Walter Dubislav's Philosophy of Science and Mathematics[1]

> "Dubislav … knew more about mathematical and logistical [logical] matters and the 'theory of theories' [philosophy of science] than anybody I met."
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> Karl Korsch, letter to Morton Wurtele,
> February 19, 1953 (Korsch 2001, ix, 1500)

Summary

Walter Dubislav (1895–1937) was a leading member of the Berlin Group for Empirical / Scientific Philosophy. This "sister group" of the more famous Vienna Circle emerged around Hans Reichenbach's seminars at the University of Berlin in 1927 and 1928. Dubislav was to collaborate with Reichenbach, an association that eventuated in their conjointly conducting university seminars. Sadly, the political changes in Germany in 1933 proved ruinous to Dubislav. He published scarcely anything after Hitler came to power and in 1937 committed suicide under tragic circumstances. The intent here is to pass in review Dubislav's philosophy of logic, mathematics, and science. The point of this exposition is to shed light on some seminal yet hitherto largely neglected currents in the history of philosophy of science.

1. Exact Philosophy by Kant and Fries

[1] For a profile of Dubislav's life in connection with his works, see the author's. … Our Acknowledgements go to the participants on two seminars that we hold at the University of

Unlike the majority of the logical empiricists, Dubislav was a distinguished historian of logic and philosophy.[2] He had explored the logical theories of Blaise Pascal, Leibniz, and Bernard Bolzano, among others.[3] But his interest centered above all on Kant and Jacob Friedrich Fries. Dubislav regarded these philosophers as especially important because both did original and highly influential work in the substantiation (*Begründung*) of human knowledge and in concept-formation, topics central to his formalist philosophy of mathematics and science. That said, while Dubislav followed Kant and Fries on some points, he rejected many of their claims.

Kant held that while in mathematics we start with the evident, most simple and most clear data, in philosophy we begin with what is vague (*dunkel*) and complicated. While mathematical method is progressive (constructing concepts from elements we know via intuition), philosophical method is regressive.[4] The task of philosophy, as Kant saw it, is to reveal how our knowledge is substantiated; in other words, its project is to uncover the first principles from which our knowledge can be deduced, and "with the help of which we make successful propositions about future events" (1929a, 22).

[2] An exception is perhaps Heinrich Scholtz of Münster (cf. Scholtz 1931) who was close to the Berlin Group.

[3] On March 24, 1932, Dubislav even delivered a radio lecture on Albertus Magnus.

[4] It is interesting to note here that many years after Kant, this dictum was repeated by Bertrand Russell almost *mot-à-mot*: "While mathematics, starting from comparatively simple propositions, seeks to build up more and more complex results by deductive synthesis, philosophy, starting from data which are common knowledge, seeks to purify and generalize them into the simplest statements of abstract form that can be obtained from them by logical analysis" (Russell 1914, 19).

In order to do this, philosophy makes use of a "regressive method of showing [*Aufweisen*]," [5] or demonstrating, the originating moments—the foundations—of our knowledge. We reveal those epistemological foundations through disclosive debate (*Erörterung*). Lastly it judges what is true or false by means of a critical, "Socratic" procedure of assessment (1929a, p. 20). We exercise critical judgments by way of abstraction; but since we do not employ inferences in the process, this method is not inductive—though it is related to induction. In this regard, explains Dubislav, Kant counted it a great mistake in Aristotle to identify critical judgment with induction, after Socrates had articulated it in its pure form.

Jacob Friedrich Fries followed all these points of Kant's critical philosophy. But he broke with Kant, Dubislav found, when he (Fries) asserted that the very *practice* of science and mathematics, as well as our moral *practice*—not merely science and ethics as such—is underpinned by first principles.[6] Their explicitation (*Herausschälen*) is effected with the help of an "inductive–empirical" method.

Fries contended that while we *know* the basic principles of "human understanding," this knowledge is vague (*dunkel*). We can make such knowledge explicit through "psychological analyses," thus "clarifying our consciousness" (1929a, 20). But this sort of exposition and clarification of what is at first obscure does not, observes Dubislav, commit the mistake of the psychologism with which it is so often confused, since we do not employ exogenous psychological propositions as basic elements of our knowledge. What

[5] While Fries and the neo-Friesians, as well as Dubislav till 1931, spoke about *Aufweisung*, in 1931 Dubislav spoke about *Exposition* (1931, 12; 16 f.).

[6] This point shows Jacob Friedrich Fries as the source of the great role practice played in Dubislav's and Reichenbach's exact philosophy. On Fries influence on Reichenbach and Dubislav see (Milkov 2013, 13 ff.).

this process of explication entails is simply analyzing the knowledge—including scientific and mathematical knowledge—that we already possess.

Fries held that first principles, even those of science and mathematics, change all the time.[7] Kant didn't follow this path simply because principles determined in this way cannot substantiate the practical propositions of his ethics. To be sure, moral practice doesn't follow any categorical imperative—although we can, in Dubislav's view, render explicit ever-changing imperatives that are implicit in it (Dubislav 1937).

Another salient matter on which Fries took issue with Kant—and here Fries was followed by Leonard Nelson[8] and Dubislav—is the contention that we must *justify* our knowledge. Fries, however, asserted that human knowledge is only to be substantiated, not justified. Indeed, he regarded reason as self-sufficient enough not to require justification. He pointed out, moreover, that every attempt to justify knowledge leads to circular inferences. This is the case because we already know the first principles of our knowledge: they also substantiate our skeptical explorations of knowledge. The task of philosophy is simply to articulate them.

2. Philosophy of Mathematics

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[7] This had been a defining element of Jacob Friedrich Fries' epistemology, one that Leonard Nelson later adopted and that Reichenbach subsequently appropriated as well. Decades after Dubislav, Michael Friedman rediscovered this point (Friedman 2001) while examining Reichenbach's book *Theory of Relativity and A Priori Knowledge* (1920).

[8] Leonard Nelson (1882–1927) was a Neo-Frisian and Neo-Kantian philosopher who massively influenced not only the leading members of the Berlin Group (cf. Milkov 2013) but also Karl Popper (cf. Milkov 2012).

Philosophy of mathematics was a leading focus of Dubislav's theoretical interest. He made several attempts to establish grounds for an approach to the foundation of mathematics as it had evolved by the beginning of the twentieth century. In these undertakings he took cues from David Hilbert's formalism.

In "On the Relation between Logic and Mathematics" ("Über das Verhältnis der Logik zur Mathematik") (1925/26) Dubislav distinguished three approaches in philosophy of mathematics. The logicists, led by Bertrand Russell,[9] insist that mathematics can be reduced to logic, while the intuitionists, centering on the work of L. E. J. Brouwer, defend the view that logic is based on mathematics. Following Hilbert, the formalists (Dubislav among them) hold that while the principles of mathematics are independent of logical principles, mathematics nevertheless depends upon logic in that the latter helps mathematics by formulating its proofs.

Decades before Hilbert, however, Fries had also argued that the subject-matter of mathematics is different from that of logic. Fries approached mathematics as a system of truths constructed with the help of axiomatic systems that are nothing but synthetic a priori judgments gained through pure intuition. He asserted, in addition, that definitions can fix the meaning of the newly introduced signs (symbols).

As against Fries, Dubislav declared that there are no truths in mathematics: the latter is only a game of calculating. In fact there are two alternative formal sciences: mathematics and logic. The task of logic is to produce true inferences from true premises. If we call the sum of all true statements "the true," then we can define logic as "science of the true." The "science of mathematics," on the other hand, begins with axiomatic systems that are logical in character and are free of contradictions; its aim is to derive the statements that

[9] Although Dubislav discusses Frege extensively (e.g. in 1926, 14–15; 1931, 30 ff.; see also § 6 bellow) he did not conceived Frege as a logicist.

in Leibniz's sense are "potentially included" in those systems. Mathematics is consequently the science of the correct (*vom Richtigen*), not of the true (1925/26, 207–8).

In "On the So-called Object of Mathematics" ("Über den sogenannten Gegenstand der Mathematik") (1930) Dubislav paired Plato's and Kant's philosophies of mathematics, classifying them as *old intuitionists*. Both Plato and Kant held that mathematics is about abstract objects that we grasp with the help of our "pure intuition." And since objects of just that sort are what we also perceive when we cognize our environment (the external world), mathematics is the queen of all sciences.

Dubislav tells us that Kant approached mathematics as a set of true judgments substantiated by way of their interrelation (1930, 30). As distinguished from the judgments of logic, which are tautological, or analytic (Dubislav employed the two terms as synonyms), the judgments of mathematics are synthetic a priori. As it turned out, Kant's belief that the axioms of mathematics (geometry, in particular) are truths was discredited beginning the publications of Lobachevsky as early as in 1826 and ultimately by Einstein's theory of relativity.[10]

In opposition to the old intuitionists, Brouwer, Dubislav's *new intuitionist*, taught that there are no mathematical truths. Rather, in Brouwer's view mathematics is a system of constructions based upon immediate intuitions. Judgments in mathematics that are not so constructed are meaningless. Hence while Kant and Fries defended the position that we can both discover and *show* the objects of mathematics, its primitive truths, Brouwer advanced a radical constructivism from the standpoint of which everything that exists in mathematics is constructed. Mainstream mathematics, however, would seem to have posed a problem for Brouwer since it is based on unrestricted logical growth, in particular,

[10] By way of substantiating this claim, Dubislav referred to Einstein's theory of relativity in interpretation of his friend Hans Reichenbach.

on growth predicated upon the law of the excluded middle. Dubislav explains that Brouwer understood this procedure as not justified since logic, as he conceived it, is "nothing but a system of rules abstracted from the mathematics of finite sets" (1930, 34). In other words, it is simply a "language".

The *empiricism* that Dubislav associates primarily with J. S. Mill portrays mathematics as a science that explores the general structure of objects in the world. It is a sum, not of axioms but of hypotheses or "experimental truths" or abstractions. Following the lead of Bolzano, Dubislav dismisses empiricism of this sort, declaring that variables better articulate abstractions than do hypotheses.[11] He contends, further, that empiricism in philosophy of mathematics employs psychological concepts, such as "ideas," that contemporary psychology (of Dubislav's day)—Gestalt psychology, for example—reveals to be problematic.

Moving on to *conventionalism*, Dubislav turns to Henry Poincaré as the leading representative, according to whom the axioms of mathematics are definitions in the sense of norms (*Forderungen*) introduced *ad hoc* as the occasion warrants (1930, 38). As such norms, the axioms of mathematics are neither true nor false. Thus, similarly to the neo-intuitionists, conventionalists hold that mathematics has no independent subject matter and, further, that it is no science. Moreover, conventionalists make no claim with respect to the truth of mathematical propositions, which they conceive as only systems of arbitrary (*willkürlich*) norms. What this entails is that mathematics is synthetic—conventions are to be *created*—and consequently stands opposed to logic, which is analytic and hence tautological.[12] Moreover, despite the fact that the axioms of mathematics are neither true

[11] Cf. n. 23, bellow.

[12] This position is closely related to Wittgenstein's philosophy of mathematics as expressed after 1928, in particular in *Remarks on the Foundations of Mathematics* (Wittgenstein 1956)

nor false, we can nonetheless deduce from them all of the fundamental laws (*Grundgesätze*) of mathematics.

Norms, however, are always norms of something, and this led critics to challenge the cogency of a conventionalism that denies to mathematics any subject-matter and which thus has nothing to say about any content associated with its own norms. The *formalists* addressed this challenge by asserting that such norms are about signs (*Zeichen*),[13] although Dubislav proposed that instead of signs we do better to adduce variables as the content of mathematical norms. The formalists, notes Dubislav, also introduced the idea of recasting the system of axioms as a system of propositional functions (1930, 40).

Formalism sprang from axiomatics. It holds that pure logic and pure mathematics are not sciences pursuing truths, but rather calculi. In Dubislav's words, "the pure mathematics is nothing but a calculus we shall call 'mathematical calculus', in which there is neither truth nor falsehood, and consequently is arbitrary from logical perspective" (1930, 46). Pure mathematics and logic are simply "formal games." They start from certain arbitrarily chosen initial formulas, and their only objective is to derive other formulas by following particular, arbitrarily chosen rules. Of course, we can attach to these calculi systems of truths. In fact, scientific theories are nothing but calculi with attached rules of interpretation.[14] Understandably enough, an indispensable element of the formalists' task

according to which mathematics advances (invents) new calculi that are not reducible to logic.

[13] According to David Hilbert, "in mathematics, in particular, what we consider is the concrete signs themselves, whose shape is immediately clear and recognizable" (Hilbert 1925, 379; see also Hilbert 1927, 464).

[14] This is a central point of Dubislav's formalist philosophy of science we are going to discuss in § 4.

is to prove that the systems of signs they advance are free of contradiction. Toward this end, the formalists apply the technique of the new logic ("logistic"). To this program Dubislav himself contributed in (1929b).

With *Contemporary Philosophy of Mathematics* (*Die Philosophie der Mathematik in der Gegenwart*) (1932) Dubislav returned to the three-pronged discussion of the foundations of mathematics, specifically to the *criticism* of Kant and the Neo-Kantians (including the neo-Friesians), of *logicism*, and of *intuitionism*.[15] A reviewer in *The Bulletin of the American Mathematical Society* at the time found that "Dubislav's comment on these various doctrines is both pregnant and … correct. … All these theories are rejected—though with cordial recognition of their valuable contributions—in favor of formalism, for which mathematics is but a game with certain marks and rules. Logic is a part of this game, but comparatively elementary one, for it lacks the infinity and selection axioms" (Allen 1933, 330).

3. Criticism of Russell and Wittgenstein

Some years prior to *Contemporary Philosophy of Mathematics* Dubislav launched a sharp criticism, in "In the Relation of Logic and Mathematics" (1925/26), of Russell and Whitehead's logicism, particularly as it appeared in the second edition of *Principia Mathematica*, which had just been published. His first complaint was that *PM* does not undertake to demonstrate that the whole of mathematics is reducible to logic, but that only part of it is so. More generally, he contended that reducing mathematics to logic is impossible in principle. Dubislav insisted that there is a system of statements in the arithmetic of whole numbers that is isomorphic with Russell and Whitehead's system of logical axi-

[15] Part of this discussion was published in (1931/32).

oms. But we cannot prove every statement of arithmetic with the help of only *some* of them, which shows that mathematics is not reducible to logic (1925/26, 199 ff.).[16]

Dubislav also offered the following as specific corrections to *PM*. He:

- took as primitive the concept of implication, to the cost of the concept of negation;
- eliminated the distinction between propositions and propositional functions, as well as the distinction between statements and propositions;
- introduced new primitive concepts in logic, such as the variables "every" and "some" (1925/26, 203).

In his later writings, however, Dubislav tempered somewhat his criticism of logicism. In "On the So-Called Subject-Matter of Mathematics" (1930), for example, he opined that the sense of asserting that we can reduce mathematics to logic "depends" on how we understand "reduce." Indeed, he concedes that "one can construct [*aufbauen*] all formulas of the mathematical calculi with the help of the logical calculi, but one cannot deduce them from the starting formulas of logic" (1930, 48). This advocacy of the relative supremacy of mathematics over logic found further expression in (1932) and (1937).

In *Contemporary Philosophy of Mathematics* (1932) Dubislav asserted that the close relatedness of pure mathematics to pure logic follows from the circumstance that the two disciplines have identical principles of concept formation. More specifically, they are both

[16] Kurt Grelling, among others, deemed this proof unsuccessful (Grelling 1928, 105). Be this as it may, some years later Gödel followed similar lines of argument although much more successfully. Interestingly enough, Dubislav was the first to communicate Gödel's argument in print (1931, 96). The story goes that Dubislav discussed it with Gödel by their return journey from Königsberg to Berlin after the Königsberg 2nd Conference of Epistemology of Exact Sciences in September 1930 (Thiel 2002, 389).

calculi that help to derive formulas from initial formulas, following specific operator rules (instructions), and so have nothing to do with the truth (1932, 41). The formalists hold, in addition, that with the help of the logical calculi we can construct all of the formulas of mathematics; which means that every proof in mathematics is a purely logical procedure, i.e. it is a tautological transformation (1932, 39). This, however, does not mean that mathematics can be reduced to logic.

Dubislav also attacked Wittgenstein's "dogmatic apriorism", according to which tautologies are always true. He poses his initial challenge by questioning the nature of "this strange truth … that cannot be verified" (1933, 32). He next points out that the calculi (i.e., the tautologies) are not discovered but *created*, which means that they can't be senseless.[17] The claim that tautologies are not senseless is also supported by the fact that sometimes we are not sure whether a particular proposition is or is not a tautology. Dubislav concluded that tautologies express thoughts, and that consequently we may verify them. More specifically, he held that tautologies express facts about our knowledge, not about the world; so they are "knowledge of the second order" (1933, 37).

4. Formalist Theory of Science

Dubislav was a formalist not only in his philosophy of mathematics but also as a philosopher of science. As we have seen, formalism takes as its leading concern the substantiation of our knowledge. Dubislav maintained that we substantiate scientific knowledge in three ways, namely by recourse to logic, to probability, and to experience. Besides substantiation, the issue of definition is also essential to formalist epistemology. And while substantiation is indispensable to deductive modes of knowing (i.e., in derivation of knowledge), definition plays a central role in concept formation.

[17] Cf. n. 10.

Dubislav defined scientific theories, at first tentatively, as systems of statements expressed in a language and interrelated in "nets of substantiation." He also argued that "objects" (which included facts and events in his terminology), concepts, and axioms figure as fundamental truths (*Grundwahrheiten*) in scientific theories. The formalists unequivocally attach (*koppeln*), and so co-ordinate (*zuordnen*), objects and concepts of science to calculi, supplying by means of this procedure rules of interpretation (1930, 47).[18] Relative to this, Dubislav also remarked that to this purpose, scientists may employ different (alternative) formal systems.

Operating with the foregoing account of scientific theorizing, the formalist articulates a system of statements about the objects and concepts of science with specific relations between them, replacing in this way the scientific theory with a calculus, with a system of signs. In general, to formalize a scientific theory or discipline means to disregard whatever cannot be captured by the theory of relations.[19] The formalist's task, more exactly, is to construct such a network with the aid of formal logic (1930, 44).

We can also say that the initial system of fundamental statements (*Grundbehauptungen*) that the formalist derives from a scientific theory corresponds to systems of propositional functions that we attach to the calculus, while the objects of the fundamental state-

[18] In his *Philosophy of Nature* (1933) Dubislav criticized this formulation. Following the latest publications of Carnap (1932), he now maintains that this is only a "realistic way of talk" (*Redeweise*) that, strictly speaking, is incorrect. In truth, we connect statements with statements that are nothing but formulae. We so connect formula to formula. The formulae of science, however, can't be substantiated through logic or mathematics since they have "external constants" (1933, 86 f.).

[19] Dubislav explicitly refers in this connection to Russell's doctrine of relations here. Cf. § 7 bellow.

ments are cast as variables in the formulas of calculi (1931, 71). In the new physics, for example, the formalist philosophy of science assumes the task of axiomatically examining (*durchmustern*) the theory of relativity (1931, 107).[20] It is in this way, concluded Dubislav, that pure logic and pure mathematics contribute to our knowledge of the objects (i.e. facts) in our environment and their "behavior."

This line of analysis led Dubislav to embrace the radical position that scientific theories are nothing but elements of logical–mathematical calculi (1932, 47). Insofar as there are different calculi that are free of contradiction, as well as different kinds of arrangements of objects of the world captured in theories, there are alternative scientific theories that are intrinsically isomorphic.

The first step in formalizing a scientific discipline is, for Dubislav, to turn it into a hypothetic-deductive system that is free of gaps—that is, without presuppositions that aren't explicated. Dubislav calls this "an axiomatic construction [*Aufbau*] of the discipline" (1933, 22). The next step is to take the system of the principles that ground the discipline and distil it into a system of relations, not a system of truths. Lastly one spells out the system with the help of the calculi of mathematics and logic. Among other things, this method substantiates Leibniz's claim that the universal character of logical/mathematical formalism enables us to express any thought whatsoever.

Dubislav's formalist theory of science would seem to bracket all structures that fail to exhibit any relational character. Unquestionably this is to render the content of science quite austere. For all of its austerity, however, this formalist philosophy of science exactly affirms the inter-logical relations in scientific theories, and that is what is of interest in epistemology of science. An essential proviso that bears on the limits of this theory is Du-

[20] This was the main objective of Reichenbach (1920, 1924, 1928).

bislav's insistence that the scientific theories we venture to formalize are to be restricted only to those that lend themselves to the formalizing procedure.

5. Definitions

Theory of definition is the kernel of analyticity and also of analytic philosophy. A. J. Ayer, for one, maintained that the main business of the then new analytic philosophy was to provide definitions-in-use, or implicit definition (Ayer 1936, 80). This was also the purport of Russell's theory of descriptions—"that paradigm of philosophy" (Ramsey). It argued for replacing (i. e. translating *salva veritate*) propositions about entities with which we are not acquainted with propositions about those with which we are.

What has been largely forgotten, however, is that Dubislav was the first philosopher to make the theory of definition his prime theoretical concern. This focus reflected Dubislav's orientation as a radical formalist in philosophy of mathematics and science—as a matter of fact his work on definitions was prompted by his investigations into Hilbert's axiomatic method.

Typically, Dubislav initiated his study of definitions in his *magnum opus* (1931) with a historical review of the subject. Aristotle, he noted, had explored definitions as explanations of objects. The important modern developments in theory of definition originated with distinguished French mathematicians and philosophers. In *De L'Esprit Géométrique et de l'art de persuader* (1658) Blaise Pascal introduced the Cartesian inspired precept that every expression in a definition must have a clear and unambiguous meaning. Pascal further stipulated that every statement in a definition needs to be based on statements that we know to be true. Later, it was the French mathematician J. D. Gergonne who introduced into logic (in 1818/19) the concept of implicit definitions, or definitions-in-use. To remind the reader, in implicit definitions, the newly introduced sign receives its meaning through its relation to the signs already available in the system (1931, 40).

The theory of definitions attributable to Kant and Fries has two sides: in mathematics and in the mathematical sciences we have concept formation, while the philosophical side features concept analysis or criticism (1931, 12 ff.). Dubislav explained that in both cases concepts are not given but rather derive from a process which he terms "concept determination." Concept determination in mathematics and the mathematical sciences is synthetic or progressive, affirmed Dubislav, while in philosophy it is analytic or regressive (1931, 113).[21]

In order better to understand the notion of concept determination, however, one needs first to be clear about the nature of concepts as such. Empiricists regard concepts as general ideas. On the other hand, idealists such as Bolzano, Lotze and Husserl took concepts to be ideal objects that, like values, cannot be thought in more fundamental terms: we simply discover and grasp them.[22] Lastly, the formalists (Dubislav included) treated the concept as a sign that takes the form of a propositional function with one variable (1931, 113–16).[23]

Beyond the notion of concept determination, Dubislav's theory of definitions also includes, as we've seen, a program of replacing scientific theories with formal systems of

[21] For Kant's conception of "criticism", and of "progressive" and "regressive" knowledge see § 1.

[22] Similar conception was also embraced by Frege.

[23] Criticizing Bolzano for claiming that concepts are ideal objects, at the same time Dubislav followed him when maintain that concepts are to be expressed as propositional functions with one variable. Cf. Dubislav (1929c, 364 f.)

signs. The guiding requirement is to apply only those instructions for substitution that prove correct operationally in a formal calculus.

Dubislav's original contribution to the theory of definitions was the "calculation criterion of definitions," which introduced pure "combination games" into the process of definition (1931, 81). The purpose of such games of definition is to generate novel constellations of "game pieces" from initial ones, proceeding strictly according to the rules of the game. Ultimately, what Dubislav argued for was "fairly played [formal] games" (*spielgerecht spielende Spiele*) of definition, games with respect to which the problems of truth and contradiction have no bearing. It is from this standpoint that he spoke of "game theory" of definition.

6. Criticism of Frege's Theory of Definition

Reflecting his doctrine of definition is Dubislav's rather critical review of Frege's theory of definitions, which as it turned out was one of the very first evaluations of Frege's logic to appear in print. Historically, Dubislav's criticism of Frege's theory of definition needs to be seen against the background of the Hilbert–Frege controversy (cf. Blanchette 2012). While for Frege the thought was fundamental in logic, Hilbert held that the calculus is the fundamental element of logic, the position Dubislav adopted.

The main thrust of Dubislav's criticism is that in contrast to the formalist theory of definition, Frege's logic reduces the role of definitions to merely regulating the linguistic form of statements and not statements themselves. On a more general level, Frege posited that each discipline consists both of a constellation of mutually substantiating statements and of the language in which those statements are articulated. Regarding language Frege distinguished descriptions (*Kennzeichen*) from simple signs, discriminating "complete" (*vollständige*) from "incomplete" language signs. Unfortunately, Dubislav falsely identifies them as "saturated" and "unsaturated" signs (1931, 31).[24] In the process he introduced his classic distinction between the sense and reference (*Bedeutung*) of language signs. Frege taught that the object of a complete sign is its reference, while the way that a complete language sign signifies that object is its reference (1931, 31). However, Dubislav tells nothing about the difference between sense and reference of propositions, as different from names, by Frege.

Dubislav took Frege to task for failing to clarify how we are to employ signs to construct the formulas of the definiens, and also for failing to clarify the language articulating such formulas (1931, 68). One cannot say the same, by the way, of either David Hilbert or C. I. Lewis, both of whom utilized simple and unequivocal game signs, or marks, that are not necessarily part of natural language.

On the positive side, Dubislav expressed his conviction that Frege's theory of definition newly reset the terms of the foundations of a science (of arithmetic, in particular), the concepts of which need to be well defined. Every definition of a sign that a science deduces (derives) amounts to a reduction of the sign to "fundamental sign," or *Grundzeichen* (1931, 33). Historically, philosophers of logic have postulated a whole array of alternative fundamental signs. For instance, space, time, movement, the concept "all" count among the *Grundzeichen* that Pascal introduced.

Another contribution of Frege was the fruitful distinction between senseless propositions and propositions with sense. Dubislav interpreted the latter as propositions that we could regard as formulas in logical calculi (1931, 117). We cannot, on the other hand, impose strict criteria for what counts as senselessness. Put more precisely, when we make scientific discoveries, we utilize concepts—"atom," for instance—which, says Dubislav,

[24] In Frege's logic "saturatedness" means something completely different (cf. Weiner 2004, 74–77).

are strictly speaking senseless (1933, 27). Cogent scientific language is something achieved only after science has already accomplished its main work.[25]

7. Structural Theory of Truth

By 1930 Dubislav was also devoting his energies to the truth of scientific *theories*—this by contrast with the members of the Vienna Circle at the time, who preoccupied themselves with the truth merely of *propositions* of science. This sharp difference in the focus of their research projects traces back to what Dubislav took as the starting point of his exact philosophy: the axiomatic method and the theory of definitions. Dubislav conceived the truth of scientific theories to be a function of the theory of definitions. Indeed, instead of replacing simply signs with other signs, to Dubislav the task of science simply was to replace "objects" (under which Dubislav also includes events and facts), of the external world with sign systems, or theories.

In this connection Dubislav calls attention to Leibniz, who long since had shown that in the same way in which we explore natural numbers with the aid of digits, we can investigate the objects of science with the help of a theory. Dubislav maintains that the concepts of the theory and the "objects" of its realm relate in the same way.

Turning to a nineteenth-century thinker, Dubislav notes how in the Preface to his *Mechanics*, Heinrich Hertz contended that the objective of scientific theory is to make "pictures" of its objects, so that the possible deductions from these pictures are also pictures of what is deductively derived from the initial objects (1930/31, 35).[26] To this Dubislav add-

[25] Years later, Karl Popper will use virtually the same argument against the logical positivists.

[26] It is well-known today that Hertz was also the genealogical source of Wittgenstein's "picture theory" of languagein the *Tractatus*. The connection between Wittgenstein and Dubislav was apparently Joseph Petzoldt, who was not only Dubislav's "Habilitationsvater" (chief su-

ed that the deductions themselves occur as transformation of formulae—as calculi constructed with the help of specific rules. The scientific theories deduced from the configurations of the initial objects of science, in their turn, are nothing but the formations we gain with the assistance of these calculations. What Dubislav ultimately concludes from this is that to claim that the world is causally determined is to mean nothing more than that we can conceive it as in every detail isomorphic with the calculus (1930/31, 34 f.).

According to Dubislav's doctrine of truth proper, if a scientific theory is to be true, it must have the same structure, be isomorphic with, the states of affairs (or objects) it treats: "A theory is true if and only if there is an isomorphic relation between it and the objects explored" (1930/31, 37). The appeal to isomorphism here has its source in Russell's *Introduction to Mathematical Philosophy* (Russell 1919, 52–62). Unlike Russell, however, Dubislav took "isomorphism" to refer to an *identity* between the two sides of a relation (1933, 18).

This understanding of isomorphism might lead one to expect that Dubislav subscribed to a correspondence theory of truth. What he in fact argued for, however, was a coherence theory of truth, as is clear when he declares that "we never verify the truth of a statement independently from the already known" (1930/31, 33; see also 1933, 24). It's worth noting for the historical record that Dubislav embraced the coherence theory of truth before either Otto Neurath, in "Sociology and Physicalism" (1932), or Carnap, in "The Physical Language as Universal Language of Science" (1932).

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pervisor of his second dissertation), but also thought Wittgenstein mechanics when the latter studied at the Berlin Institute of Technology (*Technische Hochschule zu Berlin*) between 1906 and 1908. Petzoldt's lectures on mechanics, which the young Wittgenstein attended, closely followed Heinrich Hertz on this subject (cf. Graßhoff 2006).

The main thesis of Dubislav's theory of truth is that a single experiment cannot conclusively verify a scientific theory: at best it may falsify it.[27] Dubislav contended that this is the way in which scientific revolutions have often occurred, citing by way of illustration the Michelson-Morley experiment, which discredited once and for all the ether theory of light (1930/31, 34).

Moving on the twentieth century and referencing the works of Richard von Mises, but in fact clearly following Reichenbach, Dubislav further develops his doctrine of truth in light of his recognition that science operates not only with causal laws but also with statistical laws. He cites in this connection the scientific method operative in establishing Heisenberg's indeterminacy (uncertainty) principle, which established that the measuring of two related quantities in the micro-world is fundamentally undetermined.

8. General Philosophy of Science

Over the years, Dubislav's interest in philosophy of logic and mathematics expanded in the direction of general philosophy of science. This development found expression in 1933 with the publication of his last book *Philosophy of Nature* (*Naturphilosophie*).

8.1. The Alleged Pragmatism of Dubislav's Philosophy of Science

In the Preface of *Philosophy of Nature* Dubislav announced that the book was written from the standpoint of Hermann von Helmholtz's idea that as human beings we have a specific cognitive apparatus that helps us to interpret the world. On this view, the basic task of science is to adjust our actions with the aid of our knowledge so that we achieve our ends (1933, 47).

[27] Carl Hempel criticized the latter conception, defended by his teacher Dubislav, in his PhD thesis. See Hempel 1935b, 248 f.

One readily discerns Helmholtz's quasi-pragmatist doctrine in Dubislav's claim that scientific exploration is embedded in the practice of life (*Lebenspraxis*).[28] Amplifying this position Dubislav reminds us that science also makes use of ordinary language and its scheme of representation (1933, 40). Epistemologically speaking, science simply develops in a systematic way what we already knew unsystematically in our daily life. Even an elementary observation is based on some "invariants" of our routine knowledge.

This position had decisive consequences for Dubislav's epistemology. Most significantly, it rendered problematic the objects of our perception. Paradoxically enough, we cannot say that we "see" or "observe" them (1933, 43). The point is that elements of our past everyday knowledge are tacitly ingredient in the act of seeing, of observing, which we perform at any given moment.[29] Dubislav inferred from this that just as the latest theories of scientific investigation are not established "truths" but merely convincing accounts of things, so the very sources upon which scientific investigation builds are inherently vague (1933, 53).

A most telling implication of Dubislav's insight here is that the view of naïve realism in the epistemology of science is mistaken. We cannot compare, or present as correspondence relation, a particular formulation of knowledge with its object (or fact). The same can be said about the positivism, including the logical positivism, which proposes to con-

[28] Reichenbach's alleged pragmatism, as expressed in *Experience and Prediction* (1938) and elsewhere, followed Helmholtz on this point, and not Peirce or James, as it was often suggested.

[29] As today we say, our scientific observations are "theoretically laden." It is highly probable that on this point Dubislav followed the supervisor of his second dissertation (*Habilitationsschrift*) Joseph Petzoldt who maintained that "there is a theory in every 'fact' " (Petzoldt 1927, 159).

struct the world of science from the angle of a single observer—from the viewpoint, that is, of "methodological solipsism."[30] Positivism incurred a further irremediable difficulty to which Dubislav also called attention: it predicates our knowledge overall on statements that we simply accept as true—we don't and in fact can't prove that they are true (1933, 46 ff.).[31]

Dubislav also censured positivism for promulgating the notion that while common sense often regards two cases of the given in our visual perception (for example, a glass of water and a glass of liquor) as identical, science finds that, "in truth," they are different. Against this, Dubislav argued that science does not interpret the observations of common sense as false (*sie weginterpretieren*), but rather as subjective. What this means is that, for all their subjectivity, common-sense observations retain their absolute character. For this reason, asserts Dubislav, we can call common-sense perceptions absolute and construe them as ultimate indicators of fact (*Tatsachenanzeiger*). By contrast, the objective, scientific picture of the world is relative—indeed, it changes with every new scientific discovery and with every new well-grounded theory. The most certain fact of our perception would seem to be the immediately and simultaneously registered differences (different cases of perception of the same object) in it (1933, 49). Thus subjectivity and absoluteness apparently go together in philosophy of science.

At the same time Dubislav shows that in reality it is not difficult to construct scientific intersubjective systems of statements. Supporting this claim is the fact that scientific theories have practical success, which means that their theoretical expectations are confirmed

[30] This is a clear critic to Carnap's *Aufbau* project in which the method of "methodological solipsism" played a central role.

[31] Dubislav's critique and justification of human knowledge was shortly presented in § 1.

simply as a matter of course. As noted at the outset of the present section, this argument was also central to Helmholtz's concept of how science interprets reality.

8.2. Methods of Scientific Investigation

Dubislav's *Philosophy of Nature* is also an original inquiry into "the general methods of investigation of natural science" (1933, 3). What distinguished Dubislav's study from the philosophy of nature as treated by the likes of Schlick or Zilsel[32] is that it is not expressly concerned with aporiai specific to science. Instead, it systematically explores the logical and methodological problems of scientific knowledge (Hempel 1934, column 760). Thirty-three years after the appearance of *Philosophy of Nature* Dubislav's former student, Carl Hempel, published *Philosophy of Natural Science* (1966). While justly hailed a groundbreaking book in the Anglophone philosophy of science,[33] scarcely anybody has noticed that on many points Hempel's work follows Dubislav's lead (Milkov 2014).

As an exponent of the philosophy of nature (*Naturphilosophie*) Dubislav undertook to elucidate the general research methods of science, such as those applied in definition and the methods of substantiation. He also explored the role of observation, experiment and inference in science. Unlike Francis Bacon and J. S. Mill, who strove to formulate the *rules* of scientific inquiry, Dubislav scrutinized the praxis of leading scientists from the past, tracing the methodology of, for example, Isaac Newton in his formulation of the theory of gravitation—that paradigm of scientific theory.[34]

[32] Cf. Schlick (1925), Zilsel (1928).

[33] Philip Kitcher called Hempel (1966) "one of the great introductions to any field of philosophy". (Kitcher 2001, 158)

[34] Hempel worked in a similar way, exploring cases of history of science. Cf. Hempel (1966).

A problem that Dubislav addressed in this line of investigation is the difference between observation and experiment. The introduction of the experimental method was a milestone in the progress of science. Experimental praxis involves limiting the number of possible interpretations by varying the conditions of the experiment (1933, 54). But experiments, Dubislav insisted, are fruitful only when they are connected with a relevant theory. An important point that Dubislav underscored, one later reasserted by Carl Hempel, is that there are no *expermienta cruces* that serve as signposts to direct science down the right path. Dubislav insisted, however, that there are experiments that decisively falsify theories.[35] While one could hardly expect such results from thought experiments, Dubislav nonetheless assessed their nature as well and concluded that thought experiments serve the scientist as a very expedient in the process of theory formulation (1933, 60).

Dubislav held it to be a necessary condition for producing an observation statement (*Beobachtungsaussage*) that one needs to articulate it in an inter-subjective, trans-sensible, and universal language.[36] As we have seen, however, pace naïve empiricism and positivism, Dubislav rejected the view that there are pure observation statements. Moreover, he found that no strict boundary exists between observations and hypotheses, the latter being, in the most general sense, assertions that go beyond the setting of observations (1933, 62).

Dubislav maintained that observation statements derive from inspections, that they do not require substantiation, and that no subsequent inspection disproves an observation statement. With the help of statistical analyses that yield the average value of observation

[35] Cf. n. 27.

[36] On this point Dubislav shook hands with Otto Neurath and Rudolf Carnap.

statements,[37] we can formulate intermediate statements (*Zwischenaussagen*). Moreover, with the appropriate hypotheses, we can also derive from intermediate statements additional intermediate statements that, in their turn, point to statements of observation whose values are captured statistically by those additional intermediate statements (1933, 66).

By contrast with observations, hypotheses, noted Dubislav, are to be substantiated. Unlike tautologies we do not derive them, and they may serve as the very source from which the scientist educes other statements, including observation statements. Dubislav declared that if a requisite number of statements of observation based upon a hypothesis are true, we can consider the whole complex of formal notation, language, observation statements, observations, intermediate statements, experiments, and hypothesis verified (1933, 65). In fact, scientific theories are nothing but such formal notational complexes. Dubislav cautions, however, that we can never verify with absolute certainty the propositions of science. Consequently, no simple criteria are available for determining the truth or falsehood of scientific theories (1933, 68–9).

In his discussion of the methods of scientific investigations Dubislav also introduced the concept of "logical behaviorism," a notion that his pupil Carl Hempel would take up in his own writings (Hempel 1935a, 381).[38] But what Dubislav meant by "behaviorism" differs fundamentally from the familiar psychological sense of term made famous by John B. Watson. Dubislav's behaviorism is rather what Neurath and Carnap had called "physicalism," which Dubislav understood as the claim that all propositions of science are intersubjective (1933, 69).

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[37] Dubislav follows here Reichenbach's interpretation of probability as the limit of relative frequencies.

[38] The introduction of the concept "logical behaviorism" is often mistakenly credited to Hempel.

To be sure, Dubislav was aware that some things simply cannot be expressed intersubjectively—our experiences, for example. These Dubislav classed as ineffable (*unsagbar*) in principle, as incommunicably private. On the other hand, he claimed that whatever we *can* communicate we can express in the language of physicalism (1933, 71). This position mirrored that of the Vienna Circle (and, ultimately, of Frege and Wittgenstein) and this is one of the few points on which Dubislav followed his Vienna friends Schlick, Neurath and Carnap.[39]

Finally, prefiguring Hempel on another aspect of scientific methodology, Dubislav maintained that, in principle, the natural sciences and the humanities have joint methods of investigation and that they both are reducible to the given. However, since they are at different stages of development, Dubislav insisted that it is not appropriate to assign the methods of natural science to those of the complexly evolved humanities, which achieve methodological exactitude in their theoretical ventures only step by step (1933, 99).

8.3. Description and Explanation

Besides scientific method, Dubislav also subjected to detailed analysis the problem of scientific explanation, a theme that would became prominent in Carl Hempel's philosophy of science.[40] The positivists maintain that science simply describes facts of reality, a position Dubislav associates with the Germanophone physicist G. R. Kirchhoff and Ernst Mach. Mach, in particular, held that science, following the "principle of economy," aims to *describe* the given (1933, 93). According to Pierre Duhem, too, a theory of physics doesn't produce explanations, but rather descriptions.

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[39] Cf. n. 37.

[40] To be true to the historical record, the problem of explanation and description in science was discussed long before Dubislav. Cf. Rickert (1896, 78–91).

Dubislav characterized descriptions as statements about the given, about the "this exactly" (*dies da*) of observation statements and intermediate statements. In contrast, the explanation of a process, event, or phenomenon is an epitome (*Inbegriff*) of statements that treat a phenomenon's "behavior," and gets formulated on the basis of selected observations. Explanations can be useful, argues Dubislav, if they derive from a calculus and are accompanied by rules linking them (via *Kopplungsvorschriften*) to phenomena (1933, 94). In fact, the natural sciences employ both methods—description and explanation. Rather than the principle of simplicity (or economy), the sciences employ instead the principle of truth, according to which scientific statements must be verifiable (*prüfbar*) at least in principle.

9. Epilogue

Walter Dubislav was a leading member of the Berlin Group who played a defining role as well in the work of the Society for Empirical / Scientific Philosophy in Berlin. Dubislav produced original work in philosophy of mathematics, logic and science, consequently following David Hilbert's method of axiomatic. This brought him to defend formalism in these disciplines as well as to exploring the problems of substantiating human knowledge. Dubislav also developed elements of general philosophy of science. In contrast to other logical empiricists, he showed intensive interest into history of exact philosophy, trying to explicate important points of it and to develop them further.

At the height of his career Dubislav enjoyed celebrity status; and yet, unaccountably, the decades since his death have seen his formative contribution to the philosophy of science virtually consigned to oblivion. The chief aim of this conspectus has been twofold: to call attention to the distinguished level of Dubislav's achievement and to prompt wider recognition of his seminal achievements as a philosopher of science.

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