Argle Victorious
A Theory of Holes as Hole-Linings

Leona Mollica*

October 19, 2022

In their [Lewis and Lewis, 1970], David and Stephanie Lewis offered an account of the metaphysics of holes, and in so doing inaugurated a small but thriving literature on the topic (see e.g. [Meadows, 2015] [McDaniel, 2010]). Posterity has been less kind, however, to the substantive view offered in the paper, placed in the mouth of the fictional interlocutor Argle, according to which a hole is nothing over and above its *lining*, the film of matter surrounding it. A consensus has emerged that Casati and Varzi, in their [Casati and Varzi, 1997], have dealt fatal blows to the holes/hole-linings identity thesis, and that we correspondingly must look elsewhere for a proper analysis (see [Wake et al., 2007] [Meadows, 2015] and of course [Casati and Varzi, 1997] itself). I argue that this consensus is misguided: both the main arguments that Casati and Varzi offer explicitly and stronger arguments to which they point the way can, I contend, be accommodated by a theorist who takes holes to be mere hole-linings.

The Lewis and Lewis paper begins with Argle avowing nominalistic materialism: according to him, all that exist are material concreta. But, objects Bargle, holes are not material concreta, as they indeed precisely "result from the absence of matter." By identifying holes with their linings, Argle hopes to maintain their status as material objects, evading Bargle's criticism.

One natural response to Bargle's initial argument that the existence of holes undermines nominalistic materialism is that he, Bargle, is cheating. If holes are not material objects, this is so only on a technicality: they seem of the material world in a way that abstracta and spirits do not; there is nothing to holes being as they are beyond holed material concreta being as they are. Even if we reject nominalistic materialism, our basis for this rejection should not be something so apparently near to the concrete and material as the holes of material bodies. The more broadly popular views of holes in the literature–according to which they are immaterial bodies [Casati and Varzi, 1997], spatiotemporal regions [Wake et al., 2007], or properties [Meadows, 2013] [Meadows, 2015]–sit uncomfortably with this intuition. On a broadly Argelian view, to the contrary, holes are every bit as much a part of the concrete material order as they seem, since they themselves are just material concreta of a specific sort. Even when these theories have it that the properties of holes are entirely fixed by

*I would like to thank Harvey Lederman for extensive feedback on several drafts of this paper.

the properties of material holed objects, this requires one to accept such an additional claim of supervenience or dependence, whereas Argle secures this result immediately. I hope to demonstrate that much can be said for such a theory.

I begin by recounting and briefly motivating the holes-as-linings theory, as expounded in [Lewis and Lewis, 1970], including both its account of holes themselves and (more importantly) its account of hole *sameness*, and remark on some unorthodox, but to my mind nonetheless palatable, consequences. I shall then look at what I take to be Casati and Varzi's strongest arguments against Argle's view, arguing that the analysis of hole-linings offered in the previous section undermines their case. Nevertheless, serious challenges will turn out to face it, more elaborate than Casati and Varzi's but similar in spirit. I conclude with a new account of hole identity that overcomes these problems, and rebut objections that it gives up on the promises of nominalism and materialism. If successful, this will be a story of the triumph of holes-as-linings, a case of Argle victorious.[1]

1 Argle's Theory

1.1 The Motivation

I have already offered the gist of Argle's account: any hole is surrounded by a material lining, and the hole just *is* the lining. It is worth noting a couple different benefits of such a theory, in addition to that discussed in the introduction. The first I have already noted: the view is straightforwardly consistent with materialist nominalism in a way that, say, Casati and Varzi's theory of holes as immaterial bodies, or Meadows' theory of holes as properties, is not [Casati and Varzi, 1997] [Meadows, 2015]. It would be a mistake, however, to think that the benefits of such an account accrue only to materialists and nominalists. Its success would be more generally be of interest to work on the broader category of *absences* (cf. [McDaniel, 2010] [Sorensen, 2008]), (quasi-)objects like gaps, holes, and shadows that seem to consist in the lack of something. Some philosophers have the feeling that absences cannot, in a deep sense, *exist*. Jackson gives good expression to this sentiment: "surely, as it stands, to say that there are holes, empty spaces etc., is to take paradigm examples of *nothings* and make them into *somethings*" [Jackson, 2009] (emphasis retained). An account of holes like Argle's renders such seemingly "negative" entities as holes into "positive" entities like hole-linings: presumably Jackson and friends will not complain about the latter. The linings account, thus, holds promise as an object lesson in dissolving metaphysical suspicions over "absences" generally.

[1]Note that this theory really is more properly Argle's than the Lewises': they disavow it as their own in [Lewis and Lewis, 1996], and thus I shall refer to the theory as Argle's and Argelian rather than, as in [Casati and Varzi, 1997], the Lewises' or Ludovician.

1.2 The Account: Linings

Argle, frustratingly, never specifies in much detail what a hole-lining *is*: we are simply told things like that it is "[t]he matter [that] surrounds the hole." This is unfortunate, for, as we shall see, the viability of Casati and Varzi's main criticism of the lining theory is hard to assess without a more exact criterion of lining-hood. I shall therefore try in this section to offer a working definition of a hole-lining that at least roughly matches our intuitive understanding.

We begin by introducing the notion of a *discontinuity*, which will play an important role in our subsequent characterisation of hole-linings. Intuitively, a discontinuity of an object is a region of space(time) that the object surrounds but from which it is entirely absent. A cup, for example, will generally have a very voluminous central discontinuity relative to its own volume, one in which liquids tend to be located while the cup is in use. It is natural here to ask why we should not simply identify holes themselves with discontinuities; why bother with linings when we can wed the concept of a hole to that of a discontinuity directly?

This is, indeed, a plausible account (or beginning of an account) of holes, and is essentially that given in [Wake et al., 2007], from which my definition of a discontinuity will draw substantially. It is, however, subject to criticisms that will not threaten my own. Chief among these is that, as the paper itself notes, the theory is only tenable if one accepts an at least limited counterpart-theoretic account of *de re* modality, which many philosophers reject as implausible. The reason is this: spati(otempor)al regions are widely thought to bear their locations essentially, whereas it is an obvious datum that the same cannot be said of holes; the most obvious way to remedy this problem is to relativise *de re* modal profiles to descriptions or sortals in the manner characteristic of counterpart theory. Thus, "This hole could be elsewhere than it actually is" and "This region could not be elsewhere than it actually is" can both come out true even if this hole is this region, given that the former shifts us to a context with a different counterpart relation than the counterpart relation of the context to which the latter shifts us.[2] This is not a knockdown argument against the holes-as-discontinuities account, of course, but it does suggest that allowing discontinuities into our theorising does not force such an account upon us.

These worries aside, let us proceed to our own definitions. We begin with a function r associating objects with regions of space (considered as sets of spatial points), so that $r(o)$ is the region of space exactly occupied by o. This should obey obvious principles, such as that if x is a part of y then $r(x) \subseteq r(y)$.[3] The

[2] Given that this account is a restricted version of supersubstantivalism, its reliance on counterpart theory is a special case of the more general appeal counterpart theory has for supersubstantivalists, see [Schaffer, 2009].

[3] The principles I am thinking of often go under the heading of "Mereological Harmony," which is given expression in e.g. [Varzi, 2007] [Uzquiano, 2011] [Leonard, 2016] and [Saucedo, 2011]. While Mereological Harmony has come under criticism on various counts (see again [Saucedo, 2011], also [McDaniel, 2007] [Hawthorne and Uzquiano, 2011] [Newman, 2002] and others), it also has adamant defenders (see e.g. [Schaffer, 2009]), and much of this discussion concerns fringe cases like colocated bosons, universals, and tropes, which the relevant examples in this paper will try to

idea of an exact location or exact occupation, in its rough outline, is intuitive: I am precisely located at just the region I fill to the brim, such that any part of it, and no part of any disjoint region, has part of me located at it.[4] We denote the *convex hull* of $r(o)$–the smallest convex region containing $r(o)$–by $r^*(o)$.

Definition 1.1. *d is a* discontinuity of *o iff d is a maximal topologically connected subregion of* $r^*(o) - r(o)$.[5] [6]

For the purposes primarily of familiarity, I will be assuming that the space in which we are working is $\mathbb{R}^3$, equipped with the standard Euclidean metric and Lebesgue measure. I do not think that this much structure–or at least, this *particular* structure, since we will indeed need some metric for many of our definitions to work–will be essential to the theory I am going to provide, but it will help to have some formal background assumptions about the space in which we are working, and $\mathbb{R}^n$ with its usual metric and measure seems as good a candidate as any.

We are now in a position to give a first pass at defining a hole-lining. We begin first by defining the notion of an object "going right up to" or "immediately

avoid. The main non-fringe assumption I will make, conflicting with e.g. [Cartwright, 1975] and [Uzquiano, 2006], is that exact locations are free to be either closed or open. This will come up later explicitly.

[4]This concept is, of course, a contested one in the literature on persistence: there is a debate between endurantists and exdurantists on the one hand, who think that ordinary material objects are exactly located at three-dimensional regions of zero temporal extent, and perdurantists on the other, who think that such objects are located instead at four-dimensional spacetime regions usually of more than zero temporal extent. (This way of framing the question is most natural in a pre-relativistic setting. For attempts to further this debate in a relativistic framework, see [Gilmore, 2008] [Balashov, 2010].) See [Sider, 2001] [Hawthorne et al., 2008] [Donnelly, 2011] [Eddon, 2010] [Gilmore, 2008] [Balashov, 2010].

I have tried, so far as is possible, to avoid framing my arguments in such a way as to presuppose any of these views of persistence. In particular, the main examples to be considered involve atemporal three-dimensional worlds, in which varying views about persistence should converge, and I have tried to leave my definitions general enough to suit any competitor in this debate. The main difficulty posed by my definitions will be for the endurantist, because in accordance with Mereological Harmony I take it that each material object has a *unique* location, and this is sometimes denied by endurantists who think that an object will have multiple exact locations at different times. This problem, though, is easily resolved by indexing all claims in my definitions about locations to a time. Despite the intrinsic interest of such debates, therefore, I think they will here be irrelevant.

[5]One might also plausibly stipulate that a discontinuity must have measure > 0, and that a holed object's region and its convex hull both be measurable. Given this stipulation and that we are working with the Lebesgue measure, or indeed any σ-finite measure, it follows that any object will have at most countably many discontinuities, since given such a measure no set (and in particular the convex hull of the object's region minus the object's region) with positive measure can be decomposed into uncountably many disjoint sets each with positive measure, and the discontinuities of an object will always be so disjoint from one another.

[6]According to the common mathematical definition, a region will be *connected* just in case it is not the union/fusion of two disjoint nonempty open sets, and we can think of a connected spatial object as an object whose exact location is a connected region. Intuitively, a region or body is connected if one can draw a curve between any two points in it without leaving it. This intuitive gloss does not fit the definition perfectly, but in the topological spaces considered in this paper, the differences will be irrelevant. A *maximally* connected subregion of R is just a connected subregion of R with no connected proper superregion that is also a subregion of R.

surrounding” a discontinuity.

Definition 1.2. *A part x of o* goes right up to a discontinuity d in *o iff for any real ϵ such that ϵ is the infimum of the distances between some point in $r(x)$ and all points in d, all points p in $r(o)$ such that some $\delta \leq \epsilon$ is the infimum of the distances between p and all points in d are also in $r(x)$.*

The basic idea is that a part of o goes right up to a discontinuity d in it just in case that part contains all the parts of o at a certain minimal distance from d. [7] [8] Not every part of x that immediately surrounds one of its discontinuities will thereby count as a hole-lining in x. For one, this would counterintuitively always make x itself a hole-lining. We instead will count only the *singly-perforated* surrounders as linings.

Definition 1.4. *A part x of o is a* lining of a discontinuity d in *o iff it goes right up to d in o and it goes right up to no other discontinuity e in o. x is a* lining in *o iff it is a lining of some discontinuity in o.*

I should note a consequence of this definition that conflicts with certain assumptions prevalent in the holes literature. Nothing has been made in this account of holes involving any special *contrast* between their linings and their insides; for all that has been said, the occupant of a discontinuity lined by a lining might be entirely homogenous with the lining itself.[9] This might be thought to be paradoxical: if all linings are holes and *vice versa* and being a lining involves no restrictions concerning contrast of matter, it makes for a plenitude of holes as parts of any seemingly hole-free solid object, like a smooth wall, which many have taken to be a contradiction in terms.

Now a first point to note is that, if the result is indeed unacceptable, it can be avoided by writing in by hand the desired condition about contrast. Supposing we can specify this relevant kind of contrast by saying the contents of a discontinuity *appropriately contrast* with one of its linings, we can simply define

[7] The reason for the mention of infimal distances is that for some or all points p in some linings, such as one surrounding a discontinuity that is an open ball, there may be no set of points in the discontinuity least distant from p.

[8] Note that, for the first time, we are appealing intuitively to principles in Mereological Harmony (discussed earlier in a footnote) whereby an object's location places constraints on the mereology of the object, rather than *vice versa*. More specifically, if $r(x)$ contains a point p, x must mereologically contain a part at least partially located at p. In the current setting, where we are concerned with objects composed of point particles, this definition could be rephrased thus (assuming that the location of a point-particle is a point), without relying on this component of Mereological Harmony:

Definition 1.3. *A part x of o* goes right up to a discontinuity d in *o iff for any real ϵ such that ϵ is the infimum of the distances between some point in $r(x)$ and all points in d, all point-particles p in o such that some $\delta \leq \epsilon$ is the infimum of the distances between $r(p)$ and all points in d are also parts of x.*

[9] Argle: ”A hole is a hole not just by virtue of its own shape but also by virtue of the way it contrasts with the matter inside it and around it.”

Definition 1.5. *x is a* hole *iff for some o and discontinuity d in o, x is a lining of d in o and the contents of d appropriately contrast with x.*

This notion of *appropriately contrasting* at play can be filled in by varying readers as they see suited to the nature of holes as such; none of the examples appearing in this paper will hinge on subtleties about this contrast, as in each case the discontinuity will be empty and its relevant linings composed of homogenous point particles.

But it is open to question whether this result is untoward in the first place. Argle and Bargle both seem to have assumed that, for a hole-lining to be a hole in o, it sufficed that it be a hole-lining and a part of o. But I think this is wrong: for a hole-lining to be a hole in o, on my favoured interpretation, it must be a lining *of a discontinuity d in o*. Thus, if I have a gapless ball composed of point particles, it has (by classical mereology) as a part P itself minus all the particles from some large connected subregion P' of its location, and this part is a hole, but there are nevertheless no holes *in* the ball, because it lines not a discontinuity in the ball but a discontinuity in P (namely P').

On my view, an abundance of homogeneously filled holes should be a desideratum on any theory. Suppose I am talking about a ball of point particles with radius 2, and wish to discuss that part P of it that fuses just those particles not at a distance of 1 or less from the centre (note that the existence of this part follows immediately from the axioms of classical mereology plus the existence of the ball). It is entirely natural, in describing P, to say that it has a hole in it with radius 1, though that hole is fully and homogeneously filled. Indeed, it can be useful in discussing objects like P to talk of the holes in them: when talking, for example, of the part of an unperforated object that will remain if I perforate it in a specific way, or of the part of a sheet of paper visible while it is partially obscured by an object resting atop it, or of the part of a material object known for sure to exist when our knowledge of the overall object is fragmentary. As with arbitrary undetached parts generally, they are helpful to have around when needed and can otherwise remain conveniently ignored.

Here is another way to put the point. Whether an object (like P) has any holes in it strongly supervenes on its shape: two objects, across worlds, with the same shape will have the same number and relative locations of any holes in them. But P retains its shape whether or not the matter in the discontinuity it lines is deleted, as its shape is intrinsic to it.[10] Thus, it follows it will have as many holes when it exists alone as when it exists (as in actuality) as part of a homogeneous ball. Since it plainly contains a hole in it in the former case, it does also in actuality, meaning there exists a homogeneously filled hole (in P); moreover, as a hole in P is (according to us) just a hole-lining in P, a hole-lining in P a part of P, and any part of P a part of the ball, the ball must contain this hole as a part despite the absence of holes *in* it. That is, it must contain the lining as a part even though the lining is a lining of no discontinuities in

[10] Even those who disagree that shape is intrinsic do not usually contest such independence of a material object's shape from the existence or location of other material objects, like the mereological complement of P in the ball, mereologically and spatially disjoint from them [Skow, 2007].

the ball itself (as the ball, unlike some of its proper parts, lacks discontinuities altogether).

An analogy with shape predicates may be helpful. Take another ball of homogeneous point particles; it is natural to say of the ball, given its shape and homogeneity, that it has no rectangular prisms in it. This in spite of the fact that, according to classical extensional mereology, it contains uncountably many rectangular prisms as proper parts, say a filled cube with each vertex touching the ball's surface. This is, like with the "filled in" holes of the last two paragraphs, because the cube does not suitably contrast with the remainder of the ball, leading us to ordinarily ignore it when discussing the ball as such. The ball, similarly, contains holes (that is, hole-linings) as parts, and yet because of their lack of contrast with the contents of the discontinuities they line, we suitably ignore them in ordinary contexts. This should not lead us to deny, however, that these cubic or lining parts are still rectangular prisms or holes respectively.[11]

But, as Definition 1.5 demonstrates, this additional commitment to a plenitude of holes is entirely optional. Those unpersuaded by the above arguments for filled holes' innocence are free to reject them while still identifying each hole with a lining.

1.3 The Account: Sameness

When are holes x and y the same hole as one another? The most obvious account, where x is the same hole as y just in case x is a hole-lining and is identical with y, will not do: almost any hole will be surrounded by a large host of linings of varying degrees of thickness. On pain of accepting that any even merely singly perforated object contains a huge array of holes,[12] we instead need to give a weaker criterion. Here is Argle's:

Definition 1.6. *x is* the same hole as *y iff x and y are both hole-linings of some object o and contain some common part z that is also a hole-lining of o.*[13]

The intuition behind this definition is fairly straightforward. Two hole-linings should count as the same hole when they surround the same gap in their host object; the definition, at least in simple cases, captures this idea. The most obvious application of this notion is to counting holes. Ordinarily in counting Fs, of course, there will be at least n Fs iff there are n Fs all distinct from each other, and exactly n Fs just in case there are n F's and any F must

[11] In this I differ from Argle himself, who declares, "The same is true of other shape-predicates; I wouldn't say that any part of the cheese is a dodecahedron, though I admit that there are parts – parts that do not contrast with their surroundings – that are *shaped like* dodecahedra" [Lewis and Lewis, 1970]. This pair of opinions – that filled linings are not holes and non-contrasting dodecahedrally shaped parts are not dodecahedra – is entirely natural; one's views on the one question should mirror one's views on the other.

[12] Drawing on our earlier discussion, that is: holes *in* it rather than *parts of* it.

[13] We will assume, again mainly for purposes of familiarity, the standard axioms of classical mereology.

be one of those n. It is easy to generalise these definitions to other counting relations:

Definition 1.7. *There are at least n Fs according to R, where n is a natural number and R is a relation, iff* $\exists x_1 \ldots \exists x_n (Fx_1 \wedge \ldots \wedge Fx_n) \wedge (\bigwedge_{0<i<j\leq n} \neg Rx_i x_j)$.

Definition 1.8. *There are exactly n Fs according to R, where n is a natural number and R is a relation, iff* $\exists x_1 \ldots \exists x_n (Fx_1 \wedge \ldots \wedge Fx_n) \wedge (\bigwedge_{0<i<j\leq n} \neg Rx_i x_j) \wedge (\forall y (Fy \rightarrow \bigvee_{0<i\leq n} Ryx_i))$.

This is not the only discussion in (David) Lewis' corpus on counting by relations beyond identity. The topic also surfaces in [Lewis, 1976], where he suggests that we should count persons having property F at time t by a relation he calls identity-at-t, weaker than genuine identity. Without entering into the extensive literature these examples, such counting by non-identity relations is arguably familiar from other cases, like counting (by "type" rather than "token") four letters in a printed instance of the word *potato* (where the relevant relation would be one of alphabetic co-typicality), or counting mathematical objects in a category up to isomorphism rather than by identity strictly. Lewis notes two important criteria such a counting relation must meet: it must be an equivalence relation, and it must "be an *indiscernibility* relation; not for all properties whatever, as identity is, but at least for some significant class of properties. That is, it ought to be that two related things have exactly the same properties in that class" (emphasis retained). Both are important.

First, it is crucial that the relation be an equivalence relation, and in particular transitive. For suppose we were counting the natural numbers between one and three inclusively by the symmetric, reflexive relation of being at most one number distant from. We shall then have it that there are at least two such numbers, by 1.7 (since 1 and 3 will both be such numbers and neither of them is related to the other by the relation of unit distance), and exactly one such number, by 1.8 (since 2 will bear the relation to every number with the property). But, of course, this is absurd: there cannot be exactly n Fs and at least m Fs where $n < m$, and this consequence is due to the non-transitivity of the relation.[14]

Second, it is likewise important that it be an indiscernibility relation. Otherwise, one and the same hole, say, could be both F and non-F, where the property of being F is important to our treatment of it *as* a hole. While I will not attempt to specify in advance what properties must be preserved under hole-sameness, there are some pretheoretically obvious examples. It should not be, for example, that x is the same hole as y but has a volume of one litre whereas y has a volume of two litres (and not also of one litre, if a hole can have multiple volumes). Of course, if we think that holes are hole-linings, in a sense we must have cases of divergent volumes: the measure of the regions of the linings themselves must sometimes differ, given the possibility of varying thickness. The relevant kind of volume is different: two similar holes should have the same *e-volume*, meaning that the volume of the discontinuities they

[14]Thanks to Erica Shumener for help on this point.

line must be identical. If I have a hole that can carry two litres and a hole that can only carry one, I have two holes.

2 Casati and Varzi contra Argle

So much for Argle's theory; we turn now to its criticism at the hands of Casati and Varzi. They offer two main arguments against the view: that it requires us to give strange extra definitions for various expressions when applied to holes, and that it gives wrong counts of holes in various perforated objects.

In their first objection, Casati and Varzi note that Argle must take ordinary predicates to have very different meanings when applied to holes as such than they usually do. Enlarging a hole, for example, can consist in destroying it, that is, destroying one of its hole-linings, even though one usually cannot enlarge something by destroying it. And as Argle noted, when talking about holes as such, "is identical to" and "surrounds" become something like synonymous, which is not generally true: my room's walls surround me but are not me. Casati and Varzi object that these ambiguities are very strange, strange enough to give us reason to reject Argle's theory.

Some of these ambiguities I find commonplace and harmless. For example, Casati and Varzi object that "inside" as applied to a hole will be ambiguous between "within the region of the hole-lining", since the lining is the hole and that is what we usually mean by "inside" when talking about non-holes (there is chocolate inside the cookie just in case there is chocolate within the region of the cookie itself), and "within the discontinuity surrounded by the hole", which picks out what we usually talk about as being inside a hole (if I put a marble inside a hole, I am not cramming it into the hole-lining but *surrounding* it by the lining). But the exact same is true of, say, jugs or bottles: what is inside a bottle (according to ordinary speech) is what is within the discontinuity surrounded by it, not what is within its own thin region.

More fundamentally, I disagree because I do not think capturing the natural language meaning of hole vocabulary was ever Argle's purpose. Argle is willing to grant that ordinary talk about holes is riddled with immaterialist falsehood; what he proposes is a way of *reinterpreting* holes talk so as to satisfy his exacting ontological standards. We should thus predict that the reinterpreted sentences will have meanings that vary from the ones we would pretheoretically assign them. As Argle says of hole-sameness, "I find it convenient to use 'same' meaning 'co-perforated' wherever a man of your persuasion would use 'same' meaning 'identical.' You know my reason for this trickery: my sentences about sameness of holes will be true just when you wrongly suppose your like-sounding sentences to be" [Lewis and Lewis, 1970]. The polysemy Casati and Varzi note is just a byproduct of this reinterpretation, and is to be expected.

Here is their second objection. Suppose we have an index card punctured just twice by a card-puncher, leaving two centimetre-wide gaps on either side of the card. There will be two hole-linings *a* and *b*, thin strips of paper surrounding the relevant punctures, in the card, and the card *c* itself (or some large connected section of it containing *a* and *b*), with both the thin strips as proper parts. How many holes are *a*, *b*, and *c*? Casati and Varzi allege that these are all hole-linings. Accordingly, they should say that the answer is counterintuitive in exactly the same way as counting the first three natural numbers by unit distance: since *a* and *b* are not the same hole as one another, there at least are two such holes; since *c* contains them both as parts, there is just one such hole.[15] But this is absurd. We should, therefore, give up on Argle's theory.

This argument only has plausibility if we grant the central claim that the card itself (or a large *a*-and-*b* encompassing portion thereof) is a hole-lining. This claim is not intuitively plausible: we have no independent inclination to call the entire doubly-perforated card, or any doubly-perforated part thereof, a hole-lining; accordingly, our definitions forbid doubly-perforated holes explicitly. Even if one found the claim intuitive, however, this would not affect the core of the argument, for what is ultimately in question is not whether we can identify holes with "hole-linings" as intuitively understood, but whether there are *any* such material parts of holed objects that can be naturally and fruitfully identified with holes. If the linings of our definitions are not the "linings" of folk intuition, that does not prevent them from playing the desired theoretical role. So, Argle's view is immune to the criticism given by Casati and Varzi as it stands.

3 Onion World

I think, though, that we can give more powerful anti-Argelian arguments along the lines suggested by Casati and Varzi. Consider first a timeless, three-dimensional, Euclidean world featuring no material objects disjoint from a single ball of point particles with radius 2. There are, of course, no holes in this world yet, but consider the world that we obtain by taking this sphere and removing just the point particles in the closed ball of radius 1 around the centre of the original ball.[16] This world has just one hole in the resulting maximal material object, a hole with a(n e-)volume of $\frac{4}{3}\pi$. So far, so simple.

Consider, though, the world resulting from the following alteration of this newly holed sphere. We remove, first of all, all the point particles at a distance between 2 and $1\frac{1}{2}$ from the centre of the sphere. We remove, second of all, all the point particles at a distance between $1\frac{1}{4}$ and $1\frac{1}{8}$ from the centre of the sphere, leaving the band of particles at a distance between $1\frac{1}{2}$ and $1\frac{1}{4}$ from the centre of the sphere. We continue this process infinitely, so that starting from a point at a distance of 2 from the centre of the sphere and moving directly toward the centre, we will travel first uninterrupted through a point particle-free line of length $\frac{1}{2}$, then uninterrupted through a line fully occupied by point particles with length $\frac{1}{4}$, then through a point particle-free line of length $\frac{1}{8}$, and so on, with

[15]Strangely, Casati and Varzi actually object that Argle should count *three* holes, but this is apparently to beg the question against his claim to count holes otherwise than by strict identity.

[16]It is important that this ball of removed particles be closed, since were it open there would remain a point-thick skin around it that we could identify as its only lining.

each maximal line segment either totally filled or totally unfilled by particles being half the length of the previous one, until we reach the central hole of e-volume $\frac{4}{3}\pi$. Question: how many *spherical* holes *with e-volume* $\frac{4}{3}\pi$ are there in the maximal material object, i.e. the fusion of all the material shells or bands, which surrounds the central, ball-shaped hole with e-volume $\frac{4}{3}\pi$?

The intuitive answer is: one. The answer given our old working definition of a hole-lining, however, is: zero. For any part of the mutilated sphere going right up to the central discontinuity of volume $\frac{4}{3}\pi$ will contain all the point particles at a distance of ϵ or less from the discontinuity, for some ϵ, and thus will contain all the shells with a radius less than $1+\epsilon$. Any such part, therefore, will surround not one but infinitely many discontinuities: both the central discontinuity and all the gaps between the aforementioned shells.

Our ammended definition begins with the notion of the *punctuation* of such a part:

Definition 3.1. *Let x be a part of o that goes right up to some discontinuity in o. The* degree of punctuation of x in *o is the number of discontinuities d in o such that x goes right up to d in o.*

The idea here is fairly straightforward: a part's degree of punctuation is just the number of discontinuities it surrounds. We can now give an improved definition of a hole-lining:

Definition 3.2. *A part x of o is a* lining of a discontinuity d in *o iff it goes right up to d in o and there is no other part y of o that goes right up to d such that the degree of punctuation of y in o is strictly less than the degree of punctuation of x in o. x is a* lining in *o iff it is a lining of some discontinuity in o.*

The idea is that a hole-lining is a part of an object surrounding (going right up to) a discontinuity where any other part of the object surrounding (going right up to) the same discontinuity would have to be just as large, in the sense of having as great a degree of punctuation.[17] It is thus a *minimal* surrounder of the discontinuity, in that it minimises how many (other) discontinuities it surrounds. We can clearly see that the entire card in the Casati and Varzi example given above (or any doubly perforated part thereof) will not count as a lining of any discontinuity in the card, since the thin linings around the individual discontinuities will always improve on the card (or doubly perforated part) with

[17] It might be objected that the argument thus far of this section rests on a mistake. Holed objects should be connected, as the fusion of Matryoshka-like spheres is not, or at least hole-*linings* should be. We thus have no reason to give up our earlier definition of a lining and accept this new one.

Suppose we accept a definition like 3.2 but build in the requirement that any lining must be connected. Then consider a sort of hybrid between the ball with just the unit radius hole in the centre and the onion object. One hemisphere is indiscernible from a hemisphere of the former, one indiscernible from a hemisphere of the latter. Thus, one half will be filled with point particles save for an empty unit half-ball at the centre, while the other half will consist in an infinite series of concentric domes of diminishing breadth joined to the first half as their base and surrounding an empty unit half-ball at the centre. This connected object will pose, with minor adjustments, all the problems for the connectedness-requiring definition as the Matryoshka object does for the official definition, and the same response I will offer later for my own stated problems will resolve these versions of the problems, too.

respect to minimising degree of punctuation. For any maximal band of matter in the onion world, though, the fusion of it and all maximal bands nearer than it to the centre does count as a lining of the central, closed, spherical discontinuity of volume $\frac{4}{3}\pi$, since each such fusion has the same degree of punctuation, namely $\aleph_0$, as each surrounds that many gaps and no surrounder of the central hole surrounds fewer discontinuities.[18]

Our troubles, however, are not over. For, while such linings do count as linings, we get an implausible number for some subset of them, still: exactly one, though also at least two. For let e be some maximally connected band of matter, f the fusion of e and all bands within (i.e. nearer the centre than) e, g the fusion of all bands within e, and h the fusion of e with the next-closer band i. f and g will then both be the same hole as one another, since they share g as a common part and both line the central discontinuity. f will also be the same hole as h, since h is a part of f and a lining of the discontinuity that is the gap between e and i. h, however, is not the same hole as g, since their largest common part i lines no discontinuities in the maximal material object whose central discontinuity g and f both line, nor does it contain any such linings as parts. Thus, transitivity fails, and with it intelligible counting of the hole-linings f, g, and h: these three are exactly one hole but at least two.

There is a way of resolving this problem without revising our definition of being the same hole.[19] Instead of counting directly by R in the manner of 1.7 and 1.8 when R is non-transitive, count instead by R^*, the intersection of all the maximal transitive subrelations of R. By definition, R^* is guaranteed to itself be transitive, and thus to avoid the problems that plague non-transitive counting relations. I am inclined to think, however, that the problems posed by Argle's account of hole identity go beyond the difficulties non-transitive relations pose for just counting.

We saw above that, if x is the same hole as y, the e-volume of x should be the e-volume of y, too. This is not guaranteed by Argle's definition, however. Take again the onion-like mutilated ball and its parts f and h. The e-volume of f will be $\frac{4}{3}\pi$, since it lines the central hole, and yet the same will not in general be true of h. Supposing that the outermost band in f is the absolutely outermost band, for example, the e-volume of h will be just over $2\frac{1}{5}$. Being the same hole as, thus, fails to preserve exactly the properties we would want such a relation to preserve; it is a form of identity in name only.

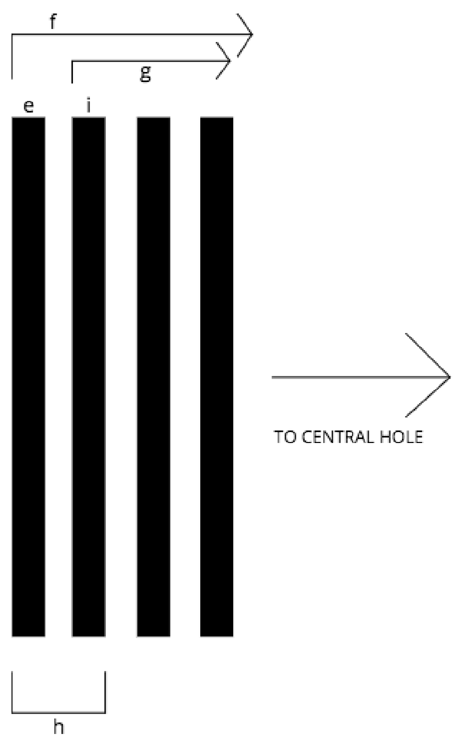
The intuitive problem is obvious: f and h are being counted as the same hole despite lining different discontinuities. But this suggests a straightforward solution: define being the same hole as directly in terms of lining the same discontinuities.

Definition 3.3. *x is* the same hole as *y iff there is some o such that x is a lining*

[18]As noted in previous footnote, this definition also solves the problem mentioned above, that the old definition allowed linings that did not go "all the way around" the discontinuity. By 1.2, a lining must contain *all* the points at a distance to the discontinuity less than any other point in it, and thus cannot leave out any such points.

[19]I owe this point to Harvey Lederman.

Figure 1: A partial cross-section of the onion-like object, not to scale.



in o and y is a lining in o and for every p such that x and y are linings in p, every discontinuity d in p that x lines in p y lines in p and vice versa.

The relation thus defined is manifestly an equivalence relation, and thus sidesteps the worries presented above without giving up on the intuitive definitions of counting in finite cases offered earlier. f will, under this new definition, clearly not be the same hole as h, since h lines the gap between the two outermost bands and f does not (since its degree of punctuation is far too large). Thus, we do not get the counterintuitive result that there are at least two but at most one holes in a certain class, nor that e-volume can vary under the same hole. Argle, it seems, has found the definition he needed at last.

I would like, next, to discuss a potential objection to the characterisation of holes I have given in this paper. In spite of my pretensions to give a theory of holes that respects their materiality, the objection goes, I have availed myself essentially of spati(otempor)al regions, precisely the sort of seemingly immaterial object reviled by Argle. How, then, does my account improve along this dimension on the sort of account offered in [Wake et al., 2007], according to which holes just are regions?

I have three responses. First, while for convenience I have defined holes in terms of substantivalist regions, it is not clear I could not give equivalent relationalist definitions. The basic trick would be as follows. To begin, we define in a relationalistically kosher way a preorder $\preccurlyeq_o$ on the parts of a body o, where $x \preccurlyeq_o y$ corresponds intuitively to y lining (*not* merely surrounding) at least all the discontinuities in o that x lines (and possibly more). We then identify the holes of o with its $\preccurlyeq_o$-minimal parts. x and y would then be the same hole just in case, for all z, $x \preccurlyeq_z y$ and $y \preccurlyeq_z x$. The definition of $\preccurlyeq_o$ would be a nontrivial task, but I see no reason as of yet to think it impossible.

Second, while the definitions appeal to regions, the objects themselves falling under these definitions do not obviously ontologically depend upon regions. Here are a couple of ways to bring out this point. First, consider computers, of the material, silicon-and-plastic kind. The most natural ways to define such computers will appeal to immaterial abstracta; they will, say, require the computer to be isomorphic in its causal workings to a certain kind of set-theoretic construct. This does not make the computer *itself* into any sort of abstract object, though: it is a material concretum like any other. The same is true *mutatis mutandis*, of my definitions. Second, a theory like mine will yield different verdicts regarding *de re* counterfactuals (or counterpossibles) about holes. It is true on a literal (see next paragraph) reading of my definitions that, were there no regions, there would be no holes, just as with the theory presented in [Wake et al., 2007]. Of any particular hole h, however, it does not follow that, were there no regions, h would not exist. The hole h is just a bit of matter, and there is no reason to think that this bit of matter would not exist were the world relationalist. Assuming plausibly that regions are essentially regions, we do not get this verdict from Wake et al.: for them, were there no regions, h (as a region) would not exist.[20] So, while the property of *being a hole* getting instantiated

[20]What do I mean by "essentially" here? As a first pass: if x is essentially F, then were x not F, a

might, on the most substantivalist reading of my definitions, depend on there being regions, the holes themselves do not.

Third, a literal reading of my substantivalist definitions is not required. The definitions could instead be read in a fictionalist spirit: thus read, for example, 1.2 would say that a part x of o goes right up to a discontinuity in o[21] just in case *according to the fiction of substantivalism*, for some discontinuity d in o, the region of x is such that, if some point p in it is at a distance ϵ from d, every point p' in the region of o at a distance equal to or less than ϵ from d is also in the region of x.[22] In essence, we define holes as certain *non*-fictional objects that bear certain properties *according to* the fiction of substantivalism, which involves such fictional objects as regions. Wake et al. cannot do this: the objects with which they identify holes are essentially denizens of the fiction of substantivalism, and thus they cannot specify as holes any non-fictional (according to the relationalist) objects that have certain properties according to the fiction of substantivalism. I thus take it that my theory does or can treat holes as genuinely material in ways that the theory of [Wake et al., 2007] cannot.

Conclusion

This paper has had three aims. First, I have tried to undermine a widely accepted argument in the literature on holes, Casati and Varzi's against Argle. Second, I have given, as far as I know, the first attempt at rigorously characterising the notion of a hole-lining. This notion is of independent philosophical interest, even if you reject the holes-as-linings view; at the very least, it should help opponents of the view state clearly exactly what it is they are rejecting. Third, I have offered the first account since [Lewis and Lewis, 1970] itself that makes holes out to be genuinely material, and I have rebutted charges that this achievement is illusory. If successful, these constitute a strong challenge to the general suspicion that any theory of holes along the lines suggested by Argle is without serious merit.

References

[Balashov, 2010] Balashov, Y. (2010). *Persistence and Spacetime*. Oxford University Press.

[Cartwright, 1975] Cartwright, R. (1975). *Scattered Objects*, pages 153–171. Springer Netherlands, Dordrecht.

[Casati and Varzi, 1997] Casati, R. and Varzi, A. (1997). Holes and other superficialities. *Philosophy and Phenomenological Research*, 57(3):734–736.

contradiction $\perp$ would be true.

[21] Where *going-right-up-to-a-discontinuity-in* is regarded as a two-place relation between a part and a whole, not a three-place relation between a part, whole, and actual discontinuity; the reference to "a discontinuity" is merely superficial and not ontologically committing.

[22] For more on fictionalism, see [Kalderon, 2005].

[Donnelly, 2011] Donnelly, M. (2011). Endurantist and perdurantist accounts of persistence. *Philosophical Studies*, 154(1):27–51.

[Eddon, 2010] Eddon, M. (2010). Why four-dimensionalism explains coincidence. *Australasian Journal of Philosophy*, 88(4):721–728.

[Gilmore, 2008] Gilmore, C. (2008). Persistence and location in relativistic spacetime. *Philosophy Compass*, 3(6):1224–1254.

[Hawthorne et al., 2008] Hawthorne, J., Sider, T., and Zimmerman, D. W. (2008). *Three- Dimensionalism vs. Four-Dimensionalism*, volume Contemporary debates in philosophy, pages 263–282. Blackwell Publ, Malden, MA.

[Hawthorne and Uzquiano, 2011] Hawthorne, J. and Uzquiano, G. (2011). How many angels can dance on the point of a needle? transcendental theology meets modal metaphysics. *Mind*, 120(477):53–81.

[Jackson, 2009] Jackson, F. (2009). *Perception: A Representative Theory*. Cambridge University Press.

[Kalderon, 2005] Kalderon, M. E. (2005). *Fictionalism in Metaphysics*. Oxford University Press UK.

[Leonard, 2016] Leonard, M. (2016). What is mereological harmony? *Synthese*, 193(6).

[Lewis, 1976] Lewis, D. (1976). Survival and identity. In Rorty, A. O., editor, *The Identities of Persons*, pages 17–40. University of California Press.

[Lewis and Lewis, 1970] Lewis, D. and Lewis, S. (1970). Holes. *Australasian Journal of Philosophy*, 48(2):206–212.

[Lewis and Lewis, 1996] Lewis, D. and Lewis, S. (1996). Casati and varzi on holes. *Philosophical Review*, 105:77–79.

[McDaniel, 2007] McDaniel, K. (2007). Extended simples. *Philosophical Studies*, 133(1):131–141.

[McDaniel, 2010] McDaniel, K. (2010). Being and almost nothingness. *Noûs*, 44(4):628–649.

[Meadows, 2013] Meadows, P. J. (2013). What angles can tell us about what holes are not. *Erkenntnis*, 78(2):319–331.

[Meadows, 2015] Meadows, P. J. (2015). Holes cannot be counted as immaterial objects. *Erkenntnis*, 80(4):841–852.

[Newman, 2002] Newman, A. (2002). *The Correspondence Theory of Truth: An Essay on the Metaphysics of Predication*. Cambridge University Press.

[Saucedo, 2011] Saucedo, R. (2011). Parthood and location. In Zimmerman, D. and Bennett, K., editors, *Oxford Studies in Metaphysics Vol. 6*. Oxford University Press.

[Schaffer, 2009] Schaffer, J. (2009). Spacetime the one substance. *Philosophical Studies*, 145(1):131–148.

[Sider, 2001] Sider, T. (2001). *Four Dimensionalism: An Ontology of Persistence and Time*. Oxford University Press.

[Skow, 2007] Skow, B. (2007). Are shapes intrinsic? *Philosophical Studies*, 133(1):111–130.

[Sorensen, 2008] Sorensen, R. A. (2008). *Seeing Dark Things: The Philosophy of Shadows*. Oxford University Press.

[Uzquiano, 2006] Uzquiano, G. (2006). Receptacles. *Philosophical Perspectives*.

[Uzquiano, 2011] Uzquiano, G. (2011). Mereological harmony. In Bennett, K. and Zimmerman, D., editors, *Oxford Studies in Metaphysics*. Oxford University Press.

[Varzi, 2007] Varzi, A. C. (2007). *Spatial Reasoning and Ontology: Parts, Wholes, and Locations*, pages 945–1038. Springer Netherlands, Dordrecht.

[Wake et al., 2007] Wake, A., Spencer, J., and Fowler, G. (2007). Holes as regions of spacetime. *The Monist*, 90(3):372–378.