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Triangular functions of Al-Tememe Acceleration Methods of First Kind for Improving the Values of Integrals Numerically

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Abstract: *The main aim of this work is to introduce acceleration methods called a Trigonometric acceleration methods which are of series of numerated methods. In general, these methods named as AL-Tememe's acceleration methods of first kind to his discoverer "Ali Hassan Mohammed". These are very beneficial to acceleration the numerical results for definite integrations with continuous integrands which are of 2nd order main error, with respect to the accuracy and the number of the used subintervals and the fasting obtaining results. Especially, for acceleration the results which are obviously obtained by trapezoidal and midpoint methods. Moreover, these methods could be enhancing the results of the ordinary differential equations numerically which are of 2nd order main error.*

1. INTRODUCTION

Electricity the driving force of modern civilization, is indispensable in our day to day life. There are two basic types of electricity generation. One of which is through conventional energy resources which will get extinct in near future, hence demanding an alternative arrangement. Therefore, it is of great urgency to go for non-conventional energy resources. The non-conventional

There are numerical methods for calculating single integrals that are bounded in their integration intervals such as:

1. Trapezoidal Rule
2. Midpoint Rule
3. Simpson's Rule,

which are called "Newton-cotes formulas".

In this paper, we introduce two methods which are trapezoidal and midpoint methods for finding an approximate values of single Integrals which integrands are continuous in interval of integration using triangular acceleration methods which are part of a series of AL-Tememe's acceleration methods of first kind. We will make a comparison of these methods as an accuracy and fasting reaching of its values with the real values (analytic) for those integrals.

Consider the integral J defined as:

$$J=\int_{x_0}^{x_m} f(x)dx \qquad \ldots(1)$$

Such that, f(x) is a continuous function defined on $[x_0,x_m]$. We need to calculate the integral J approximately. In general we can write Newton-cotes formula as:

$$J=\int_{x_0}^{x_m} f(x)dx = f(x)dx = G(h) + E_{G(h)} + R_G \qquad \ldots(2)$$

Here, G(h) is Lagrangian approximation to the value of the integral J, (the letter G symbolizes the rule type), E_G is the remainder and related to amputation after the use of certain terms of $E_G(h)$, h= $\frac{x_m-x_0}{m}$; m is number of sub intervals used and the general form of G(h) is :

$$G(h) = h(w_0f_0 + w_1f_1 + w_2f_2 + \ldots + w_2f_{m-2} + w_1f_{m-1} + w_0f_m) \qquad \ldots(3)$$

Where $f_r = f(x_r)$ and $x_r = x_0 + rh$; r = 0,1,2,…,m and weight coefficients w_r take the sequence ($w_0, w_1, w_2, \ldots, w_2, w_1, w_0$).

To simplify the formula (3), we write weights by w_0 such that w_1=2(1-w_0), w_2=2w_0, we note that when $w_0 = \frac{1}{2}$ we get the trapezoidal rule and refer for G (h) by T(h) where T(h) = $\frac{h}{2}$ (f_0+2f_1+2f_2+…+2f_{m-1}+f_m). When w_0=0, we get the midpoint rule and we refer to it by M(h) where M(h)=h(f_1+f_3+...+f_{2i-1}); i=1,2,…,m. The general formula of E_G(h) is the following:

1- For Trapezoidal rule:

$$E_T(h) = \frac{-1}{12}h^2(f_m{}'-f_0{}') + \frac{1}{720}h^4\left(f_m{}^{(3)} - f_0{}^{(3)}\right) - \frac{1}{30240}h^6\left(f_m{}^{(5)} - f_0{}^{(5)}\right) + \cdots \qquad \ldots(4)$$

2- Midpoint rule:

$$E_M(h) = \frac{1}{6}h^2(f_m{}' - f_0{}') - \frac{7}{360}h^4\left(f_m{}^{(3)} - f_0{}^{(3)}\right) + \frac{31}{15120}h^6(f_m{}^{(5)} - f_0{}^{(5)}) - \ldots \qquad \ldots(5)$$

In these methods when the integrands is a continuous function and their derivatives are in each point of integration points interval $[x_0, x_m]$ the error formula can be written as:

$$\left.\begin{aligned} J - T(h) &= A_1h^2 + A_2h^4 + A_3h^6 + \cdots \\ J - M(h) &= B_1h^2 + B_2h^4 + B_3h^6 + \cdots \end{aligned}\right] \qquad \ldots(6)$$

Where $A_1, A_2, A_3, \ldots$ and $B_1, B_2, B_3, \ldots$ are constants that do not depend on h but on the values of their derivatives at the end of integration interval.

[1]

2. DERIVATION AL-TEMEME'S ACCELERATION OF TRIANGULAR FUNCTIONS:

A series of acceleration methods of Al-Tememe's are introduced and we will call it triangular accelerations. Due to the similar error of both trapezoidal and midpoint methods regarding h basics, we will deal with the error for trapezoidal method to derive our acceleration methods following the same way to derive these methods as for the midpoint method.

In Trapezoidal rule:

$$J = \int_{x_0}^{x_m} f(x)dx = h\left[\frac{1}{2}f_0 + f_1 + f_2 + \cdots \ldots + f_{m-1} + \frac{1}{2}f_m\right] + E(h) \qquad \ldots(7)$$

Where

$E(h) = A_1h^2 + A_2h^4 + \ldots$; $A_1, A_2, \ldots$ are constants ...(8)

Since $\sin(h) = h - \frac{h^3}{6} + \frac{h^5}{120} - \ldots$ [2]

So ,we can write

$$E(h) = h(A_1h + A_2h^3 + A_3h^5 + \ldots) \cong h\sin(h) \qquad \ldots(9)$$

So , we will have:

$$J \cong \frac{h}{2}[f_0 + f_1 + \cdots + f_m] + h\sin(h) \qquad \ldots(10)$$

Therefore ,

$$E = J - T(h) \cong h\sin(h) \qquad \ldots(11)$$

We assume that $T_1(h_1)$ represents the value of the above mentioned integration numerically when $h=h_1$, also, $T_2(h_2)$ represents the value of integration numerically when $h=h_2$, So;

$J - T_1(h_1) \cong h_1\sin(h_1)$ ...(12)

$J - T_2(h_2) \cong h_2\sin(h_2)$ ...(13)

From the equations (12) and (13) we get:

$$A^F_{sin} \cong \frac{h_2\sin(h_2)\,T_1(h_1) - h_1\sin(h_1)T_2(h_2)}{h_2\sin(h_2) - h_1\sin(h_1)} \qquad \ldots(14)$$

The formula (14) is called triangular acceleration sine law for Al-Tememe of the first kind. We refer to it by (A^F_{sin}).

Also it is possible to write the equation (11) by the formula:

$E = J - T(h) \cong h^2\cos(h)$; since $\cos(h) = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \ldots$ [2]

Following the same above method, we get;

$$A^F_{cos} \cong \frac{{h_2}^2\cos(h_2)T_1(h_1) - {h_1}^2\cos(h_1)T_2(h_2)}{{h_2}^2\cos(h_2) - {h_1}^2\cos(h_1)} \qquad \ldots(15)$$

The formula (15) is called triangular acceleration cosine law for Al-Tememe of the first kind. In the same way, we can conclude the third triangular acceleration law that we will call as the triangular acceleration tangent law, referred to as A^F_{tan}. The fourth triangular acceleration law is called the triangular acceleration secant law that referred to by A^F_{sec}:

It is possible to write the equation (11) in the following formulas:

$E = J - T(h) \cong h\tan(h)$, since $\tan(h) = h + \frac{h^3}{3} + \frac{2h^5}{15} + \ldots$

$E = J - T(h) \cong h^2\sec(h)$, since $\sec(h) = 1 + \frac{h^2}{2} + \frac{5h^4}{24} + \ldots$ [2]

So,

$$A^F_{tan} \cong \frac{h_2\tan(h_2)T_1(h_1) - h_1\tan(h_1)T_2(h_2)}{h_2\tan(h_2) - h_1\tan(h_1)} \qquad \ldots(16)$$

$$A^F_{sec} \cong \frac{{h_2}^2\sec(h_2)T_1(h_1) - {h_1}^2\sec(h_1)T_2(h_2)}{{h_2}^2\sec(h_2) - {h_1}^2\sec(h_1)} \qquad \ldots(17)$$

Now, we will derive the fifth triangular acceleration law ; since the error is:

$$E(h) = J - T(h) = A_1h^2 + A_2h^4 + \ldots \cong \cos(h) - 1 = -2\sin^2\left(\frac{h}{2}\right) \qquad \ldots(18)$$

The law of acceleration will take the following form:

$$A^F_{sin^2} \cong \frac{\sin^2\left(\frac{h_2}{2}\right)T_1(h_1) - \sin^2\left(\frac{h_1}{2}\right)T_2(h_2)}{\sin^2\left(\frac{h_2}{2}\right) - \sin^2\left(\frac{h_1}{2}\right)} \qquad \ldots(19)$$

Formula (19) is called the triangular acceleration square sine law. To derive the sixth triangular acceleration law,

$$E(h) = J - T(h) = A_1h^2 + A_2h^4 + A_3h^6 + \ldots$$

$$= h^2(A_1 + A_2h^2 + A_3h^4 + \ldots) \cong h^2(1 + \cos h) = 2h^2\cos^2\left(\frac{h}{2}\right) \qquad \ldots(20)$$

The sixth triangular acceleration law takes the following formula;

$$A^F{}_{\cos^2} \cong \frac{{h_2}^2\cos^2\left(\frac{h_2}{2}\right)\ T_1(h_1)-{h_1}^2\cos^2\left(\frac{h_1}{2}\right)\ T_2(h_2)}{{h_2}^2\cos^2\left(\frac{h_2}{2}\right)-{h_1}^2\cos^2\left(\frac{h_1}{2}\right)} \quad \ldots (21)$$

The formula (21) is called the triangular acceleration square cosine law.

3. EXAMPLES:

We will review some integrals that have continuous integrands on the interval of integration using triangular acceleration methods of Al-Tememe to improve the results numerically:

3.1: $\int_1^2 \ln(x)\,dx$ and its analytic value is 0.38629436111989 is rounded to 14 decimal .

3.2: $\int_0^1 e^x dx$ and its analytic value is 1.71828182845904 is rounded to 14 decimal.

3.3: $\int_1^2 \sqrt{x}\,dx$ and its analytic value is 1.21895141649746 is rounded to 14 decimal .

We will compare the values of the acceleration methods with values of trapezoidal rule and midpoint rule. The priority of the acceleration methods can be calculated based on n values, n=1,2,3,...; The results we adopted in Mat lab. Program through putting Eps=10^{-10} that represents (the absolute error of the subsequent value- previous value).

4. THE RESULTS:

The integrands of integration $\int_1^2 \ln(x)\,dx$ is continuous in the integration interval [1,2] and the formula of the correction terms for the (trapezoidal and midpoint rules) identical for the formula in the equation (8).

1- Regarding trapezoidal rule with the triangular acceleration methods, we note in table (1) the following: when n=45,46,47,48,49, the values of triangular sine method $A^F{}_{\sin}$ are correct for 9 decimal places while the value of trapezoidal method without acceleration was correct for 4 decimal places only at n=49. The same accuracy we got through accelerating triangular acceleration cosine $A^F{}_{\cos}$ and square sine $A^F{}_{\sin^2}$ and square cosine $A^F{}_{\cos^2}$ with same (n), but the same accuracy of triangular tangent method $A^F{}_{\tan}$ and triangular secant method $A^F{}_{\sec}$ we got when n=46,47,48,49.

2- With respect to midpoint rule with triangular acceleration methods are concerned, we note the following through table (2): when n=40,41,…,48 the values of triangular sine method $A^F{}_{\sin}$ are correct for 9 decimal places but we note that the value of midpoint method without acceleration was correct for 4 decimal places only at n=48. Also, the same accuracy we got for the same n values of all other triangular acceleration rules.

In order to find the integration value $\int_0^1 e^x dx$ numerically , we note that integrands is continuous in the integration interval [0,1] , the formula of correction error (trapezoidal and midpoint rules) respectively are as in the formula (8).

1. Regarding Trapezoidal rule with triangular acceleration methods are concerned, we note the following through table (3) : when n=36,37 , the values of accelerating $A^F{}_{\sin}$ is correct for 8 decimal places. But we note that the value of trapezoidal method without acceleration method was correct for 3 decimal places only at n=37. We got the same accuracy for the another of triangular acceleration rules for the same values of n.
2. With respect to midpoint rule with triangular acceleration methods are concerned, we note the following through table (4): when n= 21,22,…,36 , the values of accelerating $A^F{}_{\sin}$ is correct for 8 decimal places. But we note that the value of Trapezoidal method without acceleration method was correct for 4 decimal places only at n=36. We got the same accuracy for the another of triangular acceleration rules for the same values of n.

To find the values of integration $\int_1^2 \sqrt{x}\,dx$ numerically where integrands is continuous in the integration interval [1,2] and the formula of correction errors for the rules of (trapezoidal and midpoint) respectively as mentioned above (equation No.8).

1- For trapezoidal rule with the triangular acceleration methods, we note in table (5) the following: when n=27,28, the values of acceleration of triangular sine method $A^F{}_{\sin}$ are correct for 9 decimal places while the value of trapezoidal method without acceleration was correct for 4 decimal places only at n=28. We got the same accuracy for the another of the triangular acceleration rules for the same value of n.

2- With respect to midpoint rule with triangular acceleration methods are concerned, we note the following through table (6): when n=27,28, the value of accelerating $A^F{}_{\sin}$ is correct for 9 decimal places, but we note that the value is correct for five decimal places at n=28 without acceleration for midpoint rule. We got the same accuracy for the another of triangular acceleration rules for the same values of n.

5. TABLES:

n	Values of trapezoidal rule	$A^F{}_{\sin}$	$A^F{}_{\cos}$	$A^F{}_{\tan}$	$A^F{}_{\sec}$	$A^F{}_{\sin^2}$	$A^F{}_{h^2\cos^2}$

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1	0.34657359027997						
2	0.37601934919407	0.38583510050374	0.38583609735117	0.38583360550779	0.38583310726435	0.38583485132826	0.38583534968713
3	0.38169376216592	0.38623335016163	0.38623346540382	0.38623317730754	0.38623311969247	0.38623332135230	0.38623337897123
4	0.38369950940944	0.38627834184134	0.38627837093677	0.38627829819933	0.38627828365235	0.38627833456764	0.38627834911508
5	0.38463153556860	0.38628847622029	0.38628848673572	0.38628846044740	0.38628845518984	0.38628847359147	0.38628847884912
6	0.38513881300058	0.38629171859639	0.38629172327903	0.38629171157249	0.38629170923121	0.38629171742573	0.38629171976704
7	0.38544502685801	0.38629300488986	0.38629300728163	0.38629300130224	0.38629300010637	0.38629300429192	0.38629300548780
8	0.38564390995210	0.38629359539924	0.38629359674554	0.38629359337980	0.38629359270665	0.38629359506267	0.38629359573582
9	0.38578032684952	0.38629389675309	0.38629389756789	0.38629389553088	0.38629389512348	0.38629389654939	0.38629389695679
10	0.38587793674575	0.38629406340633	0.38629406392798	0.38629406262387	0.38629406236305	0.38629406327592	0.38629406353674
11	0.38595017408175	0.38629416157068	0.38629416191996	0.38629416104676	0.38629416087212	0.38629416148336	0.38629416165800
12	0.38600512622621	0.38629422241179	0.38629422265439	0.38629422204789	0.38629422192659	0.38629422235114	0.38629422247244
13	0.38604789773836	0.38629426173521	0.38629426190893	0.38629426147463	0.38629426138777	0.38629426169178	0.38629426177864
14	0.38608183925352	0.38629428806039	0.38629428818804	0.38629428786893	0.38629428780511	0.38629428802848	0.38629428809231
15	0.38610922387841	0.38629430621872	0.38629430631461	0.38629430607490	0.38629430602696	0.38629430619475	0.38629430624269
16	0.38613163774487	0.38629431907039	0.38629431914380	0.38629431896026	0.38629431892356	0.38629431905203	0.38629431908874
17	0.38615021487207	0.38629432837225	0.38629432842941	0.38629432828651	0.38629432825793	0.38629432835796	0.38629432838654
18	0.38616578342773	0.38629433523854	0.38629433528370	0.38629433517079	0.38629433514820	0.38629433522724	0.38629433524983
19	0.38617895962484	0.38629434039596	0.38629434043212	0.38629434034173	0.38629434032365	0.38629434038693	0.38629434040501
20	0.38619020963221	0.38629434433041	0.38629434435970	0.38629434428648	0.38629434427183	0.38629434432309	0.38629434433773
21	0.38619989137125	0.38629434737394	0.38629434739792	0.38629434733798	0.38629434732599	0.38629434736795	0.38629434737994
22	0.38620828333449	0.38629434975805	0.38629434977786	0.38629434972832	0.38629434971841	0.38629434975309	0.38629434976300
23	0.38621560483485	0.38629435164698	0.38629435166350	0.38629435162220	0.38629435161394	0.38629435164285	0.38629435165111
24	0.38622203047880	0.38629435315919	0.38629435317307	0.38629435313836	0.38629435313142	0.38629435315572	0.38629435316266
25	0.38622770067230	0.38629435438133	0.38629435439308	0.38629435436371	0.38629435435783	0.38629435437840	0.38629435438427
26	0.38623272935532	0.38629435537770	0.38629435538771	0.38629435536268	0.38629435535767	0.38629435537520	0.38629435538020
27	0.38623720977007	0.38629435619653	0.38629435620511	0.38629435618365	0.38629435617936	0.38629435619438	0.38629435619867
28	0.38624121881413	0.38629435687448	0.38629435688188	0.38629435686337	0.38629435685967	0.38629435687262	0.38629435687632
29	0.38624482036166	0.38629435743965	0.38629435744606	0.38629435743002	0.38629435742681	0.38629435743804	0.38629435744125
30	0.38624806782323	0.38629435791382	0.38629435791941	0.38629435790544	0.38629435790264	0.38629435791242	0.38629435791522
31	0.38625100613726	0.38629435831403	0.38629435831892	0.38629435830669	0.38629435830425	0.38629435831281	0.38629435831525
32	0.38625367333297	0.38629435865370	0.38629435865800	0.38629435864725	0.38629435864510	0.38629435865263	0.38629435865478
33	0.38625610176691	0.38629435894349	0.38629435894728	0.38629435893780	0.38629435893590	0.38629435894254	0.38629435894443
34	0.38625831910892	0.38629435919194	0.38629435919530	0.38629435918690	0.38629435918522	0.38629435919110	0.38629435919278
35	0.38626034913373	0.38629435940592	0.38629435940891	0.38629435940144	0.38629435939995	0.38629435940518	0.38629435940667
36	0.38626221236087	0.38629435959103	0.38629435959369	0.38629435958703	0.38629435958570	0.38629435959036	0.38629435959170
37	0.38626392657522	0.38629435975181	0.38629435975419	0.38629435974823	0.38629435974704	0.38629435975121	0.38629435975240
38	0.38626550725298	0.38629435989199	0.38629435989413	0.38629435988878	0.38629435988771	0.38629435989146	0.38629435989253
39	0.38626696791213	0.38629436001468	0.38629436001661	0.38629436001179	0.38629436001083	0.38629436001420	0.38629436001516
40	0.38626832040247	0.38629436012242	0.38629436012415	0.38629436011981	0.38629436011894	0.38629436012198	0.38629436012285
41	0.38626957514664	0.38629436021734	0.38629436021891	0.38629436021498	0.38629436021419	0.38629436021695	0.38629436021773
42	0.38627074134152	0.38629436030123	0.38629436030266	0.38629436029909	0.38629436029838	0.38629436030088	0.38629436030159
43	0.38627182712726	0.38629436037561	0.38629436037690	0.38629436037366	0.38629436037301	0.38629436037528	0.38629436037593
44	0.38627283972970	0.38629436044172	0.38629436044290	0.38629436043995	0.38629436043935	0.38629436044142	0.38629436044201
45	0.38627378558084	0.38629436050065	0.38629436050173	0.38629436049903	0.38629436049849	0.38629436050038	0.38629436050092
46	0.38627467042132	0.38629436055332	0.38629436055431	0.38629436055184	0.38629436055135	0.38629436055307	0.38629436055357
47	0.38627549938765	0.38629436060050	0.38629436060141	0.38629436059915	0.38629436059869	0.38629436060028	0.38629436060073
48	0.38627627708696	0.38629436064288	0.38629436064371	0.38629436064163	0.38629436064122	0.38629436064267	0.38629436064309
49	0.38627700766113	0.38629436068102	0.38629436068178	0.38629436067987	0.38629436067949	0.38629436068083	0.38629436068121

Table (1) to calculate integration $\int_1^2 \ln(x)dx = 0.38629436111989$ by using trapezoidal rule with the triangulation acceleration methods of Al-Tememe of the first kind

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n	Values of midpoint rule	$A^F{}_{\sin}$	$A^F{}_{\cos}$	$A^F{}_{\tan}$	$A^F{}_{\sec}$	$A^F{}_{\sin^2}$	$A^F{}_{h^2\cos^2}$
1	0.40546510810816						
2	0.39137966962482	0.38668428508256	0.38668380823857	0.38668500021691	0.38668523855266	0.38668440427616	0.38668416588518
3	0.38858386383525	0.38634719081485	0.38634713403457	0.38634727598075	0.38634730436792	0.38634720500933	0.38634717662025
4	0.38758831049475	0.38630830612209	0.38630829168057	0.38630832778382	0.38630833500422	0.38630830973240	0.38630830251177
5	0.38712433792291	0.38629949517789	0.38629948994320	0.38629950302980	0.38629950564707	0.38629949648654	0.38629949386923
6	0.38687143945183	0.38629666903214	0.38629666669765	0.38629667253384	0.38629667370106	0.38629666961576	0.38629666844852
7	0.38671865164902	0.38629554636764	0.38629554517425	0.38629554815771	0.38629554875440	0.38629554666598	0.38629554606929
8	0.38661936553764	0.38629503057108	0.38629502989898	0.38629503157922	0.38629503191527	0.38629503073910	0.38629503040306
9	0.38655124000595	0.38629476721261	0.38629476680571	0.38629476782297	0.38629476802643	0.38629476731434	0.38629476711089
10	0.38650248251866	0.38629462152148	0.38629462126092	0.38629462191234	0.38629462204262	0.38629462158663	0.38629462145634
11	0.38646639258722	0.38629453568362	0.38629453550912	0.38629453594537	0.38629453603262	0.38629453572725	0.38629453563999
12	0.38643893473140	0.38629448247277	0.38629448235155	0.38629448265460	0.38629448271521	0.38629448250307	0.38629448244247
13	0.38641756097228	0.38629444807634	0.38629444798953	0.38629444820656	0.38629444824996	0.38629444809804	0.38629444805464
14	0.38640059837474	0.38629442504711	0.38629442498332	0.38629442514280	0.38629442517469	0.38629442506306	0.38629442503116
15	0.38638691176804	0.38629440916091	0.38629440911298	0.38629440923279	0.38629440925675	0.38629440917289	0.38629440914893
16	0.38637570892107	0.38629439791659	0.38629439787990	0.38629439797163	0.38629439798998	0.38629439792577	0.38629439790742
17	0.38636642334577	0.38629438977766	0.38629438974909	0.38629438982051	0.38629438983480	0.38629438978480	0.38629438977051
18	0.38635864129400	0.38629438376953	0.38629438374695	0.38629438380340	0.38629438381468	0.38629438377518	0.38629438376389
19	0.38635205488111	0.38629437925652	0.38629437923844	0.38629437928363	0.38629437929267	0.38629437926103	0.38629437925200
20	0.38634643117274	0.38629437581356	0.38629437579892	0.38629437583552	0.38629437584284	0.38629437581722	0.38629437580990
21	0.38634159131179	0.38629437315015	0.38629437313816	0.38629437316813	0.38629437317412	0.38629437315315	0.38629437314715
22	0.38633739612490	0.38629437106376	0.38629437105385	0.38629437107862	0.38629437108357	0.38629437106624	0.38629437106129
23	0.38633373600778	0.38629436941067	0.38629436940241	0.38629436942306	0.38629436942719	0.38629436941274	0.38629436940861
24	0.38633052369511	0.38629436808725	0.38629436808031	0.38629436809766	0.38629436810113	0.38629436808899	0.38629436808552
25	0.38632768901183	0.38629436701767	0.38629436701179	0.38629436702648	0.38629436702942	0.38629436701914	0.38629436701620
26	0.38632517500880	0.38629436614566	0.38629436614066	0.38629436615317	0.38629436615567	0.38629436614692	0.38629436614441
27	0.38632293508066	0.38629436542903	0.38629436542473	0.38629436543546	0.38629436543761	0.38629436543010	0.38629436542795
28	0.38632093079064	0.38629436483569	0.38629436483198	0.38629436484123	0.38629436484309	0.38629436483661	0.38629436483476
29	0.38631913021093	0.38629436434104	0.38629436433783	0.38629436434585	0.38629436434746	0.38629436434184	0.38629436434024
30	0.38631750664346	0.38629436392603	0.38629436392324	0.38629436393022	0.38629436393162	0.38629436392673	0.38629436392533
31	0.38631603762467	0.38629436357575	0.38629436357331	0.38629436357942	0.38629436358065	0.38629436357637	0.38629436357514
32	0.38631470414445	0.38629436327847	0.38629436327632	0.38629436328169	0.38629436328277	0.38629436327901	0.38629436327793
33	0.38631349002810	0.38629436302483	0.38629436302294	0.38629436302768	0.38629436302863	0.38629436302531	0.38629436302436
34	0.38631238144355	0.38629436280738	0.38629436280570	0.38629436280990	0.38629436281074	0.38629436280780	0.38629436280696
35	0.38631136650578	0.38629436262009	0.38629436261859	0.38629436262233	0.38629436262307	0.38629436262046	0.38629436261971
36	0.38631043495691	0.38629436245807	0.38629436245674	0.38629436246007	0.38629436246074	0.38629436245841	0.38629436245774
37	0.38630957790604	0.38629436231735	0.38629436231616	0.38629436231914	0.38629436231973	0.38629436231765	0.38629436231705
38	0.38630878761635	0.38629436219465	0.38629436219358	0.38629436219625	0.38629436219679	0.38629436219491	0.38629436219438
39	0.38630805732989	0.38629436208727	0.38629436208631	0.38629436208872	0.38629436208920	0.38629436208751	0.38629436208703
40	0.38630738112265	0.38629436199297	0.38629436199210	0.38629436199427	0.38629436199471	0.38629436199319	0.38629436199275
41	0.38630675378404	0.38629436190989	0.38629436190910	0.38629436191107	0.38629436191146	0.38629436191008	0.38629436190969
42	0.38630617071623	0.38629436183645	0.38629436183574	0.38629436183752	0.38629436183788	0.38629436183663	0.38629436183628
43	0.38630562784966	0.38629436177136	0.38629436177071	0.38629436177233	0.38629436177266	0.38629436177152	0.38629436177120
44	0.38630512157186	0.38629436171349	0.38629436171290	0.38629436171438	0.38629436171467	0.38629436171364	0.38629436171334
45	0.38630464866719	0.38629436166191	0.38629436166137	0.38629436166272	0.38629436166299	0.38629436166204	0.38629436166177
46	0.38630420626565	0.38629436161581	0.38629436161532	0.38629436161655	0.38629436161680	0.38629436161594	0.38629436161569
47	0.38630379179926	0.38629436157451	0.38629436157405	0.38629436157518	0.38629436157541	0.38629436157462	0.38629436157439
48	0.38630340296469	0.38629436153742	0.38629436153700	0.38629436153804	0.38629436153825	0.38629436153752	0.38629436153731

Table (2) to calculate integration $\int_1^2 \ln(x)\, dx = 0.38629436111989$ by using midpoint rule with the triangulation acceleration methods of Al-Tememe of the first kind

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n	Values of trapezoidal rule	A^F_{sin}	A^F_{cos}	A^F_{tan}	A^F_{sec}	$A^F_{sin^2}$	$A^F_{h^2cos^2}$
1	1.85914091422952						
2	1.75393109246483	1.71885937131169	1.71885580957160	1.71886471293859	1.71886649316452	1.71886026161674	1.71885848097839
3	1.73416246012343	1.71834735351874	1.71834695203573	1.71834795571124	1.71834815643177	1.71834745388526	1.71834725315130
4	1.72722190455752	1.71829828277689	1.71829818209701	1.71829843379285	1.71829848413026	1.71829830794635	1.71829825760732
5	1.72400561978279	1.71828776203994	1.71828772575276	1.71828781646988	1.71828783461293	1.71828777111162	1.71828775296823
6	1.72225749247148	1.71828446778652	1.71828445164966	1.71828449199156	1.71828450005983	1.71828447182070	1.71828446375233
7	1.72120308298745	1.71828317568305	1.71828316744731	1.71828318803659	1.71828319215440	1.71828317774198	1.71828317362413
8	1.72051859216430	1.71828258649193	1.71828258185840	1.71828259344220	1.71828259575894	1.71828258765031	1.71828258533355
9	1.72004924448417	1.71828228711022	1.71828228430685	1.71828229131527	1.71828229271695	1.71828228781106	1.71828228640938
10	1.71971349138931	1.71828212203514	1.71828212024081	1.71828212472662	1.71828212562378	1.71828212248372	1.71828212158656
11	1.71946505517880	1.71828202500430	1.71828202380306	1.71828202680616	1.71828202740678	1.71828202530461	1.71828202470399
12	1.71927608944639	1.71828196495874	1.71828196412450	1.71828196621009	1.71828196662721	1.71828196516730	1.71828196475018
13	1.71912902392084	1.71828192619507	1.71828192559775	1.71828192709104	1.71828192738970	1.71828192634440	1.71828192604574
14	1.71901232834587	1.71828190026825	1.71828189982939	1.71828190092654	1.71828190114597	1.71828190037796	1.71828190015853
15	1.71891818200045	1.71828188239763	1.71828188206799	1.71828188289209	1.71828188305691	1.71828188248004	1.71828188231522
16	1.71884112857999	1.71828186975695	1.71828186950457	1.71828187013553	1.71828187026172	1.71828186982005	1.71828186969385
17	1.71877726754298	1.71828186061215	1.71828186041565	1.71828186090689	1.71828186100514	1.71828186066127	1.71828186056302
18	1.71872375064165	1.71828185386446	1.71828185370920	1.71828185409735	1.71828185417498	1.71828185390328	1.71828185382565
19	1.71867845876929	1.71828184879778	1.71828184867349	1.71828184898422	1.71828184904637	1.71828184882886	1.71828184876671
20	1.71863978892522	1.71828184493366	1.71828184483299	1.71828184508468	1.71828184513503	1.71828184495884	1.71828184490850
21	1.71860651041164	1.71828184194522	1.71828184186281	1.71828184206885	1.71828184211005	1.71828184196583	1.71828184192462
22	1.71857766569422	1.71828183960476	1.71828183953665	1.71828183970693	1.71828183974099	1.71828183962179	1.71828183958773
23	1.71855250074870	1.71828183775075	1.71828183769396	1.71828183783592	1.71828183786431	1.71828183776494	1.71828183773655
24	1.71853041528076	1.71828183626671	1.71828183621899	1.71828183633828	1.71828183636213	1.71828183627863	1.71828183625477
25	1.71851092659363	1.71828183506750	1.71828183502712	1.71828183512808	1.71828183514827	1.71828183507760	1.71828183505741
26	1.71849364298888	1.71828183408995	1.71828183405554	1.71828183414157	1.71828183415877	1.71828183409856	1.71828183408135
27	1.71847824393067	1.71828183328666	1.71828183325716	1.71828183333091	1.71828183334566	1.71828183329404	1.71828183327929
28	1.71846446507697	1.71828183262166	1.71828183259622	1.71828183265982	1.71828183267253	1.71828183262801	1.71828183261529
29	1.71845208685947	1.71828183206731	1.71828183204527	1.71828183210039	1.71828183211141	1.71828183207283	1.71828183206180
30	1.71844092568213	1.71828183160227	1.71828183158307	1.71828183163108	1.71828183164069	1.71828183160708	1.71828183159747
31	1.71843082707411	1.71828183120978	1.71828183119297	1.71828183123500	1.71828183124340	1.71828183121398	1.71828183120558
32	1.71842166031633	1.71828183087669	1.71828183086191	1.71828183089885	1.71828183090624	1.71828183088038	1.71828183087300
33	1.71841331418995	1.71828183059253	1.71828183057949	1.71828183061209	1.71828183061861	1.71828183059579	1.71828183058927
34	1.71840569358704	1.71828183034892	1.71828183033738	1.71828183036625	1.71828183037202	1.71828183035181	1.71828183034603
35	1.71839871678904	1.71828183013910	1.71828183012883	1.71828183015450	1.71828183015963	1.71828183014167	1.71828183013653
36	1.71839231326689	1.71828182995762	1.71828182994846	1.71828182997136	1.71828182997594	1.71828182995991	1.71828182995533
37	1.71838642189159	1.71828182980000	1.71828182979181	1.71828182981230	1.71828182981639	1.71828182980205	1.71828182979796

Table (3) to calculate integration $\int_0^1 e^x dx = 1.71828182845904$ by using trapezoidal rule with the triangulation acceleration methods of Al-Tememe of the first kind

n	Values of midpoint rule	A^F_{sin}	A^F_{cos}	A^F_{tan}	A^F_{sec}	$A^F_{sin^2}$	$A^F_{h^2cos^2}$
1	1.64872127070013						
2	1.70051271665021	1.71777740848232	1.71777916181358	1.71777477896944	1.71777390262089	1.71777697021348	1.71777784676506
3	1.71035252481953	1.71822447126884	1.71822467110643	1.71822417152840	1.71822407162004	1.71822442131155	1.71822452122659
4	1.71381527977109	1.71826741839504	1.71826746862585	1.71826734305074	1.71826731793660	1.71826740583759	1.71826743095254
5	1.71542136299584	1.71827663112212	1.71827664924248	1.71827660394200	1.71827659488209	1.71827662659209	1.71827663565217
6	1.71629468642129	1.71827951641898	1.71827952448058	1.71827950432671	1.71827950029599	1.71827951440360	1.71827951843437
7	1.71682157370428	1.71828064823795	1.71828065235335	1.71828064206491	1.71828064000724	1.71828064720911	1.71828064926680
8	1.71716366499569	1.71828116437212	1.71828116668785	1.71828116089856	1.71828115974071	1.71828116379320	1.71828116495105
9	1.71739825679913	1.71828142664211	1.71828142804331	1.71828142454032	1.71828142383973	1.71828142629181	1.71828142699241
10	1.71756608646113	1.71828157125809	1.71828157215500	1.71828156991273	1.71828156946427	1.71828157103386	1.71828157148232
11	1.71769027620964	1.71828165626468	1.71828165686517	1.71828165536396	1.71828165506372	1.71828165611456	1.71828165641480
12	1.71778474111514	1.71828170887000	1.71828170928704	1.71828170824444	1.71828170803592	1.71828170876574	1.71828170897426
13	1.71785826205691	1.71828174283083	1.71828174312944	1.71828174238291	1.71828174223361	1.71828174275617	1.71828174290548

14	1.71791660180808	1.71828176554546	1.71828176576485	1.71828176521636	1.71828176510666	1.71828176549061	1.71828176560031
15	1.71796366936380	1.71828178120212	1.71828178136692	1.71828178095492	1.71828178087252	1.71828178116092	1.71828178124332
16	1.71800219205266	1.71828179227682	1.71828179240300	1.71828179208755	1.71828179202446	1.71828179224528	1.71828179230837
17	1.71803411963110	1.71828180028876	1.71828180038700	1.71828180014140	1.71828180009228	1.71828180026420	1.71828180031332
18	1.71806087589213	1.71828180620057	1.71828180627819	1.71828180608413	1.71828180604532	1.71828180618116	1.71828180621997
19	1.71808352017054	1.71828181063960	1.71828181070174	1.71828181054639	1.71828181051532	1.71828181062407	1.71828181065514
20	1.71810285381891	1.71828181402508	1.71828181407542	1.71828181394957	1.71828181392440	1.71828181401250	1.71828181403766
21	1.71811949208411	1.71828181664333	1.71828181668453	1.71828181658152	1.71828181656091	1.71828181663303	1.71828181665363
22	1.71813391366157	1.71828181869388	1.71828181872793	1.71828181864279	1.71828181862577	1.71828181868536	1.71828181870239
23	1.71814649551206	1.71828182031824	1.71828182034663	1.71828182027566	1.71828182026146	1.71828182031115	1.71828182032534
24	1.71815753774546	1.71828182161846	1.71828182164231	1.71828182158268	1.71828182157075	1.71828182161249	1.71828182162442
25	1.71816728168269	1.71828182266912	1.71828182268932	1.71828182263884	1.71828182262874	1.71828182266408	1.71828182267417
26	1.71817592315244	1.71828182352559	1.71828182354279	1.71828182349978	1.71828182349118	1.71828182352129	1.71828182352989
27	1.71818362240715	1.71828182422937	1.71828182424411	1.71828182420724	1.71828182419986	1.71828182422568	1.71828182423305
28	1.71819051160602	1.71828182481202	1.71828182482474	1.71828182479294	1.71828182478658	1.71828182480884	1.71828182481519
29	1.71819670052411	1.71828182529769	1.71828182530871	1.71828182528115	1.71828182527564	1.71828182529493	1.71828182530045
30	1.71820228095233	1.71828182570515	1.71828182571476	1.71828182569075	1.71828182568594	1.71828182570275	1.71828182570755
31	1.71820733012053	1.71828182604901	1.71828182605742	1.71828182603640	1.71828182603220	1.71828182604691	1.71828182605111
32	1.71821191338386	1.71828182634084	1.71828182634822	1.71828182632976	1.71828182632606	1.71828182633899	1.71828182634268
33	1.71821608634821	1.71828182658981	1.71828182659633	1.71828182658003	1.71828182657678	1.71828182658818	1.71828182659144
34	1.71821989656472	1.71828182680325	1.71828182680902	1.71828182679459	1.71828182679170	1.71828182680181	1.71828182680469
35	1.71822338489041	1.71828182698708	1.71828182699221	1.71828182697937	1.71828182697681	1.71828182698579	1.71828182698836
36	1.71822658658793	1.71828182714607	1.71828182715066	1.71828182713921	1.71828182713692	1.71828182714493	1.71828182714722

Table (4) to calculate integration $\int_0^1 e^x dx = 1.71828182845904$ by using midpoint rule with the triangulation methods of Al-Tememe of the first kind

n	Values of trapezoidal rule	$A^F{}_{sin}$	$A^F{}_{cos}$	$A^F{}_{tan}$	$A^F{}_{sec}$	$A^F{}_{sin^2}$	$A^F{}_{h^2cos^2}$
1	1.20710678118655						
2	1.21592582628907	1.21886565724291	1.21886595580008	1.21886520948958	1.21886506026499	1.21886558261452	1.21886573187368
3	1.21760058943387	1.21894041695533	1.21894045096825	1.21894036593866	1.21894034893397	1.21894040845246	1.21894042545828
4	1.21819032421508	1.21894855892532	1.21894856748003	1.21894854609359	1.21894854181645	1.21894855678669	1.21894856106397
5	1.21846394067681	1.21895037148559	1.21895037457262	1.21895036685511	1.21895036531164	1.21895037071384	1.21895037225734
6	1.21861274836675	1.21895094834889	1.21895094972252	1.21895094628846	1.21895094560165	1.21895094800548	1.21895094869229
7	1.21870253534411	1.21895117655515	1.21895117725646	1.21895117550320	1.21895117515255	1.21895117637983	1.21895117673048
8	1.21876083509464	1.21895128114369	1.21895128153834	1.21895128055172	1.21895128035439	1.21895128104502	1.21895128124235
9	1.21880081641793	1.21895133446032	1.21895133469912	1.21895133410211	1.21895133398271	1.21895133440062	1.21895133452002
10	1.21882942042945	1.21895136392338	1.21895136407625	1.21895136369408	1.21895136361765	1.21895136388517	1.21895136396160
11	1.21885058717783	1.21895138126894	1.21895138137128	1.21895138111542	1.21895138106425	1.21895138124335	1.21895138129452
12	1.21886668794481	1.21895139201531	1.21895139208639	1.21895139190869	1.21895139187315	1.21895139199754	1.21895139203308
13	1.21887921915615	1.21895139895895	1.21895139900985	1.21895139888261	1.21895139885716	1.21895139894623	1.21895139897168
14	1.21888916292988	1.21895140360632	1.21895140364372	1.21895140355023	1.21895140353153	1.21895140359698	1.21895140361567
15	1.21889718547279	1.21895140681136	1.21895140683944	1.21895140676922	1.21895140675517	1.21895140680433	1.21895140681838
16	1.21890375161135	1.21895140907940	1.21895140910091	1.21895140904714	1.21895140903639	1.21895140907402	1.21895140908478
17	1.21890919365403	1.21895141072078	1.21895141073753	1.21895141069567	1.21895141068730	1.21895141071660	1.21895141072497
18	1.21891375426980	1.21895141193227	1.21895141194550	1.21895141191243	1.21895141190581	1.21895141192896	1.21895141193558
19	1.21891761401213	1.21895141284217	1.21895141285277	1.21895141282628	1.21895141282099	1.21895141283953	1.21895141284482
20	1.21892090946531	1.21895141353626	1.21895141354484	1.21895141352339	1.21895141351910	1.21895141353412	1.21895141353841
21	1.21892374549429	1.21895141407314	1.21895141408017	1.21895141406261	1.21895141405910	1.21895141407139	1.21895141407490
22	1.21892620369026	1.21895141449368	1.21895141449949	1.21895141448497	1.21895141448207	1.21895141449223	1.21895141449513
23	1.21892834830469	1.21895141482686	1.21895141483170	1.21895141481960	1.21895141481718	1.21895141482565	1.21895141482807
24	1.21893023049043	1.21895141509358	1.21895141509765	1.21895141508748	1.21895141508545	1.21895141509257	1.21895141509460

25	1.21893189138008	1.21895141530913	1.21895141531258	1.21895141530397	1.21895141530225	1.21895141530828	1.21895141531000
26	1.21893336435237	1.21895141548486	1.21895141548779	1.21895141548046	1.21895141547900	1.21895141548413	1.21895141548559
27	1.21893467672220	1.21895141562927	1.21895141563178	1.21895141562550	1.21895141562424	1.21895141562864	1.21895141562990
28	1.21893585101609	1.21895141574883	1.21895141575100	1.21895141574558	1.21895141574450	1.21895141574829	1.21895141574937

Table (5) to calculate integration $\int_1^2 \sqrt{x}\,dx = 1.21895141649746$ by using trapezoidal rule with the triangulation acceleration methods of Al-Tememe of the first kind

n	Values of midpoint rule	A^F_{sin}	A^F_{cos}	A^F_{tan}	A^F_{sec}	$A^F_{sin^2}$	$A^F_{h^2cos^2}$
1	1.22474487139159						
2	1.22045482214110	1.21902473311972	1.21902458788575	1.21902495093059	1.21902502352131	1.21902476942291	1.21902469681537
3	1.21962490729963	1.21896096699947	1.21896095014465	1.21896099228035	1.21896100070688	1.21896097121299	1.21896096278590
4	1.21933134597420	1.21895390785520	1.21895390359679	1.21895391424265	1.21895391637175	1.21895390891978	1.21895390679062
5	1.21919490018209	1.21895232911530	1.21895232757587	1.21895233142440	1.21895233219409	1.21895232950015	1.21895232873044
6	1.21912062752287	1.21895182568184	1.21895182499624	1.21895182671024	1.21895182705304	1.21895182585324	1.21895182551044
7	1.21907579051564	1.21895162632053	1.21895162597032	1.21895162684584	1.21895162702095	1.21895162640808	1.21895162623298
8	1.21904666812806	1.21895153489674	1.21895153469960	1.21895153519245	1.21895153529101	1.21895153494602	1.21895153484745
9	1.21902669212166	1.21895148827318	1.21895148815386	1.21895148845215	1.21895148851181	1.21895148830301	1.21895148824335
10	1.21901239850118	1.21895146250200	1.21895146242561	1.21895146261658	1.21895146265477	1.21895146252109	1.21895146248290
11	1.21900182020268	1.21895144732710	1.21895144727595	1.21895144740382	1.21895144742940	1.21895144733989	1.21895144731432
12	1.21899377303605	1.21895143792426	1.21895143788874	1.21895143797755	1.21895143799532	1.21895143793314	1.21895143791538
13	1.21898750954859	1.21895143184810	1.21895143182266	1.21895143188626	1.21895143189898	1.21895143185446	1.21895143184174
14	1.21898253910230	1.21895142778101	1.21895142776231	1.21895142780904	1.21895142781839	1.21895142778568	1.21895142777633
15	1.21897852883803	1.21895142497598	1.21895142496193	1.21895142499704	1.21895142500406	1.21895142497949	1.21895142497247
16	1.21897524649001	1.21895142299089	1.21895142298014	1.21895142300702	1.21895142301240	1.21895142299358	1.21895142298821
17	1.21897252599609	1.21895142155423	1.21895142154586	1.21895142156678	1.21895142157097	1.21895142155632	1.21895142155213
18	1.21897024608107	1.21895142049380	1.21895142048719	1.21895142050372	1.21895142050703	1.21895142049545	1.21895142049215
19	1.21896831650739	1.21895141969734	1.21895141969205	1.21895141970529	1.21895141970793	1.21895141969867	1.21895141969602
20	1.21896666900938	1.21895141908977	1.21895141908548	1.21895141909620	1.21895141909835	1.21895141909084	1.21895141908869
21	1.2896525117287	1.21895141861979	1.21895141861628	1.21895141862505	1.21895141862681	1.21895141862066	1.21895141861891
22	1.21896402221512	1.21895141825166	1.21895141824876	1.21895141825602	1.21895141825747	1.21895141825239	1.21895141825094
23	1.21896295001961	1.21895141795999	1.21895141795757	1.21895141796362	1.21895141796483	1.21895141796059	1.21895141795938
24	1.21896200901661	1.21895141772649	1.21895141772446	1.21895141772954	1.21895141773056	1.21895141772700	1.21895141772599
25	1.21896117864474	1.21895141753781	1.21895141753609	1.21895141754039	1.21895141754125	1.21895141753824	1.21895141753738
26	1.21896044221831	1.21895141738396	1.21895141738249	1.21895141738616	1.21895141738689	1.21895141738433	1.21895141738360
27	1.21895978608267	1.21895141725755	1.21895141725629	1.21895141725943	1.21895141726006	1.21895141725785	1.21895141725723
28	1.21895919897666	1.21895141715287	1.21895141715178	1.21895141715449	1.21895141715503	1.21895141715314	1.21895141715260

Table (6) to calculate integration $\int_1^2 \sqrt{x}dx = 1.21895141649746$ by using midpoint rule with the triangulation acceleration methods of Al-Tememe of the first kind

6. Conclusion:

We can say that acceleration methods of Al-Tememe have the same efficiency to improve the results of integrals, which are reviewed regarding accuracy and partial periods used in addition to the speed of calculating their values.

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