

On the Mereological Structure of Complex States of Affairs

Abstract. The aim of this paper is to elucidate the mereological structure of complex states of affairs without relying on the problematic notion of structural universals. For this task tools from graph theory, lattice theory, and the theory of relational systems are employed. Our starting point is the mereology of similarity structures. Since similarity structures are structured sets, their mereology can be considered as a generalization of the mereology of ordinary sets. In general, the mereological systems arising from similarity structures turn out to be not Boolean but Heyting systems. Employing Armstrong's notion of thick particulars, similarity structures are shown to capture the mereological structure of complex "chemical states of affairs" such as "being butane" or "being isobutane", structural universals are not needed.

Key Words: Structural Mereology, Complex States of Affairs, Similarity Structures, Graphs, Lattices, Boolean Algebras, Heyting Algebras.

1. Introduction. Let us subscribe to a factual ontology according to which the world is a world of states of affairs (cf. Armstrong 1997). A factual ontology cannot be content to baldly assert that the world is a heap of opaque states of affairs, rather, it should tell us something about the structure of states of affairs and how they are related. Even if the partisans of a factual ontology agree on the issue what are atomic states of affairs, there is less than full agreement on what is to be understood by complex states of affairs and their structure. The main reason for this disagreement is the problematic notion of structural universals allegedly involved in the constitution of complex states of affairs. On the one hand, David Lewis put forward some strong arguments against structural universals (cf. Lewis 1986), contending that the alleged structural universals violate the principle of uniqueness of composition, or that they seem to require the repetition of simpler universals. On the other hand, many philosophers asserted, "any theory of universals worth its salt must be able to offer an account of structural universals" (Bigelow and Pargetter 1989, 219). This requirement may be difficult to satisfy. According to Bigelow and Pargetter, Lewis conclusively showed that mereology does not help much for the development of such

a theory, since the “is-a-part of” relation is too weak to cope with the intricate pattern of entailments that structures such as methane, butane, and similar complex states of affairs exhibit:

Given the intrinsic nature of methane, there is a complex pattern of entailments that requires explanation. If a molecule instantiates methane it necessarily follows that there are parts of the molecule which instantiate the other universals hydrogen, carbon, bonded; also that there are four times the number of instantiations of hydrogen than carbon; also that the bonded relation is instantiated four times by carbon-hydrogen pairs, never by hydrogen-hydrogen pairs, and that no hydrogen atom is bonded more than once; and so on. With such rich pattern of entailments to explain ... the chance of explaining them all using no more than the “is part of” relation is slim (Bigelow and Pargetter 1989, 220).

A result of this paper is that Bigelow and Pargetter’s verdict on the chances of mereology is untenable. A neat mereological analysis of complex states of affairs will be given that does not depend on the problematic notion of structural universals. The proposed account to be developed in this paper clearly distinguishes between two levels:

- (1) The mereological level dealing with parts of complex states of affairs.
- (2) The constitutional level dealing with particulars and basic monadic or relational universals as the constituents of atomic states of affairs.

According to (1) a complex mereological state is described as a mereological whole that has less complex states of affairs as its parts. The level (2) deals with the internal, non-mereological structure of the atomic parts of complex states of affairs, i.e., with the individuals and universals that constitute the atomic states of affairs that are parts of complex states of affairs. With respect to the constitutional level (2), we closely follow Armstrong’s account, i.e. the basic building blocks of states of affairs will be individuals and (monadic and relational universals (cf. Armstrong 1997).

For the convenience of the reader let us briefly recall the basics of Armstrong’s approach. A factual ontology assumes that the world is a world of states of affairs. States of affairs are constituted by particulars a, b, c, d, \dots and monadic or relational universals F, G, B, H, C, \dots to yield states of affairs such as Ha, Gb, cBd, \dots to be interpreted as the states that a is H ,

b is G, c and d are relata of the relation B, and so on. Universals may be understood as state-of-affairs types (Armstrong 1997, 127), or “unsaturated” entities requiring to be completed by particulars in order to yield actual states of affairs. Among particulars one may distinguish between thin and thick particulars. A thin particular is a particular in abstraction from all its properties and relations, the particular *qua* particular only (cf. Armstrong 1989, 52). Thin particulars may be conceived as limiting cases that are obtained from (more or less) thick particulars such as Fa or (F&G)b after having abstracted away all their monadic properties. In our daily practice, we seldom deal with “really thick” particulars that take into account all non-relational aspects, nor are we used to be confronted with thin particulars; the particulars we usually meet are of an intermediate thickness such as Fa, (F&G)a, (F&G&H)b (cf. Armstrong 1989, 52f). In order to simplify matters as far as possible, in the following we distinguish only between two classes: thin particulars a, b, c, ... on the one hand, and thick particulars Fa, Gb, (F&G)c, on the other hand, thereby ignoring different degrees of thickness.

The special mark of universals is repeatability, i.e. one and the same universal, say H (hydrogen), can be instantiated in several states of affairs Ha, Hb, Obviously, states of affairs such as Fa, Gb lack repeatability. Hence they are to be considered as particulars. More precisely, they are thick particulars. Armstrong called this the “victory of particularity over universality” (ibidem, chapter 8). This victory has an important consequence. Conceiving universals as states-of-affairs types that yield states of affairs by being saturated through appropriate particulars it is possible that universals are completed by appropriate thick particulars. For instance, assume that a, b, and c are thin particulars, H and O are the monadic universals “hydrogen” and “oxygen”, respectively, and — denotes a binary universal “bonded”. Then we may consider the complex state of affairs Ha—Ob—Hc that obtains iff the thick particulars Ha, Ob, and Hc are bonded forming together a molecule of water (H₂O).

Armstrong and virtually all other authors who dealt with relational states of affairs that involve some kind of bonding assume without further argument that the bonding universal has to be completed by thin particulars only. There is no reason, however, to make this assumption. It makes perfectly good sense to assume that there are relational universals such as bonding that are defined for thick particulars, such that the states of affairs takes the form, say, Fa—Gb as is the case for oxygen molecules Oa—Ob. Restricting the domains of relational universals to thin particulars only, would render it impossible to combine thick particulars to complex states of affairs. This seems highly implausible. From a scientific point of view, chemical bonding obtains, or does not obtain, for a rather limited class of

particulars, namely, for atoms or molecules of some kind, i.e. for certain thick particulars such as Ha, Ob, Cc etc. This is reflected already in the familiar structure formulas of chemistry such as H₂O, SO₂, which should be interpreted as a kind of shorthand for describing complex states of affairs such as Ha—Ob—Hc and Od—Se—Of. A crucial assumption of this paper is to conceive the relational universal bonding — as a universal that is completed by thick particulars.

Replacing relational states of type x—y by states of type Fx—Gy is not just a minor formal variant of the ontological analysis of complex states of affairs. Rather it turns out to be the first step of a revision that leads to a much better mereological analysis of complex states than that of Armstrong: As will be shown, Armstrong’s mereological analysis of complex states such as “methane” or “isobutane” amounts to nothing more than counting the number (n) of atoms and the number (m) of bondings that can be found in such a molecule. Consequently, for Armstrong the resulting mereological structure of the molecule is just the Boolean algebra $P(\{1, \dots, n+m\})$, i.e. the power set of the $\{1, 2, \dots, n+m\}$. This Boolean mereology is a far cry from the specific characterization of these molecules encapsulated in their chemical structure formulas. Actually, a much better and more precise mereological analysis of complex states of affairs can be formulated in the framework of in Armstrong’s factual ontology provided we replace Boolean mereological systems by Heyting systems. This improved analysis can distinguish between isomere states of affairs as presented by the notorious case of butane and isobutane.

The shortcomings of Armstrong’s ontological analysis of complex states are avoided by employing the conceptual apparatus of mereology in a novel way. Mereology comprises more than just the traditional Boolean mereology of sets. Mereology as a general theory of parthood is more general than the elementary part of set theory as Lewis contended. In our analysis of complex states of affairs there is no need for structural universals. They turn out to be a useful *façon de parler*, but they carry no ontological weight. The mereological theory of complex states of affairs to be developed in the following offers an eliminative account of structural universals.

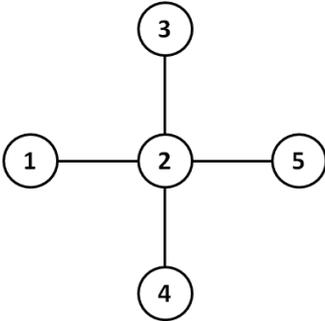
The outline of this paper is as follows: In section 2 the formal foundations for the investigations of the entire paper are laid down. In particular, a detailed analysis of the structural mereology of similarity structures is set up.¹ This provides the basis for the study of the mereology of complex states carried out in section 3. There for every complex state of affairs C a mereological system PART(C) of structural parts of C is defined. The

¹ For more general, somewhat different accounts of structural mereology see (Mormann 2009, 2010).

mereological systems PART(C) allows us to distinguish between isomeric states of affairs such as “being butane” and “being isobutane”. In section 4 our approach is compared with some recent attempts to come to terms with problems related to structural universals. First, we will deal with Pagés’s *Structural Universals and Formal Relations* (Pagés 2002), then with Kalhat’s *Structural Universals and the Principle of Uniqueness of Composition* (Kalhat 2008). In section 5 we close with some remarks on how the account of this paper can be generalized to deal with wider classes of complex states of affairs.

2. Mereology of Similarity Structures. A similarity structure (S, \sim) is defined as a set S endowed with a reflexive and symmetric binary relation $\sim \subseteq S \times S$. Two elements are said to be similar iff $(a, b) \in \sim$. As usual this is denoted by $a \sim b$. Similarity structures may be conceived as simple graphs: the vertices of the graph are the elements of S and two different vertices are similar iff they are the vertices of the same edge. For instance, if $S = \{1, 2, 3, 4, 5\}$ the graph (2.1) represents the similarity structure (S, \sim) according to which

(2.1)



each of the elements 1, 2, 3, 4, 5 is similar to itself, 1 and 2 are similar, 2 and 3 are similar, 2 and 5 are similar, 2 and 4 are similar, and no other similarities between the elements of S occur. In other words, the graph structure is to be interpreted in such a way that it gives complete information about all matters of similarity concerning its elements. If we deal with several similarity structures and have to distinguish their similarity relations, they are denoted by (S, \sim_S) and (T, \sim_T) . More formally, a similarity structure (S, \sim_S) may be characterized as a structured set or a relational systems in the sense of the mathematical theory of relational systems. The similarity relation of (S, \sim_S) may be identified with the subset $\sim_S := \{(x, y); x \sim y\} \subseteq S \times S$.

This reinterpretation of similarity structures is useful for elucidating their mereological structure. Let S be any non-empty set. According to Lewis (Lewis 1991, 1.2, 3ff) the non-

empty subsets T of a set S may be considered as parts of S and the set-theoretical inclusion \subseteq as the mereological parthood relation. In mathematics, the set of subsets of a set S is called the powerset of S and denoted by $P(S)$. As is well known $P(S)$ is a (complete) Boolean algebra with bottom element \emptyset the empty set. Hence, the mereological system of nonempty subsets of S is a Boolean algebra from which the bottom element 0 has been removed.

Lewis, as most other mereologists, excludes the empty set \emptyset from the realm of the mereology of sets. In this paper we do not follow this convention. For technical convenience it is assumed that a classical mereological system is a Boolean algebra in the ordinary mathematical sense, i.e. possessing a bottom element 0 corresponding to \emptyset . More generally, all mereological systems considered in this paper are assumed to have a bottom element 0 . This convention simplifies definitions and calculations but does not carry any philosophical weight, i.e., everything would go through without it.

From now on $\text{PART}(S)$ denotes the set of subsets of S . The relational system $(\text{PART}(S), \subseteq)$ is called a classical or Boolean mereological system (cf. Simons (1987)). Having a mereology for sets, one may ask whether there is also a mereology for structured sets, in particular for structured sets such as similarity structures (S, \sim) . Lewis did not treat this problem, and, as far as I know, philosophers never dealt explicitly with this question. There exists, however, an extended literature on the topic of parts of structured sets and relational systems in mathematics, see Lawvere and Schanuel (1996) or Lawvere and Rosebrugh (2003). In this paper we will restrict our attention to the special case of similarity structures (S, \sim) .

A natural starting point for a structural mereology is the question, what are the parts of a structured set such as a similarity structure (S, \sim) ? A trivial answer is simply to forget about the structure of S and to take the parts of the set S as the parts of the structured set (S, \sim) . A more interesting proposal is to attempt somehow to take into account the extra structure of S for the definition of the structural parts of (S, \sim) . In other words, the similarity relation \sim should play a role in the definition of the structural parts of (S, \sim) . The following definition of structural parts is a natural way to achieve this aim:

(2.2) Definition. A structural part of (S, \sim_S) is a similarity structure (T, \sim_T) such that $T \subseteq S$ and $\sim_T \subseteq \sim_S$. The set of structural parts of (S, \sim_S) is denoted by $\text{PART}(S, \sim)$. Then a partial order on $\text{PART}(S, \sim)$ is defined by $(U, \sim_U) \leq (V, \sim_V) := U \subseteq V$ and $\sim_U \subseteq \sim_V$ ($U, V \subseteq S$). (U, \sim_U) is said to be a part of (V, \sim_V) iff $(U, \sim_U) \leq (V, \sim_V)$. The set of structural parts of (S, \sim)

endowed with this partial order is also denoted by $\text{PART}(S, \sim)$. The top element of this order is (S, \sim_S) and the bottom element is the empty similarity structure $(\emptyset, \sim_\emptyset)$.♦

The definition (2.2) is a natural generalization of Lewis's definition of parts of sets (cf. Lewis 1991, 3ff). Instead of arbitrary sets S we consider appropriate ordered pairs (S, \sim_S) of sets such that (T, \sim_T) is a part of (S, \sim_S) iff T is part of S and \sim_T is part of \sim_S in the ordinary sense. Indeed, the ordinary mereology of sets may be included in our generalized approach as follows: For a set S define its diagonal $D(S)$ as $D(S) := \{(x,x); x \in S\}$. Then $(S, D(S))$ is a similarity structure and $(T, D(T)) \leq (S, D(S))$ holds if and only if $T \subseteq S$, i.e., T is a part of S in the ordinary sense. In other words, the mereology of structural parts of "discrete" similarity structures $(S, D(S))$ is isomorphic to the mereology of sets S in the usual sense.

The generalization (2.2) of the parthood relation \subseteq from sets to pairs of sets may look innocent enough, nevertheless, as we shall see, it ushers us outside the realm of classical Boolean mereology. The mereological systems $\text{PART}(S, \sim)$ defined in (2.2) turn out to be non-Boolean mereological systems. In other words, Boolean mereology is too narrow a framework to capture all kinds of reasonable mereological systems, in particular, the mereological systems arising from similarity structures (S, \sim) .

Intuitively formulated (2.2) stipulates that similarity be hereditary from a part to its whole: If $(T, \sim_T) \leq (S, \sim_S)$ and $a, b \in T$ satisfy $a \sim_T b$, then $a \sim_S b$ holds as well. On the other hand, the reverse implication between the similarity relation \sim_T and \sim_S may not hold: Intuitively this may be formulated as that it may be the case that the coherence of the whole (S, \sim_S) is strictly larger than the coherence of its parts (T, \sim_T) , i.e., it may be the case that for $a, b \in T$ one has $a \sim_S b$ but $a \sim_T b$ does not hold.

The definition (2.2) of structural parts is completely analogous to the definition of subsets as parts of sets. Thus, if one accepts subsets as parts of sets, there is no reason not to accept structural subsets as structural parts of similarity structures. For reasons that will become clear in the following, it seems expedient to single out the following special parts of similarity structures:

(2.3) Definition. Let (S, \sim_S) be a similarity structure.

- (i) A structural part (T, \sim_T) of (S, \sim_S) is discrete iff for $a, b \in T$ one has $a \sim_T b$ iff $a = b$, i.e., (T, \sim_T) is discrete iff $\sim_T = D(T)$.
- (ii) A structural part (T, \sim_T) of (S, \sim_S) is full if and only if for $a, b \in T$ $a \sim_T b$ iff $a \sim_S b$.♦

The discrete parts of (S, \sim_S) can be identified with the parts of the set S , i.e. with the subsets of S . Thus, the theory of structural parts of similarity structures (S, \sim_S) comprises as a part the theory of parts of sets S . If (S, \sim_S) itself is discrete, i.e. $\sim_S = D(S)$, then $\text{PART}(S, \sim_S) = \text{PART}(S)$.

In order to characterize $\text{PART}(S, \sim)$ structurally and to understand why it is in general not Boolean we need some definitions and results from lattice theory (cf. Davey and Priestley 1990, Grätzer 1998).

Recall that a complete lattice (L, \leq) is a partially ordered set such that every subset $P \subseteq L$ has a least upper bound (supremum) $\bigvee P \in L$ and a greatest lower bound (infimum) $\bigwedge P \in L$. The least upper bound of $P = \{a, b\}$ is denoted by $a \vee b$ and its greatest lower bound is denoted by $a \wedge b$. The least upper bound $\bigvee L$ is denoted by 1 and called the top element of L . Dually, the greatest lower bound of $\bigwedge \emptyset$ is denoted by 0 and called the bottom element of L .

Boolean algebras are well known to be lattices that can be characterized by the following requirements:

(2.4) Lemma. A lattice L is Boolean iff it satisfies the following two requirements:

- (i) L is distributive, i.e. for all $x, y, z \in L$ one has $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$;
- (ii) L is complemented, i.e. for all $x \in L$ there is an x^* such that $x \wedge x^* = 0$ and $x \vee x^* = 1$. The element x^* is called the (Boolean) complement of x .♦

The partial order $\text{PART}(S, \sim)$ is a lattice, the supremum $\bigvee K$ and the infimum $\bigwedge K$ of $K \subseteq S$ are just the set-theoretical union $\bigcup K$ and the intersection $\bigcap K$, respectively, endowed with the similarity relations inherited from S . The following simple example shows that the lattice $\text{PART}(S, \sim)$ of structural parts of a similarity structure may not be Boolean: The similarity structure

$$(2.6) \quad x \text{---} y \text{---} z$$

has as proper part the similarity structure consisting of the isolated vertices x, y , and z :

$$x \quad y \quad z$$

Obviously this substructure lacks a complement.

In other words, $\text{PART}(S, \sim)$ violates the so called law of weak supplementation (WSP) which many mereologists consider as an indispensable feature of any “reasonable” mereological

system (cf. Simons (1987)). A fortiori, $\text{PART}(S, \sim)$ is in general not a Boolean mereological system. It has to be characterized differently.

(2.7) Definition. Let (H, \leq) be a complete distributive lattice. For $a \in H$, $K \subseteq H$ define $a \wedge K$ to be the set $\{a \wedge k; k \in K\}$.

- (i) H is a Heyting algebra iff it satisfies the strong distributive law: $a \wedge \bigvee K = \bigvee (a \wedge K)$
- (ii) For $K := \{b; a \wedge b = 0\}$ the supremum $\bigvee K$ of K is called the Heyting complement of a ; it is denoted by a^* .²
- (iii) An element $a \in H$ is called regular if and only if $a = a^{**}$.♦

Evidently, every finite distributive lattice is a Heyting algebra. For infinite distributive lattices, this is no longer true, however. Due to the following lemma, the class of Heyting algebras turns out to be a generalization of the class of Boolean algebras:

(2.8) Lemma. A Heyting algebra H is a Boolean algebra if and only if $a^{**} = a$ for all $a \in H$.♦

Now we can state the first main result concerning the mereology of similarity structures (cf. Reyes and Zolfaghari (1996)):

(2.9) Theorem. Let (S, \sim) be a similarity structure.

- (i) The mereological system $\text{PART}(S, \sim)$ of structural parts is a Heyting algebra that usually is not a Boolean algebra.
- (ii) The mereological system $\text{PART}_D(S, \sim)$ of discrete structural parts of (S, \sim) is a Boolean system isomorphic to $\text{PART}(S)$.♦

Proof. Let (T, \sim) be a subgraph of (S, \sim) . Then the Heyting complement (T^*, \sim_{T^*}) is defined as follows. Let T^* be the set-theoretical complement of T in S . The restriction of the similarity relation \sim of S to T^* defines a similarity structure \sim_{T^*} on T^* . Then the Heyting complement of (T, \sim) is defined as (T^*, \sim_{T^*}) . With these definitions, $\text{PART}(S, \sim)$ is a Heyting system. Examples of non-full structural parts such as (2.6) show that $\text{PART}(S, \sim)$ is in general not Boolean.♦

² Intuitively, a^* is the largest element of H that does not overlap with a , i.e., for which $a \wedge a^* = 0$ holds. In general, $a \wedge a^* = 0$ does not suffice to ensure that $a \vee a^* = 1$. Hence, a Heyting complement need not be a Boolean complement in the sense of (2.5)(ii).

Similarity structures have appeared under many different names in philosophy, mathematics, computer science and related disciplines, e.g. as tolerance space, indifference structures, inhomogeneous sets (Carnap), or (undirected) graphs, to name a few. Moreover, as has often been noticed, to a large extent the theory of these structured sets may be developed along the lines of the theory of sets. From a modern point of view, this means to compare the category of sets with the category of graphs, the category of tolerance spaces, or however we may call the category of these structures (cf. Stout (1992), Lawvere and Schanuel (1996), Lawvere and Rosebrugh (2002)).

In the general framework of category theory, every category \mathbf{C} can be shown to come along with its specific mereology. This \mathbf{C} -specific mereology ascribes to each object S of \mathbf{C} the system of its subobjects (= parts) of S in \mathbf{C} . Depending on the structure of \mathbf{C} the structure of these mereological systems more or less differs from the structure of the mereological systems of sets, to wit, Boolean algebras.

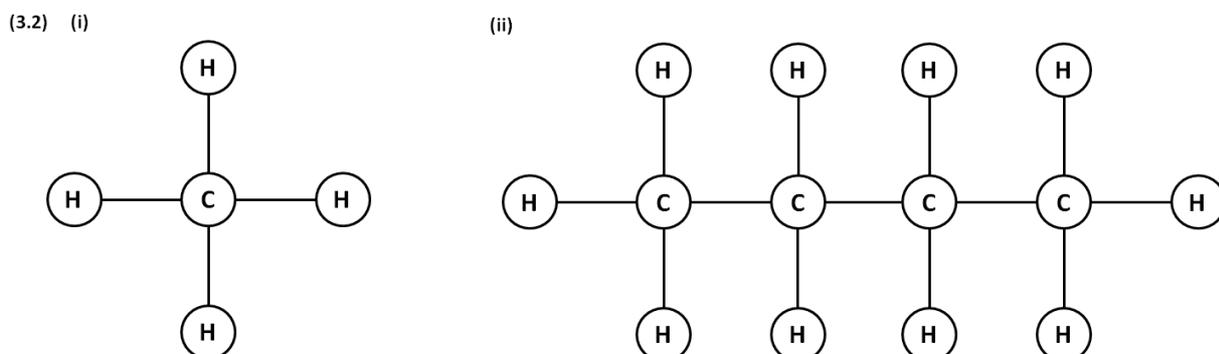
For the modest purposes of this paper the technical apparatus of category theory is not necessary. But in the long run, analytical metaphysics and formal ontology in particular would certainly profit from the conceptual tools of category theory and related developments. For some tentative steps in this direction, the reader may consult Mormann (2009, 2010).

3. Structural Mereology of Complex States of Affairs. Looking at structural formulas of elementary chemistry such as H-O-H, H-C-H, O-O, Na-O-H on the one hand and at graphs of similarity structures such as (2.1) on the other, it is more or less obvious that similarity structures and structural formulas of chemistry are closely related to each other. One further definition is necessary to make this relation explicit:

(3.1) Definition. Let S_1, S_2, \dots, S_n be non-empty, mutually disjoint sets and $S := S_1 \cup S_2 \cup \dots \cup S_n$, $n = 1, 2, 3, \dots$. Then a similarity structure (S, \sim) is called an n -sorted similarity structure. The S_i are called the sorts of (S, \sim) .♦

An ordinary similarity structure is just a 1-sorted similarity structure. In the following we will only deal with 2-sorted similarity structures but all arguments go through for n -sorted structures with $n \geq 3$.

A 2-sorted similarity structure $(S_1 \cup S_2, \sim)$ may be graphically represented as a “labeled” graph in such a way that the vertices of S_1 , are labeled, say, by the letter H and the vertices of S_2 are labeled by the letter C, respectively:



Obviously, the 2-sorted graphs of (3.2) correspond to the structure formulas of methane and butane, respectively. More complex molecules involving more than two sorts of atoms may be described analogously with the help of n -sorted graphs, $n \geq 3$.

What has been said about the mereological structure of 1-sorted similarity structures (S, \sim) in section 2 directly applies to n -sorted similarity structures since the labeling does not affect the mereological structure of a graph. In other words, n -sorted similarity structures S give rise to mereological systems $\text{PART}(S, \sim)$ of structural parts in the same way as 1-sorted structures do. In particular, theorem (2.9) also holds for n -sorted similarity structures.

Now we are ready to apply the conceptual apparatus developed so far to calculate the mereological structure of complex states of affairs in detail. As an example we calculate the mereological structure of the complex state of affairs of being a methane molecule.

(3.3) Mereological Analysis of Methane. Assume a, b, c, d, e to be thin particulars, H the universal hydrogen, and C the universal carbon. Define $S_1 = \{Ha, Hb, Hc, Hd\}$, $S_2 = \{Ce\}$ and $S := S_1 \cup S_2$. Denote by $—$ the binary bonding universal, and define a similarity relation \sim on S by

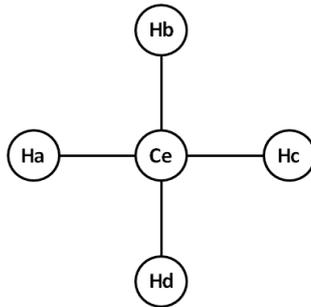
$$\sim := D(S) \cup \{Ha—Ce, Ce—Ha, Hb—Ce, Ce—Hb, Hc—Ce, Ce—Hc, Hd—Ce, Ce—Hd\}$$

Then the similarity structure (S, \sim) is the state of affairs that the particulars a, b, c, d, e instantiate a methane molecule such that $a, b, c,$ and d instantiate the universal H , e

instantiates the universal C, and the binary bonding universal — is instantiated by the set {Ha—Ce, Ce—Ha, Hb—Ce, Ce—Hb, Hc—Ce, Ce—Hc, Hd—Ce, Ce—Hd}.♦

Since — is symmetric the complex state of affairs of being methane may be displayed as

(3.4)



in the familiar way. Note that (3.4) is NOT the kind of complex states of affairs considered in (Armstrong 1997, 34 - 37), since the particulars that complete the bonding universal — are the thick particulars Ha, Hb, ..., Ce. In contrast, in Armstrong's analysis, the bonding universal — is completed by thin particulars a, b, ..., e. Hence, the partial states of affairs which according to his analysis involve the bonding universal are a—e, b—e, c—e, and d—e (cf. Armstrong 1997, 34). The important point is that these states of affairs are mereological atoms that mereologically have no further structure. In contrast, the states of affairs in (3.4) such as Ha—Ce are NOT mereological atoms. They do have a non-trivial mereological structure, namely, they have proper parts Ha and Ce.

The other atomic states that according to Armstrong's analysis appear in the complex state methane (3.4) are Ha, Hb, Hc, Hd, and Ce. Then, according to Armstrong's proposal, the mereological structure of the complex state of affairs (3.4) is the Boolean algebra $P(\{Ha, Hb, Hc, Hd, Ce, a—e, b—e, c—e, d—e\})$. In general, for a molecule with n atoms and m bondings Armstrong's analysis yields as the mereological structure of that molecule the Boolean algebra of the power set generated by n+m elements.

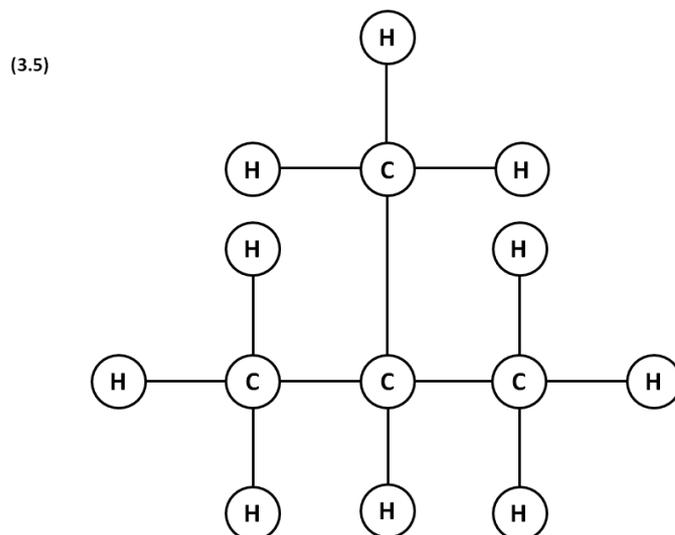
Our analysis yields a different, more complex structure. Denoting the similarity structure (3.4) by (M, ~) the lattice PART(M, ~) of its structural parts may be calculated by using the definition (2.2). First, one observes that PART(M, ~) has five atoms, to wit, Ha, Hb, Hc, Hd, Ce. In contrast to Armstrong's account, the states of affairs Ha—Ce, ..., Hd—Ce are neither atoms nor are generated by atoms. Nevertheless they are mereologically not trivial: Using (2.2) one calculates

$$H_j, Ce < H_j \& Ce < H_j-Ce$$

$$j \in \{a, b, c, d\}$$

but obviously there is no atom that makes a difference between $H_j \& Ce$ and H_j-Ce . Hence, the part H_j-Ce of M is neither an atom nor is generated by atoms. Analogously one shows that parts of M such as H_j-Ce-H_k ($j \neq k$) are neither atoms nor are generated by atoms. This evidences that $PART(M, \sim)$ essentially differs from the Boolean lattice that Armstrong obtains as the result of his mereological analysis. $PART(M, \sim)$ is a non-Boolean Heyting algebra.

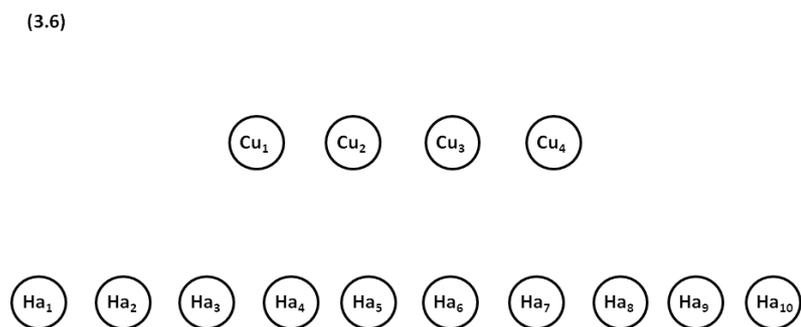
Emphasizing the difference between the two analysis is not to say that they are unrelated. The conceptual relation between Armstrong's mereological analysis and the one proposed in this paper can be succinctly described as follows: given a complex state of affairs with similarity structure (S, \sim_s) Armstrong takes the number $|S|$ of vertices (atoms) of S and the number $|\sim_s|$ of edges of (S, \sim) and ascribes the Boolean algebra $P(\{1, \dots, |S| + |\sim_s|\})$ to the state. Unsurprisingly, this procedure yields a rather poor picture of (S, \sim_s) , as is evidenced by the fact that for many essentially different molecules it gives the same mereological structure. A particularly pertinent example that has been amply discussed is the case of butane and isobutane. The structure formula of butane was given in (3.2)(ii), that of isobutane is



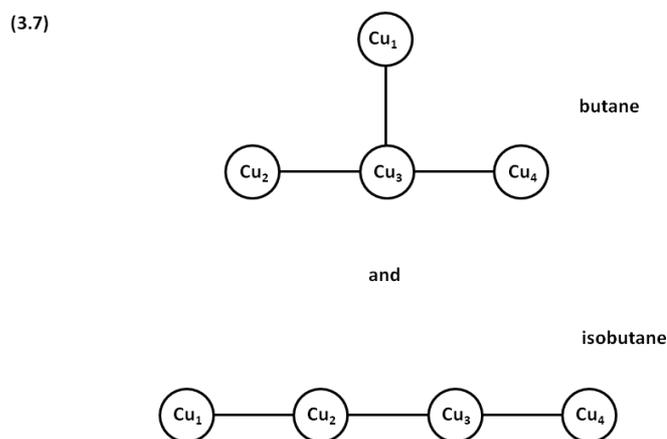
As is easily counted the sum of the number of atoms and the number of bonds of butane and isobutane is $14(\text{atoms}) + 13(\text{bonds}) = 27$. Hence, according to Armstrong's analysis

the mereological systems of these molecules are isomorphic to the Boolean algebra $P(\{1, \dots, 27\})$.

On the other hand, the 2-sorted graphs (3.2)(ii) and (3.5) of butane and isobutane are clearly non-isomorphic. Consequently the mereological systems $\text{PART}(\text{Butane})$ and $\text{PART}(\text{Isobutane})$ are different. This does not exclude that these systems have isomorphic subsystems as is actually the case for butane and isobutane. The graphs of butane and isobutane have in common the discrete 2-sorted graph consisting of four C-states and ten H-states (see (3.2)(ii) and (3.5):



In other words, $\text{PART}_D(\text{Butane}) = \text{PART}_D(\text{Isobutane}) = P(\{1, 2, \dots, 14\})$. Moreover, by definition $P(\{1, 2, \dots, 14\})$ is a subalgebra of Armstrong's algebra $P(\{1, \dots, 27\})$. Nevertheless the full mereological systems $\text{PART}(\text{Butane})$ and $\text{PART}(\text{Isobutane})$ are non-isomorphic since the full mereological systems $\text{PART}(\text{Butane})$ and $\text{PART}(\text{Isobutane})$ are non-isomorphic. This can be deduced from the fact that these systems contain different non-discrete states of affairs as parts, for instance the full subgraphs



are non-isomorphic, since the graph of the first one contains the element Cu_3 that has three different neighbors while this is not the case for the graph of the second one.

Thus, even if one concurs with Lewis that “it’s no good thinking that a structural universal is composed of simpler universals which are literally parts of it” (Lewis 1986, 212), this does not mean that it does not make perfectly good sense to think of a complex state of affairs as being composed of simpler states of affairs. These simpler states are literally parts of the complex state. Hence, there is no need to introduce structural universals for describing the structure of complex state of affairs (*pace* Bigelow and Pargetter (1989)) mereology alone suffices. The mereological structure takes into account non-discrete states of affairs such as $Fa—Gb$ as parts of complex states of affairs; these parts distinguish between different complex states of affairs having the same discrete parts - as is the case for butane and isobutane. Lewis’s alleged difficulty that different structural universals were to be composed of the same basic universals does not arise due to the different non-discrete parts that make a difference between isomere states of affairs.

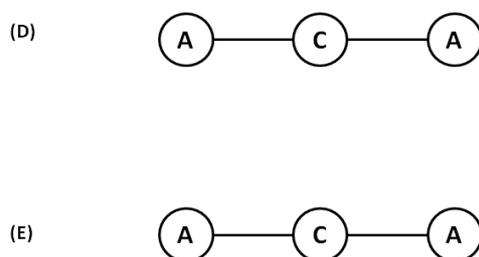
One may admit structural universals as a convenient *façon de parler* but they lack any ontological weight. Alleged structural universals such as H_2O or CH_4 that in a superficial analysis of water or methane seem to play an analogous role to the one that basic universals such as H or C play in the analysis of hydrogen atoms or carbon atoms Ha or Ce , are to be considered as a convenient shorthand of the full mereological structures that actually do the real work.

4. Formal Relations and Arrangement Universals. After having presented in some detail our proposal of how to elucidate the structure of complex states of affairs a brief comparison with some other attempts to cope with the problem of “structural universals” may be in order. Particularly pertinent in this regard are Pagés (2002) and Kalhat (2008). Both authors contend that Armstrong’s factual ontology should be enriched by some further element in order to distinguish between different structural universals that are composed of the “same” simple universals. Both take that extra element to be a relation. In contrast, the account of the present paper relies on a distinction that is already present in Armstrong’s original approach, namely, the distinction between relational states of affairs that only involve thin particulars such as Rab , and relational states of affairs that involve thick particulars such as $Fa B Gb$, respectively. Hence, even before we delve into the technical details one can say that our approach is ontologically more austere and closer to Armstrong’s original account than that of Pagés or Kalhat.

Pagés calls the extra element that he introduces to cope with the problems of structural universals, “formal relations” (cf. Pagés (2008, 218)). Formal relations are designed to take care of the numerical identity and difference of the particulars involved in the exemplifications of their constituent properties and relations. Formal relations provide the relational description of the n-sorted graphs which describe the structure of complex states of affairs can be described. Pagés uses the following elementary example to show how this recipe works: Let A and C be monadic universals that are instantiated by A-atoms and C-atoms, respectively.

Now consider two molecular states of affairs D and E represented by the 2-sorted graphs:

(4.1)



Filling in the particulars x, y, and z and representing the bonding — by the binary universal B — the states of affairs corresponding to (D) and (E) have the following relational descriptions:

(4.2)

(D) $Ax \ \& \ Cy \ \& \ Az \ \& \ B(x, y) \ \& \ B(y, z) \quad R(x,y,z)$

(E) $Cx \ \& \ Ay \ \& \ Az \ \& \ B(x, y) \ \& \ B(y, z) \quad S(x,y,z)$

Denote the triadic relation of (4.2)(D) by R and the triadic relation of (4.2)(E) by S. Then the structural properties of molecules of type D and E can be analyzed as follows:

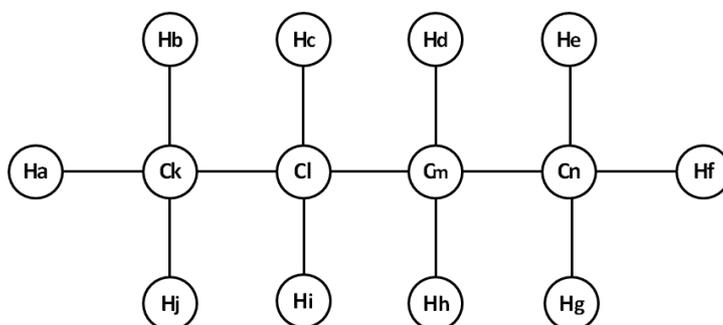
(4.3) Ontological Analysis of D- and E-molecules (Pagés 2008, 218). Let m be a molecule.

- (i) m is a D-molecule if and only if there are three particulars a_1, a_2, a_3 constituting m and different from each other such that the sequence $\langle a_1, a_2, a_3 \rangle$ instantiates the first order formal relation R .
- (ii) m is an E-molecule if and only if there are three particulars a_1, a_2, a_3 constituting m and different from each other such that the sequence $\langle a_1, a_2, a_3 \rangle$ instantiates the first order formal relation S .♦

Evidently, appropriate “formal relations” can be used to describe more complex states of affairs such as methane, butane, or isobutane and their differences. Equally evident is that Pagés’s formal relations are just implicit descriptions of n -sorted graphs. In contrast to the present approach, however, in Pagés’s account the mereological aspects of complex states of affairs remain in the dark. Moreover, as Kalhat remarks (Kalhat 2008, 75), one may doubt, whether formal relations are firmly rooted in the conceptual praxis of chemistry.

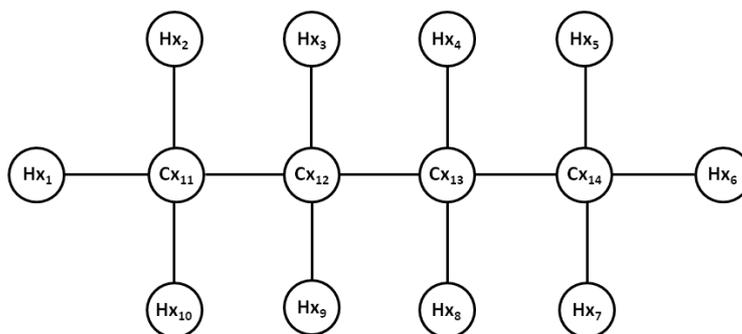
Now let us have a closer look on Kalhat’s attempt to come to terms with structural universals and their role in the constitution of complex states of affairs (cf. (Kalhat 2008)). First of all, one should note that Kalhat ignores the repetition problem, i.e. the problem of how one can make sense of the assumption that in the alleged structural universals of methane, butane, and the like simple universals such as hydrogen and carbon can appear several times. Granted this, for Kalhat the road to structural universals is quite short (cf. Kalhat 2008, 67). Starting from a complex state of affairs

(4.4)



the structural universal of being butane is obtained by replacing the particulars a, b, \dots, m, n of (4.4) by variables x_1, \dots, x_{14} , and the chemical bonding between the atoms of (5.4) by some kind of abstract bonding:

(4.5)



Kalhat proposes to consider structures such as (4.5) as structural universals, but he does not give any explanation what the ontological status of these entities might be. As it seems, it is not the same as the one that ordinary universals enjoy. On the one hand, there are ordinary monadic universals such as H or C, on the other hand, according to (4.5) there are objects such as Hx, Hy, Cz and the like, which, of course, are to be identified somehow with H or C, respectively. Be this as it may, Kalhat remains silent about what the parts of a structural universal such as (4.5) may be. Perhaps, just the Hx_i and $Cx_j, i, j = 1, 2, \dots$. Since he explicitly does not want to distinguish between parts and constituents of states of affairs, another option may be that the parts of (4.5) are H, C, and the $x_i, i = 1, 2, \dots$, (cf. Kalhat 2008, 75). But then he runs into Lewis's difficulty that one and the same part, such as H and C, appear several time in one and the same whole. Thus, the mereological structure of his structural universals remains unclear. Things become worse, when he introduces as a further part of (4.4) what he calls the "arrangement universal" being butane-wise arranged. In our terminology, this is just the abstract 2-sorted graph $(S_1 \cup S_2, \sim)$ corresponding to (4.5), obtained by replacing the universals H and C by abstract labels. Finally, he postulates that the similarity structure $(S_1 \cup S_2, \sim)$ is to be considered as a part of the structural universal being butane.

In sum, then, the mereological structure of Kalhat's structural universals remains quite obscure. His "arrangement parts" suspiciously resemble Aristotelian forms, and in particular his approach is of no help to elucidate the formal theory of complex states of affairs with general structural theories such as mereology, graph theory or the theory of similarity structures.

5. Concluding Remarks. The aim of this paper was to elucidate the mereological structure of complex states of affairs, or, more modestly, to elucidate the mereological structure of some complex states of affairs that occur in elementary chemistry. Employing conceptual tools from graph theory, lattice theory, and the theory of relational systems it has been shown that we don't need structural universals for the constitution of complex states of affairs. Rather, non-classical Heyting mereology suffices. That is to say that not all parts of complex states of affairs are joins (suprema) of discrete atomic states. Complex states of affairs use to have non-discrete states as parts like $Fa-Gb$. The existence of these states strongly influences the mereological structure of complex states. It ushers us outside the realm of classical Boolean mereology. For instance, already the mereological system $PART(Fa-Gb)$ is a non-Boolean Heyting algebra of five elements.

In this paper we only considered complex states of affairs with one binary bonding universal — whose mereology turned out to be isomorphic to the mereology of similarity structures or, equivalently, simple graphs. Our approach may be easily generalized to complex states of affairs that involve more than one binary bonding universal. This amounts to the investigation of graphs with different sorts of edges. Further generalizations dealing with n -ary ($n \geq 3$) instead of binary bonding universals are possible leading to the study of hypergraphs instead of ordinary graphs.

Since graphs *qua* similarity structures are structured sets, their mereology can be developed along the lines of the mereology of sets (cf. Lewis 1991). The outcome is that the mereological structures of complex states of affairs are Heyting mereological systems. Structural universals have no essential role in this approach. Formulas such as H_2O or CH_4 that apparently refer to them, are useful “incomplete symbols” somewhat in Russell's sense, but they do not belong to the basic ontological furniture of a world of states of affairs. A factual ontology can do without them, provided one is prepared to go beyond the confines of classical Boolean mereology admitting more general mereological structures such as Heyting algebras.

References:

Armstrong, D.M., 1986, In Defence of Structural Universals, *Australasian Journal of Philosophy* 64, 85 – 88.

Armstrong, D.M., 1997, *A World of States of Affairs*, Cambridge, Cambridge University Press.

- Bigelow, J., Pargetter, R., 1989, A Theory of Structural Universals, *Australasian Journal of Philosophy* 67(1), 1 – 11.
- Davey, B.A., Priestley, H., 1990, *Introduction to Lattices and Order*, Cambridge, Cambridge University Press.
- Forrest, P., 1986, Neither Magic Nor Mereology, *Australasian Journal of Philosophy* 64, 89 – 91.
- Forrest, P., 1986, Ways Worlds Could Be, *Australasian Journal of Philosophy* 64, 15 - 24.
- Grätzer, G., 1998, *General Lattice Theory*, Second edition, Basel and Boston, Birkhäuser.
- Kalhat, J., 2008, Structural Universals and the Principle of Uniqueness of Composition, *Grazer Philosophische Studien* 76(2008), 57 – 77.
- Lawvere, F., Rosebrugh, R, 2003, *Set Theory for Mathematicians*, Cambridge, Cambridge University Press.
- Lawvere, F., Schanuel, P., 1996, *Conceptual Mathematics*, Cambridge, Cambridge University Press.
- Lewis, D., 1986, Against Structural Universals, in *Papers on Metaphysics and Epistemology*, Cambridge, Cambridge University Press, 78 – 107, originally, *Australasian Journal of Philosophy* 64(1), 25 – 46.
- Lewis, D., 1991, *Parts of Classes*, Oxford, Basil Blackwell.
- Mormann, T., 2009, Updating Classical Mereology, *Proceedings of the XIII. International Conference on Logic, Methodology and Philosophy of Science*, edited by C. Glymour, W. Wang, D. Westerwstahl, Beijing 2007, King's College Publications, London, 326 – 343, 2009.
- Mormann, T., 2010, Structural Universals as Structural Parts: Toward a General Theory of Parthood and Composition, *Axiomathes* 20(2 – 3), 229 – 253.
- Pagés, J., 2002, Structural Relations and Formal Relations, *Synthese* 131(2), 215 – 221.
- Reyes, G.E., Zolfaghari, H., 1996, Bi-Heyting Algebras, *Toposes and Modalities*, *The Journal of Philosophical Logic* 25, 25 – 43.
- Simons, P., 1987, *Parts. A Study in Ontology*, Oxford, Oxford University Press.
- Stout, L. N., 1992, The Logic of Unbalanced Subobjects in a Category with Two Closed Structures, in S.E. Rodabaugh, E.P. Klement, U. Höhle (eds.), *Applications of Category Theory to Fuzzy Subsets*, Dordrecht, Kluwer, 73 - 105.