



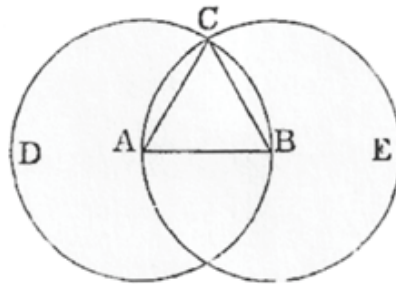
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'What the Tortoise Said to Achilles' Lewis Carroll's Paradox of Inference



- (A) Things that are equal to the same are equal to each other.
(B) The two sides of this triangle are things that are equal to the same.
(Z) The two sides of this triangle are equal to each other.

Edited by

Amirouche Moktefi and Francine F. Abeles

Essays and Analyses by

*Francine F. Abeles, Pascal Engel,
George Englebretsen, Clare Imholtz, Mathieu Marion,
Amirouche Moktefi and John Woods*



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Guest editors: Amirouche Moktefi and Francine F. Abeles.

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What the Tortoise Said to Achilles: Introduction

Francine F. Abeles & Amirouche Moktefi

‘What the Tortoise said to Achilles’ (WTSA), sometimes known as Carroll’s paradox of inference, appeared in the leading British journal *Mind, A Quarterly Review of Psychology and Philosophy*, in 1895. This journal published articles by such well-known figures as Charles Darwin and William James. Unlike Carroll’s earlier publication, ‘A logical paradox’ (1894), commonly known as the barbershop paradox, which immediately attracted responses from serious logicians, none for WTSA was received in Carroll’s lifetime. However, WTSA has since been widely discussed among philosophers and is currently considered as a classic text in the philosophy of logic. What is more remarkable is that in the articles that have appeared in journals and books for over 120 years, there has been no accepted resolution to the problem Carroll posed in WTSA. Many scholars even believe that Carroll did not write his paper with a specific purpose in mind. In addition to philosophers, this paper also resisted Carrollian bibliographers who have long thought that it actually appeared in 1894. It is true that little is known on the genesis and the writing of this fascinating article. Hence, confusion and mystery have long surrounded the reception of Carroll’s WTSA. Mystery might never vanish, but it is the aim of this volume to lessen confusion.

Here we offer a set of papers providing key elements to the history and purpose of this enigmatic piece that will contribute to Carrollian studies and more generally, to philosophy. Indeed, the following articles explore what Carroll’s WTSA teaches us both about logic and about Carroll himself. The volume naturally opens with a reprint of Carroll’s original paper ‘What the Tortoise said to Achilles’ which is the object of all subsequent pieces. The next item reproduces Carroll’s correspondence with George Frederick Stout, the editor of the journal *Mind*. These letters include the only known source where Carroll commented on his paper and its purpose. This material is followed by five articles and a selective bibliography.

In the first article, ‘The making of “What the Tortoise said to Achilles”’: Lewis Carroll’s investigations toward a workable theory of hypotheticals’, the editors consider the place of Carroll’s WTSA within his long interest on the subject of hypotheticals. The narrative of WTSA is first shown to reflect Carroll’s engagement with both geometry and logic. Then, the editors trace the path Carroll travelled that led to the two

articles on hypotheticals that he published in *Mind*. Particular attention is paid to the controversy that Carroll had in the period 1892–1894 with John Cook Wilson, the Wykeham Professor of Logic at Oxford University. Finally, several clues are discussed and observations are made on what might have been Carroll’s purpose in writing WTSA.

The second article, ‘Lessons from Lewis Carroll’s paradox of inference’, is written by Mathieu Marion, a world authority on the philosophy of Cook Wilson, the Oxford logician and contemporary of Carroll. In his paper, Marion first discusses two early interpretations of WTSA by Cook Wilson and Bertrand Russell, before addressing the better-known interpretation offered by the philosopher Gilbert Ryle. All three readings were significant and influential, the latter view being introduced by the author as a synthesis of the former views. Then, Marion applies the lessons he learned from Carroll’s paradox to two topics: the place of logic in our “web of belief” and some widespread misconceptions about the history of ancient Greek logic.

In the third article, ‘What did Carroll think the Tortoise said to Achilles’, George Englebretsen considers Carroll’s WTSA as a multi-faceted piece from which there is more than one lesson to be learned. The author first revisits the early opinions he held on WTSA, two being published in the forerunner of this journal, *Jabberwocky*, in 1974 and 1994, and the reasons why he believed them. After reviewing many interpretations offered by other philosophers throughout the twentieth-century, some of which he holds himself, Englebretsen offers a further lesson that he has learned from WTSA about the *dictum de omni* and its role in syllogistic reasoning.

Pascal Engel, who devoted many essays to Carroll’s WTSA, is the author of the fourth article in this collection: ‘The philosophical significance of Carroll’s regress’. Engel views WTSA as a mirror for epistemology. He observes that it led to multiple interpretations in connection with various philosophical problems. Then he discusses four of the most prominent problems in that literature, namely the nature of logical inference, our understanding of logical rules, the justification of those rules, and the nature of normative guidance in reasoning. The author considers what Carroll’s WTSA might teach us about these issues and suggests some contemporary answers and developments.

The last article in this collection, ‘Required by logic’, is by John Woods who published a short piece on WTSA titled, ‘Was Achilles’ “Achilles’ heel” Achilles’ heel?’, in 1965. The author repudiates two

claims he made in that early note and offers in the present essay a new examination of Carroll's puzzle. After discussing some opinions held about the purpose of the story in relation to rules and premises, the author views WTSA as an anticipation of ideas developed much later by the American philosopher Gilbert Harman. Woods concludes that Carroll had the right instinct but lacked the logic skills to establish the point he was making.

This volume concludes with a selective bibliography prepared by the research librarian, Clare Imholtz, in collaboration with the first editor. The bibliography is divided into two sections: the first section lists reprints and translations of WTSA, either in selected and collected works by Carroll or in other works, while the second section lists studies and citations devoted to WTSA. Although the bibliography is far from exhaustive, it demonstrates the permanence and the variety of interests expressed in Carroll's piece.

WTSA might be Carroll's most famous article. However, it should not be viewed as an accidental contribution. Contrary to the common belief that Carroll regarded mathematics as a recreational pastime after retiring from his Lectureship in 1881, a look at his work demonstrates that he was seriously investigating a number of mathematical subjects in the last years of his life. In addition to his contributions to *Mind* and to the journal *Nature*, Carroll also published many mathematical pieces in *The Educational Times*, a monthly London periodical. Interestingly, one of the many versions of Carroll's barbershop paradox is Question 14122 posthumously published in *The Educational Times* in 1899. Unlike all his earlier pieces where he used his name, C.L. Dodgson, next to this question's number, the editors of the journal appended: The late "Lewis Carroll." In addition to his contributions to periodicals, Carroll also published several books in the last decade of his life, notably *A New Theory of Parallels* (1888), *Pillow problems* (1893) and the first part of *Symbolic Logic* (1896). Moreover, it is known that he worked on several other books that unfortunately did not reach publication. WTSA was not an isolated and lucky flash of genius from a retired mathematical lecturer. Rather, it should be understood as the product of a man at work during a period of intellectual excitement.

The editors resolved to prepare this volume in order to offer a new set of articles by well-known scholars on the enigmatic WTSA from a fresh perspective. We hope that the volume provides an accessible account of the paradox, its history and its impact to both Carrollians and

philosophers. The preparation of this volume benefited from the help and support of many contributors to whom the editors would like to express their gratitude. First, we would like to thank the four leading philosophers who accepted our invitation to address Carroll's paper and what might be learned from it: Pascal Engel, George Englebretsen, Mathieu Marion and John Woods. We would like also to thank Clare Imholtz for her valuable contribution and Mark Richards, the executive editor of *The Carrollian*, for his help throughout the preparation of the volume. The editors also benefited from the expertise of many scholars who will remain unnamed but with whom we discussed issues concerning this volume directly or indirectly. In particular, we would like to thank Edward Wakeling and the late Professor Ivor Grattan-Guinness.

* * *

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