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THE DEBATE ON *BEGRIFFSTHEORIE* BETWEEN CASSIRER,
MARC-WOGAU – AND SCHLICK

0. INTRODUCTION

The aim of this paper is to reconstruct a peculiar debate between Ernst Cassirer and the Swedish philosopher Konrad Marc-Wogau on *Begriffstheorie* that took place in the late thirties of the 20th century. This debate may be conceived as sort of ersatz of the discussion between Cassirer's Neokantian *Begriffstheorie* on the one hand, and logical empiricist accounts on the other, in particular Schlick's *Begriffstheorie* as presented in his *Allgemeine Erkenntnislehre*.¹ Although Schlick did not participate *in personam* in the discussion that took place between Cassirer and Marc-Wogau, one may consider him as a “virtual” participant of the debate, since his *Begriffstheorie* played an important role in the background, in particular for Cassirer. More precisely, I'd like to show that the debate can be read as a dispute on the feasibility of a “rich” account of *Begriffstheorie*, favored by Cassirer, and the necessity of restricting *Begriffstheorie* to an “austere” approach whose protagonists were Marc-Wogau and Schlick, although in a quite different ways.

More generally, the debate on *Begriffstheorie* exemplifies the complex interactions – and non-interactions – between three important currents of scientific philosophy, namely, the Marburg Neokantianism of Cassirer, the scientifically minded philosophers of the Uppsala School, and, indirectly, the Logical Empiricism of the Vienna Circle.

To set the stage, first let us recall briefly some biographical details of the protagonists. After National Socialism had come to power in January 1933, Cassirer left Germany in April of the same year. First he went to England, in 1934 he settled down in Uppsala. When in 1941 a German invasion of Sweden seemed imminent, he went to the U.S. where he lived until his death in April 1945. Konrad Marc-Wogau (1902 – 1991) was Professor of Philosophy in Uppsala from 1946 till his retirement in 1968. During Cassirer's stay in Sweden he and Cassirer were engaged in a lively debate that mainly took place in the then newly founded journal *Theoria*. From 1936 to 1940 their exchange in *Theoria* comprises at least seven items. Moreover, already in 1936 Marc-Wogau had published the monograph *Inhalt und Umfang des Begriffs* in which he dealt with a variety of *Begriffstheorien*,

1 Another important current of *Begriffstheorie* flourishing in the Vienna Circle was the one put forward by Carnap in the *Aufbau*. For reasons of space I cannot deal with it here.

among them Cassirer's. He found all of them wanting, since they all led to "dialectical", i.e., inconsistent concepts of concepts.

For Cassirer *Begriffstheorie*, i.e., the philosophical theory of the formation of scientific concepts, was not just one philosophical topic among others. Rather, he considered *Begriffstheorie* as a truly central point of philosophy *überhaupt* (cf. Cassirer 1928, 163). Marc-Wogau agreed with Cassirer on the importance of the *Begriffsproblem*. He was well aware of the fact that he did not attack some minor point of Cassirer's approach, but launched an assault against the very center of Cassirer's philosophy. Moreover, he conceived his attack not only as directed against Cassirer's theory, but against the traditional philosophical account of concepts as a whole.

In the background of the debate on *Begriffstheorie* between Cassirer and Marc-Wogau, Schlick and his *Allgemeine Erkenntnislehre* played an important role. Marc-Wogau claimed that Cassirer's *Begriffstheorie* could not survive logical analysis in that it led to an inconsistent notion of the concept. Although in *Inhalt und Umfang* he did not deal with Schlick's account of *Begriffstheorie* as elaborated in *Allgemeine Erkenntnislehre* it transpires from his criticisms of the other theories of concepts treated that he would have judged Schlick's account as "dialectical", i.e., as inconsistent, as well.

On the other hand, Cassirer held Schlick's *Allgemeine Erkenntnislehre* in high esteem as a step in the right direction (cf. Cassirer 1927), but he criticized Schlick in that he took *Begriffe* as merely conventional symbols, laying "stress only on the negative moment of the function of "denoting" and its "conventional" character." According to him, "a sharper analysis of this function discovers immediately another, more positive aspect. (Cassirer 1927, 136). This "more positive aspect" of the symbolic character of scientific concepts he claimed to have unfolded in his constitutive account of *Begriffstheorie* as presented in *Substance and Function* (Cassirer 1910) and later in *The Philosophy of Symbolic Forms* (Cassirer 1923 – 1929). Complementarily, Schlick, in *Allgemeine Erkenntnislehre*, criticized (Neo-) Kantian accounts of *Begriffstheorie* as overstating the power of thinking without mentioning Cassirer by name. He pithily asserted:

Thinking does not create the relations of reality... [R]eality does not obtain form and regularity first from consciousness; on the contrary, consciousness is only a section cut out of reality. ... There are no synthetic judgments *a priori*. (GTK, §40)

For Schlick, *Begriffe* were merely conventional symbols. In this sense, he subscribed to a rather austere version of *Begriffstheorie*. But even this kind of modest theory Marc-Wogau would have blamed as untenable and logically flawed since it got involved in the pernicious dialectics of *Inhalt* and *Umfang*.

The outline of this paper is as follows. In the next two sections we recall the basics of the accounts of *Begriffstheorie* of Schlick and Cassirer. This requires dealing with Helmholtz's theory of concepts in some detail. In section 3 we will

deal with Marc-Wogau's critique of Cassirer's *Begriffstheorie* and Cassirer's counter-critique in some detail. In section 4 I put forward some arguments from modern *Formal Theory of Concepts* which show that Marc-Wogau's objections to Cassirer's *Begriffstheorie* are untenable. This does not mean that Cassirer's rich constitutive account of concepts was without problems, but at least it shows that there is no reason to suspect that every theory of concepts that subscribes to some kind of relation between *Inhalt* and *Umfang* is per se inconsistent. In section 5 we conclude with some general remarks on the complex relations between the Logical Empiricism of the Vienna Circle, the Marburg Neokantianism, and the Uppsala School as they show up in the debate on *Begriffstheorie*.

1. KNOWLEDGE AS COORDINATION: HELMHOLTZ AND SCHLICK

The term *Begriff* is probably one of the most vague terms ever-used in philosophy, psychology, and other disciplines (cf. Weitz 1984, Marc-Wogau 1936). One cannot start with a neat and comprehensive definition. In this paper I propose to conceive *Begriffstheorie* as a result of two complementary influences: On the one hand, it may be understood as a result of post-Kantian epistemology, which no longer accepted Kantian "pure intuitions" as an important apriori ingredient for scientific knowledge. On the other hand, *Begriffstheorie* may be seen as a philosophical reaction of the conceptual evolution of the sciences, i.e., it was an attempt of philosophy to come to terms with the new conceptual developments of the sciences, in particular with those of logic, mathematics, the mathematized empirical sciences. Also insights of physiology and psychology that concerned the ways of human conceptualization required the attention of philosophy.

A convenient starting point is Helmholtz's "semiotic" theory of knowledge (cf. Helmholtz 1921). Helmholtz considered himself as a (Neo)Kantian, moreover he was a first-class scientist with an immense expertise in physics, physiology and other disciplines. Helmholtz's epistemology may be characterized as a rather special version of a "scientifically corrected" Kantianism. According to it, on one side there is the world *W* of Kantian things-in-themselves, on the other side there is the domain *S* of one's sensations. Things and sensations are correlated to each other in a 1-1-way in such that sensations are to be interpreted as signs of objects:

Our sensations are precisely effects produced by external causes in our organs, and the manner in which one such effect expresses itself depends, of course, essentially on the type of apparatus which is affected. Insofar as the quality of our sensation gives us information about the peculiarity of the external influence stimulating it, it can pass for a sign – but not for an image. For one requires from an image some sort of similarity with the object imaged: ... A sign, however, need not have any type of similarity with which it is a sign for. The relations between the two are so restricted that the same object, taking effect under equal circumstances, produces the same sign, and hence unequal signs always correspond to unequal effects." (Helmholtz 1878, 347)

Knowledge, then, is based on a mapping $W \xrightarrow{f} S$ of the world W of things-in-themselves into the domain S of sensations satisfying the requirement $f(x) \neq f(y) \Rightarrow x \neq y$. According to Helmholtz, this weakly “structure-preserving” relation between an outer world and a domain of inner sensations is sufficient to ensure that we are able to know the lawful structure of reality (cf. Helmholtz 1878, 348). In his *Allgemeine Erkenntnislehre* Schlick, in general, faithfully followed Helmholtz’s semiotic approach.² A point where he deviated from Helmholtz was that he replaced “sensations” by “concepts”. The reason was that he considered “sensations” or “mental images” as too vague and undetermined as that they could fulfil the symbolic role that Helmholtz had provided for them. In order to ensure stability and determinateness of our thought, he proposed to replace Helmholtz’s sensory images (*Empfindungen*) by “concepts”. Concepts were distinguished from images by the fact that they were completely determined and had nothing uncertain about them (cf. GTK, §5, 20). One may ask, how natural minds like ours with their continuously changing sensory images can handle such ideal entities as concepts as Schlick defined them. Schlick offered an answer apparently inspired by Vaihinger’s *Philosophie des Als Ob*: Strictly speaking, concepts do not exist, what is important is their functional role:

We operate with concepts as if they were (sensory) images (*Vorstellungen*) with exactly delineated properties that can always be re-cognized with absolute certainty. Their properties are called the characteristics or features (*Merkmale*) of the concept, and are laid down by means of specific stipulations which in their totality constitute the definition of the concept. In logic, the totality of the characteristics of a concept is called its “intension” (or “content”); the set of objects denoted by the concept is called its “extension.

...

Accordingly, a concept plays the role of a sign for all those objects whose properties include all defining characteristics of that concept.” (GTK, §5, 20)

For later use it will be expedient to comment briefly on this piece of traditional concept logic to which Schlick subscribes here. In traditional logic a concept has two complementary components: on the one hand, its *Inhalt* (intension), given as the set of its defining characteristics, and on the other hand its *Umfang* (“extension”), given as the set of all objects whose properties include all its defining characteristics. This duality suggests the so-called “law of reciprocity” (cf. Marc-Wogau 1936, 10ff) according to which the following “reciprocity” between the *Inhalt* and the *Umfang* of a concept holds: the larger the *Inhalt* of a concept, the smaller its *Umfang*, and vice versa. This time-honoured “law” of traditional logic

2 In an approving comment on Helmholtz’s *The Facts of Perception* Schlick explicitly characterized his *General Theory of Knowledge* as an attempt “to show that forming such a mapping of what is lawlike in the actual, with the help of a sign system, altogether constitutes the essence of all knowledge, and that therefore our cognitive process can only in this way fulfil its task and needs no other method for doing so.” (Schlick and Hertz 1921, 166, endnote 15)

appears in various forms in virtually every logical treatise of the 19th and early 20th century. As will be discussed later in more detail, it is the target of Marc-Wogau's incisive criticism put forward in *Inhalt und Umfang*. More precisely, he contended that all accounts of *Begriffstheorie* that hold some version of the reciprocity law were doomed to be inconsistent.

Before we come to this, let us note that the project of defining concepts by characteristic features is threatened by two complementary dangers, either by infinite regress or ending up in some features that lack exact definitions but instead are grounded in some murky empirical intuition that undermined the exact character of concepts so defined. According to Schlick, it was Hilbert's account of implicitly defined concepts that provided a way out of this impasse. According to it the basic concepts of mathematical theories are just defined as entities that satisfy the axioms specified for them. Hence, there seem to be at least some concepts that can be defined in a completely precise and unambiguous way. Let us assume, for the sake of the argument, that we possess concepts in Schlick's sense that are coordinated in a 1-1-way with objects. What is the purpose in coordinating concepts to objects? Schlick's answer in *Allgemeine Erkenntnislehre* is that coordinations enable us to make judgments about objects, and only 1-1-coordinations enable us to make true judgments (cf. GTK, § 10). This answer is unsatisfying in that it still allows a "Lagadonian coordination"³ of concepts and objects that coordinates each object with one concept in a 1-1-way in some arbitrary fashion. Such a Lagadonian conceptual system would allow us to make true judgments in a trivial manner. In order to exclude such undesired conceptual systems, Schlick hastened to add that the real aim of coordinating objects and concepts is not simply to enable us to formulate true judgments but to get knowledge, which depends on very special coordinations:

Knowledge is more – much more – than mere truth. Truth requires nothing but uniqueness of coordination; as far as truth is concerned, it does not matter what sign is used for that purpose. Knowledge, on the other hand, means unique coordination with the help of certain definite symbols, namely, those that have already found applications elsewhere. ... Hence if we were to coordinate a special sign to each fact and object in the world, we should have nothing but isolated truths, each of which would have to be learned separately. ... Our truths would be nothing but discrete points, so to speak; they would not form a coherent system. Yet it is only in such a system that knowledge is possible, since the finding anew of one thing in another presupposes a pervasive interconnection. (GTK, 66, 67, dt. 97)

Thus, an essential point of a Schlickian *Begriffstheorie* would have been to distinguish between "good" and "not so good" conceptual coordinations. This issue, however, remained underdeveloped in *Allgemeine Erkenntnislehre*. Schlick was content to give some vague hints pointing at a sort of Machian thought economy

3 On the philosophical appeal of Lagadonian languages see D. Lewis *On the Plurality of Worlds* (Lewis 1986, p. 145).

by which we could single out “good” parsimonious from “bad” Lagadonian concepts.

Summarizing we may say that Schlick’s coordinative account of knowledge is characterized by two complementary features: on the one hand it was based on a rich notion of structured reality that did not only recognize “simple” objects as real, but even the most “theoretical” relations; on the other hand, it ascribed a rather austere role to the conceptualizing activity of the subject: for Schlick, concepts were nothing but conventional 1-1 coordinations that allow easy “syntactical” manipulations. Thereby his account claimed a neat separation between factual and conventional components of knowledge. As we shall see, Cassirer’s *Begriffstheorie* pulled in the opposite direction: although based on coordination as well, it emphasized the active role of the conceptualizing subject against that of reality “out there”.

2. CONCEPTUAL CONSTITUTION: CASSIRER’S *BEGRIFFSTHEORIE*

Cassirer’s philosophy of science is concept-oriented *par excellence*: “The theory of the concept becomes a cardinal problem of systematic philosophy. It becomes the nub around which logic, epistemology, philosophy of language and cognitive psychology are rotating” (Cassirer 1928, 163). Since *Substance and Function* (Cassirer 1910) he conceived philosophy of science as a theory of the formation of scientific concepts. His theory was naturalistic in the sense that according to him philosophy should not decree what scientific concepts were and how they worked. Rather, since scientific concepts evolved in the history of science, it was the task of philosophy of science to study this conceptual development of science and to make philosophical sense of it, not to legislate it according to some preconceived philosophical ideas.

The role concepts played in the evolution of scientific knowledge science according to Cassirer’s *Begriffstheorie* may be described in telegram style as follows. Scientific knowledge does not cognize objects as ready-made entities. Rather, knowledge is organized objectually in the sense that in the continuous stream of experience invariant relations are fixated. The unity of a concept is not to be found in a fixed group of properties, but in a rule, which lawfully represents the mere diversity of experiences as a sequence of elements. The meaning of a concept depends on the system of concepts in which it occurs. It is not completely determined by one single system, but rather by the continuous series of systems unfolding in the course of history. Scientific concepts and conceptual systems do not yield pictures of reality, rather, they provide guide lines for the conceptualisation of the world. The fundamental concepts of theoretical physics are blueprints for possible experiences. Factual and theoretical components of scientific knowledge cannot be neatly separated. In a scientific theory „real“ and „non-real“ components

are inextricably interwoven. Not a single concept is confronted with reality but a whole system of concepts. Our experience is always conceptually structured. There is no non-conceptually structured „given“. The „given“ is an artifact of a bad metaphysics.

The concepts of mathematics and the concepts of the empirical sciences are essentially of the same kind. In a similar way, as the objects of a mathematical theory are constituted by a system of concepts, the objects of an empirical theory are constituted by theory's concepts. With this account of scientific concepts and their role in the ongoing evolution of science Cassirer goes beyond Helmholtz's and Schlick's structural realist accounts of knowledge and coordination. He emphasized the "constitutive" character of symbolic representation:

... we do not know "objects" as if they were already independently determined and given *as objects*, – but we know *objectively*, by producing certain limitations and by fixing certain permanent elements and connections within the uniform flow of experience. The concept of the object in this sense constitutes no ultimate limit of knowledge, but is rather the fundamental instrument, by which all that has become its permanent possession is expressed and established. The object marks the logical possession of knowledge, and not a dark beyond forever removed from knowledge. "(SF, 303f)

Instead of conceiving knowledge as a structure-preserving map between a world of transcendent things on the one hand and a domain of sensory images (Helmholtz) or conventional symbols (Schlick), for Cassirer knowledge as coordination meant the coordination of different areas or stages of knowledge. This led to a new "internal" account of coordination or representation that described the coordination between thought and reality not as a relation between two ontologically different spheres but as a relation between different areas of knowledge. Thus, if one graphically represents Schlick's and Helmholtz's account by a simple relation $W \text{-----} \rightarrow S$ between a world W of transcendent things and a domain of internal symbols S , Cassirer's account could perhaps be represented by an unending chain of conceptualizations evolving in the history of science:

$$\dots \text{-----} \rightarrow C_i \text{-----} \rightarrow C_{i+1} \text{-----} \rightarrow C_{i+2} \text{-----} \rightarrow \dots$$

Here, the C_i should not be interpreted as "mere conceptualizations". Rather, the C_i are always thought to aim at empirical confirmation and corroboration. In modern terms, they may perhaps be conceived as interpreted models of reality. Thus, a scientific object is never "given as such", independently of all the C_i , it always appears in a lawful conceptual context by which it is constituted. In a similar way as a mathematical object such as a geometrical point cannot be thought outside a geometrical system, an object of physics or of any other science cannot be thought outside its theoretical context to which it belongs. Indeed, Cassirer considered it as the essential task of critical idealist philosophy of science to make clear "that the same foundational syntheses (Grundsynthesen) on which logic and mathematics rest also govern the scientific construction of experiential knowledge ..." (Cassirer

1907, p. 44). This contention did not imply that empirical and mathematical objects and concepts are one and the same thing. In *Substance and Function* he pointed out there was an important difference between mathematics and empirical concepts:

In contrast to the mathematical concept, however, in empirical science the characteristic difference emerges that the construction which within mathematics arrives at a fixed end, remains in principle *incompleteable* within experience. But no matter, how many „strata“ of relations we may superimpose on each other, and however close we may come to all particular circumstances of the real process, nevertheless there is always the possibility that some co-operative factor in the total result has not been calculated and will only be discovered with the further progress of experimental analysis. Cassirer (1910/1953, p. 254)

In a nutshell, then, for Cassirer the difference between mathematical and empirical concepts resided in the fact that the latter are open (“incompleteable”) while the former are closed: the implicit definition of a point in Euclidean geometry fixes the meaning of this concept once and for all. In contrast, the meaning of a concept such as “atom” is never fixed by a single conceptual system. Cassirer claimed that the key concepts of empirical science had a “serial form” (“Reihenform”) in that their meaning was not fixed once and for all by a single theoretical framework. Rather, it emerged in a series of theoretical stages in the ongoing evolution of scientific knowledge. Thus concepts comprise two complementary moments: on the one hand they are rules for further investigations, on the other hand they are devices for determining the objects of scientific knowledge. The feasibility of this complex relation of the two components is at stake in the debate between Cassirer and Marc-Wogau.

3. MARC-WOGAU’S CRITICISM AND CASSIRER’S DEFENSE

The aim of Marc-Wogau’s treatise *Inhalt und Umfang des Begriffs. Beitrag zur Theorie des Begriffs* (1936) was to clarify the essence of the concept of concept. According to him, such a clarification was urgently needed, since virtually all extant accounts of *Begriffstheorie* were fatally flawed. As an expedient starting point for such a clarification he considered the problem of the relation between the *Inhalt* and the *Umfang* of a concept: “It seems to me that the nature of concept can best be clarified at this problem.” (Marc-Wogau 1936, 5). He pointed out that in the theory of concepts one easily runs into logical difficulties. For instance, the “concept of concept” (*der Begriff des Begriffs*) immediately leads to well-known paradoxes of a class that contains itself as an element. Hence, in order to avoid such pitfalls one had to be extremely careful in the choice of the basic assumptions on which to build a consistent *Begriffstheorie*. Consequently, Marc-Wogau was prepared to recognize only those “determinations” (*Bestimmtheiten*) as concepts

which were non-contradictory (*widerspruchlos*) or unequivocal (*eindeutig*) and could be grasped by a uniform (*einheitlich*) thought (ibid., 7). Thereby he hoped to exclude inconsistent expressions such as “round square” from the realm of *Begriffstheorie*. Although it is rather plausible not to admit openly contradictory concepts such as “being red and non-red all over at the same time” Marc-Wogau’s requirement of uniformity is more tricky, in particular, since he considered non-uniformity as the main source of “dialectical”, i.e., inconsistent concepts. According to him, virtually all theories of concepts sinned against the command of uniformity and fell prey to inconsistency. The main entrance door for non-uniformity (and thence inconsistency) was that virtually all theories of concepts subscribed to a correlation between *Inhalt* and *Umfang* of a concept. Marc-Wogau claimed that this correlation could not be grasped in a “uniform thought” and therefore led to an inconsistent “double thought” (*Doppelgedanken*).

He attempted to show that all accounts of *Begriffstheorie* endorsed a very strong version of the reciprocity law according to which the *Inhalt* uniquely determined the *Umfang* and the *Umfang* uniquely determined the *Inhalt*. This claim may well be doubted. For instance, a non-extensional *Begriffstheorie* readily allows for the existence of concepts having the same *Umfang* but different *Inhalte*. Fortunately, we need not go into the details of these quibbles when we wish to grasp the essence of Cassirer’s and Marc-Wogau’s dispute. Cassirer intended to refute Marc-Wogau principally, i.e., he readily admitted that he did subscribe to a “double thought” approach of concept. But he denied that this led to contradiction. Hence he argued that even if *Inhalt* and *Umfang* determined each other in the strict way that Marc-Wogau assumed, even then this fact would not lead to inconsistency. In the following I want to show that Cassirer was right, even if the argument he presented for this thesis, was less than convincing.

In some sense, Marc-Wogau’s arguments against the “double thought” hidden in the standard approaches of *Begriffstheorie* resemble those of the British idealists such as Bradley who claimed that the concept of relation was “unintelligible” and even “contradictory”. As Marc-Wogau put it:

If one relatum of this relation (between *Inhalt* and *Umfang*) is thought, thereby the other is thought as well. Consequently, the relata coincide. If A is to be related to B in such a way that A obtains its determination (or determinateness) only through B, then it is impossible to distinguish between A and B. They coincide.” (Theoria 2, 291ff)

Let us call this thesis Marc-Wogau’s identity thesis. Cassirer’s counter-argument against the identity thesis was to give a counter-example, i.e., he presented a decent, scientifically recognized relation whose relata strictly determined each other but nevertheless were not identical. Thus his strategy was based on the naturalist assumption that it is not the task of philosophy to decree what is possible and what is not possible but to understand the conceptual evolution of the sciences. According to him, there was no reason to assume that “thinking together” *Inhalt* and

Umfang led to contradiction since the conceptual evolution of science had shown that relations whose relata are different but nevertheless strictly determine each other, do not lead to contradictions. In other words, he accused Marc-Wogau of being caught in the trap of some unfounded philosophical prejudice refuted by the conceptual evolution of science.

In order to refute Marc-Wogau's identity thesis Cassirer relied on Schlick's *Allgemeine Erkenntnislehre*, and pointed out that systems of implicitly defined concepts as considered by Schlick refute Marc-Wogau's thesis:

In an implicitly defined conceptual system there is given a totality of concepts that stand in strict correlation to each other and have no independent content outside this correlation. None of them is meaningful "for itself", each is defined only with respect to the other, or, better said, with respect to the whole system. Nevertheless this mutual dependence cannot be considered as a flaw; rather it lays the foundation for a certain highly characteristic advantage. One cannot say that, due to the fact that none of the system's basic concepts can be explained or used meaningfully outside the system, their meaning disappears or becomes ambiguous. Each has its well-determined place in the system and thereby it distinguishes itself from any other concept of the system. (Cassirer 1938, 226)

He concluded that thereby Marc-Wogau's thesis was "directly refuted" (ibid.).

Even if from a formal point of view Cassirer's argument against Marc-Wogau seems flawless, one may consider it not as fully convincing: firstly, it is an abstract argument in the sense in that it has nothing to do with the specifics of the reciprocity law that correlates *Inhalt* and *Umfang*. It simply gives an example showing that there exist relata that strictly determine each other without being identical. Secondly, the argument based on implicitly defined concepts does not provide any positive evidence for Cassirer's own version of a constitutive *Begriffstheorie*. In the next section I want to show that today we have powerful formal tools that allow one to refute Marc-Wogau's thesis on his own ground. That is to say, there are consistent theories of concepts that satisfy a strong version of the reciprocity law.

4. FORMAL THEORY OF CONCEPTS AND ADJOINT SITUATIONS

Cassirer's refutation of Marc-Wogau's identity thesis by invoking implicit definitions may not be considered as fully adequate, since the implicit definition of concepts has nothing to do with the problematic of the relation between *Inhalt* and *Umfang* that occupies centre stage in Marc-Wogau's *Begriffstheorie*. In other words, Cassirer's argument is too general than to be really convincing.

Fortunately, today better and more specific arguments are available to back up Cassirer's arguments against Marc-Wogau. I'd like to mention two different approaches. First, the so-called *Formal Concept Analysis* (FCA) inaugurated in the 1980s by the German mathematician Rudolf Wille and his collaborators. Sec-

only, on a more general level, the theory of adjoint situations that belongs to the core of the foundational discipline of category theory founded in the late 1940s by the American mathematicians Saunders Mac Lane and Samuel Eilenberg. Both approaches offer mathematical models of (generalized) concepts that are better suited to refute Marc-Wogau's identity thesis than Cassirer's vague allusion to Schlick's equally vague theory of implicit definitions in *Allgemeine Erkenntnislehre*.

FCA starts with the reciprocity law. A concept is determined by its *extent* ("Umfang") and its *intent* ("Inhalt"). The *extent* consists of all objects belonging to the concepts, while the *intent* is the collection of all attributes shared by the objects. As it is often difficult to list all the objects and usually impossible to list all its attributes, it is natural to work within a specific context in which the sets of objects and attributes are fixed.⁴

Then a context is defined as a triple (G, M, F) where G and M are sets and $F \subseteq G \times M$. The elements of G are called objects, and the elements of M are called attributes. If $(g, m) \in F$ this is to be interpreted as the fact that in M the object g has the attribute m , or, put it differently that the attribute m is instantiated by g . For $A \subseteq G$ and $B \subseteq M$ define

$$I(A) := \{m \in M; \text{for all } g \in A (g, m) \in F\}$$

$$U(B) := \{g \in G; \text{for all } m \in B (g, m) \in F\}$$

Informally, $I(A)$ is the set of attributes common to all the objects in A , and $U(B)$ is the set of objects having all the attributes in B . Denoting the power set of A and B by PA and PB , the operators I and U just defined above may be conceived as mappings

$$PA \xrightarrow{I} PB \quad \text{and} \quad PB \xrightarrow{U} PA$$

These mappings have some interesting properties. For instance, they satisfy the following requirements (cf. Ganter and Wille 1999, chapter 0.4, Definition 16, 11):

- (1) $A_1 \subseteq A_2 \Rightarrow I(A_1) \supseteq I(A_2)$
- (2) $B_1 \subseteq B_2 \Rightarrow U(A_1) \supseteq U(B_2)$
- (3) $A \subseteq U(I(A))$ and $B \subseteq I(U(B))$

The pair (I, U) is called a Galois connection, and the maps I and U are called dually adjoint to each other. As is well known a pair of maps $PA \xrightarrow{I} PB$ and $PB \xrightarrow{U} PA$ is a Galois connection if and only if it satisfies the equivalence

$$(4) \quad A \subseteq U(B) \Leftrightarrow B \subseteq I(A)$$

Now we are ready to define concepts of a context (G, M, F) as pairs $(A, B) \in PG \times PM$ that are "balanced" in the sense that $I(A) = B$ and $U(B) = A$. The *Umfang*

4 If this is not done, one runs into difficulties, as is discussed in detail by Marc-Wogau (1936).

of the concept (A, B) is A while its *Inhalt* is B . The set of concepts $C(G, M, F) := \{(A, B); I(A) = B\}$ has the structure of a complete lattice (cf. Theorem 3, p. 20, Ganter and Wille 1999).

By definition, *Umfang* and *Inhalt* of a concept strictly determine each other. Nevertheless they are different. But given the *Inhalt* I one can calculate the *Umfang* U , and, vice versa, given the *Umfang* U , one can calculate the *Inhalt* I . For $A \in PG$ and $B \in PM$) one obtains:

$$U(B) = \cup\{A; B \leq I(A)\} \quad \text{and} \quad I(A) = \cup\{B; A \leq U(B)\}$$

In sum, the Galois connection (I, U) neatly disproves Marc-Wogau's identity thesis according to which strict mutual determination implies identity. Moreover, the refuting example is directly concerned with *Inhalt* and *Umfang* as key concepts of *Begriffstheorie*.

By briefly mentioning FCA and the theory of *Galois connections* I only scratched at the surface of what may be characterized as a modern version of traditional *Begriffstheorie*. In this direction much more has to be done in order to find out if traditional *Begriffstheorie* could indeed be fruitfully related to contemporary strands of research in category theory, computer science and cognitive science.

It would be a gross underestimation of the theory of Galois connections to take it just as an abstruse calculus that is useful for some special theory such as FCA. Rather, Galois connections are a very special case of so called Adjoint Situations. Adjoint situations are, according to the assessment of Saunders Mac Lane, one of the founding fathers of category theory, THE fundamental concept of category theory. There is no time to explain this contention in any detail. Be it sufficient just to state that in adjoint situations the rather austere structures PG and PM are replaced by appropriate, much more richly structures categories, and the role of the mappings I and U is taken over by appropriate functors these categories. Then one of the fundamental theorems of category theory, the so-called Adjoint Functor Theorem, ensures that under certain conditions something like a generalization of the reciprocity law holds.

Painting it with a broad brush we may contend that in this way the allegedly obsolete *Begriffstheorie* of the early 20th century, centering on the notorious "law of reciprocity", has found an unexpected comeback in the guise of category theory. The fact that adjoint situations are one of the core concepts of category theory, and the fact that category theory is one of the most successful contemporary foundational theories suggest that even today *Begriffstheorie* may deserve more than mere philosophico-historical interest.

5. CONCLUDING REMARKS

Begriffstheorie may be considered as hidden meeting point for a variety of philosophical currents more or less closely related to some sort of "scientific" philoso-

phy, in particular Logical Empiricism, among them Schlick's empirio-criticism of the early 20s, Cassirer's critical idealism, and the logical philosophy of the Uppsala School. More generally, as Marc-Wogau's treatise *Inhalt und Umfang* shows the issue of *Begriffstheorie* was a common ground for the various currents of analytic and continental philosophy that in the following decades became neatly separated. *Begriffstheorie* was a topic where philosophers of quite different orientations met. It exemplifies that once upon a time philosophers, who today are classified as belonging to allegedly quite different traditions, were engaged in discussing similar problems. *Begriffstheorie* shows in particular that it would be a serious distortion to characterize the continental tradition as anti-logical, and the analytical tradition as pro-logical.

From Marc-Wogau's perspective the theories of concepts put forward by philosophers such as Cassirer, Frege, Husserl, Kant, Rickert, Russell, and others, all appeared to be rather similar, since they all suffered from similar defects. He treated them as united in the common endeavor of elucidating the nature of (scientific) concepts. Implicitly he thereby defied the sharp distinction between continental and analytic philosophy that later became current. This feature of *Begriffstheorie* would have deserved more attention as I could give to it in a short paper like this. Rather, I concentrated on *Begriffstheorie* as a common ground of the more closely related currents of Cassirer's critical idealism and Schlick's early logical Empiricism as presented in his *Allgemeine Erkenntnislehre*.

From Marc-Wogau's rigid conception of logic that did not allow one "to think together" the complementary aspects *Inhalt* and *Umfang*, both Cassirer's and Schlick's accounts of *Begriffstheorie* were untenable. Logically, Marc-Wogau's criticism is refuted by the existence of concepts that mutually determine each other without being identical. Pragmatically, Marc-Wogau's account of *Begriffstheorie* is unacceptable, since it is hard to see how scientific concepts could do the work they are designed to do without assuming the existence of some kind of relation between two components of concepts that more or less resemble the classical aspects of *Inhalt* and *Umfang*. Summing up one may say that Cassirer's *Begriffstheorie* survives Marc-Wogau's assault since his argument against "dual" accounts of concepts is fatally flawed. This is not to say that Cassirer's rich "constitutive" account of *Begriffstheorie* did not suffer from its own problems. But that is another story.

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