**International Journal of Engineering and Information Systems (IJEAIS)**
**ISSN: 2000-000X**
**Vol. 3 Issue 4, April – 2019, Pages: 50-54**

# Triangular Acceleration Methods of Second Kind for Improving the Values of Integrals Numerically

**[1]Ali Hassan Mohammed and [2]Shatha Hadier Theyab**

[1]Faculty of Education for Girls, Department of Mathematics, University of Kufa, Iraq.
Prof. ali57hassan@gmail.com
[2]Faculty of Education for Girls, Department of Mathematics, Iraq. Shathahaider93@gmail.com

***Abstract:*** *The aims of this study are to introduce acceleration methods that are called triangular acceleration methods, which come within the series of several acceleration methods that generally known as Al-Tememe's acceleration methods of the second kind which are discovered by (Ali Hassan Mohammed). These methods are useful in improving the results of determining numerical integrals of continuous integrands where the main error is of the forth order with respect to accuracy, partial intervals and the fasting of calculating the results specifically to accelerate results come out by Simpson's method. Also, it is possible to make use of it to improve the results of solving differential equations numerically of the main error of the forth order*

There are numerical methods for calculating single integrals that are bounded in their integration intervals.

**1.**Trapezoidal Rule
**2.**Midpoint Rule
**3.**Simpson's Rule

It is called Newton–Cotes formulas.

This study will introduce Simpson's method to find approximate values of single integrals of continuous integrands through using triangular acceleration methods, which fall within Al-Tememe's acceleration series- the second kind. We will compare these methods in respect to accuracy and the fasting of approaching these values to the real value (analytical) of those integrals.

Let's assume the integration J:

$$J = \int_{x_0}^{x_{2n}} f(x)dx \qquad (1)$$

Such that f(x) is a continuous integrand Lies above x - axis in the interval $[x_0, x_{2n}]$.Generally, Newton–Cotes formula for integration can be written in the following form:

$$\int_{x_0}^{x_{2n}} f(x)dx = G(h) + E_G(h) + R_G$$

Such that G (h) represents (Lagrangian – Approximation) of integration value J, G refers to the type of the rule, $E_G$ (h) is the correction terms that can be added to G(h) and $R_G$ is the remainder .

the Simpson 's rule value G(h) will referred by S(h) and it is givenby:

S(h)=$\frac{h}{3}[f(a) + 4f(a+h) + 2f(a+2h) + 4f(a+3h) + \cdots + 2f(a+(2n-2)h) + 4f(a+(2n-1)h) + f(b)]$;h=$\frac{b-a}{2n}$ and n=1,2,…

And the general formula for $E_G$(h) is given by :

$(E_S(h)=E_G(h))$ $\quad (f_{x_{2n}}{}^{(3)} - f_{x_0}{}^{(3)})$+... $f_{x_{2n}}{}^{(3)} - f_{x_0}{}^{(5)}$)+ $\frac{1}{1512}$ $(E_S(h)=\frac{1}{180}$

Fox [3]

So, when integrands of integration is a continuous function and their derivatives are in each point of integration intervals$[x_0, x_{2n}]$, it is possible to write error formula as:

$E=J-s(h)=A_1h^4+A_2h^6+A_3h^8+ \ldots$.

Such that $A_1$ ,$A_2$ ,$A_3$,…… are constants that their values do not depend on h but on the values of the derivatives of integrand in the end of the integration interval.

**2.Al-Tememe's triangular acceleration of second kind**

We will introduce six rules of Al-Tememe's triangular acceleration, which come within Al-Tememe's acceleration series of the second type.

It is mentioned above that the error in Simpson's rule can be written as the following:

$$E= A_1h^4+ A_2h^6+\ldots =h^3(A_1h+A_2h^3+\ldots\ldots) \qquad (3)$$

$$\cong h^3\sin h \quad \text{since}(\sinh=h-\frac{1}{6}h^3 + \frac{1}{120}h^5 - \frac{1}{5040}h^7 + \cdots) \qquad [2]$$

If we assume that J value calculated numerically based on Simpson's rule as $S_1(h_1)$ when $h= h_1$ and $S_2(h_2)$, when $h=h_2$, so:

$$J-S_1(h_1) \cong h_1^3 \sin h \quad (4)$$

$$J-S_2(h_2) \cong h_2^3 \sin h \quad (5)$$

From the equations (4) and (5), we get:

$$A^S_{Sin} \cong \frac{(h_1^3 \sin h_1)\, S_2(h_2)-(h_2^3 \sin h_2)\, S_1(h_1)}{h_1^3 \sin h_1 - h_2^3 \sin h_2} \quad (6)$$

The formula (6) is called Al-Tememe's sine triangular acceleration of the second kind that referred to by ($A^S_{sin}$)

Similarly, the second triangular acceleration rule can be written. The error E can be written as :

$$E=h^4(A_1h+A_2h^2+A_3h^4+\ldots) \cong h^4\cosh$$

Based on the same sine method mentioned above, we get the following:

$$A^S_{COS} \cong \frac{(h_1^4 \cos h_1) S_2(h_2)-(h_2^4 \cos h_2)\, S_1(h_1)}{h_1^4 \cos h_1 - h_2^4 \cos h_2} \quad (7)$$

Since $(\cosh=1-\frac{1}{2}h^2+\frac{1}{24}h^4-\frac{1}{720}h^6-\ldots)$ [2]

We call the formula (7) as Al-Tememe's cosine triangular acceleration of the second kind that referred to by ($A^S_{COS}$).

Similarly, we find the third triangular acceleration rule that we will call Al-Tememe's tangent triangular acceleration of the second kind, which is referred to by ($A^S_{Tan}$) and Al-Tememe's forth triangular acceleration rule of the second kind that we will call it Al-Tememe's secant triangular acceleration of the second kind, which is referred to by($A^S_{Sec}$). These laws are:

$$A^S_{Tan} \cong \frac{(h_1^3 \tan h_1)\, S_2(h_2)-(h_2^3 \tan h_2)\, S_1(h_1)}{h_1^3 \tan h_1 - h_2^3 \tan h_2} \quad (8)$$

Since$(\tanh=h+\frac{1}{3}h^3+\frac{2}{15}h^5+\frac{17}{315}h^7+\frac{62}{2835}h^9+\ldots)$ [2]

$$A^S_{Sec} \cong \frac{(h_1^4 \sec h_1)\, S_2(h_2)-(h_2^4 \sec h_2)\, S_1(h_1)}{h_1^4 sec\ h_1 - h_2^4 \sec h_2} \quad (9)$$

Since$(\text{sech}=1+\frac{1}{2}h^2+\frac{5}{24}h^4+\frac{61}{720}h^6+\frac{277}{8064}h^8+\ldots)$ [2]

Now, we will derive the fifth triangular acceleration : since the error is

$$E(h)=A_1h^4 + A_2h^6 + \ldots = h^2(A_1h^2 + A_2h^4 + \ldots\ldots) \quad (10)$$

$$\cong h^2 (\cosh-1) \quad (11)$$

$$=-2 \sin^2\left(\frac{h}{2}\right) \quad (12)$$

So, the fifth triangular acceleration rule will be:

$$A^S_{Sin^2} \cong \frac{\left(h^2{}_2 sin^2\left(\frac{h_2}{2}\right)\right) S_2(h_2)-\left(h_1^2 \sin^2\left(\frac{h_1}{2}\right)\right) S_1(h_1)}{h_2^2 \sin^2\left(\frac{h_2}{2}\right)-h_1^2 \sin^2\left(\frac{h_1}{2}\right)} \quad (13)$$

Formula (13) is called the triangular acceleration square sine rule. To derive the sixth triangular acceleration law,

Also the error it can be written as $E=2\cos^2\frac{h}{2}$ . Similarly, the sixth triangular acceleration law of the second kind is given by

$$A^S_{COS^2} \cong \frac{\left(h_2^2 cos^2\left(\frac{h_2}{2}\right)\right) S^2(h^2)-\left(h^2{}_1 \cos^2\left(\frac{h}{2}\right)\right) S_1(h_1)}{h_2^2 \cos^2\left(\frac{h_2}{2}\right)-h_1^2 \cos^2\left(\frac{h_1}{2}\right)} \quad (14)$$

The formula (14) is called the triangular acceleration square cosine rule.

## 3.Examples:

We will review some integrals that have continuous integrands on the interval of integration using triangular acceleration methods of Al-Tememe to improve the results numerically:

**3.1:** $I = \int_1^2 ln(x)dx$ and its analytical value is 0.3862943611989 which is rounded for 14 decimal.

**3.2:** $I = \int_0^1 e^x dx$ and its analytical value is 0.71828182845904 which is rounded to 14 decimal.

**3.3:** $I = \int_1^2 \sqrt{x} dx$ and its analytical value is 1.21895141649746 which is rounded to 14 decimal.

**3.4:** $I = \int_1^2 \frac{1}{x}dx$ and its analytical value is 0.69314718055995 which is rounded to 14 decimal.

## 4.the Results

The integrand of integration $I = \int_1^2 ln(x)\, dx$ is continuous in the integration interval [1,2], and the formula of correction terms of Simpson's rule as above mentioned (equation 3).
We put EPS=$10^{-12}$ (represents the absolute error of the subsequent value-previous value). We get the results shown in table (1). We get corret value through accelerating $A^S_{COS}$ and other accelerations to 11 decimal when n=30,32,34,36,38,40 while by using Simpson's method without acceleration was corret to 7 decimal when n=40 while acceleration of $A^S_{COS^2}$, we get the same accuracy when n=30,32,34,36,38.

the integrand of integration $I = \int_0^1 e^x(x)\, dx$ is continuous in the integration interval [0,1], and the formula of correction terms of Simpson's rule as above mentioned (equation 3).
We put EPS=$10^{-12}$ (represents the absolute error of the subsequent value- previous value). We get the results in table (2). We get corret value through accelerating $A^S_{COS}$ to 11 decimal when n=22,24,26,28 and also for the other accelerations. While the value by using Simpson's method without acceleration was corret to 7 decimal except $A^S_{Sin}$ acceleration, we get the same accuracy when n=18,20,22,24.

the integrand of integration $I = \int_1^2 \sqrt{x}\ \ dx$ is continuous in the integration interval [1,2], and the formula of correction terms of Simpson's rule as above mentioned (equation 3).
We put EPS=$10^{-12}$ (represents the absolute error of the subsequent value- previous value). We get the results in table (3). We get correct value through accelerating $A^S_{COS}$ to 11 decimal when n=22,24,26,28,30 and also for all other accelerations. While the value by using Simpson's method without acceleration was correct to 8decimal.

To find the values integrand of integration $I = \int_1^2 \frac{1}{x}\ \ dx$ is continuous in the integration interval [1,2], and the formula of correction terms of Simpson's rule as above mentioned (equation 3).
We put EPS=$10^{-12}$ (represents the absolute error of the subsequent value- previous value). We get the results in table (4). We get correct value through accelerating $A^S_{COS}$ to 11 decimal when n=42,44,46,48,50 and also for all other accelerations. While the value by using Simpson's method without acceleration was correct to 7 decimal except $A^S_{Cos^2}$ acceleration, we get the same accuracy when n=42,44,46,48.

## 5.Conclusion:

We conclude from the mentioned tables that these acceleration methods have the same efficiency and give high accuracy of results during limited number of partial intervals.

| n | Values of Simpson's rule | $A^S_{cos(h)}$ | $A^S_{sin(h)}$ | $A^S_{tan(h)}$ | $A^S_{sec(h)}$ | $A^S_{sin^2(\frac{h}{2})}$ | $A^S_{cos^2(\frac{h}{2})}$ |
|---|---|---|---|---|---|---|---|
| 2 | 0.38583460216543 | | | | | | |
| 4 | 0.38625956281457 | 0.38628789438754 | 0.38628789381218 | 0.38628789294917 | 0.38628789266151 | 0.38628837116822 | 0.38628936824786 |
| 6 | 0.38628716327880 | 0.38629395728400 | 0.38629395725415 | 0.38629395720938 | 0.38629395719446 | 0.38629398180820 | 0.38629403149164 |
| 8 | 0.38629204346631 | 0.38629430230208 | 0.38629430229800 | 0.38629430229188 | 0.38629430228985 | 0.38629430564686 | 0.38629431238295 |
| 10 | 0.38629340380481 | 0.38629434756405 | 0.38629434756313 | 0.38629434756176 | 0.38629434756131 | 0.38629434831256 | 0.38629434981677 |
| 12 | 0.38629389730141 | 0.38629435696707 | 0.38629435696680 | 0.38629435696638 | 0.38629435696625 | 0.38629435719284 | 0.38629435764608 |
| 14 | 0.38629411005202 | 0.38629435957687 | 0.38629435957677 | 0.38629435957661 | 0.38629435957656 | 0.38629435965999 | 0.38629435982676 |
| 16 | 0.38629421367579 | 0.38629436046092 | 0.38629436046088 | 0.38629436046081 | 0.38629436046079 | 0.38629436049621 | 0.38629436056699 |
| 18 | 0.38629426895381 | 0.38629436080728 | 0.38629436080726 | 0.38629436080723 | 0.38629436080722 | 0.38629436082396 | 0.38629436085740 |
| 20 | 0.38629430059436 | 0.38629436095891 | 0.38629436095890 | 0.38629436095888 | 0.38629436095888 | 0.38629436096747 | 0.38629436098464 |
| 22 | 0.38629431975207 | 0.38629436103135 | 0.38629436103134 | 0.38629436103133 | 0.38629436103133 | 0.38629436103605 | 0.38629436104547 |
| 24 | 0.38629433189634 | 0.38629436106849 | 0.38629436106849 | 0.38629436106848 | 0.38629436106848 | 0.38629436107121 | 0.38629436107667 |
| 26 | 0.38629433989431 | 0.38629436108867 | 0.38629436108867 | 0.38629436108867 | 0.38629436108867 | 0.38629436109033 | 0.38629436109364 |
| 28 | 0.38629434533433 | 0.38629436110019 | 0.38629436110019 | 0.38629436110019 | 0.38629436110019 | 0.38629436110124 | 0.38629436110332 |
| 30 | 0.38629434913817 | 0.38629436110705 | 0.38629436110705 | 0.38629436110705 | 0.38629436110705 | 0.38629436110773 | 0.38629436110909 |
| 32 | 0.38629435186233 | 0.38629436111128 | 0.38629436111128 | 0.38629436111128 | 0.38629436111128 | 0.38629436111173 | 0.38629436111264 |
| 34 | 0.38629435385453 | 0.38629436111397 | 0.38629436111397 | 0.38629436111397 | 0.38629436111397 | 0.38629436111428 | 0.38629436111491 |
| 36 | 0.38629435533858 | 0.38629436111573 | 0.38629436111573 | 0.38629436111573 | 0.38629436111573 | 0.38629436111595 | 0.38629436111639 |
| 38 | 0.38629435646235 | 0.38629436111691 | 0.38629436111691 | 0.38629436111691 | 0.38629436111691 | 0.38629436111706 | 0.38629436111738 |
| 40 | 0.38629435732589 | 0.38629436111771 | 0.38629436111771 | 0.38629436111771 | 0.38629436111771 | 0.38629436111783 | |

Table no.(1) to calculate integration $I = \int_1^2 \ln(x)\, dx = 0.38629436111989$ by simpson's rule with the triangnlation methods of Al-tememe

| n | Values of simpson's rule | $A^S_{cos(h)}$ | $A^S_{sin(h)}$ | $A^S_{tan(h)}$ | $A^S_{sec(h)}$ | $A^S_{sin^2(\frac{h}{2})}$ | $A^S_{cos^2(\frac{h}{2})}$ |
|---|---|---|---|---|---|---|---|
| 2 | 1.71886115187659 | | | | | | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 4 | 1.71831884192175 | 1.71828268682341 | 1.71828268755765 | 1.71828268865897 | 1.71828268902607 | 1.71828207838376 | 1.71828080596898 |
| 6 | 1.71828916992083 | 1.71828186599555 | 1.71828186602764 | 1.71828186607577 | 1.71828186609182 | 1.71828183963070 | 1.71828178621831 |
| 8 | 1.71828415469990 | 1.71828183336278 | 1.71828183336697 | 1.71828183337325 | 1.71828183337535 | 1.71828182992545 | 1.71828182300297 |
| 10 | 1.71828278192482 | 1.71828182953748 | 1.71828182953840 | 1.71828182953979 | 1.71828182954025 | 1.71828182878213 | 1.71828182726417 |
| 12 | 1.71828228843802 | 1.71828182878149 | 1.71828182878177 | 1.71828182878218 | 1.71828182878232 | 1.71828182855573 | 1.71828182810250 |
| 14 | 1.71828207679867 | 1.71828182857717 | 1.71828182857727 | 1.71828182857742 | 1.71828182857747 | 1.71828182849448 | 1.71828182832858 |
| 16 | 1.71828197405189 | 1.71828182850904 | 1.71828182850908 | 1.71828182850915 | 1.71828182850917 | 1.71828182847405 | 1.71828182840386 |
| 18 | 1.71828191936081 | 1.71828182848262 | 1.71828182848264 | 1.71828182848267 | 1.71828182848268 | 1.71828182846612 | 1.71828182843304 |
| 20 | 1.71828188810386 | 1.71828182847114 | 1.71828182847115 | 1.71828182847116 | 1.71828182847117 | 1.71828182846268 | 1.71828182844572 |
| 22 | 1.71828186919936 | 1.71828182846567 | 1.71828182846568 | 1.71828182846569 | 1.71828182846569 | 1.71828182846104 | 1.71828182845174 |
| 24 | 1.71828185722555 | 1.71828182846288 | 1.71828182846289 | 1.71828182846289 | 1.71828182846289 | 1.71828182846020 | 1.71828182845481 |
| 26 | 1.71828184934489 | 1.71828182846137 | 1.71828182846138 | 1.71828182846138 | 1.71828182846138 | | 1.71828182845648 |
| 28 | 1.71828184398734 | 1.71828182846051 | 1.71828182846051 | 1.71828182846052 | 1.71828182846052 | | 1.71828182845743 |

Table no.(2) to calculate integration $I = \int_0^1 e^x \; dx = 1.71828182845904$ by simpson's rule with the triangnlation methods of Al-tememe

| n | Values of simpson's rule | $A^S_{cos(h)}$ | $A^S_{sin(h)}$ | $A^S_{tan(h)}$ | $A^S_{sec(h)}$ | $A^S_{sin^2(\frac{h}{2})}$ | $A^S_{cos^2(\frac{h}{2})}$ |
|---|---|---|---|---|---|---|---|
| 2 | 1.21886550798991 | | | | | | |
| 4 | 1.21894515685709 | 1.21895046694332 | 1.21895046683548 | 1.21895046667373 | 1.21895046661982 | 1.21895055630463 | 1.21895074318373 |
| 6 | 1.21895013467771 | 1.21895135999548 | 1.21895135999009 | 1.21895135998202 | 1.21895135997933 | 1.21895136441848 | 1.21895137337903 |
| 8 | 1.21895100538783 | 1.21895140840331 | 1.21895140840259 | 1.21895140840149 | 1.21895140840113 | 1.21895140900008 | 1.21895141020192 |
| 10 | 1.21895124701366 | 1.21895141464592 | 1.21895141464575 | 1.21895141464551 | 1.21895141464543 | 1.21895141477887 | 1.21895141504605 |
| 12 | 1.21895133447083 | 1.21895141593250 | 1.21895141593245 | 1.21895141593237 | 1.21895141593235 | 1.21895141597251 | 1.21895141605283 |
| 14 | 1.21895137212513 | 1.21895141628804 | 1.21895141628802 | 1.21895141628799 | 1.21895141628798 | 1.21895141630275 | 1.21895141633226 |
| 16 | 1.21895139045026 | 1.21895141640816 | 1.21895141640815 | 1.21895141640814 | 1.21895141640813 | 1.21895141641440 | 1.21895141642691 |
| 18 | 1.21895140022042 | 1.21895141645514 | 1.21895141645514 | 1.21895141645513 | 1.21895141645513 | 1.21895141645809 | 1.21895141646400 |
| 20 | 1.21895140581060 | 1.21895141647568 | 1.21895141647568 | 1.21895141647568 | 1.21895141647568 | 1.21895141647719 | 1.21895141648023 |
| 22 | 1.21895140919440 | 1.21895141648549 | 1.21895141648549 | 1.21895141648549 | 1.21895141648548 | 1.21895141648632 | 1.21895141648798 |
| 24 | 1.21895141133897 | 1.21895141649051 | 1.21895141649051 | 1.21895141649051 | 1.21895141649051 | 1.21895141649099 | 1.21895141649196 |
| 26 | 1.21895141275111 | 1.21895141649324 | 1.21895141649324 | 1.21895141649324 | 1.21895141649324 | 1.21895141649353 | 1.21895141649412 |
| 28 | 1.21895141371149 | 1.21895141649480 | 1.21895141649480 | 1.21895141649480 | 1.21895141649480 | 1.21895141649498 | 1.21895141649535 |
| 30 | 1.21895141438295 | 1.21895141649573 | 1.21895141649573 | 1.21895141649572 | 1.21895141649573 | 1.21895141649585 | 1.21895141649609 |

Table no.(3) to calculate integration $I = \int_1^2 \sqrt{x} dx = 1.21895141649746$ by simpson's rule with the triangnlation methods of Al-tememe

| n | Values of simpson's rule | $A^S_{cos(h)}$ | $A^S_{sin(h)}$ | $A^S_{tan(h)}$ | $A^S_{sec(h)}$ | $A^S_{sin^2(\frac{h}{2})}$ | $A^S_{cos^2(\frac{h}{2})}$ |
|---|---|---|---|---|---|---|---|
| 2 | 0.69444444444444 | | | | | | |
| 4 | 0.69325396825397 | 0.69317460075693 | 0.69317460236873 | 0.69317460478635 | 0.69317460559220 | 0.69317326511316 | 0.69317047191437 |
| 6 | 0.69316979316979 | 0.69314907301252 | 0.69314907310356 | 0.69314907324011 | 0.69314907328562 | 0.69314899821935 | 0.69314884669627 |
| 8 | 0.69315453065453 | 0.69314746627120 | 0.69314746628395 | 0.69314746630308 | 0.69314746630945 | 0.69314745581059 | 0.69314743474382 |
| 10 | 0.69315023068893 | 0.69314724751064 | 0.69314724751352 | 0.69314724751785 | 0.69314724751929 | 0.69314724514463 | 0.69314724038989 |
| 12 | 0.69314866220910 | 0.69314720125415 | 0.69314720125502 | 0.69314720125634 | 0.69314720125677 | 0.69314720053659 | 0.69314719909607 |
| 14 | 0.69314798387509 | 0.69314718829010 | 0.69314718829042 | 0.69314718829091 | 0.69314718829107 | 0.69314718802508 | 0.69314718749334 |
| 16 | 0.69314765281942 | 0.69314718387251 | 0.69314718387265 | 0.69314718387285 | 0.69314718387292 | 0.69314718375976 | 0.69314718353362 |
| 18 | 0.69314747598101 | 0.69314718213501 | 0.69314718213508 | 0.69314718213518 | 0.69314718213521 | 0.69314718208167 | 0.69314718197469 |
| 20 | 0.69314737466512 | 0.69314718137238 | 0.69314718137241 | 0.69314718137246 | 0.69314718137248 | 0.69314718134496 | 0.69314718128998 |
| 22 | 0.69314731327821 | 0.69314718100734 | 0.69314718100735 | 0.69314718100738 | 0.69314718100739 | 0.69314718099227 | 0.69314718096208 |
| 24 | 0.69314727434432 | 0.69314718081991 | 0.69314718081992 | 0.69314718081994 | 0.69314718081994 | 0.69314718081117 | 0.69314718079366 |
| 26 | 0.69314724869303 | 0.69314718071793 | 0.69314718071794 | 0.69314718071795 | 0.69314718071795 | 0.69314718071263 | 0.69314718070201 |
| 28 | 0.69314723124018 | 0.69314718065968 | 0.69314718065969 | 0.69314718065969 | 0.69314718065970 | 0.69314718065634 | 0.69314718064964 |
| 30 | 0.69314721903355 | 0.69314718062500 | 0.69314718062500 | 0.69314718062501 | 0.69314718062501 | 0.69314718062282 | 0.69314718061846 |
| 32 | 0.69314721028982 | 0.69314718060360 | 0.69314718060360 | 0.69314718060360 | 0.69314718060360 | 0.69314718060214 | 0.69314718059921 |
| 34 | 0.69314720389440 | 0.69314718058997 | 0.69314718058997 | 0.69314718058997 | 0.69314718058998 | 0.69314718058897 | 0.69314718058696 |
| 36 | 0.69314719912962 | 0.69314718058106 | 0.69314718058106 | 0.69314718058106 | 0.69314718058106 | 0.69314718058035 | 0.69314718057894 |
| 38 | 0.69314719552111 | 0.69314718057508 | 0.69314718057508 | 0.69314718057508 | 0.69314718057508 | 0.69314718057458 | 0.69314718057356 |
| 40 | 0.69314719274796 | 0.69314718057099 | 0.69314718057099 | 0.69314718057099 | 0.69314718057099 | 0.69314718057062 | 0.69314718056988 |
| 42 | 0.69314719058850 | 0.69314718056813 | 0.69314718056813 | 0.69314718056813 | 0.69314718056813 | 0.69314718056786 | 0.69314718056731 |
| 44 | 0.69314718888677 | 0.69314718056610 | 0.69314718056610 | 0.69314718056610 | 0.69314718056610 | 0.69314718056589 | 0.69314718056548 |
| 46 | 0.69314718753113 | 0.69314718056463 | 0.69314718056463 | 0.69314718056463 | 0.69314718056463 | 0.69314718056447 | 0.69314718056416 |
| 48 | 0.69314718644045 | 0.69314718056356 | 0.69314718056356 | 0.69314718056356 | 0.69314718056356 | 0.69314718056344 | 0.69314718056320 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 50 | 0.69314718555495 | 0.69314718056276 | 0.69314718056276 | 0.69314718056276 | 0.69314718056276 | 0.69314718056266 | |

Table no.(1) to calculate integration I $= \int_1^2 \frac{1}{x}\ dx = 0.69314718055995\ by$ simpson's rule with the triangnlation methods of Al-tememe

## References

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