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Synthetic Geometry and *Aufbau*

1. Introduction

From antiquity to the beginnings of the 20th century philosophers took geometry as a paradigmatic example of science. Geometry defined what was to be considered as scientific knowledge. "More geometrico" was a sign of quality for philosophical and scientific argumentation. Philosophy and science had to make a great conceptual effort to get rid of this overwhelming and sometimes depressing epistemic ideal of geometry. Today this aim has been achieved to a large extent. Geometry as a philosophical topic is of secondary importance at most. Certainly, it does not occupy centre stage in the contemporary discussion of epistemology and philosophy of science. Not even in philosophy of mathematics, geometry is considered as a hot topic. For instance, in Tymoczko's (already somewhat dated) anthology *New Directions in the Philosophy of Mathematics* (Tymoczko, 1985) no contribution deals with geometry in its classical or modern form. A priori there is no need to deplore this situation, geometry may belong to those topics that for good reasons are no longer on the agenda of contemporary philosophy. Be this as it may, this situation is markedly different from that of the beginning of the 20th century. For philosophers such as Russell, Cassirer or Carnap, to name but a few, the philosophical problems posed by geometry played a central role in their investigations. It may be sufficient to mention just a few bio-bibligraphical facts: (i) Russell started his philosophical career in 1897 with the dissertation *The Foundations of Geometry*. Somewhat later he published *The Foundations of Mathematics* in which he treated geometry at great length; (ii) In *Substanzbegriff und Funktionsbegriff* Cassirer dedicated a central chapter to the topic of concept formation in geometry, considered by him as a paradigmatic case for concept formation in science in general (Cassirer, 1985 (1910)). Throughout his life, he considered *Klein's Erlanger Programm* as a guideline for his "critical idealism"; (iii) Carnap started his philosophical career with the dissertation *Der Raum, Ein Betrag zur Wissenschaftslehre* (Carnap, 1922). Although this work was largely ignored by Carnap-scholars, it is an important early work that contains the germs of many of the ideas to be unfolded and elaborated later.

Der Raum is a rather eclectic work dealing with a lot of topics, often only touching the problems and not giving them an in-depth treatment. In a first

T. Bonk (ed.), Language, Truth and Knowledge, 00–00.

approximation, it may be characterized as a Neo-Kantian work although the influences of Husserlian phenomenology and Russellian logical constructivism cannot be overlooked. In this paper, I do not aim at the probably rather difficult task to give a comprehensive interpretation of it. Rather, I'd like to concentrate on one specific topic which was to play a crucial role in Carnap's philosophical thought as a whole. This topic is concerned with the role of geometry for what later was to become Carnap's theory of constitutional systems. More precisely, I'd like to put forward the following thesis:

Geometry, as synthetic geometry, was an important source of inspiration of Carnap's philosophical thought. The constitutional theory of *Der Logische Aufbau der Welt* is inspired, to a large extent, by the relational systems of synthetic geometry treated in *Der Raum*. Moreover, since the *Aufbau* program was decisive for Carnap's philosophy in general, geometry had a substantial influence on his philosophy in general.

This thesis may be generalized: Geometry was of utmost importance not only for Carnap, but for the epistemology and philosophy of science of the early 20th century in general. Not only Logical Empiricism, but also rival philosophical currents such as the Marburg Neokantianism, or Husserl's phenomenology were deeply engaged in wrestling with the problems geometry presented to philosophy.

At that time, philosophers used to have a much richer idea of geometry than today. The philosophy of geometry was not restricted to the worn out topic of Euclidean vs. Non-Euclidean geometry. For whatever reasons, since then the philosophical community has suffered a sort of amnesia with respect to geometry. If, as recently has been claimed by various authors, a more profound understanding of the philosophy of the beginings of the 20th century is crucial for contemporary epistemology and philosophy of science, and if geometry is an essential ingredient in the philosophy of that period, then this amnesia is a serious obstacle for contemporary philosophy. Philosophers will have to make some efforts to regain the lost territory of geometry for the philosophical discourse of the 21st century.

Now, let us be more specific and concentrate on the role geometry played in Carnap's philosophy, particularly in the *Aufbau*. With respect to this topic I contend that the constitutional theory of the *Aufbau* follows the patterns of the construction (or constitution) of geometric systems in synthetic geometry. The fact that essential motives for the constitutional theory of the *Aufbau* can be found in synthetic geometry is not only of interest for the history of philosophy. A better understanding of the geometric backdrop of the *Aufbau* may help defuse some of the criticisms of the feasiblity of the *Aufbau* program. Many of them are based on misunderstandings or neglect the geometric background of the *Aufbau*. Two examples are the influential objections of Goodman (Goodman, 1963, 1977) and Quine (Quine, 1951) against the *Aufbau* program.

The outline of this article is as follows: To set the stage, in the next section *Synthetic Geometry as a Theory of Ordnungsgefüge* the basics of synthetic geometry are recalled in an informal way. In section 3 *Conceptual Geometry in Der Raum* it is argued that the roots of what was to become in the *Aufbau* the theory of constitutional systems can already be found in *Der Raum* under the heading of a geometrical theory of "conceptual geometries." "Conceptual geometries" may be considered as the earliest (and somewhat primitive) forerunners of what was to be called constitutional systems and, later, linguistic or ontological frameworks. At this early stage the geometric origins of these systems are still quite visible while later they tend to be obscured. The subsequent development of conceptual geometries is manifest in a number of unpublished manuscripts written in the interim period between *Der Raum* and *Aufbau*, in particular in *Quasizerlegung* – which is dealt with in section 4. In this section it is shown how Carnap's famous – or notorious – method of quasianalysis fits into the framework of synthetic geometry. In section 5 *From Synthetic Geometry to Constitutional Theory* it is argued that some of the objections against the feasibility of the *Aufbau* program are unfounded since they are based on inadequate understanding of the geometrical background of this work.

2. SYNTHETIC GEOMETRY AS A GENERAL THEORY OF *ORDNUNGSGEFÜGE*

Having lamented about the amnesia contemporary philosophy suffers with respect to geometry and its relevance for philosophy it would be expedient to go on with a refresher course "Synthetic geometry for philosophers." For obvious reasons, this is not possible. Instead, I'd like to explicate one leitmotif of synthetic geometry from which two characteristics of the synthetic-geometric thought emerged that became crucial for Carnap's "constitutional theory in the spirit of synthetic geometry."

The leitmotif of synthetic geometry is order. As Carnap put it, geometry is a general theory of *Ordnungsgefüge.* The expression *Ordnungsgefüge* is not a terminus technicus in mathematics. It seems that Carnap understood the term in a semi-technical sense intended to mean something like "relational structure" or "structured set." By conceiving a domain as an *Ordnungsgefüge* one imposes an order, or structure on it. This is achieved by certain stipulations of order ("Ordnungssetzungen").

Characterized as a general theory of *Ordnungsgefüge* synthetic geometry is conceived of as having a strong applicative dimension. That is to say, the *Ordnungsgefüge* synthetic geometry is dealing with are to be applied to many different cases. In *Der Raum* Carnap explains this fact for projective geometries at great length. Synthetic geometry as a general theory of *Ordnungsgefüge* offers an arsenal of possible conceptual schemes or perspectives applicable to many domains. This applicative dimension renders it deeply pluralistic: It goes without saying that no single structure can cope with all applications. Rather, the point of geometry is to study the variety of different geometries.

Mathematicians have been clearly aware of the pluralistic character of the new synthetic geometry. Early in the 19th century they had begun to study "geometries" that were quite remote from anything common sense would have considered as "geometrical." For instance, an important branch of synthetic geometry deals with "finite geometries" characterized by the fact that these structures have only finitely many points and lines. Obviously, these geometries are not candidates for the "correct" geometry of the world. In a similar vein, *Klein's Erlanger Programm* leaves behind the alternative Euclidean vs. Non-Euclidean geometry. This does not mean that it succumbed to a shallow relativism according to which any geometry is as good as any other. Rather, it aimed at a more profound understanding of a generalized geometric thought which conceptualizes a geometrical system as located in a complex logical space of possible geometries. The leitmotif of a pluralist theory of possible (geometric) orders leads to two characteristic features of the synthetic geometry:

(1) Space in the sense of synthetic geometry is a general term which comprises many different spatial structures. Geometry has to study all of them without blinders to single one as the "true" one. In this sense geometry is abstract, not in that it is remote from applications.

(2) Synthetic Geometry is relational. Its objects are determined by a net of implicit relational definitions. The ontological status of any geometric object is determined by its relational position within a certain relational system.

External ontological determinations play no role anymore. For instance, the classical Euclidean definition "a point is what has not parts," does not make sense in the framework of synthetic geometry. Rather, a point is that entity to which the functional role of a point is ascribed. This relational character of the objects of synthetic geometry is evidenced most clearly by the famous principle of duality: Assume S to be a geometric system in which points and lines are related to each other in a certain way. Then, "automatically" there is a geometric system S* in which the points of S play the functional role of the lines in S* and the lines of S correspond to the points of S*. In other words, "points" and "lines" can only be explicated by each other: One cannot know the points of a geometric system without knowing its lines, and vice versa.

Conceptualizing geometry as a general theory of *Ordnungsgefüge* gives order stipulations a crucial role. Thus, it is expedient to deal with this concept in some detail. For this purpose, let us look at the following chaotic heap of points which hopefully does not exhibit any kind of inherent order for the naked eye:

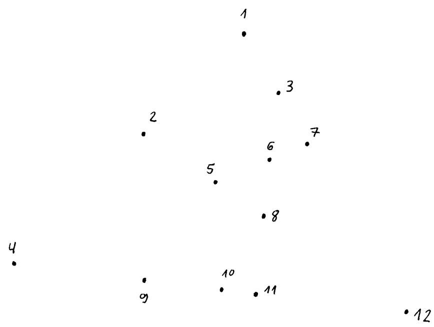

Figure 1.

What it means to establish order in this case shows the following diagram:

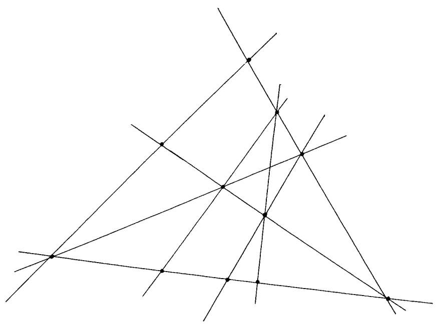

Figure 2.

To put it bluntly, the entities that establish a geometric order are lines. It is surprising that this simple idea, the imposition of order by lines, is sufficient to constitute all concepts of geometry. That is to say, points and lines are the basic building blocks for all other geometrical concepts. Of course, one has to subscribe to a rather general concept of lines in order that they can play this almost universal role of *Ordnungssetzungen*. That is to say, lines in the sense of synthetic geometry need not look like lines we are accustomed to, they have to function as lines. To rely once more on the elementary example above, there is no reason, from the point of view of synthetic geometry, to insist that lines are straight in the sense of common sense perception. One may well use other, non-straight "lines" as *Ordnungssetzungen* (cf. *Der Raum*, p. 16ff).

The upshot of all this is the following: a geometric system in the sense of synthetic geometric may be defined as a set P of points *p*, *q*, ... and set L of lines *m*, *k*, ... that are related to each other in certain ways. Formally this is described by an incidence relation I. Synthetic Geometry, then, is the theory of incidence structures $I \subseteq P \times L$. The incidence relation I determines which points are related to which lines. Intuitively stated, it determines which points are *on* which lines: $(x, m) \in I$ is to be interpreted as the fact that in the geometric system defined by I, the point x s on the line m. In the same vein, two lines m and k are said to be said to intersect if and only if there is a point x, which belongs to both of them, i.e. the ordered pairs (x, m) and (x, k) are elements of I. Depending on the axioms imposed on I different types of geometric systems are obtained. Probably the most important ones are affine and projective systems. To consider an extreme example, consider the *Fano plane* consisting of 7 points and and 7 lines, and characterized by the fact that each line has 3 points, and each point is on three lines (Figure 3).

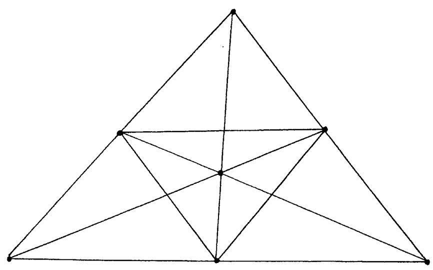

Figure 3.

This geometrical system is certainly very remote from our standard geometric intuitions. Nevertheless, it counts as a geometry in the sense of synthetic geometry.

The essence of geometry as a *general* theory of *Ordnungsgefüge* or relational spatial structures resides in the plurality of many different geometric systems and their interrelations. Here, plurality does not simply mean that instead of a single ("true") system a lot of different systems are studied. Rather, synthetic geometry as a general theory of geometric systems aims to study the complex relations between the various systems. Thus it may be characterized as a "geometry of geometries."

At this point, a remark on the relation between synthetic geometry and the general theory of relational structures may be in order. Stretching the concept, almost every relational structure may be conceived as a spatial structure "in the broad sense." Hence, it may be difficult or even impossible to draw a precise line between synthetic geometry proper and the general theory of relational structures. I think this is not necessary. Of course, everything may be treated in the more general context of relational systems, but it is often useful to maintain a more specific, geometric perspective.

3. CONCEPTUAL GEOMETRY IN *DER RAUM*

Der Raum deals with many and variegated philosophical problems that arise from geometry. It intends nothing less than to provide a base for the philosophy of geometry as a whole. Nevertheless there is a common theme to all the various strands of thought: plurality. Carnap is deeply impressed by the plurality and variety of applications of geometry. On a local level there is the plurality of different geometric systems and their applications, on a global level plurality manifests itself in the three distinct types of space corresponding to three different ways of doing geometry. According to Carnap one should distinguish between the formal, the intuitive, and the empirical space. These different types of spaces are studied by mathematics, philosophy (phenomenology), and physical science, respectively. The distinction between the disciplines which study the formal, the intuitive, and the empirical aspects of space corresponds, as Carnap asserts (cf. *Der Raum*, p. 85), to the Husserlian distinction between formal ontology, regional ontology, and factual science.

Plurality is, however, not restricted to the global level. An even more thorough-going plurality is to be found in the realm of formal geometry proper. Conceiving geometry as a general theory of *Ordnungsgefüge*, it was not too far-fetched for Carnap to consider "lines," i.e. geometric order stipulations, and "concepts" as essentially one and the same thing. This, I contend, was the basic insight of the essentially geometric character of a theory of constitutional systems (*Konstitutionssysteme*) conceived as a theory of concept systems. The first steps in the development of this idea are made in *Der Raum*. Carnap proposes to consider concepts as order stipulations in the sense of synthetic geometry. This can be spelt out in the following way: given a class P of objects (*Gegenstände*) and a class C of concepts (*Begriffe*), an incidence relation I between objects and concepts is defined by the stipulation that a concept c and an object p instantiate the incidence relation I if and only if p can be subsumed under c, or, in other words, if and only if p is a case of c. In this way we have defined a "conceptual geometry," i.e. an incidence relation $I \subseteq P\ x\ C$ that determines the relations between objects and concepts. This is a more profound analogy than one might think at first sight. In 20th century mathematics it leads to the development of lattice theory in which geometry and logic are merged in a deep and fruitful way. This topic cannot be pursued further in this paper. Instead, let us consider more closely what these conceptual geometries are. Carnap axiomatically introduces the following conceptual geometry and notes that it is isomorphic to the real projective space of three dimensions (*Der Raum*, p. 14, 16):

"Let us assume that the objects P1, P2, ... fall under the concept P such that the following conditions are satisfied: there is a concept G, under which not objects are subsumed but concepts g1, g2, g3, ... such that the following requirements are satisfied:

(1) At least three P-objects fall under any g-concept, but not all P-objects can be subsumed under one g-concept.

(2) For two different P-objects there is always one and only one g-concept under which they fall, their "common" concept.

(3) If P1, P3, P′2 fall under g1, P2, P3, P′1 under g2, and g1 and g2 are different, then there exists an object P4 that falls under the common concept of P1 and P′1 and under the common concept of P2 and P′2; moreover there is a concept g3, which subsumes P1, but no object of g2."

The first two axioms of this system of conceptual geometry are easily understood, even if they may not appear very plausible: (1) asserts that each concept comprises a sufficiently large number of cases, to wit, at least three; moreover, there is no concept that applies to all objects of the domain in question. (2) is self-explaining but certainly not particularly plausible as a condition required for concepts. The third appears as hopelessly abstruse. A picture may help to reveal its meaning:

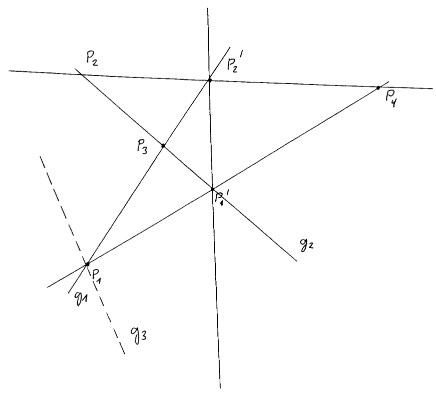

Figure 4.

At first sight this "projective conceptual geometry" may appear to be nothing more than an amusing idea devoid of any deeper meaning. This, however, would be a misunderstanding, or so I want to argue. I contend that these conceptual geometries are the precursors of the constitutional systems of the *Aufbau.* In order to collect some evidence for this claim, let us note first that according to Carnap only the structure of this system is important, not the intrinsic essence of its objects, whatever that may be. The fact that he considers conceptual geometry as just one among many other structurally equivalent examples bears this out. In *Der Raum* he mentions three, of which the first two are of particular interest for us. The first deals with a "colour system" F ("Gefüge von Farben") of the following kind:

> "Let us assume that the colours occur in certain combinations, called colour strips ("Farbstreifen"): every strip has at least three different colours, but no strip contains all colours that appear on all the other strips. Choosing two colours, there is one and only one strip, which carries both of them; it is called their carrier. [...]" (ibidem, p. 17)

And so on, that is to say, Carnap goes on to reformulate the same structural requirements he stated in the case of conceptual geometries for objects and concepts, now for colours and colour strips thereby emphasizing the structural isomorphism of the two cases. In the second example he does the same for a "judgment system" ("Urteilsgefüge") in which the role of colours is taken over by "judgments." In this way, as he points out, "not only the theory of space, but also "space" itself consists of judgments!" (ibidem, p. 17).

Carnap took the constitutional systems of the *Aufbau* and the conceptual geometries of *Der Raum* as being of the same ilk. The fact that the colour system F of *Der Raum* performs an essential role in the *Aufbau* as the basic example with the aid of which it is shown what the constitutional method of quasianalysis is and how it works (*Aufbau*, § 70, 72) supports this interpretation. Moreover, also in the intermediate manuscript *Die Quasizerlegung* of 1922/23 the guiding example by which the method of *Quasizerlegung* is explained is virtually equivalent to the colour system of *Der Raum*. More precisely, in *Quasizerlegung* (and also in the *Aufbau* as well) Carnap discusses musical chords instead of colour strips. This continuous line of examples beginning in *Der Raum,* via *Quasizerlegung* up to the *Aufbau*, furnishes strong evidence that the geometrically inspired conceptual geometries and the later constitutional theories have a common characteristic.

In the rest of this paper I'd like to substantiate this claim by a more detailed interpretation. For this purpose, *Quasizerlegung* may serve as a stepping stone. In this paper the geometric origins of Carnap's approach are still clearly visible, and, at the same time, the application of geometric structures to problems of conceptual systems is more plausible than in *Der Raum* where the geometric concept systems are fairly contrived and artificial. Obviously the projective conceptual geometries of *Der Raum* have not much to do with real conceptual systems. In *Quasizerlegung* and in the *Aufbau* more flexible and more realistic systems are introduced that are better suited for modelling constitutional systems of empirical knowledge. For instance, it is no longer assumed that conceptual geometries have to be projective ones.

4. THE QUASIZERLEGUNG OF 1922/23

The conceptual geometries of *Der Raum* treat concepts and objects on an equal footing. Thereby, a conceptual geometry has two irreducible kinds of primitive components: objects ("points") on the one hand and concepts ("lines") on the other. This is in line with the general approach of synthetic geometry. Nevertheless it is hardly plausible for conceptual systems intended to model the concepts of empirical science. In any approach of empirical knowledge the objects of knowledge have to have a certain priority. At least, objects and concepts cannot function symmetrically, as it would be the case, if one strictly followed the pattern of geometric systems for which the principle of duality holds. Hence, as models for conceptual systems of empirical knowledge, Carnap's conceptual geometries of *Der Raum* are inadaquate. Something essential is still missing to get the *Aufbau* program started. The missing link is provided by the so called *Quasianalysis* which is designed to bridge the gap between the basic objects a system and their conceptual processing in terms of concepts.

At first sight we are confronted here with an unsolvable problem, namely, the problem of structuring an unstructured "homogeneous" domain of "pointlike" objects without properties or other internal structure. Such objects cannot be conceptually structured since conceptual structuring presupposes some conceptual difference. One cannot start from a totally unstructured ("homogeneous") set of objects. Rather, any domain that is be conceptualized at all, must already be structured in some way in order to offer the cognizing agent a conceptual grip, so to speak. As a structural condition *sine qua non* Carnap proposes similarity or resemblance to serve as a minimal structure. It is the task of the constitutional method of quasianalysis to solve the problem of structuring minimally structured or "non-homogeneous" domains.

For the first time the term quasianalysis ("Quasizerlegung") appears in an unpublished manuscript of 1922/23, which has the programmatic title "Quasizerlegung – Ein Verfahren zur Ordnung nichthomogener Mengen mit den Mitteln der Beziehungslehre" ("Quasianalysis – A Method to Order Non-homogeneous Sets by Means of the Theory of Relations"). There the task of *Quasizerlegung* is described as follows:

> "Suppose there is given a set of elements, and for each element the specification to which it is similar. We aim at a description of the set which only uses this information but ascribes to these elements quasicomponents or quasi-properties in such a way that it is possible to deal with each element separately using only the quasiproperties, without reference to other elements." (*Quasizerlegung*, p.4)

As *Quasizerlegung* shows the method of quasianalysis is a purely formal method. It may be applied to any nonhomogeneous set, for instance to a set of gestaltist *Elementarerlebnisse*, as is done in the *Aufbau*, but this is not important. Any non-homogeneous set may be submitted to a quasianalysis in order to synthesize appropriate quasicomponents. First, it is important to explain more precisely what are "non-homogeneous sets." For Carnap, a non-homogeneous set is a similarity structure (E, $\sim$), i.e. a set E endowed with a binary similarity relation $\sim$. Two elements e and e* of this set related by the relation $\sim$ are said to be similar to each other. The relation $\sim$ is assumed to be reflexive – each element is assumed to be similar to itself – and symmetric – if e is similar to e*, then also e* is similar to e. The relation need not be transitive, however.

A non-homogeneous set (E, $\sim$) may be represented more perspicuously as a graph (Figure 5). Here, two distinct similar elements of E are connected by a straight line, two elements that are not similar are not directly connected by a straight line. Somewhat more generally, one may interpret a nonhomogeneous set as a relational structure, i.e. a set endowed with some basic structure, e.g. a topological structure or a similarity structure. Thus, the basic levels of conceptual systems are provided by relational structures. From this basic structure, then, the higher-order objects are constituted, analogously as in geometry all geometric objects are constructed from points and lines. Thus, the constitutional theory of the *Aufbau* may be described as the general theory that deals with the general features of conceptual constitutions in relational systems.

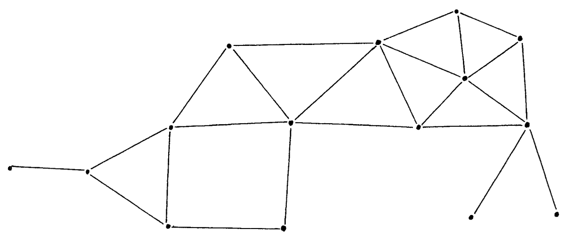

Figure 5.

As a concrete example of the abstract description of quasianalyzing a non-homogeneous set in *Quasizerlegung* Carnap discusses a non-homogeneous set of musical chords. This example is structurally equivalent with the somewhat less complicated one dealing with colours which we already know from *Der Raum*, and which appears once again in the *Aufbau*. The ascription of quasiproperties is not arbitrary, of course, but has to take into account the underlying similarity structure. How this is to be done Carnap describes in an axiomatic way as follows (*Quasizerlegung*, p. 3/4):

Definition. Let (E, $\sim$) be a non-homogeneous set. The quasicomponents (or quasiproperties) of the elements of E have to satisfy the following requirements:

(C1) If two elements are similar they share at least one quasiproperty.

(C2) If two elements are not similar they do not share any quasi-property.

(C3) If two elements are similar to exactly the same elements they have the same quasiproperties.

(C4) No quasiproperty can be removed unless (C1) – (C3) are violated.

The assignment of quasiproperties according to the requirements (C1)–(C4) is now cast in the framework of the theory of incidence structures: Denote the class of quasiproperties by Q. Then we define a relation $I \subseteq E \times Q$ by the following recipe:

$$(e, q) \in I := \text{the element } e \text{ has the quasiproperty } q.$$

the elements of E correspond to the points, and the quasiproperties, i.e. the elements of Q, play the role of lines. Analogously as in geometry lines and other geometrical objects may be considered as sets of points, in the *Aufbau* quasiproperties are considered as sets of elementary experiences. In this way, the quasianalysis of a non-homogeneous set (E, $\sim$) amounts to the construction of an incidence structure $I \subseteq E \times Q$ which fits the similarity relation $\sim$ in the sense that the requirements (C1)–(C4) are satisfied. Hence we may succinctly describe the relation between quasianalysis and synthetic geometry by the following slogan:

Quasianalysis is synthetic geometry in disguise: a quasianalysis of a similarity structure (E, $\sim$) is the construction of an incidence relation $I \subseteq E \times Q$, which fits the similarity relation by satisfying the conditions (C1)–(C4).

This conceptualization of quasianalysis has the consequence that the resources of synthetic geometry become available for constitutional theory. In principle, the *Aufbau's* "logical construction of the world" follows the constitution of the geometric systems in synthetic geometry. The task of quasianalysis, which is actually a synthetising method as Carnap remarks *en passant*, is to find for a given similarity structure, considered as the basic relational level of the world to be reconstructed, an appropriate incidence relation thereby starting the synthetisation of higher order objects analogous to that of geometry.

5. From Synthetic Geometry to Constitutional Theory

In the *Aufbau*, the geometric traces of constitutional theory have disappeared from the surface. New actors such as "gestalt psychology," "Elementarerlebnisse," "phenomenalism," the "autopsychological base," or "reductionism to the given" appear on the stage and occupy central roles, at least if one subscribes to the standard interpretations of the *Aufbau*. Complementarily, the geometric origins of the constitutional theory have become less visible, although they did not disappear totally. In particular, the geometrically motivated examples of colour strips and chords keep their essential role as intuition pumps for the constitutional theory, but Carnap no longer mentions their geometric background.

After more than seventy years since the publication of the *Aufbau*, I think it is fair to say, that, all in all, the introduction of the new "philosophical" ingredients and negligence of the original geometric motivations have obscured the true intentions of this work. I do not want to argue for this sweeping claim in this paper, rather I rely on the interpretative efforts of authors such as Friedman, Proust, or Richardson who have supplied ample evidence for this contention (cf. Friedman, 1999; Proust, 1986; Richardson, 1998). According to their interpretations the topics of phenomenalism, gestalt theory,[1] or reductionism do not lie at the heart of the *Aufbau*. Rather, the *Aufbau* was intended to exemplify a new general philosophical discipline, called constitutional theory (*Konstitutionstheorie*) which had the task of "investigating all possible forms of stepwise definitional systems of concepts" (Friedman, 1999, p. 115, *Aufbau* § 46). Carnap intended to create a scientific successor discipline of traditional epistemology and philosophy of science that remained neutral with respect to the futile metaphysical quarrels that had plagued the traditional accounts.

The "geometric account" of the *Aufbau* sketched in this paper sits well with the new interpretations. I take this as evidence that it is on the right track. Moreover, the geometric backing of the constitutional theory may provide arguments for taking up the *Aufbau*-program in a new way. In my opinion a shortcoming of many of the new interpretations is that, although they do justice to Carnap's original intentions much better than the standard empiricist interpretations, at the same time they do "relegate the *Aufbau* to the status of a monument having purely historical interest." (Goodman, 1963, p. 558). The reason is that they are not much interested in the technical details of constitutional theory, to say nothing of the more formidable task of providing a reconceptualization of the original program. There are some exceptions. Proust is one of the few who admonishes us that

> "[...] the true interest in the *Aufbau* lies not in the example of a constitutional system it offers but in the set of formal procedures that it is the function of the example to illustrate." (Proust, 1989, p. 185).

Maybe somewhat surprisingly, on this assessment, Proust essentially coincides with Goodman:

"The *Aufbau* brings to philosophy the powerful techniques of modern logic, along with unprecedented standards of explicitness, coherence, and rigor. It applies to basic philosophical problems the new methods and principles that only a few years before had thrown fresh and brilliant light upon mathematics. The potential importance to philosophy is comparable to the importance of the introduction of Euclidean deductive method into geometry. The *Aufbau* [...] is still one of the fullest examples we have of the logical treatment of problems in philosophy. But its significance in the long run will be measured less by how far it goes than by how far it is superseded." (Goodman, 1963, p. 558)

I think Goodman was essentially right – up to one point. This point concerns, as the reader may have already guessed, the complete neglect of the geometrical background of the *Aufbau*. This is a bit ironical, since Goodman's criticism concentrated exactly on the *Aufbau's* geometrical core, to wit, the constitutional method of quasianalysis. Even more, Goodman treated at great length the basic geometric example of colour strips (*Der Raum*, *Aufbau* § 70, 72, Goodman, 1977, ch. 5). A host of others authors have followed him and turned the criticism of Carnap's method of quasianalysis into a kind of industry similar to the discussions of Gettier problem in epistemology. Regrettably none of them came ever close to see the geometric origins of the method, probably because they had not read *Der Raum*.

I think, the neglect of the geometric backdrop of the *Aufbau* is a fatal flaw of Goodman's criticism. It deprived him of seeing the constitutional method of the *Aufbau* in a larger mathematical context. This would have rendered it implausible that the quasi-analytical method could fail for the mathematically rather trivial reasons Goodman and his followers advanced in their criticisms by pointing to various "difficulties" such as the "difficulty of imperfect community" or the "difficulty of companionship" (Goodman, 1977, p. 117f). Isolated from the general theoretical framework of geometry and the theory of relational systems, quasianalytical constitutional theory, confined to a bag of simple illustrative examples, is doomed to remain a curious gadget without deeper meaning. The fact that the critics of the *Aufbau* always stick to the very same simplistic toy examples instead of exhibiting the "mathematical essence" of quasianalysis is evidence for this sad state of affairs.

Before I engage in the task of elaborating this general "geometric background argument" against the standard criticism of the quasianalytical constitution program let us first consider a simple but convincing philosophical argument against Goodman's criticism of the feasibility of quasianalytical constitution (cf. Proust, 1986). For Goodman the fatal flaw of the quasianalytical constitution of qualities is that under "unfavourable circumstances" this method may happen not to yield the "correct" qualities. According to Goodman the disagreement between the constituted and the "real" qualities shows the definite failure of the quasianalytical account. For Proust, this assessment reveals a serious misunderstanding of the constitutional approach:

"Goodman's objections [...] reestablish in spite of him the fiction of an omni-scient God capable of controlling through originary intuition, that is, without construction, what the constitution derives from its extensional data." (Proust, 1984, p. 299).

Let us reconsider in the light of Proust' argument what is probably Goodman's most famous alleged counter-example against quasianalysis. Assume 1, 2, and 3 to be three objects having the following properties: 1 is green and round, 2 is round and wooden, and 3 is green and wooden. Then 1 and 2, 2 and 3, and 1 and 3. respectively, are similar, since the elements of these pairs have a common property:

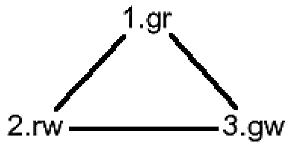

Figure 6.

Now let us quasianalyse this set of objects, i.e. let us attempt to set up a property distribution under the assumption that we only know the similarity relations that obtain. That is to say, we only know that 1, 2, and 3 resemble each other, but we do not know "how," i.e. we do not know in virtue of what properties they are similar. Diagrammatically this means we only know the similarity structure:

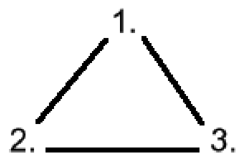

Figure 7.

If we apply the quasianalytical constitution of the *Aufbau* characterized by the axioms (C1) and (C2) this task has no unique solution. For example, we may attribute to 1, 2, and 3 only one property, say x. This distribution obviously satisfies (C1) and (C2). We could have been less parsimonious by proposing two properties x and y for each object. This distribution also satisfies (C1) and (C2). Or, we could have proposed the somewhat perverse distribution corresponding to the "real" one, i.e. we could have endowed 1 with g and r, 2 with r and w, and 3 with g and w. Having available only the rules (C1) and (C2) one cannot establish a ranking between these synthesising analyses and all the others which are also possible. Thus, a quasianalytical constitution of the *Aufbau* following the lines of (C1) and (C2) would be quite an ambiguous endeavour. For Goodman, this ambiguity shows that the quasianalytical constitution fails since it is not bound to produce the "real" property distribution. According to Proust this argument is ill-founded: If the *Aufbauer* does not have sufficient information, he cannot be expected to get the "correct" property distribution, since the correct one is distinguished from other possible distributions only by considerations that lie outside the constitution system – in God's intuition, so to speak. Hence, Goodman's criticism is ill-founded. Of course, Proust's argument does not resolve the ambiguity. Here, some technical improvements offer a way-out: if we apply instead of the *Aufbau* version the quasianalysis of *Quasizerlegung*, i.e. if we require (C1)–(C4) the ambiguity is drastically reduced. An even stronger system of axioms has been proposed in (Mormann, 1994) that disambiguates and defuses all counter-examples which Goodman and his fellow critics put forward. Nevertheless, there remain similarity structures which have essentially different property distributions. The smallest similarity structure for which this obtains is the following one (cf. Mormann, 1994):

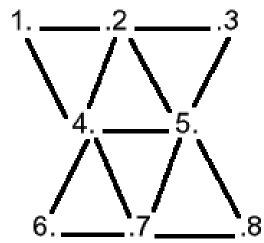

Figure 8

As is easily verified, this structure has a quasianalysis which satisfies (C1)–(C4) and attributes a common property to the elements 2, 4, 5 but not to 4, 5, 7. On the other hand, it also has a quasianalysis that attributes a common property to 4, 5, and 7 but not to 2, 4, and 5. Both property distributions are symmetric and there is no reason to prefer one to the other. Pace Goodman, this does not pose a serious problem to the constitutional account of the *Aufbau*. Relying on Proust's argument (Mormann, 1995) has pointed out that one may view a quasianalysis of a non-homogeneous set E as a sort of theory that "explains" the similarities existing between the elements of E by attributing to them hypothetical properties in an orderly fashion characterized by axiomatic rules such as (C1)–(C4). It may happen that more than one assignment meets these requirements. This means there are different "theories" which explain the same set of empirical data equally well. This is not a fatal flaw: underdetermination is a common feature of scientific theories.

6. CONCLUDING REMARKS

Summarizing I'd like to contend that the combined strengths of the proper philosophical interpretation of the quasianalytical method following the lines of (Proust, 1986) and (Mormann, 1995), and the technical emendments of this method in the vein of, say, *Quasizerlegung* and (Mormann, 1994) lay to rest Goodman's sweeping criticism. This does not suffice, of course, to reopen the case of the *Aufbau*-program. As Goodman correctly observed, the *Aufbau's* "significance in the long run will be measured [...] by how far it is superseded." (op. cit.). Hence, the defusion of criticisms such as Goodman's falls short of providing positive evidence that pursuing this program is a promising entreprise for contemporary epistemology and philosophy of science. For this task it would be necessary to carry out interesting quasianalytical constitutions of conceptual structures. In the case of space or spacetime this has been sketched in (Mormann 2000, 2002). Pace Quine's well-known criticism (Quine, 1951) it *is* possible to reconstruct spacetime starting from the austere base of an "inhomogeneous set" of elementary experiences. This shows that the quasianalytical constitution can be employed not only for the constitution of toy structures such as colour systems ("Farbgefüge") but for really interesting structures as well. As another "really interesting" structure besides geometry let me mention mereology, i.e. the theory of systems (x, $\leqslant$) where "$x \leqslant y$" is to be read as "x is a part of y." Parthood relations belong to the most basic structures and can be found almost everywhere (cf. Simons, 1987). It has been shown that mereological part-whole relations can be constituted in the framework of a quasianalytical constitution theory. That is to say, starting from appropriate similarity structures (S, $\sim$) one can constitute a (classical) parthood relations $\leqslant$ on S (cf. Mormann, 2002). In other words, classical mereology falls squarely within quasianalytical constitutional theory. This indicates that the perspectives of a quasianalytical constitution theory are not as bleak as critics of this method want to make us believe, at least, if we conceptualize this approach in the following way:

Quasianalysis should be conceived as a *general* method. There is no reason to stick to the rather anemic characterization of quasianalysis as it is to be found in the *Aufbau*. Equally important is the formally more elaborated version (based on different axiomatic requirements) in *Quasizerlegung*. Moreover, it may be fruitful to go beyond Carnap's proposals and experiment with new axioms for quasianalytical constitutions. These systems then correspond to the plurality of geometric systems studied in modern structural geometry. Quasianalysis should be exonerated from the charge of being only a philosopher's toy that has not much to do with "real" constitutional methods. Hence, instead of restricting the discussion to problems of the constitution of colour strips (which were meant as an example) one should employ the method for the constitution of really interesting objects and structures. This means going beyond the examples Carnap discusses in the *Aufbau* which all too often are motivated by philosophically obsolete and misleading (e.g. gestaltist) considerations.

Thus viewed, I contend, the *Aufbau* is more than a monument of the philosophical past.

NOTES

1. Majer disagrees with this thesis. In his contribution to this volume "Carnaps Übernahme der Gestalttheorie in den "Aufbau" im Lichte heutiger, vor allem computationaler Theorien des Sehens" he contends that gestalt theory is of central importance for the philosophy of the *Aufbau*. According to him, I and others are guilty of an "honorable perversion of history" ("ehrenwerte Geschichtsklitterung"). I think this is absurd: nobody will deny that the "Elementarerlebnisse" appearing in the *Aufbau* have their origin in gestalt theory (cf. Mormann, 2000, p. 94f). The point is, if gestalt theory is of crucial importance for the philosophy of the *Aufbau* as Majer claims. I think, his contention is fundamentally wrong. Following a proposal of the editor, I take the opportunity to give some arguments for my claim. For obvious reasons, this has to be done very briefly. As textual evidence for his claim, from the *Aufbau* Majer uses (with one exception) only quotations from the preface of the second edition (1961). For reasons I do not want to go into here this may be considered as problematic But even that preface belies Majer's claim: Although Carnap, as he explicitly asserts, still subscribes to the main philosophical outlook of his early opus magnum, in the preface he professes that now (1961) he would proceed differently than in 1928:

   > "Ich würde heute in Erwägung ziehen, als Grundelemente nicht Elementarerlebnisse zu nehmen ..., sondern etwas den Machschen Elementen Ähnliches, etwa konkrete Sinnesdaten, ... Ich habe aber auch schon im Buch die Möglichkeit einer anderen Systemform darstellt, deren Basisbegriffe sich auf physische Gegenstände beziehen (§ 59). Außer den drei als Beispiele dort angegebenen Formen einer Basis im Physischen (§ 62) würde ich vor allem auch eine Form in Erwägung ziehen, die als Grundelemente physische Dinge enthält und als Grundbegriffe beobachtbare Eigenschaften und Beziehungen solcher Dinge." (Aufbau, vii).

   This should be sufficient to make sure that at least in 1961 the author of the *Aufbau* did not consider gestalt theory as belonging to the philosophical core of that work. According to him, *Elementarerlebnisse* could be replaced by other objects totally unrelated to gestalt theory. Well, one might suspect that the later Carnap misinterpreted his early work, or at least drastically changed his interpretation. This, however, is not the case. His assessment of 1961 is essentially the same as that we find in the *Aufbau* of 1928:

   > "Was den Inhalt des dargestellten Konstitutionssystems betrifft, so sei wiederholt ausdrücklich betont, daß es sich nur um einen beispielsweisen Versuch handeln kann. Unsere Darstellung der Konstitutionstheorie hat ihren eigentlichen Zweck in der Stellung der Aufgabe eines Konstitutionssystems und in der logischen Untersuchung der zu einem solchen System führenden Methode, nicht in der Aufstellung des Systems selbst. Daß nun doch wenigstens einige Stufen des Systems hier durchgeführt werden ... geschieht mehr in der Absicht, durch dieses Beispiel die Aufgabe zu illustrieren, als den Beginn ihrer Lösung zu versuchen" (*Aufbau,* § 106).

   Hence, the gestaltist features of the constitutional system sketched in the *Aufbau*, and therefore gestalt theory itself, are surface phenomena. The philosophical core of the *Aufbau* is the logical method of quasianalytical constitution. Quasianalysis, however, is a purely structural method independent of the nature of the elements to be quasianalysed. In the constitutional system the elements are "atoms" or "points without qualities," as Carnap says. If Majer wanted to argue for the centrality of gestalt theory for the *Aufbau* he had to show that quasianalysis as a formal method was a method of gestalt theory. This would be totally misguided per se. Moreover, in § 69 Carnap explicitly states that the method of quasianalysis is derived from the Frege-Russell abstraction principle. Then he continues:

> "Es (the method of quasianalysis) ist überall da von Bedeutung, wo es sich um die Behandlung unzerlegbarer Einheiten irgendwelcher Art handelt, d.h. um Gegenstände, die .... nicht Bestandteile oder Merkmale ... aufweisen, sondern gewissermaßen nur punktuell gegeben sind, die daher nur synthetisch behandelt werden können ...."

In this quotation one should pay attention to "überall," which makes clear that quasianalysis is a general method not restricted to gestaltist Elementarerlebnisse.

In order that the constitutional process gets started for these "pointlike" elements there must be given assertions about them. Due to the pointlike character of the elements, these assertions have to be relational. As a whole, § 69 of the *Aufbau* repeats, sometimes literally, the corresponding arguments of the ms. *Quasizerlegung* of 1922/23. In order to make his statements more vivid, at the end of that section (in the "Beispiel"), Carnap once more relies on the colour example, already treated in *Der Raum* thereby invoking implicitly the geometric origins of quasianalysis. That is to say, authors such as Hilbert, Frege, Whitehead or Russell could claim to be the "grandfathers" of the quasianalytical constitutional method – I do not see a place for Wertheimer or Köhler. A final evidence for the superficial role gestalt theory played for Carnap's philosophy is that it disappears after 1928 without leaving a trace. Majer has nothing to say to this fact but that Carnap made a volte-face. By way of contrast, the orientation towards a structural approach to matters philosophical, as it comes to the fore in constitutional theory, is an invariant of Carnap's philosophy from the very beginning to the end.

Summarizing one may say that Majer's thesis that gestalt theory is central to the *Aufbau's* philosophy falls into a confusion of example and method. It contradicts (i) the explicit intentions of Carnap and (ii) it does not contribute anything to a better understanding of Carnap's philosophical development.

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