On the Coherence of Strict Finitism

Auke Alesander Montesano Montessori

Abstract

Strict finitism is the position that only those natural numbers exist that we can represent in practice. Michael Dummett, in a paper called *Wang's Paradox*, famously tried to show that strict finitism is an incoherent position. By using the Sorites paradox, he claimed that certain predicates the strict finitist is committed to are incoherent. More recently, Ofra Magidor objected to Dummett's claims, arguing that Dummett fails to show the incoherence of strict finitism. In this paper, I shall investigate whether Magidor is successful in preventing Dummett from proving the incoherence of strict finitism. Though not all the counterarguments Magidor presents are successful, she does in the end manage to corner Dummett. There remains an opportunity for Dummett to insist on the incoherence of strict finitism, but this is a very small opening. The final conclusion of this paper is that Dummett cannot logically prove the incoherence of strict finitism, even though a limited chance for success remains.

Keywords: *Strict finitism, Dummett, Magidor, Sorites Paradox*

In 1975, Michael Dummett wrote a paper called *Wang's Paradox*. In this paper, he used Wang's paradox, a variation on the Sorites paradox, to show that strict finitism is incoherent. Strict finitism is a rather peculiar constructivist view of mathematics. It maintains that only those numbers that can be represented in practice actually exist [1, pp. 302-303]. Strict finitism also holds that only proofs with a number of steps that can be checked are legitimate. Numbers which are equal to a number of steps in a proof that can be checked are called apodictic numbers. Ten and one hundred are apodictic, since proofs with ten or one hundred steps can be checked in practice. Googol (10^{100}) is not apodictic because a proof with googol steps cannot be checked in practice [3, pp. 472-473].

Kriterion – Journal of Philosophy, 2019, 33(2): 1–14.
http://www.kriterion-journal-of-philosophy.org

Dummett's claim that strict finitism is incoherent has been contested by Ofra Magidor. She has developed two strategies to undermine Dummett's argument [2, p. 404]. The goal of the current essay is to discover whether Magidor succeeded. I shall argue that, in the end, she indeed prevents Dummett from proving the incoherence of strict finitism. Dummett will not be completely defeated however. He still has some space left to insist on the incoherence of strict finitism, but Magidor does show that his chances are bleak.

We can only properly understand Magidor's objections if we understand Dummett's original arguments against strict finitism. To foster this understanding is the task of the first part of this essay. In a nutshell, Dummett shows that certain predicates the strict finitist is committed to are incoherent. Thus, strict finitism as a position is unviable.

The incoherent predicate that the strict finitist is committed to can either be 'natural number' or 'apodictic'. Part two shows that we can easily dismiss Dummett if the predicate that is attacked is 'natural number'. Parts three and four deal with the coherence of the predicate 'apodictic number'. It is in these parts that we encounter Magidor's attempt to dismantle Dummett's argumentation. Part three shows that Magidor's first counterargument against Dummett fails. However, in part four Magidor achieves her goal for the most part. It turns out that Dummett cannot after all prove the incoherence of strict finitism. Dummett could make a final effort to show that he is right by referring to certain empirical research we could do. However, this only works if the research is actually done and turns out favourable for Dummett. It is not very likely that this will occur, but Magidor did technically not succeed in completely disarming Dummett.

1 Dummett's argumentation

As said, Wang's paradox forms the basis for Dummett's attack against strict finitism. His formulation of the paradox is as follows:

> ''0 is small;
>
> If n is small, n+1 is small:
>
> Therefore, every number is small. '' [1, p. 303]

The idea here is quite simple and familiar to anybody acquainted with the Sorites paradox. There are some numbers that are small, with zero being an obvious example. Now, it seems equally obvious that a difference of one does not really matter for smallness. If we take any small number, and we add 1 to this number, then the result is still a small number.

The problem is that these two premises can be used to reach an absurd conclusion. 0+1 equals 1, so we must accept 1 as a small number. This holds on the basis of step 2 of the paradox, which states that a difference of 1 does not matter for smallness. 1+1=2 and 2 is therefore also small. The same strategy is applied to show the smallness of 3, 4, 5, *et cetera*. This chain can go on for ever, which shows us that all natural numbers are small. This conclusion is unacceptable. Certain numbers, such as 27 billion or googol, are clearly not small. We are in a situation where we believe that googol is not small, but the two premises, both of which we are inclined to believe, show that googol is small. A paradox has been reached [1, pp. 303-305].

The relationship between Wang's paradox and strict finitism is that Dummett holds that strict finitism runs into this paradox. Strict finitism is committed to certain predicates that have the same problem as 'smallness' [2, pp. 403-405]. There are multiple candidates for the problematic predicate. One option is that the paradox holds for the predicate 'natural number' as interpreted by the strict finitist.[1] For the strict finitist, the predicate 'natural number' and 'number that can be represented in practice' range over the exact same number of numbers. Magidor focuses on the predicate 'apodictic' [2, pp. 404-405]. For the majority of this paper, I will follow Magidor and critically examine the coherence of the predicate 'apodictic'. Natural numbers will be discussed in the next section and then abandoned. Regardless of what specific predicate we have in mind, the important thing to note is that Dummett believes that some predicate the strict finitist is committed to is paradoxical. The task ahead for Magidor is to prevent Dummett from reaching this conclusion.

2 Natural numbers

Before I turn to natural numbers, I will say some more on Wang's paradox. What feature of predicates like 'small', 'apodictic' or 'natural number' causes the paradox? The problem is that the sets that form the extensions of these predicates are both weakly infinite and weakly finite. A set is weakly infinite if there is some well-ordering of the set which has no last member. A set is weakly finite when, for some finite ordinal number n, there is no nth member of the set [1, p. 312]. Wang's paradox is exploiting this combination of features. The second step of the paradox is based on weak infinity. If we say about some small number that it holds that one of its neighbours is not small, then that number is the last member. Since there is no last member, we can never claim, for any number x that is next to a small number, that x is not small. But weak finity ensures that there are some numbers that are not small. Thus, the combination of weak finity and weak infinity causes the paradox [2, pp.403-408].

The strange thing about the predicate 'natural number' as interpreted by the strict finitist is that the weak finity of its extension is not problematic in the way described above. The reason for this is fairly simple. For smallness, the paradox only works because there are some numbers that are not small, like googol or 27 billion. This fails for natural numbers since there are no natural numbers that are not natural numbers! Let us spell this out explicitly. Note here that 'being a number that can be represented in practice' is the same as 'natural number'.

> 0 can be represented in practice;
>
> If n can be represented in practice, then n+1 can be represented in practice:
>
> All natural numbers can be represented in practice.[2]

To arrive at the paradox, we need to claim that there are natural numbers that cannot be represented. However, according to the strict finitist, no such numbers exist. That is exactly the fundamental premise of her position! The strict finitist would simply accept the conclusion of the above proof. So, if Dummett is trying to claim that it is the predicate 'natural number' or the predicate 'being a number that can be represented in practice' that is causing paradoxes for the strict finitist, then his claim fails. There is simply no way to make the predicate 'natural number' paradoxical.[3] If Dummett is to show that strict finitism is committed to paradoxical predicates, then his only hope is that the predicate 'apodictic' turns out to be paradoxical.

3 The weak infinity of apodictic numbers

We have seen that the natural number approach goes awry because the set of all natural numbers is not weakly finite. Attempting to do the same for apodicticness is not an option. For example, googol can be represented in practice.[4] And yet, it is impossible to check a proof that has googol steps in practice. But if we follow the paradox, we are forced to state that googol is apodictic. It seems that the extension of 'apodicitc' is straightforwardly weakly finite in a problematic way. For the predicate 'apodictic', the solution must come by contesting the weak infinity claim.

For the weak infinity claim, it is essential that there is no clear border between apodictic and non-apodictic numbers. By a border I mean a number which is not apodictic, but whose predecessor is apodictic. An opponent of Dummett can only prove that the set of apodictic numbers is not weakly infinite if she can prove that such a border indeed exists. This is so because precisely the discovery of a border proves that it does not follow from the fact that n is apodictic, that $n+1$ is apodictic as well. Note that it is not necessary to find the specific number that is the border. Opponents of Dummett only need to prove that there is a border, wherever it may be.

According to Magidor, such a border exists. She thinks that the case of apodictic numbers is akin to that of tipping the balance of a scale by adding small bits of grain to one side. Adding one bit of grain does not change much, but there is some definite point where the balance is tipped. Thus, there is a clear border between the amount of grain that tips the scale and the amount that does not tip the scale [2, pp. 404-406].

The reason Magidor thinks that this analogy works is because mathematicians cannot keep checking steps for an eternity. At some point, they physically collapse. It is not easy to discover this upper limit, but it does exist. Thus, Magidor thinks that she has shown that the extension of the predicate 'apodictic' yields a weakly infinite set [2, pp. 404-406].[5]

One might wonder how relevant this physical endpoint is for the discovery of an appropriate border between apodictic and non-apodictic numbers. If Magidor is merely saying that certain numbers are not apodictic because the mathematician will collapse at some point, then we did not learn anything new. It was already accepted that some numbers are not apodictic. This did not stop Dummett from claiming that the extension of 'apodictic' forms a weakly infinite set before and it will not do so now either. We do not need to show that some numbers are not apodictic, we need to show that there is a definite border between apodictic and non-apodictic numbers. That is, we need a number that is not apodictic, but whose predecessor is apodictic. Magidor has not shown that the number of steps checked that was reached right before collapse is apodictic. If this cannot be shown, then the empirical fact that physical collapse at some point occurs is not interesting.

In other words, Magidor must claim that the set of apodictic numbers equals the set of numbers of steps that can be checked before collapse. This comes down to translating, 'can be checked in practice' as 'can be checked before physical collapse'. It can be fairly objected that this seems a highly arbitrary translation. Presumably, the strict finitist is aiming at current practices within mathematics. And in that context, it simply does not happen that people work on one and only one proof until they physically collapse. Normally, mathematicians stop checking a proof long before that point. This means that the border for apodictic numbers, if there is one, is much lower than what Magidor suggests.

It is also possible to argue for a much higher border for apodictic and non-apodictic numbers. If we are already going as far as to presume that working until collapse counts as a fair border, then why not go even further? What if we allow multiple mathematicians to check the same proof? Then we no longer depend on the physique, or lifespan, of one person. Perhaps certain technologies could be developed to make checking proofs go smoother. In principle, we could go on checking a proof for as long as humanity exists. If we are willing to admit that this is possible, and it does seem logically and physically possible, then there is no straightforward reason why we should not consider these extreme conditions as the proper border for apodictic and non-apodictic numbers. The consequence would be that the border lies between two incredibly high numbers. Numbers like one million would probably be apodictic if we allow for multiple generations of checking mathematicians.

The point of the above exercise was not to show what insanely high number of checked steps we could reach if we wanted to. Nor am I trying to prove that we can translate the term 'apodictic' in many ways. The point is that all of the translations above, including the one by Magidor, are arbitrary. Apodicticness is a complicated notion that cannot be captured by the simple translations above. There is no reason to accept any of them as an acceptable rendition of what it is to be apodictic.

If Magidor wants to endorse her translation a correct one, she should argue for it. As far as I am aware, she does not do so. The result is that no strict border between apodictic and non-apodictic numbers has been found. The translation Magidor suggested was supposed to ensure that the number that comes right before physical collapse is apodictic. With the translation rejected as arbitrary, it might very well be that this supposedly last member of the set of apodictic numbers is not apodictic at all! The idea that for any apodictic number n, $n+1$ is apodictic as well is not refuted.

Magidor has failed to show that there is a non-arbitrary border between apodictic and non-apodictic numbers. Of course, we could assign some more or less random border if we wanted to. This is uncontroversial, since this can be done for any vague predicate. We can assign random boundaries for predicates like 'red', 'small', 'trustworthy'. It is equally uncontroversial that this is not an acceptable solution.[6] Such borders often do not represent what we normally mean when talking about such predicates. So, unless Magidor gives some convincing reason why physical collapse should be the border, we can maintain that the predicate 'apodictic' remains vague. Magidor's argument is not sufficient to prevent Dummett from insisting that the extension of the predicate 'apodictic' is weakly infinite.

Of course, all of the above should not be taken as proof of the vagueness of the term 'apodictic'. I only showed that Magidor's attempt to prove the opposite fails. She could not refute the weak infinity of the extension of the predicate 'apodictic', so Dummett is allowed enough space to claim it is weakly infinite. I did not here attempt to show that Dummett is correct in claiming this. Instead, I limited myself to showing that Magidor's counterattack fails. She did not manage to reach the negative result she sought, so I consider her first attempt to defeat Dummett a failure.

4 Preventing the paradox

It does not seem possible to prevent the set of numbers that is the extension of the predicate 'apodictic' from being both weakly infinite and weakly finite. Dummett is right that the strict finitist is committed to predicates which trigger the paradox. The only way out available to Magidor is to prevent Dummett from using weak infinity and weak finity to reach a contradiction. Should she fail in doing so, then we are forced to conclude that strict finitism is not a viable position.

As Magidor points out, and Dummett is aware of, it is actually not so easy to prove that the paradox holds. The most poignant problem is with the step from premise 2 to the conclusion. If we know that 0 is apodictic and that if n is apodictic, that $n+1$ is also apodictic, how can we then know that all numbers are apodictic? This last step requires induction, a principle the strict finitist tends to reject. Strict finitists, typically being constructivists about mathematics, do not like this type of automatic reasoning. They demand that every step is spelled out and checked. [2, pp. 406-407].

Dummett retorts that induction is not needed as a principle. We could do the reasoning by hand. First, we show that 1 is apodictic. After all, 0 is apodictic and if n is apodictic, then $n+1$ is as well. Now we go on to show that 2 is apodictic, 3 is, 4 is, *et cetera*. We will eventually reach a number that we normally hold to be non-apodictic, despite the considerable number of steps this will take. The paradox holds without induction [1, pp. 304-306].

Magidor, in response, points out that the strict finitist will block this move. Each individual step is probably acceptable, but the proof as a whole is not. Before the proof can be acceptable, each step needs to be checked. For the most obvious non-apodictic numbers, such as googol and 27 billion, this is not possible. We cannot check a googol steps in practice. Thus, the strict finitist is likely to reject the step by step approach, simply because there are too many steps involved [2, pp. 406-408].

Dummett is still not defeated. He suggests that we take some number m that is obviously not apodictic. We also take a number n that is apodictic. The final number we introduce is a constant, k. The claim by Dummett is that for some possible interpretations of m, n, and k, a contradiction can be reached. The slightly informal proof for this goes as follows: Take some non-apodictic number m for which it holds that if we subtract the constant k, we get an apodictic number. At the same time, for some apodictic number n, it holds that $n + k$ yields an apodictic number. Note that this second premise requires that constant k is an apodictic number as well. Given the weak infinity of the set of apodictic numbers, if $n + k$ is apodictic, then $n + 1 + k$ is also apodictic. Next, we can prove that $n + 2 + k$ is apodictic. In this manner, we can keep on increasing the size of the number next to n. At some point, n plus this number becomes equivalent to $m-k$. This number is, given the presumption we started with, apodictic. In other words, we can prove that $m - k + k$ is apodictic. This is the same thing as saying that m is apodictic. This result contradicts our starting presumption that m is not apodictic. So, given the premises of Wang's paradox and the existence of some appropriate n, m, and k, we can derive a contradiction without using a proof that has too many steps to check [1, 306-308].

Magidor responds by doubting the existence of some appropriate n, m and k. Dummett should provide concrete suggestions for the values of n, m and k. The easiest one to fill in is n, since any apodictic number will do. Zero or ten would both be fine candidates. It is more difficult to find concrete numbers for k and m. It is important to note that the strict finitist will only accept concrete numbers for k and m. Again, this is because of the constructivist background of strict finitism [2, pp. 407-410].

What Dummett needs is essentially this:

> ''There is a number k and a number M, such that M is not apodictic, k is apodictic, and $M - k$ is apodictic." [2, p. 408]

There is a way to show that m and k indeed exist.[7] Take any apodictic number as our starting number. Let us call this starting number p. Next, double the starting number. The result is then $2p$. Is $2p$ apodictic or not? If we say that $2p$ is apodictic as well, then all is well for the moment. However, if we say that $2p$ is not apodictic, then we have reached a contradiction. After all, because of the weak infinity of the set containing all apodictic numbers, we can prove that $2p$ should be apodictic. It is given that if n is apodictic, then $n + 1$ is also apodictic. If we fill in p for n, we can prove that $p+1$, $p+2$, $p+3$ and so on are all apodictic. After some time, we reach the point where we can claim that $p + p$ is apodictic. It would take exactly p steps to prove this. This is a number of steps that should be allowed by strict finitist. We established at the beginning that p is an apodictic number, so a proof containing p steps is acceptable. Thus, we are forced to say that $2p$ is apodictic.

When it has been established that $2p$ must be apodictic, we can repeat our trick. If we accept that $2p$ is apodictic, we can show that $4p$ must be apodictic as well. If the strict finitist wants to deny this, we can simply present a proof with $2p$ steps that shows that $4p$ is apodictic. A proof with $2p$ steps must be accepted, since $2p$ is an apodictic number. The strict finitist is now forced to acknowledge $4p$ as an apodictic number as well.

With this method in hand, we can continue on to prove the apodicticness of $8p$, $16p$, $32p$, $64p$ and so on. If we insert 2 as our p, then we reach 2^{100} after a mere 100 repeats. Surely, the result that 2^{100} is apodictic is enough to show that the predicate cannot fulfil the role strict finitists imagine for it.

As Magidor convincingly shows, the problem with the above is that it presumes that all numbers are either apodictic or not. For more classically inclined mathematicians, this makes a lot of sense. With the law of the excluded middle in the back of their minds, classical mathematicians would defend the idea that for any number x, 'x is apodictic' is either true or false. However, the strict finitist is likely to deny this. Most strict finitist are constructivists and tend to deny the law of the excluded middle. Thus, they would see no reason to play along with the strategy I explained above. When asked whether $2p$ is apodictic or not, they would simply reply that they do not know. They would not consider themselves to be in the right position to judge whether $2p$ is apodictic. With the law of the excluded middle out of the way, there indeed seems to be some space to neither claim that $2p$ is apodictic nor to claim that $2p$ is not apodictic. With the apodicticness of $2p$ having become uncertain, we can no longer go on to prove that $4p$ is apodictic. Thus, our trick for showing in 100 steps that, for example, 2100 is apodictic, is not going to work. Instead, we are again forced to look for concrete candidates for m and k [2, pp. 409-410].[8]

It does not seem *prima facie* likely that there are candidates available that are obviously correct. The vagueness of the predicate 'apodictic' is such that we cannot distinguish good examples from bad ones. We need to define 'apodictic' more sharply if we are to succeed. Of course, if this is attempted, then one should watch out not to create a sharp boundary between apodictic and non-apodictic numbers. Otherwise, the extension of the predicate 'apodictic' will no longer be weakly infinite. We have seen in section three that if that is allowed, then Wang's paradox will not trigger. This would make strict finitism a viable position.

Thus, the new definition of 'apodictic' has two hard to combine tasks. There must be no sharp boundary, and yet the definition must be sharp enough for us to find appropriate candidates for k and m. The difficulty of non-arbitrarily doing so is considerable.[9] We saw in the last section that Magidor was not able to change the definition of 'apodictic' in her favour. It seems to me that Dummett has a better chance though. He does not need to find any precise border like Magidor. Magidor had to essentially dissolve the vagueness of the term 'apodictic', whereas Dummett only needs to limit this vagueness somewhat or at least map it our more precisely.

I suggest that the best way for Dummett to do this would be empirical research. It can be researched what number of steps within a proof most mathematicians would find acceptable to actually check by hand. All numbers for which this holds are apodictic in a fairly uncontroversial manner. Next, with Magidor's first tactic against Dummett in mind, we could investigate at what point mathematicians start to physically collapse. Let us say, for the sake of the argument, that we limit the amount of time to just 24 hours. It could be that if we double the number of acceptable steps to check in one day, we would reach a number at which most mathematicians would collapse. It can be argued that the number at which collapse occurs is not apodictic, even if it is not a border. If we could successfully show this, then we would have some plausible candidates for k and m. k would be the highest number acceptable to mathematicians and m would be the number of mass physical collapse. Also, m would equal or be less than 2k.

Technically speaking, such a feat is not impossible. Or if this particular 24-hour set-up fails, we could try do devise other experiments. Perhaps if we take a years' time, it could be that the collapse-number is either equal or less than twice the acceptable-number. The point is that any research will do that would yield an uncontroversial apodictic number x and an uncontroversial non-apodictic number y for which it holds that y is equal to or less than $2x$.

In this way, a small opening has formed for Dummett. If the required research can be pulled off, he would have the required concrete numbers for k and m. Still, this opening is of limited value. The first problem is that as long as the research has not been done, it provides nothing more than a promissory note. The incoherence of strict finitism can only be shown if the research has been done and has proven favourable for Dummett. Until such times, the strict finitist is not refuted.

A more serious threat to Dummett is that the research might very well yield a dead end for him. Sustained empirical investigations could simply show that there are no cases that provide acceptable k's and m's. The difficulty of thinking of intuitively plausible cases is already an early bad sign. On the top of our heads, we cannot think of a good case. Further research might show that this is caused by the fact that no good cases exist. At the very least, good cases have proven hard to find so far. If Dummett were to bet on these empirical experiments, then he would essentially be relying on luck. Unless he actually does perform experiments and then proceeds to get lucky, the strict finitist need not worry.

All in all, Dummett does have some space left to argue for the incoherence of strict finitism. But he has to rely on empirical experiments. Given the fact that success has evaded us so far, Dummett should consider himself very lucky if he does succeed to find a case that can provide him with what he needs. The position of the strict finitist is thus significantly more secure than that of Dummett, but Dummett, *pace* Magidor, still has a slight fighting chance.

5 Conclusion

The goal of this paper was to assess the struggle between Michael Dummett and Ofra Magidor concerning the incoherence of strict finitism. At the centre was Dummett's argument, that was based on Wang's paradox. This argument shows that the term 'apodictic', which is a term the strict finitist is committed to, leads to contradiction. This argument by Dummett relies on the vagueness of the term 'apodictic'. Therefore, if the vagueness of the predicate 'apodictic' could be removed, then Dummett's argument would collapse. Magidor attempted this by arguing that there is a strict border between apodictic and non-apodictic numbers by pointing to the physical collapse of mathematicians. However, this move just comes down to translating 'apodictic' as 'can be checked before collapse'. This translation was rejected as arbitrary, thereby terminating Magidor's first attempt to refute Dummett's argument.

Magidor's second attempt to curb Dummett's argument fared better. Strict finitism, given its radical constructivist views, does not allow for much in terms of proofs. There are multiple perfectly fine classical proofs that Dummett can produce to prove the incoherence of strict finitism. Magidor successfully shows that the strict finitist rejects all of them on principal grounds. It is exactly its strictness that saves strict finitism. This strictness also, partially, shows why the view is not very popular. The incredibly small room for movement causes immediate feelings of claustrophobia. We would appreciate it if our logic could do more than strict finitism can deliver.[10] Still, Magidor uses these limits in an ingenious way, with the result that Dummett cannot prove the incoherence of strict finitism with exclusively logical means.

The only way out for Dummett is aid from empirical research. Dummett needs to find some apodictic number k for which it holds that $2k$ is not apodictic. If Dummett is extremely lucky, such a case might be found if we investigate the mathematicians at work. However, until such research has been done, this is no more than a promissory note. It is therefore fair to say that Magidor has prevented Dummett from proving the incoherence of strict finitism. Still, Magidor did not succeed in completely defeating Dummett. It is not likely that the last opening that remains to him will result in victory, but it is an opening nonetheless. Contrary to Magidor's claims, Dummett still has a fighting chance.

Acknowledgements
I would like to thank three anonymous reviewers for comments that made the paper more clear.

Notes

1 This possibility has been pursued by Crispin Wright, though he looked at integers [8].

2 This presumably also holds for non-natural numbers.

3 Amit Saad has once made similar claims to the ones I make here [4, pp. 457-459].

4 10, 000, 000, 000, 000, 000, 000, 000, 000, 000, 000, 000, 000, 000, 000, 000, 000, 000, 000, 000, 000, 000, 000, 000, 000, 000, 000, 000, 000, 000, 000, 000, 000, 000

5 Magidor bases this first rebuttal of Dummett on Wright [8].

6 A philosopher who does endorse a solution close to this is Timothy Williamson. He claims that every vague predicate has some hard border, even if we do not know what this border is. Magidor is aware of this position, but does not want to use it as a premise ([2, p. 405] and [3, p. 472]). With good reason, since his theory is not uncontroversial enough to unproblematically invoke as a premise. Thus, we should appreciate Magidor's attempt to find a hard border for its own merits, without relying on Williamson's theory. For Williamson's account, see [7].

7 The following proof has been taken from Magidor [2, p. 409].

8 Magidor also argues that adopting a cut-free logic would help strict finitism resist certain versions of Dummett's argument [3, pp. 484-486]. However, I do not discuss cut-free logic in this paper for reasons of scope.

9 Some hope for Dummett comes from Jean Paul van Bendegem. He suggests that rather than looking for a border between apodictic and non-apodictic numbers, we should imagine a grey area of numbers for which it is unclear whether they are apodictic. If this grey area can be made small enough, Dummett might be able to find concrete numbers for m and k [6].

10 Unsurprisingly, not all agree. One example of a philosopher who is enamoured by strict finitism, despite its flaws, is Jean Paul van Bendegem [5].

Auke Alesander Montesano Montessori
University Utrecht
Department of Philosophy and Religious Studies
Domplein 29
3512 JE Utrecht, Netherlands
<aukemontessori@gmail.com>

References

[1] Dummett, Michael. "Wang's Paradox". In: *Synthese* 30 (1975), pp. 301-324.

[2] Magidor, Ofra. "Strict Finitism Refuted?" In: *Proceedings of the Aristotelian Society* 102 (2007), pp. 403-411.

[3] Magidor, Ofra. "Strict Finitism and the Happy Sorites". In: *Journal of Philosophical Logic* 41 (2012), pp. 471-491.

[4] Saad, Amit. "On the Coherence of Wittgensteinian Constructivism". In: *Acta Analytica* 31 (2016), pp. 455-462.

[5] Van Bendegem, Jean Paul. "Strict Finitism as a Viable Alternative in the Foundations of Mathematics". In: *Logique & Analyse* 145 (1994), pp. 23-40.

[6] Van Bendegem, Jean Paul. "A Defense of Strict Finitism". In: *Constructivist Foundations* 7 (2012), pp. 141-149.

[7] Williamson, Timothy and Peter Simons. "Vagueness and Ignorance". In: *Proceedings of the Aristotelian Society* 66 (1992), pp. 145-177.

[8] Wright, Crispin. "Strict Finitism". In: *Synthese* 51 (1982), pp. 203-282.