THOMAS MORMANN

MATHEMATICAL METAPHORS IN NATORP'S NEO-KANTIAN EPISTEMOLOGY AND PHILOSOPHY OF SCIENCE

Abstract. A basic thesis of Neokantian epistemology and philosophy of science contends that the knowing subject and the object to be known are only abstractions. What really exists, is the relation between both. For the elucidation of this "knowledge relation ("Erkenntnisrelation") the Neokantians of the Marburg school used a variety of mathematical metaphors. In this contribution I'd like to reconsider some of these metaphors proposed by Paul Natorp one of the leading members of the Marburg school. It is shown that Natorp's metaphors are not unrelated to those used in some currents of contemporary epistemology and philosophy of science.

Keywords: Metaphor, Mathematics, Neokantianism, Marburg school, Natorp.

1.INTRODUCTION

Since some time "postpositivist" philosophy of science has become interested in its history and evolution. In order to understand science, not only history of science but also history of philosophy of science has become an important topic for philosophy of science. As a result of this attitude Neokantian philosophy is being re-evaluated as a hitherto unduly neglected source of philosophy of science and epistemology. For instance, the investigations of Coffa, Friedman and others have shown that Neokantian philosophy played an eminent role for the emergence of the Logical Empiricism of the Vienna Circle. This holds in particular for the Marburg School, whose most important members were Cohen, Natorp and Cassirer (cf. Coffa 1991, Friedman 1999, 2000). It goes without saying that a short paper like this is not the appropriate place to present a detailed account of the Neokantian philosophy of science and its relation to modern philosophy. The aim of this contribution is more modest. Provisionally accepting Rorty's thesis that "it is pictures rather than propositions, metaphors rather than statements, which determine most of our philosophical convictions" (Rorty 1979, 12), I want to take a shortcut reconsidering some of the core metaphors that guided Neokantian epistemology and philosophy of science.[1]

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Remarkably, the guiding metaphors of Neokantian epistemology and philosophy of science have their origin in science itself, in particular in mathematics.[2] This points at a rather complex relation between science and philosophy of science that is not adequately described by the standard 2-level account according to which philosophy of science is a sort of metascience dealing with the sciences as its object.

Dealing with Natorp's metaphors, I'd like to show two things: first, the Neokantian metaphors are surprisingly modern. They may still deserve to be taken into consideration by contemporary philosophy of science. Secondly, a closer look at the metaphorical apparatus of a gone-by philosophical stance may help sharpen our own sensitivity for the often murky metaphorical ground on which many of our own basic philosophical convictions are based.

More precisely I want to concentrate on some metaphors that Natorp used for the elucidation of a basic thesis of Neokantian epistemology put forward by virtually all authors of the Marburg school and most other Neokantians. According to this thesis the true issue of epistemology is neither the knowing subject nor the known object, but the "knowledge relation" ("Erkenntnisrelation") by which subject and object are related. Subject and object are mere abstractions. Hence, strictly speaking, only the "knowledge relation" exists (cf. Cassirer 1910, Rickert 1915, Natorp 1903, 1912). The K-relation, as I want to call it, has a privileged status with respect to its relata, to wit, the knowing subject on the one hand, and the known (or knowable) object on the other. Rival epistemological approaches such as empiricism, positivism, and non-critical versions of idealism like Hegelianism, are accused by the Neokantians to commit a reductive fallacy falling back on some apparently simpler "monistic" position that eliminates the K-relation in favour of one of its relata. In the end, all these positions are claimed to be unable to characterize the true nature of science as an ongoing process of knowledge acquisition.[3]

For the elucidation of the K-relation, Neokantian philosophy used a variety of pictures, analogues, and metaphors. For the Marburg School the paradigm of knowledge was scientific knowledge, more precisely, mathematics and mathematical physics. Hence it is not surprising that in the Marburg account mathematical metaphors played an important role. Maybe the first of these guiding metaphors was due to Hermann Cohen, the founder of the school. According to him, the essence of the formation of mathematiced empirical science was to be found in the concept of the infinitesimal (Cohen 1863).[4] Cohen's mathematical erudition was not very profound, and he presented this thesis in a rather obscure way. Hence, his account did not gain much real influence, even among the members of the Marburg school. Cassirer's "functional approach" of critical idealism became better known one or two generations later. In *Substance and Function* (1910) Cassirer put forward a "functional" or "relational" account of scientific concepts in which he contended that the essence of the modern science resided in the concept of mathematical function.

Cassirer's "function" was by no means the only mathematical metaphor that guided Neokantian epistemology. In this paper I'd like to consider some of the lesser known mainly due to Natorp that served as guiding lines for the Marburg Neokantianism in general. In his lifetime Natorp was one of the most influential members of the Marburg school. Before Cassirer became prominent he was a kind of official spokesman of the Marburg school whose sober and relatively accessible treatises (compared with the writings of Cohen) taught generations of students the basics of the schooI's doctrines (cf. Natorp 1903, 1910, 1929).[5]

Natorp took his metaphors seriously. For him, they were more than embroideries, rather, he used them as "intuition pumps" to develop his account of scientific knowledge. He attempted to draw contentful conclusions from them, considering them as models that could be used for the description of the sciences, their methods and development. True, Natorp's metaphors are no longer ours, and sometimes they appear strange and contrived. Nevertheless, even contemporary epistemology and philosophy of science can hardly be said to be an area free of metaphors, as will be briefly discussed in the last section.

The outline of this paper is as follows. In section 2 the Neokantian transformation of the Kant's original epistemological position is discussed.[6] This sets the stage for the detailed analysis of some of Natorp's core metaphors in section 3. In particular, we will deal with his "equational account" of knowledge according to which knowing (cognizing) may be characterized as an activity analogous to solving a mathematical equation. The paper concludes with some general remarks on the problematic of metaphors in philosophy comparing Neokantianism with some post-positivist authors.

2. THE NEOKANTIAN REFORMULATION OF KANT'S EPISTEMOLOGY

The Neokantian approach to epistemology and philosophy aimed to be faithful to the spirit but not to the letter of Kant's philosophy. For Natorp this meant to restitute the "transcendental method" as the true core of the Kantian approach, and to give up all of ingredients of Kant's system that did not sit well with that method. The transcendental method deals with the problem of the possibility of experience. The NeoKantians interpreted Kant as contending that the object of experience is determined by the laws and methods of the knowing subject. Thereby the object no longer is something given ("gegeben") but something "posed" ("aufgegeben") (cf. Kinkel 1923, p. 405). Conceiving Neokantian philosophy as based on the transcendental method has two implications:

(i) Philosophy recognizes the historical, societal and scientific context in which it exists. It is aware that it is rooted in the specific theoretical and practical experiences of its time and refuses to build up "high towers of metaphysical speculations" (cf. Natorp 1912, p.195, Kinkel 1923, p. 402/403).

(ii) Philosophy accepts the facts of science, morality, art and religion. The task of philosophy is to carry out a *deductio iuris* of these facts, i.e., it has to provide a kind of "logical analysis" which shows the reasons why these facts are possible thereby revealing what is the "quid iuris" of them. In still other words, and going beyond the epistemological sphere, philosophy has to show the lawfulness and reasonableness of the cultural achievements of mankind.

Thereby the philosophy of critical idealism is lead to a "genetic" epistemology and theory of science that regards the ongoing process of scientific and cultural creation as essential, not its temporary results. These are to be considered as being of secondary importance. As Natorp put it with respect to scientific knowledge: knowledge is always "becoming" and is never "closed" or "finished". There never is something "given" that is not transformed in the ongoing and strictly speaking infinite process of cognition. The "fact of science" is, according to Natorp to be understood as a "fact of becoming" ("Werdefaktum").

The rejection of a non-conceptual given in any form brings the Marburg brand of Neokantianism in open conflict with some of the corner-stones of Kant's epistemology, to wit, the dualism of "scheme" and "intuition", and related dualisms such as that of "spontaneity" and "receptivity" of thinking: "Maintaining this dualism of epistemic factors (receptivity and spontaneity, T.M.) is virtually impossible if one takes serious the core idea of the transcendental method." (Natorp 1912, 9).

Subscribing to a "genetic" account of knowledge that emphasises the process character of knowledge gives the K-relation priority over its relata, to wit, the knowing subject and the object of knowledge. Both are constituted in the ongoing process of knowledge. Taken for themselves they are just abstractions from the more basic K-relation. Although it may sometimes be expedient to treat the subject of knowledge and the object of knowledge separately this separation is to be considered as a methodological device by which one may distinguish between two complementary accounts: one in which the object occupies centre stage, and one which emphasizes the role of the cognizing subject. Speaking in a Kantian framework, object-oriented accounts emphasize the role of receptivity of cognition, in particular perception, while subject-oriented, epistemic account are inclined to lay stress upon the constructive aspects of cognition. According to the Neokantian doctrine both accounts are mistaken. For the Neokantianism, ontology and epistemology are two sides of the same coin. Ontology without epistemology would be some kind of magic, which leaves unexplained how knowledge gets access to its object, while epistemology without ontology would be without content, since it denies the objectual character of cognition. Expressed in Kantian language, object-oriented approaches tend to emphasize the receptivity of cognition. According to them, cognition is essentially a passive and receptive behaviour. The thinking mind is confronted with something outside and independent of the sphere of reason. Ignoring more subtle differences this amounts to some kind of "copy-theory" or "mirror-theory" of knowledge. Subject-oriented approaches, on the other hand, emphasize the spontaneity of cognition. According to them, cognizing is essentially to be considered as a creative activity. Such a conception does not admit a "given" as a mind-independent presupposition of the cognizing process. Rather, the given ("das Gegebene") is to be conceived of as the product ("das Ergebnis") of the immanent determination of thought. Thereby, subject-oriented approaches are in danger of underestimating the resisting power of the real world in favor of the unrestricted creative power of the knowing mind. According to Natorp, employing the "transcendental method" as a guide-line, critical idealism overcomes the shortcomings and deficits of both the subject-oriented and the object-oriented accounts.

3. NATORP'S MATHEMATICAL METAPHORS

Natorp's metaphorical frame for elucidating the "relational" account of Marburg Neokantian epistemology and philosophy of science was based on two groups of metaphors, one taken from algebra and the other taken from geometry. Let us begin with his basic algebraic metaphor. According to it, knowing as the determination of the object of knowledge ("Erkenntnisgegenstand") is analogous to the process of solving a numerical equation. In order to be specific, the reader may have in mind a numerical equation like $x^2 + 2x + 1 = 0$. In other words, the object of knowledge may be considered as the "x of the K-equation":

> If the object is to be the x of the equation of knowledge, it has to be completely determined by the perspective of knowledge, although it is that what one is looking for. In the same way as the X, Y etc. of an equation have meaning only for and in the equation, due to the meaning of the equation itself, … the X of knowledge becomes meaningful only in the context of the inquiry. (Natorp 1910, p.39)

Hence, for Natorp, as for all his fellow-philosophers of the Marburg school, the object of knowledge was not an unproblematic starting point of the ongoing process of scientific investigations, but rather as its limit.[7] The object was a problem to be solved. In various versions this equational account of knowledge can be found in virtually all of Natorp's epistemological writings. For instance, in his *Philosophischer Propädeutik* (Natorp 1903), which may be considered as a compendium of the basic doctrines of the Marburg school, he maintained that the equational metaphor expresses "the very idea of the critical or transcendental method of philosophy (ibidem, § 7, p.10). Against a one-sided and naive realism, the Critical Idealism of the Marburg School insisted that the object of knowledge was not to be considered as "given" ("gegeben") but as a problem "posed" ("aufgegeben"[8]) to the scientific investigation as suggested by the equational metaphor of knowledge quoted above. Being engaged in a solution of an equation, at the same time one does "have" and does "not have" the object represented by "x":[9] On the one hand, one does have the object, since x occurs in an equation that (hopefully) determines it completely, on the other hand, one does not have the object, since one does not know the precise value of x. In a sense, the equation promises to deliver the object but has not yet delivered it, since also the problem-solver, i.e., the scientist has to fulfil his part of the contract.

In order to bring to the fore more clearly the philosophical content of the equational metaphor it is expedient to dwell upon the mathematical or logical form of equations in some more detail. This is in line with Natorp's own approach. An equation in the sense of Natorp has the general form $F(x) = 0$. Strictly speaking, this formula is not an assertion that can be evaluated to be true or false. In order to render the formula a proposition the free variable x has to be it has to be bounded by a quantifier. It is sufficient to consider the existential quantifier ∃x (there is at least one x).[10] Thereby we obtain ∃x(F(x) = 0). In other words, Natorp's equational model of inquiry amounts to the introduction of variables and quantifiers. The introduction of quantifiers is tantamount to entering the realm of ontology. According to him, the objects a theory is referring to are just the values of its quantified variables (Quine 1976, § 26). I do not assert that Natorp had a clear idea of the concepts of variable, range, and quantification in the sense of modern logic. But at least his equational model may be considered as an implicit and informal precursor of Quine's thesis that ontological questions appear when one has to consider quantified theoretical statements whose parameters are determined by appropriate theoretical premises and whose "solutions" - if there are any - may be conceived as the objects the theory is referring to. Numerical equations such as ∃x (F(x) = 0) may be considered as a kind of simplified model for them. Thereby, Quine's slogan "To be is to be the value of a variable" may be translated in Natorp's terms as the thesis that the object of knowledge exists exactly if it can be conceived as a "root" of a valid K-equation.

Conceiving Natorp's "K-equations" as quantified sentences, it is natural to ask on what sort of quantification they are based: substitutional, objectual quantification, or perhaps some intermediate form. According to the substitutional conception a variable is nothing but a slot in which one may insert just any constant. Such variables do not contend to refer to objects as their values. In the objectual interpretation the variable refers to some entities as its values, and one need not be able to characterize them by a name or a description (cf. Quine 1976, § 26). As Quine points out the substitutional and the objectual interpretation of variables are opposite to each other. In the following I'd like to consider substitutional variables and objectual variables as the two extreme poles of a spectrum. I will argue that such a "variable conception" of variables fits the dynamics of the process-oriented Neokantian account best. The dynamic of the object's development in the ongoing knowledge process may be described as an ontological move that starts from the substitutional pole and advances towards the objectual pole. To be specific, let us consider the equation $x^2 + 1 = 0$ to be interpreted as the task of determining the truth-value of the proposition

$$(*) \qquad \exists x\, (x^2 + 1 = 0).$$

Whether this proposition is true or not, depends on the range V over which the variable x is running. If one assumes that V is the domain of real numbers **R**, there is no object in this domain which satisfies this equation. In this situation the inquirer has two options: either he sticks to the traditionally established domain of number objects, considering therefore (*) as false, or he attempts to enlarge the range V in such a way that the equation (*) may come out as true for some object of the new domain. As is well-known modern mathematics has chosen the latter option by accepting "imaginary" numbers $\pm\, i := \pm\sqrt{-1}$ as solutions. Without doubt, this outcome will have pleased Neokantian epistemology which always sympathized with conceptual progress of the sciences, in particular mathematics.

As is indicated already by their traditional name the ontological status of the new "imaginary" numbers +i and -i, and more generally of complex numbers a + ib, was at first considered as rather dubious. Imaginary numbers were considered as mere ficticious (but useful) constructs. They were something like theoretical terms (cf. Carnap 1974) by which the theory of numerical equations could achieve a greater unity and coherence. For instance, admitting complex numbers one could assume that every quadratic equation $x^2 + ax + b$ always had two formal solutions even if these solutions did not always define real numbers. In this stage, complex number objects had a purely substitutional character. It took some time before these constructs were recognized as genuine mathematical objects having the same ontological status as that of the familiar "real" numbers. An important step on this road to full recognition was the insight that the fundamental theorem of algebra, according to which every equation of n^{th} degree has n (possibly complex) solutions, was valid only for the enlarged domain **C** of complex numbers. Another argument for their growing ontological respectability offered Gauss's representation of complex numbers as points of the Euclidean plane. Summarizing we may say that in the course of the historical and conceptual development of mathematics the ontological status of the "imaginary" substitutions changed: they got rid of their purely instrumental status and gained recognition as fully accepted mathematical entities.[11]

Natorp's attempt to explicate objectual knowledge with the metaphorical K-equation can be conceived as an intuitive generalization of Hilbert's program of the constitution of mathematical objects by implicit definitions (cf. Hilbert 1899). In Hilbert's *Foundations* geometric objects such as points, lines, and planes are defined by implicit axioms which stipulate that certain relations exist between them. Outside the system, it does not make much sense to speak of points. Inside the system, for the determination of a point as an object of Euclidean geometry, it is necessary to determine all other kinds of geometrical objects as well. Something is a point in the context of Euclidean geometry, if and only if it fits into the relational structure of Euclidean geometry. In the metaphorical language of Natorp's K-equation this fitting may be expressed as the assertion that the conceptual object "point" may be considered as a solution of a structural K-equation. For the objects of modern structural mathematics this account has some plausibility, it appears more problematic for the objects of empirical science, at least from a modern point of view. From a Neokantian stance, things may have looked different. In contrast to modern philosophy of science the NeoKantian philosophy of science assumed that there is a profound similarity between mathematics and mature empirical science such as physics (cf. Cassirer 1910). For the philosophers of the Marburg school it even became difficult to draw a line between the two kinds of knowledge. Of course, they could not deny that there is a difference: otherwise they could be accused of succumbing to an unrestricted Hegelian rationalism that neglected the object of knowledge in favour of an unrestricted conceptual activity of the knowing subject. This objection also threatened Natorp's equational model: it might have been plausible to assert that a point as an object of geometry can be considered as the "solution" of some "relational equation". It is harder to understand how this approach can work for the objects of empirical science. Physical objects such as "atoms", "electrons" or "quarks" do not go into the framework of a physical theory without remainder. In this respect mathematical and physical theories are essentially different. Natorp did not ignore this fact, and complemented his equational account in such a way that it no longer fell a prey to this objection. Elaborating the equational model he pointed out that the object of knowledge - as a solution of the K-equation - was not simply a problem but an infinite task (“unendliche Aufgabe“) that could be solved in finite time only approximately. Otherwise, the knowing subject would possess completely the empirical object to be known which would amount to an Hegelian rationalism that Natorp strictly rejected:

> Although we conceive, similarly as Hegel does, the object of knowledge (= X) only in relation to the functions of knowledge itself, and consider it ... as the X of the equation of knowledge, ... we understand that this “equation“ is of such a kind that it leads to an infinite calculation. This means that the X is never fully determined by the equation’s parameters A, B, C etc. Moreover, the series of the parameters A, B, … is to be thought not as closed but may be extended indefinitely. In contrast, Hegel allows that the irrational is completely dissolvable in the rational, to wit, the lawlike determinations of thought. (Natorp1912, 19 - 20)

The metaphor of the K-equation is flexible enough to incorporate “infinite calculation“ and approximative solvability. Natorp's “infinite calculation“ already occurs in rather elementary examples: consider an equation like $x^2 - 2 = 0$ having only irrational solutions, in our case $+\sqrt{2}$ and $-\sqrt{2}$. The effective calculation of the decimal series of these numbers is a “supertask“ and cannot be carried out by a finite subject in finite time. Every effective solution remains approximative.[12]

Another more sophistated example of an equation that leads to an “infinite calculation“ is provided by recursive equations such as the one that is used for the calculations of the Fibonacchi numbers: $x_0 = 0$, $x_1 = 1$, $x_{n+2} = x_{n+1} + x_n$. In this way, one may define an infinite K-equation in the sense of Natorp as a <u>series</u> (e_n) of equations in which the parameters of the n^{th} equation are calculated as solutions of the previous equations. These examples should suffice to make clear the point Natorp wanted to make. In order to take into account the undeniable fact that the empirical realm does not go into the domain of conceptual activity of the thinking subject without remainder, the inexhaustibility of the empirical object is re-interpreted as the impossibility for the knowing subject to obtain complete knowledge of the object to be known in finite time. If this can be considered as an acceptable substitute of the inexhaustibility of the empirical object is not to be discussed here. At least, the philosophers of Marburg school believed to have countered successfully the objection that their account of the “methodically progressing“ scientific knowledge was just a disguised version of Hegel’s absolute knowledge.[13] For them, absolute knowledge was not something that we, as finite creatures, could ever aspire to get. Rather, the object as fully known was “the point at infinity which can never be reached but which is nothing but another expression for the always identical direction of the infinite, infinite road of knowledge.“ (Natorp 1910, p.34). Here, then, we are entering the realm of geometric metaphors the philosophers of the Marburg School used to elucidate the unending quest for scientific knowledge. For them, the "illusion of the point at infinity" was an argument against the realist conception of knowledge according to which cognizing was to be conceived as an activity directed to some goal located outside the K-relation. Not so, they claimed, the point of infinity is an illusion caused by misunderstanding the methodological unity that intrinsically constitutes the uncompleatable object of scientific knowledge.

Summarizing we may say that Natorp's epistemology is characterized by a net of tightly interrelated metaphors and analogues mainly taken from algebra and geometry. These metaphors were designed to defend the epistemology of Neokantianism against two complementary threats: on the one hand, the critical philosophy of Natorp's neokantianism is directed against a "dogmatic" epistemology that assumes some kind of non-conceptual given as a base of knowledge. On the other hand, it is directed against a Hegelian conception of knowledge that hands over the objectual part of the knowledge relation without rest to the free-wheeling conceptual activity of the knowing subject.

4. CONCLUDING REMARKS

Once upon a time Berkeley admonished philosopher's to keep away from metaphors: "a metaphoribus autem abstinendum philosopho" but few philosophers have followed his advice. In particular, in the realm of epistemology and philosophy of science the use of metaphors is flourishing as the following brief list suffices to show:

(i) In *Conjectures and Refutations* (Popper 1963) Popper proposed to base the theory of truth approximation of theories on the spatial metaphor that "truth [is] located somewhere in a kind of metrical or at least topological space..." (p. 232). More precisely, he pleaded to conceptualize the notion of truthlikeness as a distance from truth.

(ii) Probably the most influential metaphor dealing with matters epistemological in the last decades has been Rorty's "mirroring metaphor" in *The Mirror of Nature* (Rorty 1989). More precisely, Rorty blames the so called representationalists as being captivated by the profoundly misleading mirroring metaphor.

(iii) In *Evidence and Inquiry* (Haack 1993) the author bases her "foundherentist" epistemology on the metaphor of the "crossword puzzle". It is not difficult to show that this metaphor has some similarity with Natorp's K-equation. Or, the other way round, Natorp's may be characterized as a foundherentist account *avant la lettre*.

(iv) McDowell's *Mind and World* (McDowell 1994) is thoroughly informed by spatial metaphors dealing with the topography of the "space of concepts" and the "space of reasons".

I think it would be too simple to dismiss all these approaches simply because they heavily depend on metaphors. The philosophical and linguistic investigations of the last decades have shown that, pace Berkeley, metaphors may well be cognitively meaningful and legitimate in philosophy and even in science (cf. Steinhart 2001). This does not mean that metaphorical assertions are exempt of criticism. Some may be better than others. The metaphors that frame Natorp's epistemology and philosophy of science are no longer ours, and his account of science has many features that appear to be obsolete from a contemporary perspective. Nevertheless, it may still be interesting to take notice of his metaphorical framework not the least as a means to better understand our own metaphorical presuppositions.

Department of Logic and Philosophy of Science
University of the Basque Country UPV/EHU
P.O. Box 1249
200.80 Donostia-San Sebastián, Spain

NOTES

[1] For the following nothing depends on the term “metaphor“. Instead of “metaphor“ one may use terms such as “analogue“, “picture“, or “model“. The only point I want to insist on is that “metaphors“ are more than rhetorical ornaments but play an important cognitive role. For a modern account of the “logic of metaphors“, see Steinhart 2001.

[2] Still, metaphors are assumed to be grounded in informal and common sense experiences. For philosophical purposes, other kinds of metaphors that may be called "theory-constitutive" (Steinhart 2001, p.7) may be more interesting. For a thorough discussion of this kind of metaphors, the reader may consult Steinhart's book.

[3] As a modern analogue of this epistemological debate one may consider the discussion of a viable “middle way“ between coherentism and foundationalism (cf. Haack 1993, McDowell 1994).

[4] For Cohen, the key for understanding the applicability of mathematics to empirical science was the concept of the infinitesimal. He rightly considered standard logic as useless for this endeavour and set about formulating a “transcendental logic“ to achieve this (cf.(Cohen 1968, p. 43ff).

[5] According to Carnap’s own testimony, Natorp was the Neokantian who had had the greatest influence on him.

[6] I think it is still necessary to emphasize that Neokantian epistemology can in no way be characterized as an epigonal rehearsal of Kant's account. Quite the contrary, the various Neokantian schools profoundly modified the very foundations of the Kantian edifice.

[7] How the concept of "limit" is to be understood precisely, will be dealt with later in more detail.

[8] The Marburg school, in particular Natorp, made a lot of this intricate relation between “gegeben“ and “aufgegeben“. For them, it was more than just a pun depending on a contingent linguistic feature of German.

[9] As is shown by the discussions to be found in Sellars and McDowell, the problem of the given is still on the agenda of contemporary philosophy (cf. Sellars 1956, McDowell 1994).

[10] Analogous considerations obtain for the universally quantified assertion $\forall x\ (F(x) = 0)$.

[11] Using the Kantian distinction between receptivity and spontaneity, one may say that the substitutional conception of variables gives spontaneity an important role: according to this approach the possible values of variables are certain symbolic constructs, whose invention takes place in the sphere of spontaneity. If these constructs turn out to be successful they are “reified“, and the “hypothetical“ or “fictitious“ roots of the knowledge equation obtain the status of fully recognized scientific objects.

[12] Natorp's concept of approximation may be said to be based on somewhat old-fashioned idea of “external“ approximation as one may call it: considering the decimal approximation of □2 we may con-

ceive it as a converging series 1, 1.4, 1.41, 1.414 of rational numbers converging to the limiting point √2. Then clearly √2 is not among the elements of this series. Hence, against his intentions, Natorp's model suggests that the object of knowledge remains outside the approximation process. Later, Cassirer took up the analogue of numerical approximation to construe an analogy that fitted much better the basic idea of Neokantian epistemology. Cassirer based his considerations on what may be called "internal approximation". According to this modern concept the converging Cauchy series (a_n) is itself a representant of its limit √2. Using this conception of a limit of a convergent series one obtains a really compelling mathematical example for the basic Neokantian claim that "the road is the end", and this is what Natorp intended.

[13] For instance, the Neokantian Siegfried Marck belonging to the South-West school of Neokantianism, considered Natorp's attempt to avoid the Scylla of Hegelianism as unsuccessful. According to him, the alleged unity of science and philosophy, and the continuity between science, philosophy, and life as propagated by the Marburg School lead to an egalitarian "methodologism" by which the critical character of philosophy was abandoned (cf. Marck 1913, p. 386).

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