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Hyperbolic Functions of Al-Tememe Acceleration Methods for Improving the Values of Integrations Numerically of First Kind

[1]Ali Hassan Mohammed and [2]Asmahan Abed Yasir

[1]Dep. of Mathematics / Faculty of Education for Girls / University of Kufa / Iraq
prof.ali57hassan@gmail.com
[2]Dep. of Mathematics / Faculty of Education for Girls / University of Kufa / Iraq
asmhan1983@gmail.com

Abstract: *The main aim of this work is to introduce acceleration methods called a hyperbolic acceleration methods which are of series of numerated methods. In general, these methods named as AL-Tememe's acceleration methods of first kind discovered by (Ali Hassan Mohammed). These are very beneficial to acceleration the numerical results for definite integrations with continuous integrands which are of 2^{nd} order main error, with respect to the accuracy and the number of the used subintervals and the fasting obtaining results. Especially, for acceleration the results which are obviously obtained by trapezoidal and midpoint methods. Moreover, these methods could be enhancing the results of the ordinary differential equations numerically which are of 2^{nd} order main error.*

1. INTRODUCTION

Electricity the driving force of modern civilization, is indispensable in our day to day life. There are two basic types of electricity generation. One of which is through conventional energy resources which will get extinct in near future, hence demanding an alternative arrangement. Therefore, it is of great urgency to go for non-conventional energy resources. The non-conventional

There are numerical methods for calculating single integrals that are bounded in their integration intervals such as:

1.Trapezoidal Rule
2.Midpoint Rule
3.Simpson's Rule

Which are called "Newton–Cotes formulas".

In this paper, we introduce two methods which are trapezoidal and midpoint methods for finding an approximate values of single Integrals which integrands are continuous in interval of integration using hyperbolic acceleration methods which are part of a series of AL-Tememe's acceleration methods of first kind. We will make a comparison of these methods as an accuracy and fasting reaching of its values with the real values (analytic) for those integrals.

Consider the integral J defined as:

$$J = \int_{x_0}^{x_m} f(x)dx \qquad \dots (1)$$

Such that, f(x) is a continuous function defined on $[x_0, x_m]$. We need to calculate the integral J approximately. In general we can write Newton-cotes formula as:

$$J=\int_{x_0}^{x_m} f(x)dx = G(h) + E_G(h) + R_G \qquad \dots (2)$$

Here, G(h) is Lagrangian approximation to the value of the integral J, (the letter G symbolizes the rule type), E_G is the remainder and related to amputation after the use of certain terms of $E_G(h)$ and $h=\frac{x_m-x_0}{m}$; m is number of sub intervals used and the general form of G(h) is :

$$G(h) = h(w_0 f_0 + w_1 f_1 + w_2 f_2 + \dots + w_2 f_{m-2} + w_1 f_{m-1} + w_0 f_m) \qquad \dots (3)$$

Where $f_r = f(x_r)$ and $x_r = x_0 + rh$; $r = 0,1,2,\dots,m$ and weight coefficients w_r take the sequence ($w_0, w_1, w_2, \dots, w_2, w_1, w_0$). To simplify the formula (3), we write weights by w_0 such that $w_1=2(1-w_0)$, $w_2=2w_0$, we note that when $w_0 = \frac{1}{2}$ we get the trapezoidal rule and refer for G (h) by T(h) where $T(h) = \frac{h}{2}(f_0+2f_1+2f_2+\dots+2f_{m-1}+f_m)$. When $w_0=0$, we get the midpoint rule and we refer to it by M(h) where $M(h)= h(f_1+f_3+\dots+f_{2i-1})$; $i=1,2,\dots,m$. The general formula of $E_G(h)$ is the following:

1- For the trapezoidal rule:

$$E_T(h) = \frac{-1}{12}h^2(f_m{}' - f_0{}') + \frac{1}{720}h^4\left(f_m{}^{(3)} - f_0{}^{(3)}\right) - \frac{1}{30240}h^6\left(f_m{}^{(5)} - f_0{}^{(5)}\right) + \cdots \qquad \dots (4)$$

2-For midpoint rule:

$$E_M(h) = \frac{1}{6}h^2(f_m{}' - f_0{}') - \frac{7}{360}h^4\left(f_m{}^{(3)} - f_0{}^{(3)}\right) + \frac{31}{15120}h^6\left(f_m{}^{(5)} - f_0{}^{(5)}\right) - \dots \qquad \dots (5)$$

This is when the integral is a continuous function and its derivatives exist at each point of the integration interval $[x_0, x_m]$. we can

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write error formulas as following:

$$\begin{aligned} J - T(h) &= A_1h^2 + A_2h^4 + A_3h^6 + \cdots \\ J - M(h) &= B_1h^2 + B_2h^4 + B_3h^6 + \cdots \end{aligned} \quad \ldots (6)$$

Where $A_1, A_2, A_3, \ldots; B_1, B_2, B_3, \ldots$ are constants and J the real value (exact) for integration. [1]

2. Acceleration of Hyperbolic Functions for Al-Tememe:

A series of acceleration methods of Al-Tememe's are introduced and we will call it hyperbolic accelerations. Due to the similar error of both trapezoidal and midpoint methods regarding h basics, we will deal with the error for trapezoidal method to derive our acceleration methods following the same way to derive these methods as for the midpoint method.

In Trapezoidal rule:

$$J=\int_{x_0}^{x_m} f(x)dx = h\left[\frac{1}{2}f_0 + f_1 + f_2 + \cdots + f_{m-1} + \frac{1}{2}f_m\right] + E(h) \quad \ldots (7)$$

Where:

$$E(h) = A_1h^2 + A_2h^4 + \ldots ; \quad A_1, A_2, \ldots \text{ are constants} \quad \ldots (8)$$

$$= h(A_1h + A_2h^3 + A_3h^5 + \ldots\ldots) \cong h\sinh(h); \ \sinh(h) = h + \frac{h^3}{6} + \frac{h^5}{120} + \ldots \quad [2] \quad \ldots (9)$$

So we have:

$$J \cong \frac{h}{2}[f_0 + f_1 + \cdots + f_m] + h\sinh(h) \quad \ldots (10)$$

We assume that T(h) is the approximate value for integration for trapezoidal rule:

$$E = J - T(h) \cong h\sinh(h) \quad \ldots (11)$$

Suppose that $T_1(h_1)$ represents the value of the above integration numerically when $h=h_1$. Also $T_2(h_2)$ represents the integration numerical value when $h=h_2$ then:

$$J - T_1(h_1) \cong h_1\sinh(h_1) \quad \ldots (12)$$

$$J - T_2(h_2) \cong h_2\sinh(h_2) \quad \ldots (13)$$

From equations (12) and (13) we obtain:

$$A^F_{\sinh} \cong \frac{h_2\sinh(h_2)T_1(h_1) - h_1\sinh(h_1)T_2(h_2)}{h_2\sinh(h_2) - h_1\sinh(h_1)} \quad \ldots (14)$$

The formula (14) is called Al-Tememe's sine hyperbolic acceleration rule of the first kind that referred to by ($A^F_{\sinh}$). Equation (11) can also be written as:

$$J - T(h) \cong h^2\cosh(h) ; \ \cosh(h) = 1 + \frac{h^2}{2} + \frac{h^4}{24} + \ldots \quad [2]$$

Following the same above method, we get;

$$A^F_{\cosh} \cong \frac{{h_2}^2\cosh(h_2)T_1(h_1) - {h_1}^2\cosh(h_1)T_2(h_2)}{{h_2}^2\cosh(h_2) - {h_1}^2\cosh(h_1)} \quad \ldots (15)$$

We call the formula (15) as Al-Tememe's cosine hyperbolic acceleration of the first kind that referred to by ($A^F_{\cosh}$).

Similarly, we find the third hyperbolic acceleration law that we will call Al-Tememe's tangent hyperbolic acceleration of the first kind, which is referred to by ($A^F_{\tanh}$) and Al-Tememe's forth hyperbolic acceleration rule of the first kind that we will call it Al-Tememe's secant hyperbolic acceleration of the first kind, which is referred to by (A^F_{sech}). These rule are:

$$A^F_{\tanh} \cong \frac{h_2\tanh(h_2)T_1(h_1) - h_1\tanh(h_1)T_2(h_2)}{h_2\tanh(h_2) - h_1\tanh(h_1)} \quad \ldots (16)$$

$$A^F_{\mathrm{sech}} \cong \frac{{h_2}^2\,\mathrm{sech}(h_2)T_1(h_1) - {h_1}^2\,\mathrm{sech}(h_1)T_2(h_2)}{{h_2}^2\,\mathrm{sech}(h_2) - {h_1}^2\,\mathrm{sech}(h_1)} \quad \ldots (17)$$

For the possibility of writing equation (11)by formulas:

$$E = J - T(h) = h\tanh(h); \ \tanh(h) = h - \frac{h^3}{3} + \frac{2h^5}{15} - \ldots \quad [2]$$

$$E = J - T(h) = h^2\,\mathrm{sech}(h); \ \mathrm{sech}(h) = 1 - \frac{h^2}{2} + \frac{5h^4}{24} - \ldots \quad [2]$$

Now we will derive the fifth hyperbolic acceleration rule: since the error is:

$$E(h) = Ah^2 + Bh^4 + \ldots \cong \cosh(h) - 1 = 2\sinh^2\left(\frac{h}{2}\right) \quad \ldots (18)$$

The rule shall be in the form:

$$A^F_{\sinh^2} \cong \frac{\sinh^2\left(\frac{h_2}{2}\right)T_1(h_1) - \sinh^2\left(\frac{h_1}{2}\right)T_2(h_2)}{\sinh^2\left(\frac{h_2}{2}\right) - \sinh^2\left(\frac{h_1}{2}\right)} \quad \ldots (19)$$

We call the formula (19) as Al-Tememe's quadratic sine hyperbolic acceleration, is referred to by ($A^F_{\sinh^2}$).

In order to derive the sixth hyperbolic acceleration law:

$$E(h) = A_1h^2 + A_2h^4 + A_3h^6 + \ldots$$

$$= h^2(A_1 + A_2h^2 + A_3h^4 + \ldots) \cong h^2(1 + \cosh h) = 2h^2\cosh^2\left(\frac{h}{2}\right) \quad \ldots (20)$$

The sixth hyperbolic acceleration rule becomes by the following formula:

$$A^{F}{}_{\cosh^2} \cong \frac{h_2{}^2\cosh^2\left(\frac{h_2}{2}\right)T_1(h_1)-h_1{}^2\cosh^2\left(\frac{h_1}{2}\right)T_2(h_2)}{h_2{}^2\cosh^2\left(\frac{h_2}{2}\right)-h_1{}^2\cosh^2\left(\frac{h_1}{2}\right)} \quad \dots(21)$$

We call the formula (21) as Al-Tememe's quadratic cosine hyperbolic acceleration, is referred to by ($A^{F}{}_{\cosh^2}$)

3.Examples:

Below are some of the integrations whose integrands are continuous in the integration period and we use the hyperbolic acceleration methods to improve their results.

3.1 : $\int_3^4 \frac{dx}{\sqrt{x}}$ and its analytic value is 0.535898384862246 is rounded to 14 decimal.

3.2: $\int_2^3 xe^{-x}dx$ and its analytic value is 0.20685757623838 is rounded to 14 decimal .

3.3: $\int_0^1 \frac{dx}{1+e^x}$ dx and its analytic value is 0.37988549304172 is rounded to 14 decimal .

We will compare the tables for the acceleration methods with the trapezoidal rule and again with the midpoint rule. The calculation of the preference for the method is based on the values of n, n =1,2,3,... and the results we have adopted in the mat lab. program by putting Eps=10^{-10} which represents (the absolute error of the subsequent value- previous value).

4.The results:

The integrand of integration $\int_3^4 \frac{dx}{\sqrt{x}}$ is continuous in the integration interval [3,4], and the formula of correction terms of (trapezoidal and midpoint rules) is similar to the formula in equation(8).

1-For the base trapezoidal rule with hyperbolic acceleration methods we observe from table (1): that when n =27,...,34 the values in the $A^{F}{}_{\sinh}$ method are valid for nine decimal order, while the trapezoidal value without acceleration was valid for only five decimal order when n=42 also the same accuracy is obtained for all remaining acceleration methods with a simple variation in n values.

2-For the base midpoint rule with the hyperbolic acceleration methods, we observe from table (2): When n =22,...,30 the values in the $A^{F}{}_{\sinh}$ method are correct for nine decimal order but we note that the value in the midpoint method without acceleration was valid for only five decimal order when n=37 also the same accuracy is obtained for all remaining acceleration methods with a simple variation in n values.

To find the value of integration $\int_2^3 xe^{-x}dx$ numerically, we note that the integrand is continuous in the integration interval [2,3] and that the formula of the correction terms for the two (trapezoidal, midpoint) rules, respectively, is as in formula equation(8).

1-For the base trapezoidal rule with hyperbolic acceleration methods, we observe the following in Table (3): when n =26,...,37 the values of the $A^{F}{}_{\sinh}$ acceleration is valid for nine decimal order, but note that the trapezoidal value without acceleration was valid for only five decimal order when n=52 and we obtained the same accuracy for the other hyperbolic acceleration rules with a simple variation in n values.

2-For the base midpoint with the hyperbolic acceleration methods we observe from table(4): that when n =19,...,29 the values of the $A^{F}{}_{\sinh}$ acceleration is correct for nine decimal order, but note that the value in the midpoint method without acceleration was correct for only five decimal order when n=46 and we obtained the same accuracy for the other hyperbolic acceleration rules with simple variations in n values.

To find the value of integration $\int_0^1 \frac{dx}{1+e^x}$ numerically, where that the integrand is continuous in the integration interval [0,1] and that the formula of the correction terms for the two (trapezoidal, midpoint) rules, respectively, is as it he formula in equation(8).

1- For the base trapezoidal rule with hyperbolic acceleration methods, we observe the following in table (5): When n =29,...,38 the values of the $A^{F}{}_{\sinh}$ acceleration is valid for nine decimal order, but note that the trapezoidal value without acceleration was valid for only five decimal order when n=49, and we obtained the same accuracy for the other hyperbolic acceleration rules with a simple variation in n values.

2- For the base midpoint with the hyperbolic acceleration methods we observe from table(6): when n =27,...,32 the values of the $A^{F}{}_{\sinh}$ acceleration is correct for nine decimal order, but note that the value in the midpoint method without acceleration was correct for only five decimal order when n=43 and we obtained the same accuracy for the other hyperbolic acceleration rules with simple variations in n values.

5.tables:

	Values of trapezoidal rule	$A^{F}{}_{\sinh}$	$A^{F}{}_{\cosh}$	$A^{F}{}_{\tanh}$	$A^{F}{}_{\mathrm{sech}}$	$A^{F}{}_{\sinh^2}$	$A^{F}{}_{\cosh^2}$
1	0.53867513459481						
2	0.53659880920983	0.53600734842036	0.53613469439500	0.53569452905836	0.53551910562557	0.53596097059415	0.53604967180475
3	0.53621021998902	0.53591177826991	0.53593314087899	0.53587406496496	0.53585926223743	0.53590571019534	0.53591767438519
4	0.53607389853480	0.53590180943773	0.53590770969690	0.53589221175837	0.53588874646778	0.53590023700128	0.53590335980340
5	0.53601074525074	0.53589962843561	0.53590183772772	0.53589614972820	0.53589493381336	0.53589905451682	0.53590019754074
6	0.53597642486603	0.53589893991886	0.53589994119853	0.53589738868185	0.53589685507509	0.53589868311647	0.53589919528835

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7	0.53595572567718	0.53589866873749	0.53589918530909	0.53589787580381	0.53589760550426	0.53589853721472	0.53589879973671
8	0.53594228905999	0.53589854478134	0.53589883737574	0.53589809823653	0.53589794687116	0.53589847062412	0.53589861871732
9	0.53593307602744	0.53589848169968	0.53589865952062	0.53589821135885	0.53589812006381	0.53589843676845	0.53589852652672
10	0.53592648553407	0.53589844688126	0.53589856105483	0.53589827376982	0.53589821546215	0.53589841809364	0.53589847561549
11	0.53592160906439	0.53589842640004	0.53589850300990	0.53589831047018	0.53589827149657	0.53589840711361	0.53589844565722
12	0.53591789996692	0.53589841371883	0.53589846701382	0.53589833318839	0.53589830615394	0.53589840031746	0.53589842710326
13	0.53591501333179	0.53589840552888	0.53589844373849	0.53589834785801	0.53589832851877	0.53589839592941	0.53589841511808
14	0.53591272282216	0.53589840004935	0.53589842815157	0.53589835767143	0.53589834347270	0.53589839299414	0.53589840709810
15	0.53591087492168	0.53589839627153	0.53589841739736	0.53589836443651	0.53589835377749	0.53589839097075	0.53589840156810
16	0.53590936252781	0.53589839359877	0.53589840978441	0.53589836922229	0.53589836106509	0.53589838953941	0.53589839765531
17	0.53590810907727	0.53589839166487	0.53589840427331	0.53589837268484	0.53589836633640	0.53589838850386	0.53589839482394
18	0.53590705866270	0.53589839023770	0.53589840020465	0.53589837523996	0.53589837022543	0.53589838773971	0.53589839273433
19	0.53590616968918	0.53589838916596	0.53589839714822	0.53589837715866	0.53589837314528	0.53589838716591	0.53589839116502
20	0.53590541069241	0.53589838834850	0.53589839481633	0.53589837862205	0.53589837537192	0.53589838672828	0.53589838996802
21	0.53590475751594	0.53589838771625	0.53589839301232	0.53589837975385	0.53589837709381	0.53589838638982	0.53589838904216
22	0.53590419136513	0.53589838722106	0.53589839159907	0.53589838064028	0.53589837844225	0.53589838612474	0.53589838831698
23	0.53590369743995	0.53589838682876	0.53589839047927	0.53589838134251	0.53589837951038	0.53589838591474	0.53589838774247
24	0.53590326395783	0.53589838651473	0.53589838958276	0.53589838190461	0.53589838036530	0.53589838574665	0.53589838728257
25	0.53590288144443	0.53589838626096	0.53589838885818	0.53589838235885	0.53589838105611	0.53589838561082	0.53589838691091
26	0.53590254221155	0.53589838605409	0.53589838826743	0.53589838272913	0.53589838161921	0.53589838550010	0.53589838660793
27	0.53590223996777	0.53589838588408	0.53589838778192	0.53589838303341	0.53589838208191	0.53589838540911	0.53589838635895
28	0.53590196952466	0.53589838574334	0.53589838737994	0.53589838328531	0.53589838246495	0.53589838533378	0.53589838615282
29	0.53590172657257	0.53589838562602	0.53589838704481	0.53589838349530	0.53589838278424	0.53589838527099	0.53589838598098
30	0.53590150750796	0.53589838552759	0.53589838676363	0.53589838367147	0.53589838305210	0.53589838521831	0.53589838583682
31	0.53590130929891	0.53589838544452	0.53589838652631	0.53589838382015	0.53589838327815		0.53589838571514
32	0.53590112937972	0.53589838537402	0.53589838632489	0.53589838394633	0.53589838346999		0.53589838561188
33	0.53590096556744	0.53589838531387	0.53589838615303	0.53589838405398	0.53589838363364		0.53589838552378
34	0.53590081599532	0.53589838526231	0.53589838600570	0.53589838414626	0.53589838377394		0.53589838544825
35	0.53590067905944		0.53589838587880	0.53589838422575	0.53589838389477		0.53589838538321
36	0.53590055337547		0.53589838576902	0.53589838429451	0.53589838399930		0.53589838532694
37	0.53590043774361		0.53589838567366	0.53589838435421	0.53589838409006		0.53589838527806
38	0.53590033111982		0.53589838559053	0.53589838440628	0.53589838416921		
39	0.53590023259214		0.53589838551777		0.53589838423847		
40	0.53590014136119		0.53589838545387		0.53589838429930		
41	0.53590005672387		0.53589838539758		0.53589838435288		
42	0.53589997805974		0.53589838534781				

Table (1) to calculate integration $\int_3^4 \frac{dx}{\sqrt{x}} = 0.53589838486225$ by using trapezoidal rule with the hyperbolic acceleration methods of AL-Tememe of the first kind

n	Values of midpoint rule	A^F_{sinh}	A^F_{cosh}	A^F_{tanh}	A^F_{sech}	$A^F_{sinh^2}$	$A^F_{cosh^2}$
1	0.53452248382485						
2	0.53554898785978	0.53584139717639	0.53577843924227	0.53599605036304	0.53608277706921	0.53586432567598	0.53582047313184
3	0.53574262974304	0.53589134929942	0.53588070387842	0.53591014260352	0.53591751910261	0.53589437314394	0.53588841114570
4	0.53581067958518	0.53589658415505	0.53589363882479	0.53590137518790	0.53590310501450	0.53589736909425	0.53589581023330
5	0.53584222581739	0.53589773071956	0.53589662713733	0.53589946839818	0.53590007577011	0.53589801740260	0.53589744644103
6	0.53585937506780	0.53589809283781	0.53589759251720	0.53589886796175	0.53589913459501	0.53589822115713	0.53589796523448
7	0.53586971996714	0.53589823549507	0.53589797732645	0.53589863178206	0.53589876687052	0.53589830122662	0.53589817002518
8	0.53587643599562	0.53589830071181	0.53589815446426	0.53589852390844	0.53589859956544	0.53589833777784	0.53589826375637
9	0.53588104129796	0.53589833390356	0.53589824501653	0.53589846903836	0.53589851467385	0.53589835636326	0.53589831149595
10	0.53588433585075	0.53589835222504	0.53589829515028	0.53589843876254	0.53589846791023	0.53589836661582	0.53589833786095
11	0.53588677366587	0.53589836300270	0.53589832470437	0.53589842095766	0.53589844044111	0.53589837264426	0.53589835337577
12	0.53588862794874	0.53589836967602	0.53589834303235	0.53589840993547	0.53589842345076	0.53589837637575	0.53589836298477
13	0.53589007109131	0.53589837398598	0.53589835488349	0.53589840281791	0.53589841248636	0.53589837878513	0.53589836919196
14	0.53589121622715	0.53589837686962	0.53589836281997	0.53589839805638	0.53589840515501	0.53589838039686	0.53589837334561
15	0.53589214009424	0.53589837885775	0.53589836829578	0.53589839477383	0.53589840010286	0.53589838150790	0.53589837620971
16	0.53589289623163	0.53589838026435	0.53589837217216	0.53589839245163	0.53589839652991	0.53589838229387	0.53589837823624
17	0.53589352291338	0.53589838128211	0.53589837497833	0.53589839077146	0.53589839394547	0.53589838286251	0.53589837970269
18	0.53589404808824	0.53589838203320	0.53589837705003	0.53589838953160	0.53589839203872	0.53589838328211	0.53589838078496
19	0.53589449255045	0.53589838259724	0.53589837860633	0.53589838860055	0.53589839060713	0.53589838359721	0.53589838159776

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20	0.53589487202998	0.53589838302745	0.53589837979370	0.53589838789044	0.53589838951542	0.53589838383752	0.53589838221774
21	0.53589519860354	0.53589838336019	0.53589838071228	0.53589838734122	0.53589838867118	0.53589838402338	0.53589838269727
22	0.53589548166737	0.53589838362082	0.53589838143190	0.53589838691107	0.53589838801004	0.53589838416895	0.53589838307287
23	0.53589572862075	0.53589838382727	0.53589838200208	0.53589838657030	0.53589838748633	0.53589838428427	0.53589838337044
24	0.53589594535440	0.53589838399255	0.53589838245859	0.53589838629753	0.53589838706716	0.53589838437657	0.53589838360864
25	0.53589613660508	0.53589838412611	0.53589838282754	0.53589838607710	0.53589838672845	0.53589838445116	0.53589838380114
26	0.53589630621659	0.53589838423498	0.53589838312834	0.53589838589741	0.53589838645235	0.53589838451197	0.53589838395806
27	0.53589645733442	0.53589838432445	0.53589838337556	0.53589838574975	0.53589838622548	0.53589838456193	0.53589838408703
28	0.53589659255260	0.53589838439852	0.53589838358025	0.53589838562751	0.53589838603768		0.53589838419379
29	0.53589671402582	0.53589838446027	0.53589838375089	0.53589838552561	0.53589838588113		0.53589838428279
30	0.53589682355575	0.53589838451208	0.53589838389407	0.53589838544012	0.53589838574980		0.53589838435747
31	0.53589692265826		0.53589838401491	0.53589838536796	0.53589838563896		0.53589838442048
32	0.53589701261615		0.53589838411748	0.53589838530673	0.53589838554490		0.53589838447397
33	0.53589709452082		0.53589838420498	0.53589838525449	0.53589838546465		
34	0.53589716930562		0.53589838428000		0.53589838539586		
35	0.53589723777248		0.53589838434462		0.53589838533662		
36	0.53589730061352		0.53589838440052		0.53589838528537		
37	0.53589735842863		0.53589838444907				

Table (2) to calculate integration $\int_3^4 \frac{dx}{\sqrt{x}} = 0.53589838486225$ by using midpoint rule with the hyperbolic acceleration methods of AL-Tememe of the first kind

n	Values of trapezoidal rule	$A^F{}_{sinh}$	$A^F{}_{cosh}$	$A^F{}_{tanh}$	$A^F{}_{sech}$	$A^F{}_{sinh^2}$	$A^F{}_{cosh^2}$
1	0.21001588578841						
2	0.20761419117408	0.20693004590333	0.20707734754674	0.20656820638511	0.20636529333651	0.20687640046980	0.20697900154636
3	0.20719100468872	0.20686599180767	0.20688925639396	0.20682492077400	0.20680880011522	0.20685938347422	0.20687241287179
4	0.20704456398270	0.20685970057277	0.20686603881295	0.20684939044991	0.20684566792800	0.20685801141309	0.20686136602334
5	0.20697708015760	0.20685834385070	0.20686070463819	0.20685462660170	0.20685332730927	0.20685773057721	0.20685895198047
6	0.20694050148838	0.20685791802447	0.20685898518843	0.20685626471592	0.20685569599778	0.20685764432446	0.20685819019729
7	0.20691847271509	0.20685775079994	0.20685830055281	0.20685690693323	0.20685661927135	0.20685761082898	0.20685789021372
8	0.20690418608265	0.20685767449258	0.20685798559683	0.20685719969886	0.20685703875795	0.20685759564410	0.20685775310584
9	0.20689439617125	0.20685763570157	0.20685782465686	0.20685734843321	0.20685725142169	0.20685758795694	0.20685768333548
10	0.20688739598024	0.20685761430621	0.20685773557737	0.20685743043331	0.20685736850095	0.20685758372901	0.20685764482671
11	0.20688221795975	0.20685760172734	0.20685768307460	0.20685747862861	0.20685743724496	0.20685758124827	0.20685762217535
12	0.20687828040762	0.20685759394193	0.20685765051952	0.20685750845138	0.20685747975178	0.20685757971513	0.20685760815075
13	0.20687521651891	0.20685758891529	0.20685762947116	0.20685752770315	0.20685750717639	0.20685757872636	0.20685759909332
14	0.20687278569903	0.20685758555294	0.20685761537662	0.20685754057907	0.20685752551056	0.20685757806555	0.20685759303348
15	0.20687082481947	0.20685758323520	0.20685760565265	0.20685754945382	0.20685753814311	0.20685757761034	0.20685758885560
16	0.20686922010412	0.20685758159566	0.20685759876933	0.20685755573116	0.20685754707602	0.20685757728851	0.20685758589983
17	0.20686789023776	0.20685758040950	0.20685759378660	0.20685756027237	0.20685755353689	0.20685757705578	0.20685758376116
18	0.20686677585439	0.20685757953422	0.20685759010814	0.20685756362314	0.20685755830323	0.20685757688411	0.20685758218289
19	0.20686583279280	0.20685757887698	0.20685758734491	0.20685756613912	0.20685756188155	0.20685757675524	0.20685758099768
20	0.20686502765186	0.20685757837571	0.20685758523676	0.20685756805792	0.20685756461019	0.20685757665698	0.20685758009368
21	0.20686433479103	0.20685757798803	0.20685758360586	0.20685756954187	0.20685756672021	0.20685757658101	0.20685757939450
22	0.20686373426288	0.20685757768441	0.20685758232826	0.20685757070404	0.20685756837255	0.20685757652152	0.20685757884688
23	0.20686321036093	0.20685757744388	0.20685758131595	0.20685757162467	0.20685756968135	0.20685757647440	0.20685757841305
24	0.20686275058186	0.20685757725136	0.20685758050551	0.20685757236157	0.20685757072888	0.20685757643669	0.20685757806578
25	0.20686234487240	0.20685757709578	0.20685757985050	0.20685757295704	0.20685757157531	0.20685757640622	0.20685757778515
26	0.20686198507503	0.20685757696895	0.20685757931647	0.20685757344243	0.20685757226524	0.20685757638138	0.20685757755638
27	0.20686166451460	0.20685757686474	0.20685757887759	0.20685757384130	0.20685757283215	0.20685757636097	0.20685757736838
28	0.20686137768644	0.20685757677846	0.20685757851421	0.20685757417151	0.20685757330145		0.20685757721274
29	0.20686112001844	0.20685757670654	0.20685757821127	0.20685757444676	0.20685757369263		0.20685757708300
30	0.20686088768769	0.20685757664620	0.20685757795710	0.20685757467768	0.20685757402080		0.20685757697415
31	0.20686067747785	0.20685757659528	0.20685757774257	0.20685757487256	0.20685757429774		0.20685757688229
32	0.20686048666718	0.20685757655206	0.20685757756049	0.20685757503795	0.20685757453277		0.20685757680432
33	0.20686031294007	0.20685757651520	0.20685757740515	0.20685757517905	0.20685757473327		0.20685757673781
34	0.20686015431635	0.20685757648359	0.20685757727197	0.20685757530001	0.20685757490515		0.20685757668079
35	0.20686000909469	0.20685757645637	0.20685757715725	0.20685757540418	0.20685757505318		0.20685757663168

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36	0.20685987580672	0.20685757643282	0.20685757705802	0.20685757549430	0.20685757518123		0.20685757658920
37	0.20685975317983	0.20685757641237	0.20685757697183	0.20685757557256	0.20685757529243		0.20685757655230
38	0.20685964010660		0.20685757689668	0.20685757564081	0.20685757538939		0.20685757652013
39	0.20685953561979		0.20685757683091	0.20685757570052	0.20685757547424		0.20685757649197
40	0.20685943887152		0.20685757677316	0.20685757575297	0.20685757554875		0.20685757646725
41	0.20685934911605		0.20685757672227	0.20685757579917	0.20685757561440		0.20685757644547
42	0.20685926569535		0.20685757667729	0.20685757584000	0.20685757567241		0.20685757642622
43	0.20685918802702		0.20685757663742	0.20685757587620	0.20685757572384		
44	0.20685911559408		0.20685757660198	0.20685757590838	0.20685757576956		
45	0.20685904793634		0.20685757657038	0.20685757593706	0.20685757581031		
46	0.20685898464309		0.20685757654214	0.20685757596269	0.20685757584673		
47	0.20685892534686		0.20685757651685	0.20685757598565	0.20685757587935		
48	0.20685886971804		0.20685757649413	0.20685757600628	0.20685757590865		
49	0.20685881746030		0.20685757647368		0.20685757593501		
50	0.20685876830667				0.20685757595881		
51	0.20685872201609				0.20685757598030		
52	0.20685867837043				0.20685757599977		

Table(3) to calculate integration $\int_2^3 xe^{-x}dx = 0.20685757623838$ by using trapezoidal rule with the hyperbolic acceleration methods of AL-Tememe of the first kind

n	Values of midpoint rule	A^F_{sinh}	A^F_{cosh}	A^F_{tanh}	A^F_{sech}	$A^F_{sinh^2}$	$A^F_{cosh^2}$
1	0.20521249655975						
2	0.20647493679132	0.20683455474977	0.20675712628751	0.20702475410420	0.20713141445749	0.20686275323643	0.20680882143091
3	0.20668999828804	0.20685516839406	0.20684334543439	0.20687604051138	0.20688423295874	0.20685852671984	0.20685190523772
4	0.20676380818259	0.20685698411137	0.20685378947477	0.20686218067933	0.20686405692643	0.20685783549143	0.20685614468130
5	0.20679771180287	0.20685736447752	0.20685617842675	0.20685923200946	0.20685988476910	0.20685767258382	0.20685705895541
6	0.20681605932686	0.20685748243266	0.20685694715304	0.20685831171702	0.20685859698081	0.20685761971807	0.20685734591328
7	0.20682709868297	0.20685752846840	0.20685725296882	0.20685795135827	0.20685809551528	0.20685759861254	0.20685745860349
8	0.20683425412559	0.20685754939935	0.20685739358317	0.20685778719922	0.20685786780628	0.20685758889052	0.20685751002600
9	0.20683915553753	0.20685756001514	0.20685746541289	0.20685770383877	0.20685775240851	0.20685758391894	0.20685753616678
10	0.20684265932349	0.20685756586120	0.20685750516169	0.20685765789459	0.20685768889342	0.20685758116592	0.20685755058486
11	0.20684525056602	0.20685756929444	0.20685752858575	0.20685763089687	0.20685765160654	0.20685757954281	0.20685755906162
12	0.20684722075609	0.20685757141764	0.20685754310853	0.20685761419362	0.20685762855372	0.20685757853615	0.20685756430813
13	0.20684875363115	0.20685757278764	0.20685755249738	0.20685760341230	0.20685761368192	0.20685757788520	0.20685756769553
14	0.20684996967385	0.20685757370360	0.20685755878400	0.20685759620224	0.20685760374042	0.20685757744925	0.20685756996138
15	0.20685095055592	0.20685757433475	0.20685756312097	0.20685759123307	0.20685759689097	0.20685757714845	0.20685757152329
16	0.20685175323025	0.20685757478109	0.20685756619087	0.20685758771844	0.20685759204772	0.20685757693552	0.20685757262816
17	0.20685241839494	0.20685757510392	0.20685756841304	0.20685758517599	0.20685758854490	0.20685757678136	0.20685757342751
18	0.20685297575906	0.20685757534209	0.20685757005349	0.20685758330009	0.20685758596087	0.20685757666756	0.20685757401735
19	0.20685344742040	0.20685757552090	0.20685757128576	0.20685758189159	0.20685758402096	0.20685757658206	0.20685757446026
20	0.20685385009117	0.20685757565726	0.20685757222588	0.20685758081744	0.20685758254173	0.20685757651683	0.20685757479806
21	0.20685419659968	0.20685757576270	0.20685757295315	0.20685757998673	0.20685758139788	0.20685757646637	0.20685757505931
22	0.20685449692528	0.20685757584527	0.20685757352287	0.20685757933617	0.20685758050216	0.20685757642684	0.20685757526392
23	0.20685475892526	0.20685757591068	0.20685757397428	0.20685757882083	0.20685757979267	0.20685757639551	0.20685757542601
24	0.20685498885422	0.20685757596303	0.20685757433567	0.20685757840834	0.20685757922483	0.20685757637043	0.20685757555575
25	0.20685519174095	0.20685757600533	0.20685757462775	0.20685757807503	0.20685757876600	0.20685757635017	0.20685757566059
26	0.20685537166583	0.20685757603981	0.20685757486588	0.20685757780333	0.20685757839201		0.20685757574606
27	0.20685553196765	0.20685757606814	0.20685757506158	0.20685757758006	0.20685757808471		0.20685757581629
28	0.20685567539969	0.20685757609160	0.20685757522361	0.20685757739524	0.20685757783032		0.20685757587443
29	0.20685580424870	0.20685757611115	0.20685757535870	0.20685757724117	0.20685757761828		0.20685757592290
30	0.20685592042671		0.20685757547203	0.20685757711192	0.20685757744040		0.20685757596356
31	0.20685602554233		0.20685757556769	0.20685757700284	0.20685757729028		0.20685757599787
32	0.20685612095676		0.20685757564887	0.20685757691027	0.20685757716288		0.20685757602700
33	0.20685620782811		0.20685757571814	0.20685757683129	0.20685757705420		0.20685757605184
34	0.20685628714665		0.20685757577753	0.20685757676359	0.20685757696103		0.20685757607314
35	0.20685635976326		0.20685757582867	0.20685757670528	0.20685757688079		0.20685757609148
36	0.20685642641225		0.20685757587292	0.20685757665484	0.20685757681139		
37	0.20685648773005		0.20685757591135	0.20685757661104	0.20685757675111		
38	0.20685654427047		0.20685757594486	0.20685757657284	0.20685757669855		

39	0.20685659651721		0.20685757597419	0.20685757653942	0.20685757665257		
40	0.20685664489428		0.20685757599994	0.20685757651006	0.20685757661218		
41	0.20685668977461		0.20685757602263	0.20685757648421	0.20685757657660		
42	0.20685673148725		0.20685757604269	0.20685757646135	0.20685757654515		
43	0.20685677032345			0.20685757644109	0.20685757651728		
44	0.20685680654173				0.20685757649249		
45	0.20685684037222				0.20685757647041		
46	0.20685687202028				0.20685757645067		

Table (4)to calculate integration $\int_2^3 xe^{-x}dx = 0.20685757623838$ by using midpoint rule with the hyperbolic acceleration methods of AL-Tememe of the first kind

n	Values of trapezoidal rule	A^F_{sinh}	A^F_{cosh}	A^F_{tanh}	A^F_{sech}	$A^F_{sinh^2}$	$A^F_{cosh^2}$
1	0.38447071068500						
2	0.38100568974157	0.38001864597783	0.38023116395426	0.37949660562294	0.37920385484651	0.37994124956382	0.38008927624339
3	0.38038137848562	0.37990189909064	0.37993622046805	0.37984130852477	0.37981752631745	0.37989215006249	0.37991137184731
4	0.38016404485549	0.37988968783379	0.37989909445839	0.37987438651158	0.37986886189194	0.37988718094062	0.37989215954011
5	0.38006365318133	0.37988701623736	0.37989052824053	0.37988148630704	0.37987955342694	0.37988610390685	0.37988792091585
6	0.38000917284547	0.37988617288548	0.37988776232141	0.37988371044474	0.37988286339492	0.37988576523619	0.37988657826017
7	0.37997634103178	0.37988584073116	0.37988666008609	0.37988458302723	0.37988415429427	0.37988563211762	0.37988604851427
8	0.37995503919888	0.37988568890775	0.37988615277429	0.37988498097489	0.37988474100677	0.37988557134210	0.37988580612267
9	0.37994043803815	0.37988561164551	0.37988589346281	0.37988518319918	0.37988503851137	0.37988554043679	0.37988568268908
10	0.37992999556210	0.37988556900043	0.37988574990566	0.37988529470957	0.37988520232250	0.37988552338714	0.37988561452912
11	0.37992227019515	0.37988554391554	0.37988566528187	0.37988536025793	0.37988529851543	0.37988551336171	0.37988557442301
12	0.37991639492338	0.37988552838399	0.37988561280413	0.37988540082244	0.37988535799942	0.37988550715600	0.37988554958515
13	0.37991182288126	0.37988551835323	0.37988557887212	0.37988542701033	0.37988539637958	0.37988550314896	0.37988553354124
14	0.37990819529310	0.37988551164213	0.37988555614894	0.37988544452623	0.37988542203901	0.37988550046845	0.37988552280557
15	0.37990526886086	0.37988550701522	0.37988554047120	0.37988545659961	0.37988543971943	0.37988549862063	0.37988551540314
16	0.37990287387075	0.37988550374175	0.37988552937294	0.37988546513975	0.37988545222220	0.37988549731345	0.37988551016558
17	0.37990088901627	0.37988550137320	0.37988552133883	0.37988547131810	0.37988546126525	0.37988549636770	0.37988550637563
18	0.37989922572711	0.37988549962528	0.37988551540754	0.37988547587696	0.37988546793665	0.37988549566981	0.37988550357859
19	0.37989781811277	0.37988549831266	0.37988551095190	0.37988547930013	0.37988547294528	0.37988549514576	0.37988550147802
20	0.37989661633421	0.37988549731149	0.37988550755250	0.37988548191084	0.37988547676465	0.37988549474607	0.37988549987579
21	0.37989558213024	0.37988549653714	0.37988550492264	0.37988548392991	0.37988547971814	0.37988549443694	0.37988549863652
22	0.37989468573374	0.37988549593065	0.37988550286243	0.37988548551120	0.37988548203102	0.37988549419483	0.37988549766585
23	0.37989390370531	0.37988549545019	0.37988550123003	0.37988548676385	0.37988548386305	0.37988549400304	0.37988549689686
24	0.37989321738516	0.37988549506558	0.37988549992312	0.37988548776651	0.37988548532937	0.37988549384951	0.37988549628129
25	0.37989261176951	0.37988549475478	0.37988549886684	0.37988548857675	0.37988548651420	0.37988549372545	0.37988549578382
26	0.37989207468348	0.37988549450141	0.37988549800566	0.37988548923722	0.37988548747996	0.37988549362432	0.37988549537829
27	0.37989159616440	0.37988549429321	0.37988549729791	0.37988548977995	0.37988548827353	0.37988549354121	0.37988549504503
28	0.37989116799637	0.37988549412084	0.37988549671191	0.37988549022926	0.37988548893046	0.37988549347241	0.37988549476912
29	0.37989078335525	0.37988549397715	0.37988549622338	0.37988549060380	0.37988548947805	0.37988549341506	0.37988549453912
30	0.37989043653501	0.37988549385660	0.37988549581348	0.37988549091801	0.37988548993742	0.37988549336694	0.37988549434616
31	0.37989012273493	0.37988549375485	0.37988549546752	0.37988549118318	0.37988549032510	0.37988549332633	0.37988549418330
32	0.37988983789251	0.37988549366851	0.37988549517390	0.37988549140824	0.37988549065411	0.37988549329188	0.37988549404509
33	0.37988957855131	0.37988549359485	0.37988549492338	0.37988549160023	0.37988549093477		0.37988549392717
34	0.37988934175566	0.37988549353170	0.37988549470859	0.37988549176483	0.37988549117538		0.37988549382608
35	0.37988912496600	0.37988549347730	0.37988549452360	0.37988549190658	0.37988549138260		0.37988549373901
36	0.37988892599054	0.37988549343026	0.37988549436357	0.37988549202921	0.37988549156184		0.37988549366370
37	0.37988874292963	0.37988549338939	0.37988549422457	0.37988549213571	0.37988549171752		0.37988549359828
38	0.37988857413024	0.37988549335376	0.37988549410337	0.37988549222856	0.37988549185325		0.37988549354124
39	0.37988841814853		0.37988549399731	0.37988549230983	0.37988549197203		0.37988549349133
40	0.37988827371885		0.37988549390416	0.37988549238119	0.37988549207633		0.37988549344750
41	0.37988813972802		0.37988549382210	0.37988549244405	0.37988549216822		0.37988549340888
42	0.37988801519375		0.37988549374956	0.37988549249962	0.37988549224944		0.37988549337475
43	0.37988789924666		0.37988549368526	0.37988549254887	0.37988549232143		
44	0.37988779111502		0.37988549362810	0.37988549259266	0.37988549238542		
45	0.37988769011188		0.37988549357715	0.37988549263169	0.37988549244247		
46	0.37988759562415		0.37988549353160	0.37988549266656	0.37988549249345		
47	0.37988750710327		0.37988549349081		0.37988549253912		

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48	0.37988742405718		0.37988549345417		0.37988549258013		
49	0.37988734604357				0.37988549261704		

Table (5) to calculate integration $\int_0^1 \frac{dx}{1+e^x} = 0.37988549304172$ by using trapezoidal rule with the hyperbolic acceleration methods of AL-Tememe of the first kind

n	Values of midpoint rule	$A^F{}_{sinh}$	$A^F{}_{cosh}$	$A^F{}_{tanh}$	$A^F{}_{sech}$	$A^F{}_{sinh^2}$	$A^F{}_{cosh^2}$
1	0.37754066879815						
2	0.37932239996940	0.37982994282989	0.37972066501909	0.38009837860218	0.38024891252168	0.37986974045467	0.37979362439261
3	0.37963696720531	0.37987855904996	0.37986126578362	0.37990908838391	0.37992107135419	0.37988347122286	0.37987378607997
4	0.37974603354228	0.37988371640046	0.37987899579608	0.37989139518981	0.37989416765548	0.37988497445544	0.37988247600360
5	0.37979633794288	0.37988484742999	0.37988308763052	0.37988761837521	0.37988858690548	0.37988530458184	0.37988439411243
6	0.37982361700130	0.37988520478414	0.37988440893157	0.37988643776231	0.37988686189188	0.37988540889979	0.37988500180742
7	0.37984004955443	0.37988534558992	0.37988493549703	0.37988597507956	0.37988618966341	0.37988545000246	0.37988524159301
8	0.37985070854261	0.37988540996654	0.37988517785750	0.37988576420122	0.37988588427621	0.37988546879391	0.37988535131466
9	0.37985801341607	0.37988544273267	0.37988530174115	0.37988565708181	0.37988572946825	0.37988547835796	0.37988540718999
10	0.37986323710632	0.37988546081992	0.37988537032482	0.37988559802976	0.37988564424499	0.37988548363728	0.37988543804488
11	0.37986710127233	0.37988547146009	0.37988541075363	0.37988556332413	0.37988559420723	0.37988548674286	0.37988545620049
12	0.37987003984694	0.37988547804840	0.37988543582484	0.37988554184956	0.37988556326791	0.37988548866579	0.37988546744444
13	0.37987232648570	0.37988548230352	0.37988545203590	0.37988552798731	0.37988554330683	0.37988548990771	0.37988547470746
14	0.37987414069963	0.37988548515050	0.37988546289195	0.37988551871622	0.37988552996243	0.37988549073863	0.37988547956749
15	0.37987560420917	0.37988548711338	0.37988547038203	0.37988551232624	0.37988552076802	0.37988549131152	0.37988548291858
16	0.37987680191426	0.37988548850212	0.37988547568428	0.37988550780650	0.37988551426641	0.37988549171683	0.37988548528964
17	0.37987779449505	0.37988548950697	0.37988547952261	0.37988550453684	0.37988550956404	0.37988549201010	0.37988548700536
18	0.37987862625398	0.37988549024853	0.37988548235631	0.37988550212432	0.37988550609502	0.37988549222653	0.37988548827160
19	0.37987933014771	0.37988549080541	0.37988548448502	0.37988550031285	0.37988550349066	0.37988549238906	0.37988548922254
20	0.37987993110350	0.37988549123017	0.37988548610910	0.37988549893135	0.37988550150473	0.37988549251303	0.37988548994788
21	0.37988044825726	0.37988549155870	0.37988548736553	0.37988549786294	0.37988549996904	0.37988549260890	0.37988549050891
22	0.37988089649630	0.37988549181601	0.37988548834981	0.37988549702621	0.37988549876646	0.37988549268400	0.37988549094834
23	0.37988128754300	0.37988549201986	0.37988548912970	0.37988549636339	0.37988549781391	0.37988549274350	0.37988549129646
24	0.37988163072920	0.37988549218304	0.37988548975408	0.37988549583285	0.37988549705152	0.37988549279112	0.37988549157514
25	0.37988193355825	0.37988549231490	0.37988549025873	0.37988549540414	0.37988549643548	0.37988549282960	0.37988549180035
26	0.37988220211862	0.37988549242240	0.37988549067016	0.37988549505467	0.37988549593336	0.37988549286098	0.37988549198394
27	0.37988244139249	0.37988549251073	0.37988549100830	0.37988549476750	0.37988549552076		0.37988549213480
28	0.37988265548840	0.37988549258387	0.37988549128826	0.37988549452977	0.37988549517920		0.37988549225971
29	0.37988284781892	0.37988549264484	0.37988549152167	0.37988549433160	0.37988549489450		0.37988549236384
30	0.37988302123741	0.37988549269598	0.37988549171749	0.37988549416535	0.37988549465566		0.37988549245119
31	0.37988317814454	0.37988549273915	0.37988549188278	0.37988549402504	0.37988549445410		0.37988549252492
32	0.37988332057179	0.37988549277579	0.37988549202306	0.37988549390597	0.37988549428305		0.37988549258749
33	0.37988345024754		0.37988549214274	0.37988549380438	0.37988549413713		0.37988549264087
34	0.37988356864980		0.37988549224536	0.37988549371730	0.37988549401204		0.37988549268664
35	0.37988367704846		0.37988549233375	0.37988549364230	0.37988549390430		0.37988549272606
36	0.37988377653950		0.37988549241020	0.37988549357742	0.37988549381111		0.37988549276015
37	0.37988386807285		0.37988549247660	0.37988549352107	0.37988549373017		
38	0.37988395247506		0.37988549253451	0.37988549347195	0.37988549365961		
39	0.37988403046813		0.37988549258518	0.37988549342895	0.37988549359785		
40	0.37988410268491		0.37988549262969	0.37988549339120	0.37988549354363		
41	0.37988416968205		0.37988549266890		0.37988549349585		
42	0.37988423195070		0.37988549270354		0.37988549345362		
43	0.37988428992559				0.37988549341620		

Table (6) to calculate integration $\int_0^1 \frac{dx}{1+e^x} = 0.37988549304172$ by using midpoint rule with the hyperbolic acceleration methods of AL-Tememe of the first kind

6.Conclusion:

We can say that these methods are working with the same efficiency in improving the results of the integrals we reviewed in terms of accuracy and the number of partial intervals used and the speed of obtaining their values with a simple variation in n values.

7.References:

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