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ON THE VICISSITUDES OF IDEALISM IN 20TH CENTURY PHILOSOPHY OF SCIENCE: THE CASE OF CASSIRER'S *CRITICAL IDEALISM**

1. Idealist Philosophy of Science?

In Anglo-Saxon philosophy of science, there is a strong conviction that idealist philosophy on the one hand and serious science and philosophy of science on the other do not go well together. Often, idealism plays the role of a straw man to whom all the vices are attributed that one wants to criticize. An extreme example is Israel Scheffler's verdict about Thomas Kuhn's historicist account put forward in *The Structure of Scientific Revolutions* (1961). According to Scheffler, Kuhn's "extravagant idealism"[1] in his philosophy of science was also to be considered a threat to the moral integrity of science itself:

> The current attacks [of Kuhn on the "received" view in philosophy of science – TM] challenge the very opposition between

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[1] I. Scheffler, *Science and Subjectivity*, Indianapolis 1967, p. 19.

> science and speculative idealism, from which scientifically minded philosophies have sprung. The attacks threaten further the underlying moral motivation of this philosophy, their upholding of the ideal of responsibility in the sphere of belief as against willfulness, authoritarianism, and inertia. The issues are fundamental, indeed more fundamental than is generally realized, precisely because a powerful moral vision has implicitly been called into question[2].

According to Scheffler, Kuhn's "extravagant idealism" implies that, "[i]n place of a community of rational men following objective procedures in the pursuit of truth, we have a set of isolated monads, within each of which each belief forms without systematic constraints"[3]. Clearly, Scheffler's savage defense of the "received view" of philosophy of science against Kuhn is based on a simplistic conception of idealism that boils down to the absurd idea according to which all variants of idealism subscribe to the doctrine that "reality is fundamentally mental". This caricature of idealism is not, however, restricted to the philosophy of science of the last century. In the 21st century one still finds philosophers such as Susan Haack who propagate virtually the same caricature of idealism that Scheffler put forward almost 50 years ago:

> An idealist holds that everything there is, is mental: that the world is a construction out of our, or, in the case of the solipsist, his own, ideas – subjective idealism; or is constituted by God's ideas – theological idealism; or that the world is itself of a mental or spiritual character – objective idealism, as in Hegel[4].

Evidently, for Haack idealism is not an option to be taken seriously. For her, idealism is nothing but the bogey opponent of realism. This stance is hardly tenable, or so I want to argue in this paper. For this purpose, one should first endorse the general

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[2] *Ibidem*, pp. 7–8.

[3] *Ibidem*.

[4] S. Haack, *Realisms and Their Rivals: Recovering Our Innocence*, "Facta Philosophica" 4 (2002), p. 70.

rule not to restrict one's attention only to the most simplistic and implausible versions of a concept but to take into consideration that the concept under scrutiny may have evolved in some way or another. For the concept of realism, this move has been noted by A. Chakravartty:

> One has to appreciate the evolution of scientific realism – what it was and what it has become. "Standard" realism is not what it once was; it has been refined in response to antirealist scepticism[5].

Indeed, the debate on "realism" in the philosophy of science has led to a bewildering manifold of different concepts of realism[6]. It would be surprising if a similar phenomenon could not also be noticed for the issue of idealism in the philosophy of science, provided that this is not excluded from in advance by boldly claiming that any idealist ingredient of philosophy of science is mistaken from the outset. Instead of begging the question in this way, it would instead be advisable to ponder an option the American pragmatist C.I. Lewis formulated long ago in *Mind and the World Order* (1929):

> It may be that between a sufficiently critical idealism and a sufficiently critical realism, there are no issues save false issues which arise from the insidious fallacies of the copy-theory of knowledge[7].

As such, for a "sufficiently critical" idealism the philosophy of science, which deserves to be taken seriously even in contemporary philosophy of science, I would like to propose Ernst Cassirer's[8] "critical idealism", sometimes referred to also as "logical idealism" or "Neo-Kantian transcendental idealism".

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[5] A. Chakravartty, *A Metaphysics for Scientific Realism. Knowing the Unobservable*, Cambridge 2007, p. 44.

[6] Cf. A. Chakravartty, *A Metaphysics…*; S. French, *The interdependence of structure, objects, and dependence*, "Synthese" 175 (2010), pp. 89–109.

[7] C.I. Lewis, *Mind and the World Order. Outline of a Theory of Knowledge*, New York 1929, p. 194.

[8] This paper is an elaboration of a talk given at the congress *Granice Nauki* (*The Frontiers of Science*) that took place in Wroclaw in April 2013. This may be taken as a reason to briefly recall some biographical details concerning Cassirer.

Cassirer's most important work in the philosophy of science is *Substance and Function* (1910). Other important books dealing with issues of the philosophy of science are *Einstein's Theory of Relativity* (1921), *The Philosophy of Symbolic Forms* (1923–1929) and *Determinism and Indeterminism in Modern Physics* (1936). Less known, but particularly relevant for the purposes of the present paper, is his early paper *Kant und die moderne Mathematik* (1907), which contains many of the basic ideas of the idealist philosophy of science that were unfolded in his later works.

Evidence that Lewis's somewhat cryptic conjecture may not have been completely off the mark is provided by the fact that today various "critical" accounts of structural realism hint in significant ways at some sort of idealism. More precisely, many contemporary structural realists explicitly consider Cassirer's approach as a precursor to some version of contemporary structuralist realism[9]. Still, they dare not explicitly mention the ide-

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Ernst Cassirer was born on July 28 of 1874 in Breslau. In 1892, he did his *Abitur* at the *Johannes Gymnasium* (*Johanneum Breslau*). Quite a few pupils of the *Johanneum* became famous in mathematics, science and the humanities, including the mathematician Otto Toeplitz, the sociologist Norbert Elias, and the Nobel price winners Fritz Haber (chemistry), and Otto Stern (physics). The *Johanneum* building still exists today (ulica Stanislawa Worcella no. 3). After the matura, Cassirer studied law, literature, psychology, and philosophy at the universities of Berlin, Heidelberg, Leipzig, and München before he eventually obtained his PhD in philosophy at Marburg under the direction of Hermann Cohen, the leader of Marburg Neo-Kantianism. In the Weimar Republic, Cassirer was one of the most important philosophers and public intellectuals. He explicitly took up a position in favor of democracy and the constitutional state. A memorable document for this commitment was his 1928 Constitution Day speech as the Rector of the newly founded University of Hamburg, in which he argued that the "idea of a republican constitution is in no sense a stranger, let alone an alien intruder, in the overall context of the history of German thought and culture" (Cassirer 1929). As a Jewish liberal democrat, in 1933 he was forced to emigrate. First, he emigrated to England, then to Sweden, and eventually he settled in the USA. He unexpectedly died on April 13 of the year 1945 in New York. For a succinct biographical sketch on Cassirer's life and work, the reader may consult John Michael Krois' *Zum Lebensbild Ernst Cassirers (1874–1945)* in Cassirer (2009).

[9] Cf. B. Gower, *Cassirer, Schlick, and "Structural Realism": The Philosophy of the Exact Sciences in the Background to Early Logical Positivism*, "British Journal for the History of Philosophy" 8 (2000), p. 71–106; A. Cei, S. French, *On the Transposition of the Substantial into the Functional: Bringing Cassirer's Philosophy of*

alist origins of Cassirer's structuralism or implicitly consider them irrelevant for his structuralist stance. However, that contemporary structural realists already recognize important affinities between their own "realism-soaked" structuralism and Cassirer's "idealism-soaked" structuralism shows that Crispin Wright may have been on the right track when he described the issue of idealism in the philosophy of science as follows:

> For all the vilification and caricature which its critics have met out over the years, the idealist tradition in philosophy has proved sufficiently durable to encourage the belief that, at least locally, there are insights for which it is striving, but for which – its persistently controversial character suggests – we have yet to find definitive means of expressions[10].

The outline of this paper is as follows. The aim of section 2 is to unfold a fundamental thesis of Cassirer's critical idealism, namely, that the proper issue of contemporary philosophy of science is to be neither science nor mathematics in isolation but the connection of the two realms. In section 3, the role of ideal elements ("idealization") in mathematics and the sciences is explained. The topic of section 4 is the complex relation between real and ideal elements that is characteristic for modern empirical scientific knowledge. Section 5 provides a succinct summary of seven theses of the philosophy of science of critical idealism. We close with some remarks on the changing reputation of Cassirer's idealist philosophy of science from the 1940s until today in section 6.

2. Cassirer's Critical Idealism.

Let us begin with a fundamental thesis of Cassirer's ide-

Quantum Mechanics into the 21st Century, [in:] *Constituting Objectivity, Transcendental Perspectives on Modern Physics*, M. Bitbol, P. Kerszberg, J. Petitot (eds.), Dordrecht 2009, pp. 95–115; J. Ladyman, D. Ross, *Every Thing Must Go. Metaphysics Naturalized* (with D. Spurritt and J. Collier), Oxford 2000; A. Chakavartty, *A Metaphysics*….

[10] C. Wright, *Truth and Objectivity*, Oxford 1992, p. 3.

alism that addresses the relation of mathematics and physics, or, more generally, the relation of mathematics and empirical sciences. I would like to contend that Cassirer's thesis might be of some interest for the contemporary agenda of philosophy of science dealing with the problems of idealization and the role of mathematics in the sciences. According to Cassirer, the philosophy of science has to concentrate neither on mathematics, as an ideal science, nor on physics as an empirical science, but rather:

> If one is allowed to express the relation between philosophy and science in a blunt and paradoxical way, one may say: The eye of philosophy must be directed neither on mathematics nor on physics; it is to be directed solely on the connection of the two realms[11].

For Cassirer's philosophy of science, the point of reference was neither mathematics, as a science of ideal objects, nor physics, as an empirical science. Rather, he was looking for a common root from which both physics and mathematics spring. This common root is identified as the method of introducing ideal elements.

Before we come to the details of his approach, it may be expedient to note that characterizing Cassirer's idealist philosophy of science baldly as a version Neo-Kantian idealism may be quite misleading. Cassirer considered Kant's philosophy a promising starting point for modern epistemology and philosophy of science, not a doctrine that had to be followed literally. Like all neo-Kantians, he emphatically endorsed the maxim of 'going *with* Kant *beyond* Kant'.

The most important deviation from Kantian orthodoxy was to give up Kant's sharp separation between understanding and sensibility as two faculties of the mind. Following his teacher Hermann Cohen, Cassirer replaced Kant's two faculties of the

[11] E. Cassirer, *Kant und die moderne Mathematik*, "Kant-Studien" XII (1907), p. 48.

mind by a single comprehensive activity of "pure thought" (*reines Denken*). Pure thought primarily expressed itself in the progressive evolution of the mathematized empirical sciences. For Cassirer, the formal kernel of neo-Kantian "pure thought" was the new relational or functional logic inaugurated by Frege, Peano, Russell, and others that had emerged from the evolution of mathematics itself. For the Marburg Neo-Kantians, this was no coincidence. It revealed that the history of science played an eminent role in the philosophy of science.

According to Cohen's well-known slogan, philosophy had to take "the fact of science" as the starting point for its considerations. This attitude was but a consequence of the "transcendental method" considered by the neo-Kantians to be the core of Kantian philosophy. According to them, philosophy did not operate in empty space but had to rely on the historically established facts of science, ethics, art, and religion that provided it with its proper content[12]. The task of philosophy was to "justify" these cultural and scientific facts by elucidating their reasonableness and making real sense of them. That is, philosophy had to explicate the meaning of human culture, in particular, the meaning of science. On a larger scale, which went beyond the traditional of standard Neo-Kantianism, Cassirer carried out this program in his monumental *Philosophy of Symbolic Forms* (1923–1929, henceforth PSF).

In a nutshell, then, for the Marburg Neo-Kantians such as Cohen, Natorp, and Cassirer the task of the philosophy of science was to make explicit the method of science as "the method of an infinite and unending creative evolution of reason. Fulfilling this task was the indestructible core of Kant's philosophy"[13]. Because of its emphasis on the historical evolution of science, the Marburg school did not conceive the "fact

[12] Cf. P. Natorp, *Kant und die Marburger Schule*, "Kant-Studien" XVII (1912), pp. 196–197.

[13] P. Natorp, *Kant…*, p. 200.

of science" as something simply given. Rather, science was conceived of as a "fact in becoming" (*Werdefaktum*). This led the critical idealism of the Marburg school to a genetic epistemology that regarded the process of scientific evolution as essential, not so much the certainty and truth of its results.

Taking into account the then contemporary state of mathematics and modern (mathematized) natural sciences such as physics, Cassirer's philosophy of science treated mathematics and physics (and the other sciences) as a unit. This does not mean that he considered mathematics as an empirical science in the line of Mill and his modern naturalist followers. Rather, his approach was based on the following "idealist" thesis:

> Mathematical knowledge and physical knowledge are of the same kind. Both are characterized by the introduction of "ideal elements" which in both areas play essentially the same role.

This thesis may be dubbed the Sameness Thesis (henceforth ST). For the first time, Cassirer put forward ST in the early paper *Kant und die moderne Mathematik* (1907, henceforth KMM). The main topic of KMM was not so much Kant but the problem of how neo-Kantian philosophy of science should assess the recent developments of logic and mathematics, in particular the growing importance of the theory of relations for logic, mathematics and philosophy in general. The sameness thesis was directed against Russell's logicism as propounded in *The Principles of Mathematics*. According to Cassirer, in this work Russell set too strictly apart the realm of logic and mathematics from the domain of the empirical. More precisely, Russell claimed that the concepts of quantity and magnitude were not reducible to logical terms and therefore did not belong to the realm of pure mathematics because they contained an irreducibly empirical or perceptual component[14]. Cassirer objected that this procedure amounted to a strict separation between the "world of thought" and the "world of objects":

[14] Cf. B. Russell, *The Principles of Mathematics*, London 1903 (1992), §150 ff.

> According to the fundamental view of logistics the task of thought has ended when it has succeeded in making a strict deductive connection among its structures and creations. The problem of the lawfulness of the world of objects, on the other hand, is left completely to direct observation, which alone within its own very narrow limits is able to teach us whether there are also here regularities or whether pure chaos reigns[15].

Somewhat ironically, the Neo-Kantian Cassirer is here criticizing the Anti-Kantian Russell for sticking too closely to the traditional Kantian separation between the conceptual on the one hand and the intuitive on the other. This separation, as the neo-Kantians were eager to put forward against Kantian orthodoxy, was untenable[16]. According to critical idealism

> […] we do not know "objects" as if they were already independently determined and given as objects — but we know objectively, by producing certain limitations and by fixating certain permanent elements and connections within the uniform flow of experience. The concept of the object in this sense constitutes no ultimate limit of knowledge, but is rather the fundamental instrument, by which all that has become its permanent possession is expressed and established. The object marks the logical possession of knowledge, and not a dark beyond forever removed from knowledge[17].

Critical idealism, based on the sameness thesis ST, was meant to overcome the limitations of Russell's logicism that neatly separated the domain of the empirical from that of the mathematical. Cassirer conceived critical idealism as an improvement over Russell's logicism in that it preserved the logical and mathematical achievements of logicism while simultaneously avoiding its philosophical shortcomings, namely, to do justice to the universal applicability of mathematics in the empirical sciences:

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[15] E. Cassirer, *Kant…*, p. 43.

[16] Cf. also H.R. Smart, *Cassirer versus Russell*, "Philosophy of Science" 3 (1943), p. 168, 168

[17] E. Cassirer, *Substance and Function & Einstein's Theory of Relativity*, New York 1910 (1953), p. 303 f.

> Thus there begins a new task at the point where logistic ends. What "Critical Idealism" seeks and what it must demand is a logic of objective knowledge. *Only when we have understood that the same foundational syntheses on which logic and mathematics rest also govern the scientific construction of experiential knowledge, that only they enable us to speak of a strict, lawful ordering among appearances and therewith of their objective meaning: only then the true justification of the principles is attained*[18].

The sameness thesis ST is the core of "critical idealism", distinguishing it from Russell's logicist realism. Critical idealism aimed to develop a "logic of objective knowledge" that comprised a theory of concept formation for mathematics *and* the empirical sciences as a successor discipline of Kant's transcendental logic[19]. In modern terms, Cassirer's new transcendental logic of science aimed at a philosophical understanding of the dynamics of theories to determine in an ever more complete way the invariants of any theorizing. In contrast to Kant, Cassirer did not assert that philosophy had already discovered the general conditions of all possible experience. Rather, the "ultimate common elements of all possible forms of scientific experience" were to emerge gradually in the evolution of scientific concepts. Thus, if philosophy of science as a kind of transcendental logic of science was to accomplish the task of elucidating the evolving logical structure of science it had to study the conceptual history of science.

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[18] E. Cassirer, *Kant…*, p. 44, emphasis added.

[19] As quite a few scholars have observed, the envisaged "logic of objective knowledge" (*Logik der gegenständlichen Erkenntnis*) amounted to a far-reaching reinterpretation of the Kantian notion of transcendental logic, to put it mildly (cf. T. Ryckman, Conditio sine qua Non? Zuordnung *in the Early Epistemologies of Schlick and Cassirer*, "Synthese" 88 (1991), p. 62; A.W. Richardson, *Carnap's Construction of the World: The Aufbau and the Emergence of Logical Empiricism*. Cambridge 1998 p. 121; M. Friedman, *Reconsidering Logical Positivism*, Cambridge 1999, p. 91). Instead of deriving the principles of pure thought from the manifold of pure intuition (cf. *Critique of Pure Reason*, B 80 f.), the new transcendental logic was to explicate the meaning of scientific concepts through the philosophical understanding of the ongoing historical evolution of the sciences, in particular, of physics and mathematics.

For Cassirer, the achievements of post-Kantian physics and mathematics contributed something essentially new because they brought to light the importance of functional and relational concepts. Hence, the new relational mathematics was not only a major technical achievement but was also of utmost philosophical importance because it revealed a new type of "invariants of scientific experience" that hitherto had not found its proper philosophical formulation.

> The procedure of the "transcendental philosophy" can be directly compared at this point with that of geometry. Just as the geometrician selects for investigation those relations of a definite figure, which remain unchanged by certain transformations, so here the attempt is made to discover those universal elements of form, that persist through all change in the particular material content of experience.
> […]
> The goal of critical analysis would be reached, if we succeeded in isolating in this way the ultimate common elements of all possible forms of scientific experience; i.e., if we succeeded in conceptually defining those moments, which persist in the advance from theory to theory because they are the conditions of any theory. At no given stage of knowledge can this goal be perfectly achieved; nevertheless it remains as a demand, and prescribes a fixed direction to the continuous unfolding and evolution of the systems of experience[20].

The central task of a critical idealist philosophy of science à la Cassirer, namely, to elucidate the role of the common "foundational syntheses" on which logic, mathematics, and the empirical sciences allegedly rest. As will be explained in detail in the following sections, the said foundational syntheses amount to the introduction of "ideal" or "limiting" elements. According to Cassirer's critical idealism, ideal elements are constitutive for mathematical and empirical knowledge.

Standard wisdom has it that mathematics is the science dealing with ideal structures and/or ideal objects. Thus, there

[20] E. Cassirer, *Substance…*, pp. 268–269.

seems to be no need for idealization inside mathematics. Mathematics already is, so to speak, on the ideal side. In contrast, Cassirer emphasized that even within mathematics much idealizing work is necessary to formulate and prove interesting theorems in a sea of ephemeral phenomena. In mathematics as well as in physics and other empirical sciences, one has to prepare the ground to provide appropriate settings for conducting interesting work, namely, to carry out significant experiments or to prove non-trivial theorems. This involves a variety of idealizing constructions. To render this thesis plausible, one has to explain in some detail what is the role of ideal elements in specific domains such as geometry, algebra, arithmetic, and physics, to mention just a few. It does not suffice to speak in purely general terms about the ideal constructions. One has to consider the specific situations from which these idealizations arise[21].

3. Ideal elements in Mathematics and the Sciences

For Cassirer, a comprehensive theory of idealization has to take into account both mathematics and the empirical sciences. One should study how idealizations work in mathematics *and* the empirical sciences. This claim is in stark contrast with modern views according to which the mathematicians are, so to speak, already *inside* the sphere of ideal objects. According to the basic tenet of ST the essential factor both of mathematical and empirical knowledge is the introduction of ideal elements, i.e., idealization is idealizing completion.

For Cassirer, the paradigmatic example of a conceptual completion in mathematics was Dedekind's completion of the rational numbers $\mathbb{Q}$ to the real numbers $\mathbb{R}$. The essential point of this completion was not that some "ideal" numbers were "added" to an already existing domain of numbers solely to ease calculations but rather that the completed relational system $\mathbb{R}$

[21] Cf. *ibidem*, chapters II and III

of real numbers provided us with a new conceptual perspective to conceive more clearly the conceptual essence of the rational numbers $\mathbb{Q}$ themselves[22].

Idealization as completion is not restricted to algebra. Rather, it is a method that pervades all of mathematics, particularly geometry. Until the beginnings of the 19^{th} century, it might have been justified to conceptualize the domain of geometry as an unalterable sphere of ideal objects such as ideal points and ideal lines. Since this time, however, it became increasingly evident that Euclidean geometry was less than perfect and ideal. Seen from a mathematical perspective, it could be said to have certain conceptual defects that call for fixing. To formulate it in a somewhat paradoxical way, too many theorems one wanted to be true turned out not to be true. Perhaps the simplest example was provided by projective geometry of the plane. From a mathematical perspective, it had long been known that between points and lines there existed a certain useful *duality*: for a given theorem, it was occasionally possible to obtain a new theorem by switching the terms "point" and "line". For instance, given the proposition that every two points determine a single line, the dual proposition is that every two lines determine a point by their intersection. Or, a triangle could be defined by its three vertices as well as by its three intersecting sides.

Unfortunately, in Euclidean geometry a dual of a theorem is not always a theorem. For instance, although two points always determine a unique line, two lines do not always determine a point because two parallels do not intersect. The method of ideal elements helps fix deficiencies of this kind. By introducing new "ideal points" located on a new "ideal line" renders the originally incomplete duality perfect. Thereby, Euclidean geometry is conceived as a part in the more complete realm of projective geometry.

In physics, idealization as the introduction of ideal elements

[22] Cf. E. Cassirer, *The Philosophy of Symbolic Forms*, III, New Haven 1955, p. 392

functions in a quite similar manner. A typical example is the physical representation of the motion of material bodies:

> Motion, in the universal scientific sense, is nothing but a certain relation into which space and time enter. Space and time themselves, however, are assumed as members of this relation not in their immediate, psychological and "phenomenal" properties, but in their strict *mathematical* meaning. [...] [Motion] demands the continuous and homogeneous space of pure geometry as a foundation; continuity and homogeneity, however, never belong to the coexistence of the sensuous impression itself, but only to those forms of manifold, into which we constructively we transform it by certain intellectual postulates. In this way, from the very beginning motion is cast in a conceptual framework[23].

Quite generally, scientific concepts have no direct sensuous realizations. This intertwinement of "factual" and "theoretical" elements is characteristic of theories of modern science[24]. Briefly, motion is a fact of conception, not of perception. It is important to note, however, that the idealizing method of empirical science should not be conceived simply as a replacement of the directly observable experiences by their ideal limit cases. This would suggest that the objects empirical science is dealing with are in line with the objects of perception. Thereby, idealization could be characterized as a continuation of empirical observation.

Cassirer emphatically insists that this is not the case. The ideal elements to be introduced are not just some other things that we "add" to the domain of "real" things. Rather, the "ideal" things express certain ways we address the "real" things. Take,

[23] E. Cassirer, *Substance...*, p. 118.

[24] For empiricist currents of philosophy of science, the employment of advanced mathematics in all areas of science presents a conceptual difficulty because, according to them, scientific concepts have only the task to reproduce the given facts of perception in abbreviated form (*ibidem*, p. 148). If this were really the case, the task of the philosophy of physics would be achieved if every concept of a physical theory had been dissolved into a sum of perceptions such that this sum could be used to recover the full realm of empirical facts falling under that concept (cf. *ibidem*, p. 151). However, such a replacement of mathematical concepts by perceptual or observational concepts is impossible.

for instance, points of physical or geometrical space. In "reality", one never meets points. They are idealizing constructs. From a strictly empirical perspective, points appear to be rather contrived entities. It would be too simple, however, to consider them just as convenient fictions that "somehow" play the role they are assumed to play. This would amount to a strict separation between the domain of empirical reality on the one hand and the domain of mathematics on the other hand. As a consequence of such a separation, it would become impossible to bring them together again by somehow establishing a link between them by stipulation. This aporetic dichotomy can be avoided by conceiving of physical concepts as a continuation of mathematical concepts.

Although the processes of concept formation in mathematics and physics are similar, they are not identical. Roughly, conceptualization in mathematics may be conceived of as a "finite" version of the more open conceptualization in physics:

> In contrast to the mathematical concept, however, in empirical science the characteristic difference emerges that the construction which within mathematics arrives at a fixed end, remains in principle *incompletable* within experience[25].

"Ideal gases", "perfect fluids" and their relatives are not approximated by the more or less homogenous gases or the more or less ideal fluids found in nature. Rather, idealizing concepts such as perfect gases or perfect fluids play an epistemological role. They provide conceptual perspectives that allow the formulation of general relational laws and thereby they help make sense of reality as a manifold of experiences. The indispensability of idealization for scientific knowledge entails that the factual and theoretical components of scientific knowledge cannot be neatly separated. In a scientific theory, "real" and "non-real" components are inextricably interwoven. This enta-

[25] E. Cassirer, *Substance…*, p. 254.

ils that no single concept is confronted with reality but with a whole system of concepts.

The line between mathematics and physics should thus not be drawn such that mathematics is characterized as a realm of ideal objects while physics is confined to the sphere of non-ideal empirical objects. Both realms are soaked with idealizations. Although idealizing in the empirical realm is more open than in mathematics, in principle "the same syntheses govern both realms". Hence, to understand what role idealizations play for scientific knowledge "the eye of philosophy must be directed neither onto mathematics nor to physics; it is to be directed solely onto the connection of the two realms"[26].

Furthermore, the philosophical task of elucidating the role of idealizations in science and mathematics cannot be tackled in an a priori manner. Philosophy does not know on "a priori" grounds which idealizations are admissible and which are not. Rather, the issue of idealization has to be studied "empirically". This means that the philosophy of science has to study the history of the formation of scientific concepts to accomplish its mission, namely, to explicate and elucidate the process of the conceptual evolution of science in which ever new forms of idealizations emerge. That is, Cassirer's critical idealism subscribed to a naturalist and historicist conception of philosophy of science.

For Cassirer, one of the most important events in the modern conceptual evolution of mathematics and the sciences was the emergence of the concept of function and, more generally, the rise of relational mathematics. According to him, a philosophical understanding of these developments was required to go beyond the confines of traditional (syllogistic) logic taking but requires modern relational logic[27]. The basic reason for the superiority of relational logic over traditional syllogistic logic was

[26] E. Cassirer, *Kant…*, p. 48.
[27] Cf. E. Cassirer, *Substance…*, p. 21.

that it offered an adequate conceptual framework for dealing with the core concept of modern science and mathematics, to wit, the concept of function. Every mathematical function represents a possible universal law, which embraces all the particular cases for which it holds. Nothing is lost, so to speak, when moving from particular cases f(a), f(b), …. to the general functional concept f(x) because all particular cases can be recovered from the general functional rule f(x).

Elementary examples of unifying scientific concepts in this sense are mathematical formulas that describe arithmetical series such as 1, 3, 6, 10,... For such a series, the "construction of unity" is provided by a formula that describes their generation according to some general law. For instance, the series 1, 3, 6,... is characterized by the law that the difference of the differences of its members is always 1. This fact is succinctly expressed by the formula $a(n) = n(n + 1)/2$, $n \in N$. The members of such series do not have a common property (in any ordinary sense of property) but are to be conceived as cases of common functional laws. Consequently, Cassirer considered the formulas of mathematics, physics, and chemistry as paradigmatic examples of scientific concepts because they brought singular facts into a lawful context[28]. Algebraic equations of geometrical curves such as $x^2/a^2 + y^2/b^2 = 1$ provide slightly less elementary examples. These conic equations can be used for describing the movements of material bodies. More precisely, they are conceptual devices for embedding the individual perceived positions of a body in a continuous, even, smooth trajectory. Continuity, however, is a highly theoretical concept. This shows again that the embedding of singular data into a theoretical context such as a continuous or smooth trajectory presupposes several idealizing assumptions.

Functions enabled modern science to conceive particular facts as special cases of general laws. An elementary example for

[28] Cf. *ibidem*.

this achievement was the embedding of the isolated positions of the planets in continuous orbits. A more complex example of a functional concept in the natural sciences that played a key role for Cassirer's functional approach was the concept of energy[29]. According to Cassirer, the concept of energy does not appear as a new object alongside the already known physical objects such as light and heat, electricity and magnetism, but it signifies only an objective lawful correlation in which all of these concepts stand. The meaning of the concept of energy resides in the equations it establishes between different kinds of events and processes. Energy in the sense of modern science is not an object in the traditional sense but a unifying perspective on a manifold of experiences. This is rendered most evident by the functional identity of potential and kinetic energy through which states are identified with temporal processes:

> The two [kinds of energy] are the "same" not because they share any objective property, but because they can occur as members of the same causal equation, and thus can be substituted for each other from the standpoint of pure magnitude[30].

Thus, the concept of energy is not to be understood as the image of something empirical "out there"; rather, it is to be conceived as an order-generating principle. In this respect, it resembles the notion of number by which we make the sensuous manifold of isolated values unitary and uniform in conception[31]. The validity of physical concepts such as mass, force, or energy does not reside simply in the fact that they faithfully describe experiences already made but that they offer perspective for future experiences:

> [Scientific] concepts are valid, not in that they copy a fixed, given being, but in so far as they contain a plan for possible constructions of unity, which must be progressively verified in practice, in

[29] Cf. *ibidem*, p. 4.
[30] *Ibidem*, p. 199.
[31] Cf. *Ibidem*, p. 189.

> application to the empirical material. […] We need, not the objectivity of absolute things, but rather the objective determinateness of the method of experience[32].

"Number" is a concept of mathematics, and "energy" is a concept of the empirical sciences. If both can be characterized as order-generating principles in essentially the same manner, this is a strong argument in favor of the sameness thesis ST. The concept of energy shows that in modern science the allegedly objective "things" of common sense and traditional metaphysics are replaced by a net of mathematically formulated relations that yield objectivity to scientific knowledge. Thereby, the notorious Kantian "things-in-themselves" can be dispensed with:

> We need not the objectivity of absolute things, but rather the objective determinateness of the method of experience[33].

Characterizing scientific knowledge by idealizing functional relations reveals that it does not aim at a description of how the world "really is". The concepts of modern science are not the mental images of certain pre-existing objects; rather, they are tools that offer new unifying perspectives[34]. Functional concepts help establish order in the ever-changing stream of sensations. They do this not by collecting common properties of ready-made objects but by establishing idealizing relational laws between the limiting concepts that arise from the data of sensations. For the paradigmatic case of physical motion, this can be succinctly expressed by the dictum "Motion is not a fact of sensation, but of thought; not of »perception«, but of »conception«"[35]. This is, of course, not to be understood as the simplistic claim that physical motion and other theoretical concepts are merely "mental"[36]. Rather, it evidences

32 *Ibidem*, p. 322; E. Cassirer, *The Philosophy…*, III, p. 476.
33 E. Cassirer, *Substance…*, p. 199.
34 Cf. E. Cassirer, *The Philosophy…*, III, p. 367.
35 E. Cassirer, *Substance…*, p. 121.
36 Cf. S. Haack, *Realisms…*, p. 70.

> that in the actual structure of science a peculiar interweaving and mutual interpenetration of theoretical and factual elements prevails and calls for a logically clearer expression of the relation between principle and fact"[37].

Indeed, a most extensive chapter of SF is dedicated is to the elucidation of this "peculiar interweaving and mutual interpenetration of theoretical and factual elements" in physics[38]. I would like to contend that this chapter of SF belongs to the most original pieces of Cassirer's critical idealism, although, regrettably, it has been rather neglected in the secondary literature to date. Cassirer was well aware that the "mutual interpenetration of theoretical and factual elements" in physical theories entailed a holistic theory of meaning:

> We do not have physical concepts and physical facts in pure separation, so that we could select a member of the first sphere and enquire whether it possessed a copy in the second; but we possess the "facts" only by virtue of the totality of concepts, just as, on the other hand, we conceive the concepts only with reference to the totality of possible experience[39].

4. The Entanglement of the Real and the Ideal

Cassirer, in contrast to most modern accounts of idealization, did not propose simply to supplement our ontology of real things with some ideal counterparts: ideal gases, perfect fluids and the like are not to be conceived of as objects of some ideal world that is somehow analogous to the real world. Ideal gases are not "somehow" approximated by the more or less homogenous gases or the more or less ideal fluids that are among the objects of the real world. Rather, those idealizing concepts have an epistemological role. They serve as conceptual perspectives that allow the formulation of general, i.e., unifying relational

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[37] E. Cassirer, *Substance…*, p. 130.
[38] *Ibidem*, chapter IV *The Concepts of Natural Science*, pp. 112–233.
[39] *Ibidem*, p. 147.

laws. Thereby, they help make sense of reality as a manifold of experiences (in Kant's sense). The theoretical unification of the scattered data of sensations is an embedding of an incomplete empirical manifold of sensations in a completed conceptual manifold. It should be noted that according to Cassirer there may be such distinct unifying embeddings. Thus, in contrast to Kant, for Cassirer there were no fixed forms that determined how the idealizing processes of completion had to be carried out. Rather, the ever-growing variety of conceptual completions of our experiences is revealed in the historical evolution of science itself.

This entails that the relational laws of modern science do not deal directly with the perceptual data. Rather, the scientific representation of the world is grounded on a wealth of idealizations in which the indefinite empirical data are replaced by strict conceptual limits. This feature of scientific knowledge is often expressed by the assertion that the idealized theories of science do not apply to the actual world but to some mysterious "ideal worlds". Cassirer vigorously protested against such a reification of idealizations:

> [The] ideal concepts of natural science affirm nothing regarding a new realm of separate absolute objects, but they only want to establish the inevitable, logical lines of direction, by which alone complete orientation is gained within the manifold of the phenomena. They only go beyond the given, in order to grasp more sharply the systematic structural relations of the given[40].

Thus, Cassirer's functional account of idealizations is in stark contrast with many contemporary accounts of idealization, e.g., with Leszek Nowak's "(supra-) realism" with respect to ideal objects. Nowak boldly contended:

> [A]ll our idealizational "constructs" are not constructs but true descriptions of some existing ideal worlds. […] As it were, we are

[40] *Ibidem*, p. 128.

> unable to theoretically invent something which would not hold nowhere, in no world[41].
> […]
> Our thinking consists only in finding some thing that holds somewhere, in some world. And the idealizational thinking straightforwardly falls under this rule[42].

For Cassirer, Nowak's account of idealization amounts to an overstated idealism that does not understand the complex relation between the real and the ideal. For any kind of idealist philosophy of science that is minimally "critical" and does not endorse the desperate thesis that "everything is mental" and, therefore, that "anything goes", the issue of elucidating how the ideal and the empirical are related is of crucial importance. A simple juxtaposition of the ideal and the real, as Nowak's "supra-realism" proposes, does not do justice the aforementioned "peculiar interweaving and mutual interpenetration of theoretical and factual elements".

As a starting point for the elucidation of the complex relation between the theoretical and the real, Cassirer adopted in *SF* the vivid exposition that Karl Pearson had given in *The Grammar of Science*[43] for the role of ideal concepts in Newtonian mechanics. Following Pearson, Cassirer noted that in Newtonian mechanics (or in any other theory of physics)

> the "rigid" body of pure geometry has to be substituted for the perceptible body and its limitless changeability, if the grounding of the exact theory of motion is to be accomplished. […][44]
> […] As [Pearson] explains, it is never the contents of perceptions as such that we can use as foundations for the judgments of pure mechanics, as points of application in the expression of the laws of motions. Rather, all these laws can only be asserted with meaning

[41] L. Nowak, *Antirealism, (Supra-)Realism and Idealization*, "Poznan Studies in the Philosophy of the Sciences and the Humanities" 44 (1995), p. 236.
[42] *Ibidem*, p. 238.
[43] Cf. K. Pearson, *The Grammar of Science*, New York 1911 (2007), p. 198 ff.
[44] E. Cassirer, *Substance…*, p. 120.

> of the ideal limiting structures which we conceptually substitute for the empirical data of sense-perception[45].

Subscribing to the sameness thesis ST, the philosophical problems related to idealization i.e., to the constitution of theoretical limiting concepts are not confined to empirical concepts; they already arise for mathematics, where limiting concepts also play an essential role, as is made particularly evident by modern calculus. This did not entail, as Cassirer emphasized, that all mathematicians really understood the proper nature of limiting concepts.

Cassirer's target in this issue was the account of the role of ideal limiting concepts in science that the German mathematician Paul du Bois Reymond had put forward in *Die allgemeine Functionentheorie. Metaphysik und Theorie der mathematischen Grundbegriffe* (1882)[46]. Du Bois Reymond claimed that the concept of limit, although being indispensable for modern mathematics, particularly for infinitesimal calculus, led to a problem that could not be solved according to strict objective criteria but according to the subjective inclination of the individual scientist. Thereby, we are confronted with a dilemma that Cassirer described as follows:

> When we raise the question whether there exists an exact limit […] to the figures of a decimal fraction, such that the limit possesses the same existence as the members of the sequence themselves, the answer we give cannot be clearly determined by logical and

[45] K. Pearson, *The Grammar…*, p. 198 ff.

[46] Paul du Bois Reymond was the younger brother of the physiologist Emil du Bois Reymond, who was not only a famous scientist but also an influential intellectual of the *Kaiserreich*; in 1872 he made a famous speech *On the Limits of Our Knowledge of Nature* (*Über die Grenzen des Naturerkennens*) before the Berlin Academy of Sciences outlining seven "world riddles" some of which, he declared, neither science nor philosophy could ever solve. This thesis sparked a heated debate in the intellectual circles of Germany that lasted well into the 1930s. Following his brother Emil, the mathematician Paul du Bois-Reymong contended that the problem of understanding the mathematical concept of limiting concept (*Grenzbegriff*), was a similarly unsolvable "riddle" as those that had his brother exhibited in his speech.

> mathematical considerations alone. […] We must choose between these two views of the world: either with empiricism we must assume as existent only what can be pointed out as an individual in the real presentation, or with idealism, affirm the existence of structures which constitute the intellectual conclusion of certain series of presentations, but which can never themselves be directly presented[47].

Both alternatives are somewhat unappealing: either one subscribes to an overly strong "metaphysical" idealism that believes in the robust reality of imperceptible and unimaginable strict limits of our concepts, or one has to be content with an untenable extremally austere empiricism that does not allow to make sense of the role mathematics plays in our empirical knowledge because it accepts as real only what can be perceived. Cassirer argued that this dilemma, which Paul du Bois-Reymond claimed to be unavoidable, as in the "world riddles" of his brother Emil, arose from the fact that Paul du Bois-Reymond relied on erroneous (overstated) concepts of idealism and empiricism. In contrast, this dilemma did not appear for critical idealism, or so he argued.

Instead of conceiving idealizing concepts as necessary tools for the logical interpretation and mastery of the manifold of sensations as critical idealism does – du Bois-Reymond's account transforms them into mysterious realities behind the phenomena[48]. Critical idealism, conceiving ideal objects functionally as "logical lines of direction of orientation" or perspectives for future possible experiences, does not fall prey to this temptation. Ideal objects make sense only as ingredients of the activity of idealizing. Hence, the ontological extravagance of assuming a platonic universe of ideal objects can be avoided:

> [T]he existence of the ideal, which can alone be critically affirmed and advocated, means nothing more than the objective logical necessity of idealization[49].

[47] E. Cassirer, *Substance…*, p. 123.
[48] Cf. *Ibidem*, p. 127.
[49] *Ibidem*, p. 129.

This kind of idealization should be acceptable also for a sufficiently sophisticated, i.e., "critical", empiricist because without it the world of perception would not be merely a mosaic but a true chaos. It is a mere misunderstanding when the empiricist of du Bois-Reymond affirms that he does not recognize that the absolutely straight line and the absolutely exact plane exist but only more or less straight lines, more or less exact planes, for this very discrimination of different stages of exactitude presupposes comparison with the exact idea, whose fundamental function is thus here confirmed throughout. The "being" of the idea, however, consisted in this function and needs no other support and no other proof.

I do not contend that Cassirer's elucidation of "the complex interweaving and penetration of the real and the ideal" in SF is fully satisfying in every respect, but at least his account points at some interesting features of the role of idealization in empirical knowledge that have been largely ignored in most modern accounts of this characteristic of modern science.

5. Seven Basic Tenets of Critical Idealism

Let us take stock of and formulate the essentials of Cassirer's idealist philosophy of science succinctly in the following seven points:

(1) Scientific knowledge does not cognize objects as ready-made entities. Rather, knowledge is organized objectually in the sense that in the continuous stream of experience, invariant relations are fixated.

(2) The unity of a concept is not to be found in a fixed group of properties but in the rule that lawfully represents the diversity of experiences as a sequence of elements. The meaning of a concept depends on the system of concepts in which it occurs. It is not completely determined by one single system but rather by the continuous series of systems unfolding in the

course of history. Scientific knowledge is a "fact in becoming" (*Werdefaktum*).

(3) Concepts and conceptual systems do not yield more or less accurate pictures of reality. Rather, scientific concepts provide guidelines for the conceptualization of the world; they are blueprints for possible experiences.

(4) Factual and theoretical components of scientific knowledge cannot be neatly separated. In a scientific theory, "real" and "non-real" components are inextricably interwoven.

(5) No single concept is confronted with reality but with a whole system of concepts.

(6) Our experience is always conceptually structured. There is no non-conceptually structured "given". Rather, the "given" is an artifact of a bad metaphysics.

(7) The concepts of mathematics and the concepts of the empirical sciences are of the same kind, namely, they are relational, idealizing concepts.

The theoretical unification of the scattered data of sensations can be described as an *embedding* of an *incomplete* empirical manifold of sensations in a *completed* conceptual manifold of possible experiences. In contrast to Kantian orthodoxy, however, for critical idealism there are no fixed forms that determine how this process is to be carried out. Rather, the ever-growing variety of conceptual completions of our experiences is revealed in the historical evolution of science itself.

6. Cassirer's Critical Idealism in the Philosophy of Science after 1945.

While the emigration of the European logical empiricists to the Anglo-Saxon world may be considered – by and large – as a success story for this philosophical movement, the various currents of Neo-Kantianism that existed in the early decades of the last century in Germany and other parts of Europe did not find a continuation in the New World and fell into almost complete oblivion after 1933. It was only for a short period of time that it seemed that Cassirer's philosophy could escape this fate. For instance, in 1943 the historian of philosophy Harold R. Smart published in *Philosophy of Science*, then arguably the leading journal of the discipline, a paper in which he praised in the highest tones Cassirer's achievements in the philosophy of science:

> The importance of Cassirer's extensive contributions to the philosophical comprehension of mathematics and mathematical physics is generally recognized to be second to none[50].

For a few years, this high reputation survived Cassirer's untimely death in 1945, as is evidenced by his granting of a volume in the prestigious *Library of Living Philosophers* series (which some consider as the equivalent of the non-existing Nobel award in philosophy) in 1949. Indeed, Cassirer was the first and only dead philosopher to be so honored. However, his star faded away soon after. Less than ten years later, John Passmore in *A Hundred Years of Philosophy* (1956) openly denied Cassirer the status as a philosopher, "as that word is now commonly understood by British philosophers". Instead he characterized him condescendingly as a "recalcitrant metaphysician"[51]. In the following decades, Cassirer as a philosopher of science almost completely disappeared from the American and European scenes[52].

In the last twenty years or so, contemporary philosophy has gradually rediscovered Cassirer as an important scientific

50 H.R. Smart, *Cassirer*..., p. 167.

51 J. Passmore, *A Hundred Years of Philosophy*, London 1966, p. 318.

52 Telling evidence for Cassirer's fall into oblivion soon after 1945 was the lack of interest in his *Nachlass*. His widow Toni Cassirer had given it soon after his death to the university of Yale where it remained in the Beineke Rare Book and Manuscript library till 1989. Only then the project was started to properly archive the texts, notes, and manuscripts that he had left behind. Due to the labor of the late John Michael Krois and his many co-workers this undertaking now has not resulted in a lavish critical edition of Cassirer's Nachlass in more than 20 volumes.

philosopher – beyond the cliché of a philosopher of culture. Recent research in the history of the philosophy of science has revealed ever more clearly the important role Cassirer played in the evolution of the modern European philosophy of science, particularly as a critique and discussion partner of the logical empiricism of the Vienna circle, including, in particular, Schlick, Carnap, and Reichenbach[53].

More recently, the Anglo-Saxon debate on scientific realism took notice of Cassirer as a precursor of what today is called structural realism[54]. It must be said, however, that the reception of Cassirer's idealism in this area of philosophy of science has remained incomplete and sketchy. His functional idealism went well beyond the various contemporary currents of structural realism. Indeed, the idealist foundations of his alleged structural realism have hardly drawn notice. In sum, a comprehensive and balanced reassessment of Cassirer's idealist philosophy of science is still pending.

Idealizm w filozofii nauki: przypadek „krytycznego idealizmu" Ernsta Cassirera

Streszczenie

W artykule zarysowano koncepcję filozofii nauki Ernsta Cassirera oraz jej ewolucję na przestrzeni XX wieku. Cassirer (1874–1945) rozpoczął swą filozoficzną karierę jako przedstawiciel marburskiej szkoły neokantowskiego idealizmu zapoczątkowanego przez H. Cohena oraz P. Natorpa. W tym kontekście określał siebie zwolennikiem idealizmu krytycznego (czy też logicznego). Główna teza idealizmu Cassirera dotyczy relacji matematyki i fizyki. Według tego autora filozofia nauki nie powinna skupiać się ani na matematyce (jako nauce idealnej, teoretycznej), ani na fizyce (jako nauce empirycznej). Zadaniem filozofii

[53] Cf. for example M. Friedman, *Reconsidering*….

[54] Cf. B. Gower, *Cassirer*…, A. Cei, S. French, *On the Transposition*…, J. Ladyman, D. Ross, *Every Thing*…, S. French, *The interdependence*….

nauki winno być natomiast badanie wspólnego źródła obydwu tych nauk. Cassirer identyfikuje je pod postacią metody idealizacji, czyli wprowadzania obiektów idealnych.