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What is the Value of Geometric Models to Understand Matter?*

Francoise Monnoyeur

Introduction

The understanding of matter is a philosophical endeavor. Scientists are interested in finding or building adequate models in order to explain phenomena and find the properties of matter.[1] Models are then tested against experience and when validated lead to the formulation of laws. From a philosophical perspective models are not only tools to understand matter and discover laws of nature, but representations that reveal how we acquire knowledge about nature.[2] The philosophical perspective takes into account that models are representations of an object under the scrutiny of a viewer, and therefore analyzes the interaction between the mind and the object.

We focus on two geometrical models created with more than two millennia apart, and consider how they capture natural phenomena. The first model appears in one of Plato's dialogues when the young astronomer Timaeus, in a conversation with Socrates, introduces geometric solids to understand the basic structure of matter and interaction of the primary elements: fire, air, water, and earth. The second geometric model appeared nowadays with the development of electron microscopy and x-ray diffraction and allows physicists to discover the atomic structure of matter. Furthermore, we analyze the value of these geometric models and identify their cognitive function.

1. Function of geometric models in Plato and crystallography

1.1. Platonic models

In the *Timaeus* one of Plato's latest dialogues, Timaeus in the name of Plato gives an account of the creation of the world and origin of

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* The author acknowledges Mr. Konstantinos Bakoglidis for some of the figures from Plato and inspiring discussions.

1. Hertz 2007 and Hesse 1953, 198.
2. Monnoyeur 2000a.

matter. This story does not pretend to deliver a scientific theory, but is a possible explanation of the origin of matter when absolute truth falls short. That's why Plato proposes a model of matter and not a theory. At this stage, it seems to me necessary to draw the distinction between theory and model, as Timaeus recalls several times at the beginning of his explanation that he is telling a "story" or a "tale":[3]

> Wherefore, Socrates, if in our treatment of a great host of matters regarding the Gods and the generation of the Universe we prove unable to give accounts that are always in all respects self-consistent and perfectly exact, be not thou surprised; rather we should be content if we can furnish accounts that are inferior to none in likelihood, remembering that both I who speak and you who judge are but human creatures, so that it becomes us to accept the likely account of these matters and forbear to search beyond it.

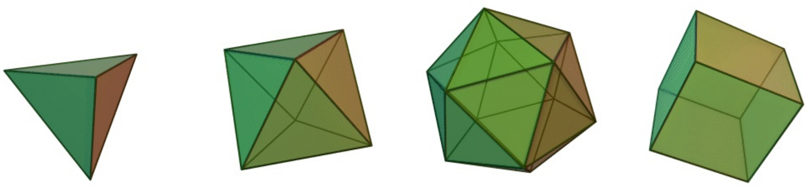
In the introduction of *Inventing the Universe*, Luc Brisson emphasizes the axiomatic structure of science in the Timaeus and writes:[4]

> In our analysis, we will endeavor to display the axiomatic structure which underlines the *Timaeus.* In fact, we will exhibit, as completely as possible, the list of axioms, or primordial assumptions, upon which Plato's cosmological system is ultimately founded.

In this article, I show how the Platonician mathematical theory of the elements, although founded on certain axioms, relates to our perception. Because we focus on physics, namely the theory of matter and the movement of the elements, a special attention is given to the relationship between mathematics and perception. Further in his introduction,[5] Brisson elaborates that Plato develops "scientific knowledge" against "scientific method of choice… directly based on the technology of experimentation." Our purpose is to present the Platonician model of the elements as hypothetical in its nature, geometrical in its reasoning, but also relying on perception in some ways. Plato proposes, therefore, a model to explain the movement and nature of matter, and chooses a geometrical model able to express the perception we have of the

3. Plato, *Timaeus* 29c-29d in Lamb 1925, 29.
4. Brisson and Meyerstein 1995, 6.
5. Brisson and Meyerstein 1995, 7.

elements. According to my interpretation, Plato's model solves the following problem: how to represent in an intelligible way the matter we sense? There are three actors involved in this problem: the moving matter we sense, the representation supporting this moving matter, and the intelligibility of this representation. Plato's main objective is to find the best representation or model of the sensible world. Previous philosophers such as Empedocles, Anaxagoras, Thales, and Leucippus used combinations of four basic elements (air, earth, water, fire) to identify primordial matter. Plato refers also to these four elements, and conceives his model to understand how the four elements move and transform into each other. This model is not a pure abstraction but relates to the sensible phenomena. For Plato, models tell us something of the essence of matter as they establish the salient and permanent relationships between the elements. According to the *analogy of the divided line* of the *Republic*,[6] these models belong to the intelligible knowledge namely the mathematical and geometric knowledge. The Platonician solution consists in establishing correspondences between the four elements and four regular polyedres (Figure 1). In the *Timaeus*, geometry is chosen because it is considered to be the everlasting or reliable representation. More precisely, the explanation of the interaction between the elements follows geometric rules when transposed in geometrical solids.



(a) *Fire (tetrahedron).* (b) *Air (octahedron).* (c) *Water (icosahedra).* (d) *Earth (cube).*

Figure 1 – The geometric model of the four elements according to Plato

The choice of geometric solids gives a basis to scientific representation and understanding of the changes in physical reality. Plato's main

6. Plato, *Republic*, Book 6, 509d-513e in Shorey 1969, 509.

objective is to represent the change of water into air through fire, and the stability of earth as it cannot be changed into any other element; on the contrary, earth remains earth when interacting with fire, water or air. The geometric explanation is as follows; the solids analyzed by Plato are made of faces, more exactly of triangles. In 54e-55d, Plato explains how we can use equilateral triangles to build three regular polyhedrons representing fire, air, and water: the tetrahedron (fire) is made of four equilateral triangles, the octahedron (air) is made of eight equilateral triangles, and the icosahedra (water) is made of twenty equilateral triangle.[7] The cube representing earth is different because it is made of four squares. Each face of the first three solids (fire, air, and water) is built of equilateral triangles. Plato describes how an equilateral triangle consists of six scalene triangles (figure 2); a scalene triangle comprises one right angle with three unequal sides (figure 3).

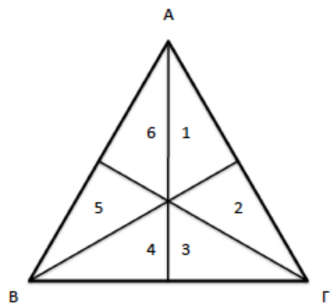


Figure 2 – The equilateral triangle from Plato is built with six scalene triangles (1 to 6)

7. Plato, *Timaeus*, 54e-55d in Lamb 1925, 54.

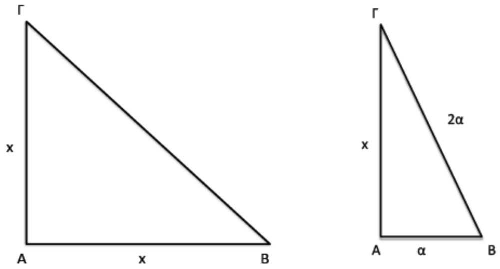


Figure 3 – Isosceles and scalene triangles

A square is made of four isosceles triangles (figure 4).

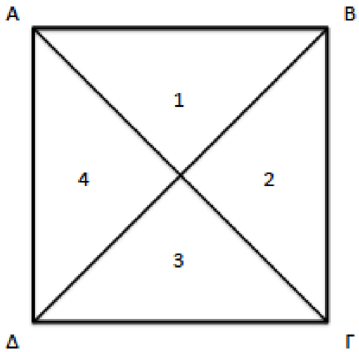


Figure 4 – A square formed by four isosceles triangles

These two right-angle triangles of figure 3, scalene and isosceles, cover the possible structures of the four primary elements. They are two presuppositions or axioms in this theory: the first is that elementary right angles triangles cannot be created or destroyed, and the second is that only the solids made of the same triangles can act on each other. Earth is identified with the square, and is the only solid made of four isosceles triangles; consequently, earth cannot act on another element or be acted upon by any other element; thus explaining the stability of the squared face. On the other hand, the faces of fire, water and air made of equilateral triangles, and can interact with each other since any change in the fundamental elements can be explained according to the division and re-composition of the fundamental triangles. The relationship between the various elements can be written with the following equation:

$$\frac{fire}{air} = \frac{air}{water} = \frac{water}{earth}$$

In the equation above, fire and earth are connected through air and water whereas water and air are the *«geometrical medians»*. Plato shows how the four elements are combined in various ratios and how they are made of material substances that interact with each other. For instance, water heated by fire dissolves and its fragments recombine and produce two corpuscles of air (vapor) and one corpuscle of fire. Plato, taking into account the number of faces, writes[8]:

$$\left\langle water \right\rangle \quad \overrightarrow{heat} \quad 2\left\langle air \right\rangle + \left\langle fire \right\rangle$$

$$20 \quad = \quad 2\text{x}8 \quad + \quad 4$$

Figure 5 – Equation for the transmutation of the elements

In this equation, Plato rationally establishes that the transmutation of fire, air, and water occurs by redistribution of the faces of these figures and writes a "chemical equation" explaining the transformation of the four elements. For Plato, the type and number of triangles determine the properties of each element. The whole system is supported with a set of relationships expressed with the geometry of the solids. Without these models, we would be unable to scientifically determine the way elements interact and move.

1.2. Comparison with the geometric models of matter used in crystallography

Crystallography, the science that examines the arrangement of atoms in solids, developed rapidly at the beginning of the twentieth century

8. Plato, *Timaeus* 56d in Lamb 1925, 56.

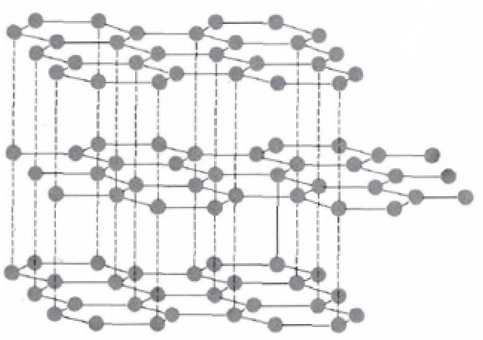
with Sir William Lawrence Bragg's discovery of the x-ray diffraction law;[9] the subsequent development of x-ray diffraction technology and transmission electron microscopy have paved the way towards exploration of the nano-structure of the materials. It is now established that matter made of atoms can only belong to seven different crystalline systems: triclinic, monoclinic, orthorhombic, tetragonal, rhombohedral, hexagonal, and cubic. In this paper, we focus solely on three crystalline structures composed of carbon atoms. The first material, graphite, is soft and composed of carbon atoms that are arranged into a hexagonal shaped structure (figure 6a). The second material, diamond, is the hardest known material and composed of carbon atoms arranged in a tetrahedral (cubic) structure (figure 6b). Finally, the third material, fullerene, is a molecule in the form of a hollow sphere that is composed of sixty atoms interconnected by pentagons and hexagons (figure 6c). Fullerene is elastic and extremely strong.[10]

The science of crystallography studies the various structures taken by the carbon atoms and identifies different materials such as graphite, diamond, or fullerene. These geometric models give scientists the ability to identify the location of atoms as well as the distances and angles between atoms. Scientists can then perform further experiments to investigate the properties of materials, for example, hardness/softness, and elasticity. We can see how each material corresponds to a specific structure with specific properties in the same way that Plato relates each of the four elements to a geometric solid with distinct properties.

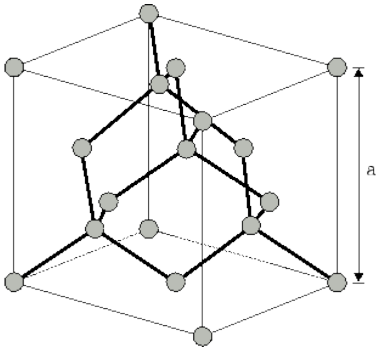
2. What is the value of these geometric models to understand matter?

There are obvious and surprising similarities between the models described by Plato to understand the transmutation of the elements and the models of the various structures of carbon atoms. Both of these models are based on Euclidean geometry, but the Platonician is chosen, and the Crystallographic is observed by electron microscopy and x-ray diffraction. Nevertheless, the identity between these two models does not happen by chance, or because Timaeus had good intuition. How

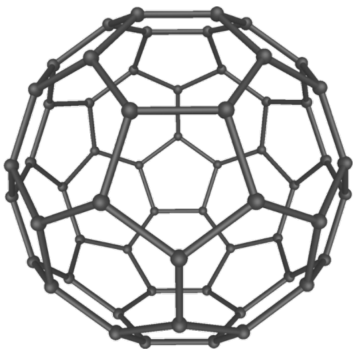
9. Giacovazzo 2002.
10. Broitman and Hultman 2014, 390-392.



(a) *Graphite.*



(b) *Diamond.*



(c) *Fullerene.*

Figure 6 – Three different crystallographic carbon structures

can we understand how these models, which were developed millennia apart, are so similar? The answer could be that both provide what is required to understand matter and correspond de facto to our cognitive functions. According to Raymond Duval:[11]

> Representation and visualization are at the core of understanding in mathematics.

We can apply this conception to the case of the geometrical models because there seems to be two distinct phases in the way we perceive these models; firstly, we conceive the mathematical explanation through visual representation, and secondly, we visualize the solids with their properties. Let's analyze how these two phases work in the Platonic and crystallography models. In the first phase, we build up a visual representation of the model that allows us to understand the properties of matter. For instance, the Platonic models identify the nature of the triangles composing the faces of the solids while crystallography perceives the atoms and connects them in order to build geometrical solids. The second phase allows us to visualize in three dimensions the solids with the properties they represent. Visual representation (two dimensions) and visualization (three dimensions) are at the core of the understanding of these models. The scientific understanding of the movement of elements and the detection of the structure of carbon atoms is realized thanks to the geometrical models that enable the visualization of the properties of the materials such as mobility, stability, hardness, softness and elasticity. Hans Reichenbach notices in the *Philosophy of space and time*[12] that:

> It seems to be much easier to make logical inferences with the help of visual representation than by means of abstract concepts.

2.1. What is the value of the Platonic geometric models?

2.1.1 Visual representation in the Platonic solids

What can we learn from the geometric decomposition of the elements? Because fire has less number of faces, the solid is more acute; it is the

11. Duval 1999, 3.
12. Reichenbach 1958, 117.

lightest, most mobile and sharpest element, which explains how it can transform water into air. Air has more faces than fire but is more mobile and lighter than water. Water has a large number of faces and is less mobile and heavier than air. Earth, the heaviest element, represents stability because its planes are squares and cannot be changed by the other faces.[13] Therefore, according to these models, there is a relationship between the number of faces and their mobility. Plato deploys geometrically the properties of the elements, captures the transformation of one element into another and then expresses this phenomenon into an equation. As geometry is a science of relations,[14] it gives the opportunity to establish relations between objects, here between the elements. Poincaré explains in the preface of his book *Science and Hypothesis* the purpose of mathematics:[15]

> Mathematicians do not study objects, but the relations between objects; to them it is a matter of indifference if these objects are replaced by others, provided that the relations do not change. Matter does not engage their attention, they are interested in form alone.

The geometric models give a scientific explanation of the properties of the elements. This means that their properties are revealed upon relationships represented geometrically and mathematically. We can see how Plato interpretes the movement and properties of the elements in terms of number and type of triangles in the faces of the solids. The triangles in the geometric solids represent the way the elements move and how they transmute into each other. Having no scientific instruments to analyze matter, we can say that the correspondence discovered between figures and the movement of the elements depends on a combination of observation and intuition. As this model is supported by mathematical equations, we can say that Plato's intuition goes beyond intuition that is delivered by senses alone. Poincaré describes as follows the kind of intuition that comes into play in this case:[16]

13. Plato, *Timaeus*, 56a-56b in Lamb 1925, 56.
14. Monnoyeur 2013.
15. Poincare 1952, 20.
16. Poincare 1914, 128.

> The principal aim of mathematical education is to develop certain faculties of the mind, and among these, intuition is not the least precious. It is through it that the mathematical world remains in touch with the real world, and even if pure mathematics could do without it, we should still have to have recourse to it to fill up the gulf that separates the symbol from reality.

Geometric spatial representation of movement relates to the properties of the elements, to the «real world». Plato's geometric solids are spatial representations of movement expressed in a proportional equation. Thanks to visual representation of triangles, the rules of division and re-composition of the triangles have been established together with the relationship between the properties and movement of the elements. The visual representation of the Platonic solids paves the way to the understanding of matter.

2.1.2 Visualization of the Platonic solids

Geometric models relate finally to the «real world» with the geometric solids that give us a three dimensional representation of the movement and properties of their elements. This visualization completes the visual representation of the relationships between the triangles and gives a picture of how fire, air, water and earth interact on each other.

Three-dimensional visualization combined with two-dimensional representation allows us to reach an understanding of the movement and properties in the elements.

2.2. What is the value of the geometric models in crystallography?

2.2.1 Visual representation in crystallography

In crystallography, the geometric solids are not chosen as in Plato, but observed with scientific instruments. Although technology helps to locate the atoms, the scientist still needs to draw the lines between the atoms in order to construct a figure that accurately represents the spatial arrangement of the atoms. This presupposes the calculation of the atoms quantity, of the respective distance between each other, and the angles they form in the structure. Thanks to the development of new technologies (electron microscopes and computers), microscopic data can be calculated.

It follows that progress in crystallography is based in the interpretation of the spatial representation detected by the scientist. The human eye and mind together with the image presented by the microscope constitutes the model to be interpreted.

2.2.2 Visualization reveals the various structures of carbon atoms

After being able to represent the atomic structure of materials, the scientist can identify the kind of material and consecutively its specific properties embedded in the different geometrical structures. The three dimensional visualization of the structure of the carbon atoms indicates their type and properties. We can reach this final stage after the building up of the visual representation of the structures. The understanding of matter happens via two phases, visual representation and visualization in three dimensions of the whole structure.

Conclusions

We have discussed how Platonic and crystallographic models are rooted in Euclidean geometry and both explain and reveal the properties of materials. In the Platonic solids, the nature and number of triangles justify their movement or stability; in crystallography the nature of the geometric structure of the carbon atoms (hexagonal, cubic, or fullerene) indicates the type of material and properties: hardness or softness and elasticity. In both cases, scientists are depending upon visual representation and visualization to be able to analyze the nature of matter and measure it. Not only these spatial models give valuable scientific information about the matter studied, but they also are the main cognitive support for scientific activity. Besides the extraordinary fact that these models have important similarities, they nevertheless differ in the way they are obtained. The Platonic models are chosen and the interpretation of matter changes relies in geometry and mathematics. On the contrary, in crystallography the physicists discover atoms and identify their position in defining the structure of these atoms. Besides technology (microscopes, diffractometers, and computers) the understanding of matter relies on fine observation, interpretation and design by the scientists of the structure itself. In both of these models, the understanding of matter is depending upon active visual representation and visualization that synthetizes the whole process of knowledge. We can conclude that the geometric models are the expression of the mind to understand physical matter.

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