Cf. with the conception of Frege–Wittgenstein for "perfect (ideal) logic".

[7] Here it is clear to see how similar Husserl's theory of manifolds is to Frege's logic. The difference between them is that while Frege analyses arithmetic to logic, Husserl analyses all deductive science to logic.

[8] Which, here is to be remembered, Wittgenstein rejects in the *Tractatus*.

[9] We developed a similar conception in our paper "Tractarian Scaffoldings": In science a formation (a theory) represents another formation (a part of the world). (see Milkov 2001a, p. 407)

[10] "Indefinables" are intuitively knowable simples. (see on them Milkov 2003, p. 95) Some authors have rightly noted that the discovery of the indefinable by Moore and Russell signalled the beginning of analytic philosophy. (see Quinn 1977, pp. 209 ff.)

[11] Here we concern the problem of truth-making. On its history see Milkov 2001b.

[12] Similar theory was developed by the so-called "realist" phenomenologists, and above all by Alexander Pfänder, Max Scheler and Adolf Reinach. See Smith 1996, pp. 186 f.

[13] Similar to that set out in Strawson 1959.

[14] This is an important distinction between the conventional (Husserlian) theory of manifolds and our radical theory of manifolds. As already noted, the former is deductive, whereas the latter is descriptive. This difference is a consequence of the fact that while the forms of the conventional theory of manifolds are analytic, the forms of the theory of pure manifolds are synthetic. Otherwise, both are a priori.

[15] As if in accordance with Russell's Theory of Descriptions which discriminates between knowledge by acquaintance and knowledge by description. The latter describes the world with pieces (elements) that we know by acquaintance.

[16] Incidentally, this claim of Husserl reminds closely Wittgenstein's general propositional form which states: "This is how things stand." (Wittgenstein 1922, 4.5) In contrast to Husserl, however, Wittgenstein didn't believe that the general propositional form is an "empty truth". This point of difference between the two philosophers signals their dissent as to are there synthetic a priori forms.

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