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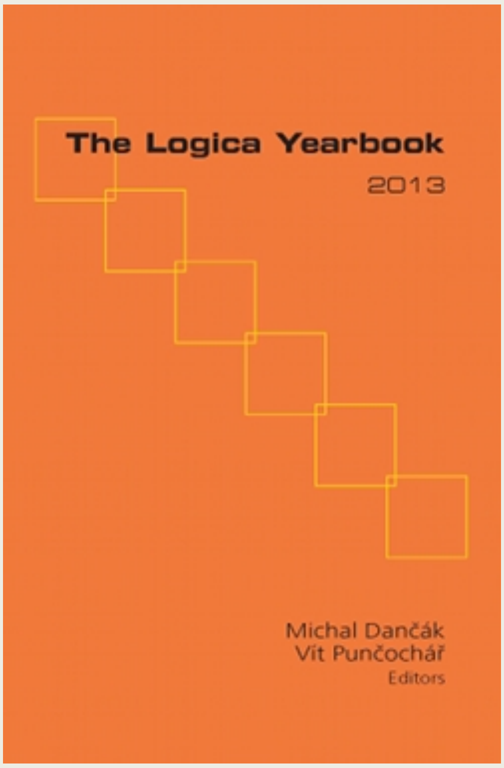

The Logica Yearbook 2013

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Modal Validity and the Dispensability of the Actuality Operator

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Abstract: In this paper, I claim that two ways of defining validity for modal languages ("real-world" and "general" validity), corresponding to distinction between a correct and an incorrect way of defining modal validity, correspond instead to two substantive ways of conceiving modal truth. At the same time, I claim that the major logical manifestation of the real-world/general validity distinction in modal propositional languages with the actuality operator should not be taken seriously, but simply as a by-product of the way in which the semantics of such an operator is usually given.

Keywords: real-world validity, general validity, actuality operator, propositional modal logics

1 Introduction

Take two modal logicians and call them "Saul" and "Max". There is some chance that what Saul and Max mean by "validity" of a formula of a modal language (with respect to a certain class of interpretations) is different. What Saul might mean is that a formula ϕ of a modal language L is valid, with respect to a certain class of interpretations, iff ϕ is true in *every actual world* of every interpretation of L; what Max might mean instead is that ϕ is valid in L iff ϕ is true in *every world* of every interpretation of L. According to Max, validity for a modal language is some sort of "super-necessity"; it is what remains necessary under every interpretation. According to Saul, validity is some sort of "super-actuality"; it is what remains (actually) true under every interpretation. For Saul, modal validity is "permutated" actuality, for Max, validity is "permutated" necessity.[1]

It could be claimed that the disagreement between Saul and Max might be resolved by invoking a pluralistic approach to logic. A logical pluralist

[1]The names "Saul" and "Max" have not been chosen arbitrarily. Saul is inspired by Saul Kripke, who introduced the first way of defining modal validity, for example in (Kripke, 1963a); Max is inspired by Max Cresswell, who usually defines modal validity in the second way (Hughes & Cresswell, 1996). In another paper of mine (Morato, 2014) I have called "Kripke validity" the former notion and "Textbook validity" the latter.

would react basically by denying the reality of this disagreement and by conceding that the difference between (what has been called) "real-world validity" and "general validity" is not, after all, to be taken seriously. The entire issue could be resolved by an appropriate disambiguation of the term "validity".[2]

My aim in this paper is twofold. On the one hand, I will show that the difference between the two notions is to be taken seriously, because it corresponds to a real, substantive distinction between two general conceptions of modality. On the other hand, I will show that the difference should not be motivated, as it is usually done, by appealing to modal propositional languages enriched with the actuality operator.

This might sound bizarre: what I will do, in effect, is to defend the plausibility and robustness of a (meta)logical distinction and to dismiss, at the same time, one of its major logical manifestations. The moral of this situation will be discussed in the final section of the paper.

2 On the genealogy of modal validity

In this section, I will show how the distinction between real-world validity (truth in every actual world of every interpretation) and general validity (truth in every world of every interpretation), far from being a distinction between a correct and an incorrect definition of validity for modal languages, could be grounded on the difference between two general conceptions of modality.[3]

In order for these two general conceptions to be made explicit, I will avail myself of a certain view on how modal logic emerges from its non-modal basis.[4]

Before doing this, however, some preliminaries are necessary. An interpretation of modal propositional logic L is a quadruple, $\langle W, R, @, V \rangle$,

[2] The kind of logical pluralism I have in mind is the one defended for example in (Beall & Restall, 2006). The "real-world"/"general" validity terminology comes from (Davies & Humberstone, 1980). Even though I will use such terminology for the rest of the paper, I am not completely happy with it, because I think it is more suited for discussions about validity in two-dimensional modal logics. (Morato, 2014) is more directly related to this topic.

[3] (Nelson & Zalta, 2012; Zalta, 1988) defend the view that general validity is not a "correct" definition of validity, because it is based on a conflation of a semantic notion (truth in an interpretation) with a metaphysical one (necessity) and because it is not in full compliance with the Tarskian approach to logical truth. I rebut both claims in (Morato, 2014).

[4] This view is inspired by and is very similar to the one presented in (Menzel, 1990).

where W is an arbitrary set of objects, $@$ is an element of W, R is a total, binary relation on W and V is a valuation function that assigns to each propositional atom, with respect to an element of W, precisely one element of the set $\{0, 1\}$. In the intended interpretation of L, the set W is a set of worlds, $@$ is the actual world, V is a function that tells us which atomic sentences are true in what world and R is a relation of accessibility among worlds. The basic metalogical notion of "truth of a formula ϕ in an interpretation I of L with respect to a world w" is defined recursively in the usual way. Other metalogical notions are "truth in an interpretation I" and "validity" (truth in all interpretations). If one defines a frame F as the pair formed by $\langle W, R\rangle$, one could define further metalogical notions such as "truth in w in a frame F", "validity in a frame F" and "validity in a class of frames".[5] The difference between real-world and general validity emerges in case one decides to use $@$ in the definition of the notion of truth in an interpretation I: those who decide to define such a notion as truth in the $@$ of I (treating truth in an interpretation as actual truth) will have, as a result, that the notion of validity (truth in all interpretations) is truth in all actual worlds of all interpretations. Those who decide to define the notion of truth in an interpretation as truth in every world of an interpretation (treating truth in an interpretation as necessary truth), will have, as a result, that the notion of validity is truth in every world of every interpretation. Those belonging to this latter group usually do not include $@$ in the definition of an interpretation.[6]

Now, all of this could be fruitfully recarved in the following way. Modal propositional logic could be seen as a *generalization* of non-modal propositional logic. While non-modal propositional truth is defined over a single interpretation—where an interpretation of propositional logic is $\langle V\rangle$ and V is an assignment of truth-values to propositional atoms—modal propositional truth is defined over a *cluster* of non-modal propositional interpretations. As far as formal semantics is concerned thus, the role of the elements in W might simply be viewed as that of *indexing* a cluster of non-modal propositional interpretations. Modal operators could then be seen as operators that quantify, in the metalanguage, over indexed non-modal propositional interpretations. A formula such as $\Box\phi$ is thus true if ϕ is true in the relevant set of indexed propositional non-modal interpretations, $\Diamond\phi$ is true

[5] See (Beall & Restall, 2006; Chellas, 1980).

[6] The specification of $@$ in a modal interpretation might be avoided if one defines validity as truth in every possible world in every interpretation, but it is essential if one wants to introduce an actuality operator defined by a clause like (2) (see page 134).

if ϕ is true in at least one relevant indexed propositional non-modal interpretation.

Given this picture, modal truth might be viewed as emerging from non-modal truth depending on how the set of indexed propositional interpretations is generated. There are two ways in which this could happen and these two ways correspond, in my view, to two ways in which the space of possibilities can be conceived.

According to the first method, in order to generate such a cluster of non-modal interpretations, you first take a specific non-modal propositional interpretation, that we could indicate with '$I^@$', and associate to $I^@$ a cluster of *alternative* non-modal propositional interpretations. These alternative non-modal interpretations could be called the "variants" of $I^@$ and are to be indexed by the members of the set W. Under this picture, a modal propositional interpretation is obtained by the association of $I^@$ with its variants. The original $I^@$ will play the role of the actual world and the variants of $I^@$ will play the role of possible worlds. What is true according to $I^@$ is true *simpliciter*, what is true according to a variant of $I^@$ is possibly true.

As far as metalogical notions are concerned, the natural choice to do, in this case, is to characterize the notion of truth in a modal interpretation (truth in I of L) as truth in the $I^@$ of the interpretation or, in other terms, as truth in the actual world of the interpretation. The notion of validity will be thus characterized as truth in every $I^@$ of every modal propositional interpretation. Given that $I^@$ is simply a non-modal propositional interpretation, the definition of modal validity and of non-modal validity will quantify over the same set of interpretations.

The second method is more straightforward. According to this method, a modal propositional interpretation is simply formed by a certain number of indexed non-modal propositional interpretations; there is no "privileged" interpretation and every non-modal propositional interpretation is, semantically, on a par.

Under this picture, the natural choice to do, as far as modal metalogical notions are concerned, is to *downgrade* non-modal meta-logical notions. Given that modal propositional logic is a generalization of non-modal propositional logic, a non-modal metalogical notion of level n will be a modal metalogical notion of level $n-1$. Non-modal truth in an interpretation will now be modal truth in an interpretation with respect to an indexed interpretation: non-modal propositional truth in I will be downgraded to the basic modal notion of "truth in w in I". Non-modal validity, namely truth in every non-modal propositional interpretation will be downgraded to modal truth in a single modal propositional interpretation: modal propositional validity is truth in every indexed non-modal propositional interpretation, namely truth in every possible world.

These two methods correspond to two different views on the nature of modal space. The first method corresponds to a view according to which the space of possibilities is seen *sub specie actualitatis*, while the second method corresponds to a view according to which the space of possibilities is seen *sub specie possibilitatis*. I will now try to clarify these two general conceptions.

According to the first conception, actuality has a privileged status over possibilities. In particular, what possibilities there are depends on what is actual. The logical space of possibilities is determined by the ways the (actual) world might have been. This idea is represented, in the first method, by the fact that possible worlds are treated as indexed *variants* of $I^@$. A consequence of an actuality-constrained conception of possibility is that possible truth does not "interfere" with plain truth: this is equally represented in the first method by the fact that what is true in a modal interpretation is what is true at the $I^@$ of the interpretation and by the fact that, in order to determine what is valid, only what is true in the $I^@$ of every interpretation is relevant. Only what is actually true in every interpretation contributes to validity.

According to the second conception, actuality has not a privileged status over possibilities. In this case, the logical space of possibilities is not constrained by what is actual, in the sense that there could be some "free-floating" (or "alien") possibilities, i.e., possibilities that are not to be conceived as simply false descriptions of the actual world. A consequence of this non-constrained conception of possibility is that possible truth and actual truth are on a par: this idea is represented in the second method by the fact that what is true in a modal interpretation is what remains true in every indexed non-modal interpretation and by the fact that, in order to determine what is valid, every truth in every indexed non-modal interpretation is relevant; it is what is necessarily true in every interpretation that contributes to validity.

As far as simple propositional modal languages are concerned (modal propositional languages with only $\Box$ and $\Diamond$), the two conceptions are extensionally equivalent: for simple modal propositional languages, one who conceives modal space *sub specie actualitatis* and one who conceives modal space *sub specie possibilitatis* will take exactly the same formulas as valid. In terms of the cluster conception developed above, this is easily seen by considering that every non-modal propositional interpretation will happen to be the privileged $I^{@}$ of a modal propositional interpretation; if this is so, what is true in every $I^{@}$ of every interpretation *just is* what is true in every indexed interpretation.

Real-world validity and general validity are thus extensionally equivalent for simple modal languages. The recarving strategy and the two conceptions of modality I have presented in this section might be taken as ways in which one can see how these two notions, while extensionally equivalent, are intensionally different.

3 The actuality operator enters the scene

In modal languages enriched with the actuality operator A, there seems to be formulas that are valid, if the notion of real-world validity is used and invalid, if the notion of general validity is used instead. In such languages, the intensional difference between the two notions seems, in effect, to make an extensional difference.

The actuality operator has been introduced in modal languages in order to satisfy the expressive need of breaking the scope of modal operators. This need is especially pressing in the first-order case, where we need to express the formal counterparts of English sentences such as:

It could happen that all those that are actually rich should be poor

where the quantifier binding those actually rich should be interpreted outside the scope of the initial modal operator ‘it could happen that’. The translation of this sentence, in a first-order modal language, is the following:

$$\Diamond\forall x(\mathsf{A}Rx \rightarrow Px) \tag{1}$$

The interpretation of such a formula depends, of course, on the semantics clause for A and on the interactions between quantification and modality. The semantics clause for A is usually given by the following clause:

$$\mathsf{A}\phi \text{ is true in } L^{\mathsf{A}} \text{ iff } V(\phi, @) = 1 \tag{2}$$

where L^{A} is a standard modal propositional language enriched with the actuality operator. The main effect of (2) is that of making A a tool to obtain *truth-value rigidity*: when applied to a formula ϕ, A sticks to ϕ the truth-value that ϕ has in the actual world and $\mathsf{A}\phi$ designates the truth-value that ϕ has in the actual world in every possible world.

It is rarely noticed that, even armed with (2), the intended reading of formula (1) is not easily captured in first-order modal logic: in particular, in a system with variable domains and actualist quantification[7], for a formula such as (1) to come out true, it is enough that there exists an interpretation I, where there is at least one world w such that no individual in w exists also in the @ of I. The intended reading of formula (1) thus can only be captured in a system with fixed or increasing domains.

If we define A by means of (2), the following formula:

$$\phi \leftrightarrow \mathsf{A}\phi \tag{3}$$

will be real-world valid, but not generally valid.

To prove that a formula is real-world valid but not generally valid, it is enough to prove that the formula is real-world valid, but not necessary. To prove that (3) is real-world valid, assume first that ϕ is true in the @ of an arbitrary interpretation I of a modal propositional language L^{A}. Given (2) (right-to-left), even $\mathsf{A}\phi$ will be true in @ and thus $\phi \rightarrow \mathsf{A}\phi$ is real-world valid. If $\mathsf{A}\phi$ is true in the @ of an arbitrary interpretation I, then again by (2) (left-to-right), ϕ will be true in @ and thus $\mathsf{A}\phi \rightarrow \phi$ is real-world valid. To prove that (3) is not necessary (and then not generally valid) it is enough to show that $\phi \rightarrow \mathsf{A}\phi$ is not necessary. Consider the following interpretation:

- $w = \{w_1, w_2, w_3\}$
- $@ = w_1$
- $V(w_1, \phi) = 0$, $V(w_2, \phi) = 1$, $V(w_3, \phi) = 1$

In w_2, the antecedent is true, but the consequent is false, and so there is at least a world where the conditional is false. Not being necessary, the formula cannot be generally valid. (3) is thus a case of a formula that is real-world valid without being generally valid.

The fact that (3) is real-world valid without being necessary implies that the rule of necessitation (RN) fails for non-simple modal languages with real-world validity. The difference between real-world validity and general validity could be thus reframed as a difference with respect to RN: general and real-world validity differ, because RN preserves the former, but it does not preserve the latter.

[7] The famous system of (Kripke, 1963b), presented also in Chapter 15 of (Hughes & Cresswell, 1996).

It might be interesting to understand what types of formulas are real-world valid without being generally valid. Given the equivalence of real-world and general validity for simple modal languages, we know that such formulas will be formulas with at least one occurrence of A. Call a formula with at least one occurrence of A an A-formula. The problem thus is to determine what kind of A-formulas are real-world valid without being necessary (and thus, *mutatis mutandis*, generally valid).

Given (2), a real-world valid A-formula governed by the actuality operator (a formula of the form $\mathsf{A}\phi$) is also generally valid. If $\mathsf{A}\phi$ is real-world valid, it is true in the actual world of every interpretation; consider such an interpretation I; in I, $\mathsf{A}\phi$ is true in the @ of I. By (2), it follows that ϕ is true in the @ of I. But if ϕ is true in the @ of I, in every world w of I it will be true that $\mathsf{A}\phi$; $\mathsf{A}\phi$ will be necessary and thus, *mutatis mutandis*, generally valid.

What about formulas governed by $\Diamond$ and $\Box$? Here the question is whether real-world valid A-formulas of the form $\Box\phi$ and $\Diamond\phi$ will also be necessary and thus generally valid.

Consider first a formula such as $\Box\phi$. If $\Box\phi$ is real-world valid, it means that $\Box\phi$ is true in any actual world of every interpretation I. In every world of I, $\mathsf{A}\Box\phi$ will be therefore true (in every world, it will be true that in the actual world $\Box\phi$ is true). But then $\mathsf{A}\Box\phi$ will be necessary and therefore generally valid. Given the necessary equivalence between $\mathsf{A}\Box\phi$ and $\Box\phi$, if $\mathsf{A}\Box\phi$ is generally valid, then $\Box\phi$ will also be generally valid.[8]

Consider now a formula such as $\Diamond\phi$. If $\Diamond\phi$ is a real-world valid A-formula, then $\Diamond\phi$ will be true at every actual world of every interpretation. From this, it follows that $\mathsf{A}\Diamond\phi$ will be real-world valid, necessary and thus generally valid. But $\Box\mathsf{A}\Diamond\phi$ is necessarily equivalent to $\Box\mathsf{A}\neg\Box\neg\phi$ which, due to the commutativity of $\neg$ and A, is necessarily equivalent to $\Box\neg\mathsf{A}\Box\neg\phi$, which is necessarily equivalent to $\Box\neg\Box\neg\phi$, which is none else than $\Box\Diamond\phi$.

From this, it follows that an A-formula that is real-world valid without being generally valid can only be a non-modal A-formula.

Assume that we only have $\neg$ and $\vee$ as primitive non-modal operators. We can immediately exclude formulas of the form $\neg\mathsf{A}\phi$, because such a formula is necessarily equivalent to $\mathsf{A}\neg\phi$ and we know already that formulas beginning with A are generally valid, if real-world valid.

We can thus conclude that real-world but not generally valid A-formulas can only be of the form $\mathsf{A}\phi \vee \psi$, $\neg\mathsf{A}\phi \vee \psi$ or $\mathsf{A}\phi \vee \neg\psi$. Where $\psi = \phi$, the first is logically false, while the other are none else than (3). What we can

[8] On the equivalence between $\mathsf{A}\Box\phi$ and $\Box\phi$ see (Williamson, 1998).

conclude, therefore, is that a formula such as (3) is the *only* case of a real-world, but not generally valid formula for a propositional language such as L^{A}.

4 On the dispensability of the actuality operator

The actuality operator was introduced in modal languages to satisfy an expressive problem, that of operating (in the propositional case) or quantifying (in the predicative case) outside the scope of modal operators. As we have seen, the problem was solved by introducing a full-fledged, scope-bearing operator, on a par with $\Box$ and $\Diamond$, whose semantics has been defined by the clause (2). The role of A is that of allowing the formula in its scope to be evaluated with respect to the actual world of the interpretation, even in those cases where the clause governed by A is in the scope of $\Box$ and $\Diamond$. The typical behaviour of A is captured by a formula such as:

$$\Diamond(\mathsf{A}\phi \wedge \psi) \tag{4}$$

where the role of A is that "protecting" ϕ from being evaluated with respect to the world selected by $\Diamond$. But a formula such as (4) could be taken as a circumvoluted way of saying that ϕ and possibly ψ are the case. What (4) expresses thus could be equally captured by a formula such as:

$$\phi \wedge \Diamond\psi \tag{5}$$

where no occurrence of A appears. The role of A in (4) is just that signalling where the scope of $\Diamond$ needs to be broken. In this sense, A seems to be more a metalinguistic indicator on how to interpret a formula than a full-fledged component of the formula.

If the contribution of A to a modal language is that of being a device for breaking the scope of modal operators, then one would expect that the role of A be completely superfluous in those formulas where there are no modal operators. The occurrence of A in (3) should thus be taken as superfluous. We have seen, however, that the (extensional) distinction between real-world and general validity is entirely based on the existence of this formula.

Admittedly, the situation is a little bit ironic. By means of languages enriched with A (semantically defined by (2)), we are able to mark the distinction between real-world and general validity; the way in which A helps to mark the distinction, however, is by means of a formula where the role of A seems to be superfluous.

The prime suspect of all this is (2). A natural reaction would be that of taking a formula such as (3) simply as an undesired by-product of an implausible way of giving the semantics of A.[9]

The reason why someone could find implausible a clause like (2) might be explained by means of the distinction, made in the Section 2. A semantic clause such as (2) makes sense only if one has a *sub specie actualitatis* conception of modal truth. Only if one believes that, when we go up in the space of possibilities, plain truth is actual truth (truth in $@$ of an I) might find plausible the idea that there is an operator by means of which one could speak of the actual world, even from other possible worlds. On the contrary, if one believes that, when we go up in the modal space, plain truth is somewhat "diluted" into possible truth (truth in a w of an I) and that all possible truths are semantically on a par—in the sense that all contribute equally to what is true in an interpretation—then what is eventually plausible is the idea of having an operator by means of which one could speak of *a* world (the one that is contextually relevant), even from another possible world.

In the case of iterated modalities, for example, in the case of a formula such as:

$$\Diamond(\phi \rightarrow \Diamond\mathsf{A}\psi)$$

the "actualist" A will interpret ψ with respect to w^*, while the "possibilist" A will interpret ψ with respect to the world selected by $\Diamond$. Note that the behaviour of a "possibilist" A is perfectly consonant with the expressive need that A was meant to satisfy (i.e., breaking the scope of modal operators). Though able to satisfy the basic need, the "actualist" A does so at the cost of what could be characterized as "semantic overreaction".

If propositional formulas containing the actuality operator are simply lazy ways of expressing contents that would otherwise be expressible without it, then one should be able to prove that the actuality operator is dispensable. In effect, some dispensability results are available, even if their philosophical significance and applications only very recently have been appreciated.[10] My aim in these final pages is to see whether these dispensability results can be of any use in the real-world/general validity issue.

[9] Those who do not accept (2) typically do not accept (3) as a theorem. See, for example, (Gregory, 2001).

[10] The first result of this kind is given in a proof-theoretic manner by Hazen (1978) (even if some traces were already present in (Crossley & Humberstone, 1977)) For another proof-theoretic result see (Stephanou, 2001). A model-theoretic version of the first result is given in

The dispensability results given in (Hazen et al., 2013) proves that every formula of a propositional language L^{A} is real-world equivalent to a formula without any occurrence of the actuality operator. Two formulas ϕ and χ are real-world equivalent iff $\phi \leftrightarrow \chi$ is real-world valid. This result is proved by proving first that every formula of L^{A} is strictly equivalent to a combination of so-called A-atoms, where ϕ is strictly equivalent to χ iff $\phi \leftrightarrow \chi$ is generally valid and A-atoms are formulas such as p, $\mathsf{A}p$, $\Box\phi$ (where ϕ is free of A) and $\mathsf{A}\Box\phi$ (where ϕ is free of A). The nice feature of A-atoms is that they are real-world equivalent to formulas without A, such as p or to $\Box\phi$.

As far as the real-world/general validity issue is concerned, what interests us here is just to see onto what A-free formula (3) gets mapped to and whether such a formula is also generally valid. If the real-world valid formula to which (3) is associated is also generally valid, then we can conclude that the real-world/general validity distinction cannot be based on the existence of a formula such as (3).

A formula such as (3) is a conjunction of the form:

$$(\neg\mathsf{A}\phi \vee \phi) \wedge (\neg\phi \vee \mathsf{A}\phi) \tag{6}$$

Given that we know that A commutes with negation, we know that (6) is (generally and thus real-world) equivalent to:

$$(\mathsf{A}\neg\phi \vee \phi) \wedge (\neg\phi \vee \mathsf{A}\phi) \tag{7}$$

We know that, by (2), $\mathsf{A}\neg\phi$ is real-world equivalent to $\neg\phi$ and that $\mathsf{A}\phi$ is real-world equivalent to ϕ. A formula such as (7) is therefore real-world equivalent to a formula such as:

$$(\neg\phi \vee \phi) \wedge (\neg\phi \vee \phi) \tag{8}$$

But a formula such as (8) is also generally valid. (3) is thus real-world equivalent to a generally valid formula. The occurrence of A in (3) could be dispensed with in favour of a formula like (8). Given that (8) is also generally valid, the distinction between real-world and general validity, as based on (3), seems therefore to be laying on quite a feeble ground.

(Hazen, Rin, & Wehmeier, 2013). A philosophical application of this dispensability result (and a tentative extension to the predicative case) is given by Meyer (2013).

5 Conclusion

In this paper, I have defended the claim that the distinction between real-world and general validity corresponds to a genuine distinction between two conceptions of modal truth, but that the manifestation of this distinction in a modal propositional language like L^{A} should not to be taken seriously. The reason is that the difference appears, in modal propositional languages, only for formulas with the actuality operator A and the actuality operator is dispensable from such languages.

As shown by Hazen (1976), the dispensability results for A in the propositional case contrast with its apparent non-eliminability in predicate modal logic. This result, however, could be viewed more as a proof of the expressive limitations of first-order modal languages than as a proof of the essentiality of A for such languages.[11]

Should then the distinction between the two notions of modal validity be approached with a pluralistic attitude? Should the difference be conceived as the result of an equivocation of the word 'valid' in the metalogical vocabulary?[12] My claim that a formula like (3) should not be taken seriously as far as the distinction is concerned, could be taken as a way of endorsing this form of logical pluralism, but it is not so. As I have claimed, the distinction seems to be grounded on two substantive views about the nature of modal truth. Choosing one notion of validity or the other, I surmise, is to choose between two substantive conceptions of modality. The real-world/general validity issue could thus be seen as another case where, as T. Williamson writes, "the contentiousness of logic is radical enough to reach the meta-logic". (Williamson, 2013, p. 20)

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[11] There is no space to pursue the matter here. Notice, however, that, where some of the expressive limitations of first-order languages are overcome, the actuality operator A seems to be dispensable; see (Meyer, 2013).

[12] Meta-logical pluralism seems in effect to be the most recent and appealing form of logical pluralism today; see (Beall & Restall, 2006).

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