

The Partial Identity Account of Partial Similarity Revisited

Matteo Morganti

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Abstract This paper provides a defence of the account of partial resemblances between properties according to which such resemblances are due to partial identities of constituent properties. It is argued, first of all, that the account is not only required by realists about universals *à la* Armstrong, but also useful (of course, in an appropriately re-formulated form) for those who prefer a nominalistic ontology for material objects. For this reason, the paper only briefly considers the problem of how to conceive of the structural universals first posited by Armstrong in order to explain partial resemblances, and focuses instead on criticisms that have been levelled against the theory (by Pautz, Eddon, Denkel and Gibb) and that apply regardless of one's preferred ontological framework. The partial identity account is defended from these objections and, in doing so, a hitherto quite neglected connection—between the debate about partial similarity as partial identity and that concerning ontological finitism versus infinitism—is looked at in some detail.

Keywords Partial identity · Partial similarity · Property · Structural · Conjunctive · Ontological finitism · Ontological infinitism

Introduction: Partial Similarity as Partial Identity and Structural Universals

Realists about universals explain exact similarities in terms of numerical identity: if two things are similar with respect to property *P*, they say, those things literally share *P*, a repeatable universal. But what if two objects resemble each other less than exactly, as in the case, say, of two green apples which are not the same shade of green? Numerical identity cannot be invoked there, but the realist cannot have recourse to brute facts either: for, if brute facts were to be introduced, it would appear preferable to just take similarity as a primitive *in general*, as nominalists

M. Morganti (✉)
Zukunftskolleg and Department of Philosophy, University of Konstanz, Konstanz, Germany
e-mail: matteo.morganti@uni-konstanz.de

would have it. The question arises, then, whether realism about universals can account for partial resemblances among properties.

One possible answer (call it the ‘high-order properties account’) invokes higher-order properties. Consider our two apples: according to the high-order properties account, the similarity between them with respect to colour is partial because their two instances of green have all the same higher-order characteristics except for the fact that one has the property of being a determinate of Greenness with intensity x (or, at any rate, some property determining its being the specific determinate that it is) and the other the property of being a determinate of Greenness with intensity y (or, at any rate, some property determining the specific determinate that it is). Against this view, it is usually pointed out (e.g., in Pautz (1997; 109) and Eddon (2007; 386)) that the higher order account of partial similarity was discarded by the foremost contemporary defender of universals, David Armstrong, on the basis that it is ontologically inflationary and, since it assumes that certain properties are instantiated necessarily by first-order universals, it conflicts with the combinatorial view of possibility that Armstrong endorses.¹ As a matter of fact, though, the problems for the high-order properties account of partial similarity are even worse: for, if universals (or, perhaps better, their instances) necessarily instantiate certain higher-order universals determining their exact ‘qualitative content’, one may wonder what qualitative content (if any) the universal has *in itself*. What is it to be green without being a specific shade of green? Moreover, in order to avoid an infinite regress, one has to regard the relevant high-order properties as partially similar² in a primitive, non-further-analysable way, with which the initial problem reappears. In view of the foregoing, it is advisable to look for an alternative to the high-order properties account.

According to another view (the ‘partial identity account’ (henceforth, PIT) suggested by Bradley (1893), Quinton (1973) and, most prominently, Armstrong (1978, 1997), two properties resemble partially because they are *complexes* of more basic properties—only some of which they have in common. The endorsement of PIT thus makes the realist committed to compound universals constituted of simpler universals. These can be of two types: *conjunctive* and *structural*. Conjunctive universals are constituted of universals exemplified by the very same object that exemplifies the conjunctive universal (e.g., a red ball exemplifies the conjunctive property denoted by the predicate ‘is red and spherical’, but also the property of being red and that of being spherical). Structural universals, instead, involve monadic universals standing in certain relations, and result from a process of mereological composition of material parts. That is, what Eddon (2007; 387) calls the ‘constituency principle’ holds, according to which a universal x is a constituent of universal y if and only if every object in every possible world that instantiates y has some proper part that instantiates x . This means that the complexity of structural universals directly mirrors that of the objects exemplifying them.

Usually, supporters of PIT take *all* partially resembling properties to correspond to structural universals.

¹ See Armstrong (1978; 105–108), (1989; 105).

² As they bestow analogous characteristics on their bearers, but differ in the specific manner in which they do so.

Structural universals have been introduced by Armstrong and others for a variety of reasons (Forrest, for example, uses them in his account of possible worlds as uninstantiated properties (1986)). Lewis (1986), however, argued that no satisfactory account can be provided of the sort of internal composition that characterises structural universals. He took this as an argument against universals in general, which is questionable.³ But independently of this it is clear that, if Lewis is correct, there is reason for scepticism towards PIT as the theory is normally understood.

The aim of this paper, however, is not (at least not primarily) to get into a discussion of the possibility of structural universals. For, one of the ideas underlying it is that the account of partial similarity based on partial identities *is not something that only realists about universals need and, therefore, need not (and should not) be based on structural universals*. Indeed, under plausible assumptions, it turns out that nominalists too have to use PIT. Consider trope theory: the trope theorist takes resemblance to be a primitive, and to also come in degrees. Therefore, it would seem that s/he doesn't need to say anything additional in order to make sense of partial resemblance. However, unless s/he also endorses an abundant approach to properties, s/he is likely to analyse at least some properties in terms of others: for example, suppose s/he regards the property of being water as *reducible* to two 'being hydrogen' tropes and one 'being oxygen' trope. Assuming, for the sake of argument, that predicates such as 'being hydrogen' have an actual, non-reducible trope-counterpart, it will then be natural for the trope theorist to use the component hydrogen and oxygen tropes, *not* the complex water property, as grounds for similarity claims involving water. For instance, being water and being heavy water will not be (partially) similar in a primitive, non-analysable way, but rather because the former has as constituents two hydrogen tropes and one oxygen trope and the latter two deuterium tropes and one oxygen trope—that is, in virtue of their sharing some constituents. Something parallel holds for resemblance nominalists, who reduce properties to classes of concrete particulars. Introducing 'partial similarity' classes in order to explain partial resemblances would complicate the resemblance nominalist's theoretical apparatus quite a lot: it would require the postulation of several (possibly, infinitely many) types of resemblance classes, each one connecting the particulars it contains via a different similarity relation, i.e., a relation of similarity to degree x (with respect to a given determinable). Sticking, instead, to the relation of exact similarity only and conjecturing that partially similar concrete particulars⁴ are composed of more basic, exactly similar ones provides a much simpler framework. In sum, both realists and nominalists have plausible reasons for endorsing PIT—the only difference being, obviously enough, that nominalists will systematically replace the realist's talk of *identity* with talk of *exact similarity*.

In view of this, it is not only possible but also advisable to assess the viability of PIT quite independently of the viability of the idea of a structural

³ This is because he argues that if there are universals at all, then there *must* be structural universals. For a counter-argument, see Williams (2007).

⁴ In the case of resemblance nominalism, clearly, what is postulated to have internal structure are concrete particulars, not properties.

(or, at any rate, complex) *universal*. This paper will do exactly this, attempting to show that, contrary to what was suggested in a number of recent arguments (none of which, as we will see, relies on the assumption that properties are universals!), the account is workable, *at least when properties are not universals*.⁵

This being the aim, PIT will be understood in what follows as a theory that accounts for partial resemblances in terms of partially identical complex properties, where identity encompasses both *numerical identity* (needed if PIT is implemented in realist contexts, where properties are universals) and *qualitative sameness* (sufficient for a nominalistic rendering of the theory).

The organization of the paper is this: “[Structural Universals?](#)” briefly addresses the question whether the notion of a structural universal is workable. The subsequent sections deal with criticisms that have been moved to PIT and offer responses to them, showing that the account is effective at least for nominalists. “[Pautz’s Objection](#)” examines an important objection moved by Pautz (1997); “[Eddon’s Objections](#)” looks at arguments presented by Eddon (2007); and “[Denkel and Gibb’s Objection\(s\)](#)” is devoted to an analysis of a criticism due to Denkel (1998) and later elaborated upon by Gibb (2007) (there, a relevant connection will be pointed out between the debate about partial similarity as partial identity and that about ontological ‘finitism’ versus ‘infinetism’). A concluding section follows, and an [Appendix](#) summarises the paper’s claims regarding how to conceive of and ‘quantify’ partial similarity in the various possible scenarios, so preparing the ground for as more technical treatment.⁶

Structural Universals?

As already mentioned, Lewis (1986) argued that structural universals are inherently problematic. He considered and rejected three possible conceptions of structural universals, which he dubbed ‘linguistic’, ‘pictorial’ and ‘magical’, respectively. To briefly recall Lewis’ well-known arguments against each one of these conceptions, consider the linguistic view first. According to it, a structural universal is a set-theoretic construction out of simpler universals. This immediately accounts for the necessary connections between constituent and constituted universals (sets have their members necessarily) but, says Lewis, is explanatory deficient: for, there is nothing that is explained by claiming that the set of the *x*s exists which cannot be explained by simply saying that the *x*s exist. According to the pictorial conception, instead, the relation between constituent universals and structural universals is that of genuine mereological composition, and so there is a clear sense in which structural universals as wholes are something over and above their constituent universals as parts. According to Lewis, however, since wholes cannot be distinct without differing with

⁵ The reasons for this caveat will become clear later.

⁶ The reason why the sketch of a more precise treatment of partial similarity as partial identity is confined to a short [Appendix](#) is that, although an increase in the mathematical precision of the claims made in the context of discussions of PIT might be deemed desirable, the present paper is intended as a ‘philosophical preliminary’ to any attempt at performing such task.

respect to their parts, there cannot be distinct structural universals with the same constituent simpler universals. But this is exactly what happens with *merely structurally different* properties, which are indeed possible—think, for instance, of chemical isomers (to be discussed in more detailed shortly). Lastly, what Lewis calls the ‘magical’ conception appears to presuppose a form of non-mereological composition that, *in some way*, makes things work as needed; but the exact nature of such composition remains mysterious. In particular, there seems to be a sort of magic thanks to which universals are repeatable and numerically identical across their instances but they also behave as particulars when it comes to composing structural universals: think, for example, of the property of being water and that of being hydrogen, and of how the latter is part of the former *twice*, yet numerically unique (Lewis famously called these magical universals ‘amphibians’).

Starting from this latter difficulty, Armstrong (1986) replied that some universals are ‘particularising universals’ (Ib.; 88) and ‘enfold particularity within themselves’, thus allowing for the sort of composition that determines structural universals without the need for any magic.⁷ This, however, doesn’t really help overcome Lewis’ perplexities, as it basically amounts to making what is problematic a primitive fact about (some) universals. A better argument seems to be the following: structural universals only exist *in rebus*, i.e., in the form of *specific instances* composed of equally specific instances of simpler universals; and in virtue of this they can include several instances of the same universals, for the latter are exemplified by *distinct* proper parts of the object that exemplifies the structural universal. Be this as it may, the more important difficulty remains concerning the form of composition that gives rise to structural universals: if it is mereological, how is one to account for merely structurally different properties? If it is not, what sort of composition relation is it?

In attempting a reply to Lewis, Armstrong points to the difference between the two states of affairs aRb and bRa for every non-symmetrical R . Since such difference cannot be accounted for in mereological terms (there is only one whole determined by the mereological sum of a , R and b), Armstrong argues that structural properties are ‘constituted by facts’, which don’t have a mereological mode of composition (1988; 312). This is certainly in harmony with Armstrong’s later ontology of states of affairs (1997), but again appears insufficient for solving the problem, as nothing is said about *how* exactly simpler properties constitute structural ones. As a matter of fact, Armstrong’s idea of giving up mereology altogether appears ill-motivated. Hawley (2010), for instance, points out that composition, even if understood mereologically, need not be unique as demanded by classical extensional mereology; for, there are alternative mereologies where it need not be assumed that two complexes with the same proper parts are the same entity. Mormann (2010) similarly suggests accounting for structural universals not in terms of ‘traditional Boolean mereology’ but on the basis of a general theory of part-hood

⁷ Armstrong’s example involves Greenness as opposed to Electron-hood: while we cannot speak of ‘a green’, we can meaningfully say ‘an electron’, and this is because Electron-hood is a particularising universal while Greenness isn’t. Indeed, we talk of the property of being green and of the property of being *an* electron.

based on category theory according to which, again, two wholes with the same parts need not be numerically identical. In both cases, mereological identity can coexist with structural diversity, and structural universals appear consequently defensible from Lewis' objections.⁸

In the next section, a couple of other suggestions will be put forward: one aiming to preserve classical mereology, the other to make sense of Armstrong's claims about the role of facts within structural properties. That will complete the discussion in this section and, at the same time, start a more specific assessment of PIT, independent on whether a realist or a nominalistic ontology is endorsed.

Pautz's Objection

Pautz (1997) puts forward the following argument against PIT:

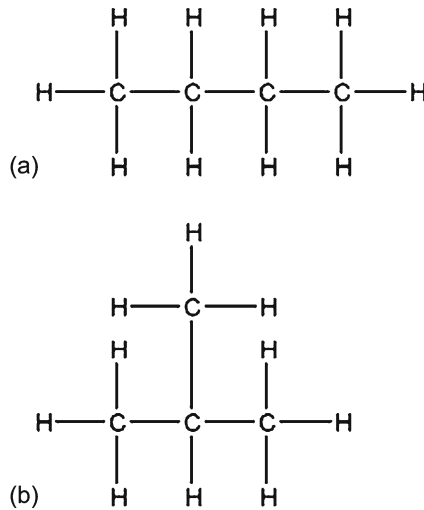
- 1) PIT is correct (assumption);
- 2) 'F and G resemble exactly' is equivalent to 'It is impossible that there are P and Q such that P and Q resemble more than F and G' (definition of exact similarity);
- 3) 'It is impossible that there are P and Q such that P and Q resemble more than F and G' is equivalent to 'It is impossible that there are P and Q such that P and Q have more constituents in common than F and G' (from 1);
- 4) Hence, 'F and G resemble exactly' is equivalent to 'It is impossible that there are P and Q such that P and Q have more constituents in common than F and G' (from 2 and 3);
- 5) 'It is impossible that there are P and Q such that P and Q have more constituents in common than F and G' is equivalent to 'F and G have all their constituents in common' (definition of maximal sharing of parts);
- 6) Hence, 'F and G resemble exactly' is equivalent to 'F and G have all their constituents in common' (from 4 and 5);
- 7) Hence, 'F and G resemble exactly' and 'F and G have all their constituents in common' must have the same entailments (from 6) and the definition of equivalence);
- 8) However, complete sharing of constituents does not entail identity, while exact resemblance does, so 7) doesn't hold;
- 9) Hence, assumption 1) is false, and PIT is incorrect.

In a nutshell, Pautz intends to show that PIT is unable to correctly distinguish between partial and exact similarity, and cannot consequently be considered the right account of partial similarity.

The crucial premise in the argument is 8) above. Indeed, it appears possible for the same set of constituent properties to give rise to non-identical structural properties: for instance, in the abovementioned case of chemical isomers. To use the by now canonical example, the properties of being butane and being isobutane are both constituted by 4 instances of being carbon and 10 of being hydrogen. Including *relations* among the constituents doesn't work: in the example just used, both being

⁸ For a recent discussion and defence of Armstrong's views, see McDaniel (2009).

butane and being isobutane include 13 relations corresponding to binary chemical bonds (see diagrams below).



(a) Butane – (b) Isobutane

Pautz's argument, however, need not be regarded as final. For, it is possible to replace symmetric dyadic relations (e.g., the bonding relations in the above example) with *monadic n-place relational properties possessed by particular objects and expressing these objects' structural roles in full detail*, and this solves the difficulty pointed out by Pautz. In abstract terms, these properties are something of the form 'is connected to a U, a W...', where 'U', 'W' etc. denote either i) specific actual constituents of the structural property in question or, more abstractly, ii) the structural features of such constituents. For a concrete illustration, consider again the butane/isobutane case. On the first construal, taking, e.g., 'R3H1C' to name the 4-place relational property of being bonded to three instances of being hydrogen and one of being carbon, it immediately turns out that butane and isobutane share 10 'R1C's and 2 'R3H1C's but *differ* in that the former includes 1 'R3C1H' and 1 'R3H1C' and the latter 2 'R2H2C'. From this it follows that the partial similarity between being butane and being isobutane can in fact, *contra* Pautz, be explained in terms of non-exact identity of constituents. One may be suspicious of 'impure' properties, that is, of properties that include an essential reference to specific entities external to their bearers. However, exactly the same result can be obtained by opting for ii) above, and making the specific relational properties encode a reference not to entities but to (non-specific) *places* in the relevant structures. This is usefully illustrated via an analogy with graph theory. Graphs are mathematical entities composed of nodes (objects) and edges (relations) connecting them, with the nodes being either labelled or unlabelled (and the edges either directed or undirected, i.e., either symmetric or asymmetric). Now, in the case of labelled graphs, the labels can be assigned to nodes as lists of n numbers, one for each of the n nodes to which the

node is connected, and each one expressing how many nodes the corresponding node is in turn connected to. This can be straightforwardly translated into relational properties conveying structural information. In our example, taking, for instance, ‘R (1,4)’ to be short for ‘is related to something that is connected to one other thing and to something that is connected to four other things’ butane and isobutane again differ structurally. For, in addition to their ‘being carbon’ and ‘being hydrogen’ properties, they have 10 R(4) and 2 R(1,1,1,4) in common, but while the former also has 2 R(1,1,4,4), the latter has 1 other R(1,1,1,4) and 1 R(1,4,4,4). Pautz’s argument is neutralised again.⁹

Whatever option one prefers between the two just described, the important thing is that if one includes relational properties rather than relations in the count of the constituents of structural properties, then complete sharing of constituents *does* entail identity. Consequently, premise 8) above is false, and conclusion 9) doesn’t follow.¹⁰

Moving one step back, one may take the plausibility of a sparse view of properties as sufficient for ruling out the ‘ontological genuineness’ of the relational properties just proposed. This would mean that such properties cannot be counted among the constituents of structural properties, and Pautz’s criticism is consequently successful. The only reaction to this available to the supporter of PIT would at that point be that exact similarity consists of the complete sharing of constituents *plus* sameness of structure, the latter defined in terms of specific relational predicates that do not denote genuine constituents, and yet *describe relevant structural facts*. On this construal, it is premise 3) above that is false, because even if one assumes PIT ‘It is impossible that there are P and Q such that P and Q resemble more than F and G’ is *not* equivalent to ‘It is impossible that there are P and Q such that P and Q have more constituents in common than F and G’. Of course, this would mean that, in order to resist Pautz’s criticism, the defender of PIT is required to modify his/her proposed theory in a much more radical way than in the previous case. Nonetheless, the suggested modification certainly doesn’t entail the collapse of the theory, for it just requires one to take more than (monadic and non-relational) constituents into account whenever structure plays a role, which is, after all, hardly surprising.

In view of the foregoing, it is legitimate to claim that [Pautz’s objection](#) is important but not fatal to PIT: it applies to Armstrong’s original version of PIT, but supporters of the theory can modify the latter as needed—either substituting relations with (appropriately characterised) *relational properties* in the set of the constituents of structural properties, or taking into account *relational facts* in addition to constituents when comparing structural properties.¹¹

⁹ Importantly, the information encoded in the relational properties being invoked is, on the second construal, purely structural.

¹⁰ One may worry that it is more natural to think that relations ground relational properties than the other way round (see, for instance, Hawley (2009)). In that case, the argument just offered would need to be backed up by additional considerations. There is no space to do this here, but their suggested role in characterising at least certain structures unambiguously might in itself count as a reason for taking relational properties to be more fundamental than relations (at least in some cases).

¹¹ One could also take structures as non-analysable primitives. However, this would force one to take partial similarities grounded in structural differences to also be primitive and non-analysable, which is clearly at odds with the basic idea that underlies PIT.

Eddon's Objections

Eddon (2007) uses classical physics as a 'testing ground' for PIT, on the basis that i) it provides very good examples of *quantitative* properties—for which, according to Armstrong (1989; 101), the theory is clearly successful and ii) it more generally constitutes a metaphysically possible scenario that supporters of PIT must be able to deal with. Let us then examine Eddon's objections to PIT, and possible ways to block them.

- 1) First, Eddon (2007; 393–396) argues that certain quantities, for example charge, are such that they admit of positive and negative, and that their positive instances may be more or less similar to negative ones; and yet, these similarities cannot be analysed in terms of sharing of components, because positive and negative instances of the same property do not share any components (assuming that they do, Eddon shows, leads to absurd consequences). Such similarities, Eddon adds, cannot be explained on the basis of their causal/nomological roles either, for this would suffice for rejecting PIT as insufficiently general.

While Eddon certainly points out an actual difficulty for PIT, his conclusion is too hasty: for, the supporter of PIT *can* in fact invoke causal/nomological role instead of identity of constituents, *provided* that s/he presents a principled distinction between similarities to which his/her theory can and should be expected to apply and similarities to which it can and should not. But one plausible way of doing this exists. It rests on the following claims:

- a) PIT can only be expected to apply in the case of determinates falling under the same determinables, for only in that case can properties have constituents in common (what constituents can be shared between, say, a colour and a shape property?);
- b) Determinates of different determinables may at best be compared on the basis of their causal features and the way in which they interact (mass and weight properties are more similar to each other than, say, properties expressing mood and properties expressing angular momentum, but certainly not in virtue of different numbers of shared constituents: rather, because weight nomologically depends on mass but the angular momentum of something isn't in any way related to whether that thing is happy or sad);
- c) The comparison of determinates of different determinables may in some cases give rise to classifications and conventions that justify the use of the same name—most notably, when two 'families' of properties interact in such a way that they allow for addition and subtraction;
- d) This is what happens for quantities admitting of positive and negative which, therefore, subsume two 'analogous', yet ontologically distinct, property-types.

Opponents of PIT may reply that the peculiar way in which quantities admitting of positive and negative interact (in particular, the fact that they can be added and subtracted to each other) suffices for taking them to be expressions of the same determinable—not just share a name. However, it is at least equally plausible that the identity of determinables is grounded in the similarity with respect to causal roles

that holds between their determinates. And, if this is so, quantities admitting of positive and negative cannot fall under the same determinable *exactly because* they can be added and subtracted to each other: for, this is the case in virtue of their having distinct (indeed, opposite!) causal powers. Hence, unless one allows for linguistic practice to mechanically determine one's metaphysical beliefs, Eddon's argument is not conclusive.

A related issue that, with this, immediately finds a solution is the following. According to Eddon (Ib.; 400–401), PIT fails to make sense of the fact that resemblances can be evaluated differently in different respects and contexts, for example when a 5 coulomb charge universal is judged to be more similar to a -5 coulomb universal than to a -1 coulomb universal because the first two have the same absolute magnitude, but also more similar to the -1 coulomb universal than to the -5 coulomb universal because less 'far apart' from the former than from the latter in the measuring scale. In response to this, the supporter of PIT can and ought to contend that, strictly speaking, it is never the case that several, equally valid similarity judgments are available, because facts about sharing of constituents are objective, well-defined and univocally determined. In the example above, in particular, (for reasons just illustrated) s/he can and ought to contend that only the -1 and the -5 coulomb properties can be compared objectively, i.e., on the basis of PIT; and that in the other cases resemblance can only be intended in a looser sense, and similarity judgments only *be based on considerations other than partial identity*, that may well be context-dependent.

- 2) Next, Eddon holds (Ib.; 396–400) that vector quantities cannot be consistently analysed in terms of partial similarity as partial identity. According to her, in particular, the spatial orientations of distinct vector properties cannot be compared in a way compatible with PIT.

There is neither the space nor the need here to consider the details of Eddon's arguments on this issue, often directed against Armstrong's specific version of PIT. What is important is that, contrary to what Eddon claims, a plausible reconstruction of vector properties can in fact be given that doesn't conflict with PIT. In particular, two options are available, depending on whether one regards vector properties as intrinsic or extrinsic to their bearers. Let us consider each one in turn.

If vector properties are considered intrinsic, then they can be analysed in terms analogous to those of *spherical coordinate systems*. In mathematics, a spherical coordinate system is a coordinate system for three-dimensional space where the position of a point is specified by three numbers: the *radial distance* of that point from an origin, its *inclination angle* measured from a fixed 'zenith' direction, and the *azimuth angle* of its orthogonal projection on a reference plane that passes through the origin and is orthogonal to the zenith, measured from a fixed reference direction on that plane. Now, vector properties might be analysed in terms of $(n+2)$ -tuples (with $n \geq 1$) of correlated *scalar* magnitudes that, together, uniquely determine the vector property (according to 'concrete' rules of composition, one may suppose, that are directly mirrored by the abstract ones that apply to mathematical vectorial spaces). In particular, n such magnitudes would 'encode' the length L of the vector (i.e., the magnitude of the property), while the other two would determine the exact

point on a sphere of radius L where the tip of the vector lies, i.e., the spatial orientation of the vector property. Despite the fact that the magnitude component and each of the direction components are not mutually comparable,¹² each one of the component-types is a scalar magnitude which is perfectly analysable in the terms of PIT.

One may not be convinced by this because s/he believes that vector properties are not, at root, intrinsic to their bearers.¹³ In that case, the supporter of PIT can modify the picture accordingly. One way of doing this is suggested by Forrest (2009), who (considering in particular the case of vector properties in General Relativity) contends that vector fields are to be analysed in terms of *relations* between scalar fields. Forrest takes vector fields to be *differentiations* of scalar fields, where a vector field is an assignment of tangent vectors to points, scalar fields are understood in the intuitive way—as continuous ensembles of scalar quantities—and differentiation is understood as the real-world counterpart of the mathematical action of a differential operator.¹⁴ Without entering the details of Forrest’s proposal, what is important for present purposes is that it suggests an analysis of vector properties in terms of monadic constituents structured together into a more complex quantity via a relation that is itself part of the complex whole. With which, one immediately obtains a structural rendering of vector properties, and the applicability of PIT even in the case in which such properties are regarded as extrinsic to their bearers immediately follows (recall the discussion structural properties in the previous section).

3) Next, Eddon considers (Ib.; 388–391) mass properties and points out that, if one supposes that mass quantities are isomorphic to the real numbers, that is, that there are uncountably many of them, then the ordering provided by the sharing of constituents doesn’t coincide with an objective measure of similarity. For, every massive object has an infinite number of constituents and, consequently, both the number of shared components and the number of non-shared components is infinite. In view of this, Eddon argues, one is forced to introduce a non-further-analysable metric expressing an ordering of distance between quantities, which is exactly what PIT was supposed to make unnecessary.

¹² The reason why the magnitude of a vector property may be encoded in more than one component is also the reason for the magnitude component not being comparable with the angle components: the magnitude of the vector *may* consist of more than one ‘determination dimension’, and the magnitude and angle components of vector properties *in fact* correspond to distinct determination dimensions. For more details on the notion of a determination dimension see below, “Denkel and Gibb’s Objection(s)”.

¹³ See, for example, Beisbart (2009). Busse (2009) voices another worry: he argues that the view just suggested fails to do justice to vectorial quantities such as those dealt with, e.g., in classical electrodynamics, which are not described as composites of several *autonomous* components. Here we cannot get into details, but it looks like Busse’s worries can be obviated by specifying *constraints* that the component scalar quantities must obey when combining into vectorial ones. To be sure, Busse doesn’t show (nor claims) that what he calls the ‘multiple quantity conception’ of vectorial properties is inconsistent.

¹⁴ Forrest points out that fields need not be composed of points, but it is not necessary to discuss this here, especially in view of the fact that Forrest shows that his point-based proposal extends quite straightforwardly to non-point-like vector quantities.

In response to this worry, the supporter of PIT may, of course, simply reject the possibility of infinitely composed quantities. However, since this would mean to rule out a priori certain possibilities (with objects and/or properties that are infinitely decomposable into simpler constituents) that appear coherent (and will in fact be taken seriously in the next section), this is not advisable. But there is another way out available to the supporter of PIT. The first thing to say is that, upon scrutiny, Eddon's objection appears based on an ambiguity concerning the work ontological analysis must do here. True, PIT should provide an ontological underpinning for similarity facts. But *not* in the sense that it must give us an exact count of *all* the basic constituents of the things being similar. Rather, in the sense that one must be able to individuate a shared and a non-shared set of constituents *in every specific actual case* of partial similarity. To see the difference between the latter and the former understanding of PIT's duties, suppose, for instance, that we have *a*—the property of having a mass of 46 g—, *b*—the property of having a mass of 47 g—and *c*—the property of having a mass of 1235 g. In this case, we *start from* the fact that *a* is more similar to *b* than to *c* (the actual weight of the objects exemplifying *a*, *b* and *c* is certainly *not* itself to be determined on the basis of PIT), and require our account of similarity to provide an ontological basis for these similarity facts (*not* for our claims about the objects specific weights). But such an explanation is readily given in terms of shared and non-shared *finite* parts within *a*, *b* and *c*: a shared part of 46 units (A) and two non-shared parts, one of 1 unit (B) and one of 1189 units (C). From a comparison of B and C, one immediately obtains the required similarity measure! Of course, further analysis might be asked for (in particular, when evaluating the partial similarity of A, B and C, and the difference between B and C) and, under the present assumptions, will in fact be required *ad infinitum*. But this is not a problem for PIT, because at each level of analysis only *finite* shared and non-shared parts need to be invoked.

To see this from a slightly different perspective, consider a geometrical line: we can and do use the hypothesis that there is an infinity of points in it to explain facts about it, *including* facts concerning segments of it and the possibility of identifying an infinity of them; but we certainly don't use the same hypothesis to ground our claims about specific lengths of segments and specific relationships between such lengths in terms of a count of points. Analogously it would be wrong to expect PIT to either provide an exact count of all constituents or employ a primitive metric in all cases: the *only* difference between finite and infinite quantities is that for the former but not the latter can PIT provide a well-defined measure of (dis-)similarity in terms of *basic* constituent properties. But PIT shouldn't be expected to individuate the fundamental constituents of all structural properties: in fact, whether there need be such constituents at all is, as we will see shortly, an open philosophical question.

- 4) Lastly, Eddon (Ib.; 391–393) formulates the following objection to PIT: given the constituency principle, the bearers of finite quantities that partially resemble each other must have constituents; however, this means that point-sized particles such as electrons, which possess, say, finite masses, in fact have proper parts, each one with smaller mass; and this a) makes it metaphysically impossible for just one object with a finite quantitative property to occupy a point at a time,

which instead appears both possible and likely to be actually the case, b) entails that we should be able to isolate the parts of point particles in the same way in which we isolate parts of spread out objects, which we are not, and c) meets with the difficulty that, unlike for ordinary objects, there is no independent justification for the assumption of internal complexity.

Incidentally, with respect to a) above, the right thing to say (if anything) is, of course, that PIT makes it impossible for point particles to possess finite quantities *and still partially resemble other entities*. Independently of this, there are two possible responses to the worries just mentioned. The quick(er) response is that the consequences of PIT that Eddon points at may well be problematic, but can (and are to) be accepted in view of the explanatory efficacy of the theory: hence, one should postulate simplicity only when one has either exact similarity or complete lack of shared constituents.¹⁵ The more elaborate, and—probably—more effective, reaction consists in modifying PIT so that the complexity of partially similar properties does not *necessarily* coincide with the mereological complexity of their bearers. True, so far we have assumed that partial identities among properties always coincide with those properties being structural properties and therefore, via the constituency principle, with the mereological complexity of the bearers of those properties. However, widespread as this assumption may be, it is essential to recall that the basic claim of PIT theorists is that properties that are partially similar cannot (all) be *simple*, not that they (at least some of them) must be structural.¹⁶ Since complex properties need not be structural—they can be conjunctive—, it follows that *PIT need not be based on structural properties and the constituency principle*, and so Eddon's objections above dissolve. Indeed, conjunctive properties are also complex and, as such, can ground partial similarities; at the same time, since they are conjunctions of simpler properties attributed to *the same* entity that exemplifies them they *do not* entail mereological complexity. This makes it possible for only one object with a finite quantitative property to occupy a point at a time even if the property in question is partially similar to other properties.

Importantly, this may be problematic for realists about universals, for they typically deny that the same universal can be instantiated many times by the same entity, while this is exactly what happens with conjunctive properties constituted by constituent parts that are all alike (which is likely, for instance, in the case of mass properties considered by Eddon). Still, recall that we are interested here in the general viability of PIT, which can also be framed (as argued at the beginning) in a nominalistic context.¹⁷ While acknowledging the importance of the problem just pointed out, therefore, we will take it that PIT can in any case survive also the last

¹⁵ Notice, incidentally, that Eddon's point b) ignores the fact that the impossibility to isolate the parts of point particles may in fact have an explanation. Consider for example the phenomenon of quark confinement: the constituent quarks of elementary particles cannot exist on their own for physical, not metaphysical, reasons.

¹⁶ The parenthetical qualifications are needed because two properties can be partially similar even if one of them is simple: the simple property will be identical—qualitatively or numerically—to one of the two or more constituents of the other property.

¹⁷ Trope theorists will argue that (some) objects can exemplify exactly similar tropes, and resemblance nominalists will distinguish between, say, the classes of similarity composed by 'P things', by 'P&P things' etc.

one of Eddon's objections (for, the answer to a) just provided also constitutes an immediate reply to b) and c), as an allegedly inevitable mereological complexity has in fact been avoided, and with it the request for explanation expressed by those additional worries).

With this, we can conclude that Eddon's objections to PIT can all be neutralised, at least if PIT is framed in a nominalistic setting. The issue concerning finite versus infinite properties and property-bearers, raised by the last two of Eddon's criticisms, deserves however further consideration. As a matter of fact, it turns out to play a crucial role in the discussion of another line of attack against PIT, which will be considered in the next section (and is the last argument that will be dealt with in this paper).

Denkel and Gibb's Objection(s)

Denkel (1998) argues as follows:

- 1) Assume PIT is correct;
- 2) Consider a case of partial similarity between two properties. These properties must have two 'aspects': an identical aspect (the shared constituents) and the non-identical aspect (the non-shared constituents, preventing the similarity from being exact) (from 1));
- 3) These two aspects must fall under the same determinable, otherwise they wouldn't form genuine structural properties (but, rather, conjunctions of two distinct properties)¹⁸;
- 4) The identical and the non-identical aspect must be identical or inexactly resembling (from 3));
- 5) Since they are related in such a way that they give rise to a non-exact resemblance, the two aspects must be inexactly resembling, hence partially identical (from 1) and 4))¹⁹;
- 6) However, this means that the identical and non-identical aspects are not simple (which would contradict 1)) and are instead complex compounds sharing only some of their constituents;
- 7) The request for an analysis in terms components arises again, ultimately *leading to an infinite regress*.

Using an abstract, schematic example, suppose properties P and Q are partially similar. According to PIT, this means that they have one or more constituents in common and one or more constituents they don't share. Putting together the former as aspect A, and the latter as aspects B and C, we get that, say, P is a structure A-B and Q is a structure A-C. But A and each one of B and C must be partially similar. Hence A must be of the form D-E, B of the form D-F and C of the form D-G. But D and each one of E, F and G must be partially similar, and so on.

¹⁸ Set aside for a moment the suggestion about conjunctive properties made at the end of the previous section.

¹⁹ This clearly assumes that if the two aspects were identical, the initial resemblance would be exact rather than partial, i.e., that the considerations about purely structural differences made earlier in connection to Pautz's objection are temporarily bracketed.

The above is deemed sufficient by Denkel for rejecting the partial identity theory as *incoherent*. Assuming for a moment that a claim of incoherence makes sense given his argument, however, counter-objections can certainly be moved against Denkel. First, Denkel assumes (5)) that whenever one has a partial similarity between two properties, the latter have an identical and a non-identical aspect *that are not exactly identical*. As shown by Gibb (2007), though, this is not the case. As Gibb explains:

[q]uantitative properties such as lengths, durations and masses are all examples of properties that Armstrong takes to be structural [... But for them,] the non-identical aspect is identical with the identical aspect (2007; 547–548).

Gibb uses the example of the properties denoted by ‘being 3T’ and ‘being 2T’, where T is a unit of duration. Since both in the identical and in the non-identical part the basic constituent is T, she argues, contrary to Denkel’s argument the non-identical aspect is in fact identical to the identical aspect (Ib.; 548).²⁰ In view of this, Gibb argues that for quantitative properties such as length or duration the analysis need not give rise to an infinite regress. Moreover, Gibb also points out the incorrectness of Denkel’s assumption (3)) that the non-identical aspect and the identical aspect must fall under the same determinable, and so be either identical or partially identical. To show why this need not be the case, she introduces the concept of a ‘determination dimension’ that was mentioned earlier. A determination dimension is something like a component of a determinable. Properties can have more than one determination dimension, as in the case (which Gibb considers in some detail) of colour properties and their three determination dimensions of hue, saturation and brightness, or of vector properties understood as complexes of scalar quantities in (the real-world counterpart of) a spherical coordinate system as suggested earlier in this paper. In such cases, properties may differ from each other with respect to one determination dimension while being identical with respect to another. Crucially, though, different determination dimensions correspond to distinct determinables and, therefore, do not (or at least need not) share constituents with each other. From this, it follows that, *pace* Denkel, partially similar properties with more than one determination dimension may be partially similar but have non-identical and identical aspects which do not fall under the same determinable and for which, therefore, the question whether they are completely or partially identical does not arise.

On the other hand, as Gibb acknowledges, Denkel’s reasoning still applies as long as i) partial similarity with respect to one particular determination dimension is concerned, and ii) the properties in question are not composed of a finite number of identical simple quantities as in the case of duration, mass, lengths and the likes. In view of this, Gibb concludes that

although Denkel has not established that the partial identity account is logically incoherent, his argument is of crucial importance, for [...] it establishes that the partial identity account is *plausible* only for those determinates whose

²⁰ Strictly speaking, this is not correct. The non-identical aspect is T while the identical aspect is 2T. But what Gibb means is clearly that the identical aspect and the non-identical aspect have themselves an identical aspect and a non-identical aspect which are identical: T. In the terms of our schematic example, this is to say that P and Q can be partially similar without being of the form A-B and A-D but, rather, of the form A-A-A and A-A. That is, one of the properties may contribute nothing to the non-identical aspect.

constituents are at bottom quantitative in nature, for it is only when a quantity is reached that the regress can be halted. Consequently, if the partial identity account is to have full coverage (which it presumably must if it is to be plausible), then every determinate property (that resembles another) *must ultimately be a quantity* (Ib.; 557; italics added).

At this point, some of interrelated questions arises: Why is it that a regress of the sort envisaged by Denkel should entail the *logical incoherence* or *implausibility* of the account? What does it mean that it is “only when a quantity is reached that the regress can be halted” and PIT requires that “every determinate property (that resembles another) must *ultimately be a quantity*”?

First of all, it must be emphasised that it is the fundamental *assumption* of PIT that *all* properties that are partially similar to each other are complex properties that are partially identical. And this clearly *entails* that all partial resemblances are rooted in exact resemblances, *even if only in the limit*. Compare with geometry: any given line is ultimately constituted of points, i.e., the elements that we obtain in the limit of a process of division of a line into parts are non-extended. Yet, the line surely has an extension and so do all the one-dimensional segments into which it can be subdivided. However, this coexistence of extension and point-likeness doesn't make our conception of a line incoherent. Rather, we consider it a natural fact of geometry that every line of finite length, although ultimately composed of non-extended parts, can always be analysed in terms of shorter components of non-zero length—an assumption that is in fact a fundamental explanatory basis for a wide range of geometrical facts.

It seems clear that the worry is with the ‘even if only in the limit’ clause. That is, the idea appears to be that an ontological analysis that never stops is unacceptable and, therefore, PIT (which, it seems, does entail a never-ending process of analysis at least in some cases) clashes with a metaphysical assumption that we (should) regard as untouchable. Perhaps, then, Denkel's charge of incoherence is too strong—what is allegedly unacceptable here in no way entails a contradiction—but Gibb's talk of implausibility can be considered justified.

This would truly be the case, however, only if one assumed ‘ontological finitism’, the thesis that there cannot be ungrounded chains of ontological (and, consequently, explanatory) dependence. Denkel and Gibb's point thus boils down to the claim that PIT cannot possibly work because it contradicts ontological finitism, while the latter is—they seem to believe—eminently plausible if not a necessary truth. Indeed, recall Gibb's other key claim, according to which the realist *must* show (or at least postulate) that every determinate property (that resembles another) must *ultimately be a quantity*. This clearly suggests that the analysis is expected to reach *in a finite number of steps* exactly similar *basic simples*, otherwise it simply fails.

Unfortunately (for Denkel and Gibb, at least), that ontological finitism should not be presupposed *a priori* has been shown quite clearly in the recently (flourishing) literature on ontological fundamentality. Schaffer (2003), for instance, claims that the intuition that there is a fundamental level, seemingly supported by common sense and science, is far from being something that philosophers should consider unquestionable. Schaffer convincingly argues that such an intuition is merely a

Newtonian heritage, and is in fact supported neither by science and its history nor by conceptual analysis. Starting from this, one might claim, more strongly, that the idea that ontological analysis must reach a basic level is essentially based on prejudice and, consequently, cannot without further argument ground a compelling argument against PIT.

Cameron (2008) considers various reasons why an infinite chain of ontological dependence relations should be discarded, and finds all of them wanting, except for a generally methodological one. Assuming that there is a fundamental ontological level not dependent on anything below it, Cameron claims, allows one to formulate more unified metaphysical explanations, as there is a collection of objects that grounds the existence of every dependent thing (Ib.; 12). This might seem a reasonable ground for taking, as Cameron does, ontological finitism to be at least contingently true. However, as Cameron himself concedes, methodological considerations do not provide conclusive arguments in this sense. In fact, there are competing methodological factors beside economy of explanation which might pull in a different direction: for instance, simplicity, intuitive plausibility and, most importantly, *explanatory scope*. With respect to the topic of this paper, surely PIT theorists can and do contend that the ability to provide an analysis of partial resemblances compensates the ‘ontological proliferation’ that follows from its central assumption of ontological complexity. Moreover, the superiority in terms of unity and economy of explanations that presuppose a fundamental level of independent entities is not obvious. For, while it is true that if there is no end to the chain of ontological dependence relations then there is no (finite) collection of *entities* that explains everything, there might be a (finite) collection of *statements* that does. Indeed, this seems to be the case for PIT and its general claim that *every* instance of partial similarity is to be analysed in terms of partial identity.²¹

In view of the foregoing, it looks as though the issue concerning the possibility of carrying out the proposed analysis of similarity facts had better be decoupled from the issue of whether ontological infinitism can (should) be accepted. In any event, the realist who wants to explain partial resemblances in terms of PIT is faced with two alternatives, *both of which are logically coherent and metaphysically respectable*:

- a) To insist that basic simples of non-infinitesimal magnitude always exist, which allows one to avoid postulating infinite complexity but requires one to hypothesise a privileged fundamental level—let’s call this the ‘Gibb strategy’;
- b) To claim that the proposed analysis of resemblance facts works, although this *might be* because the world (or parts of it) are infinitely composed and the basic simples have infinitesimal magnitude; this would allow one to be ‘agnostic’ about a supposed privileged level of fundamental properties (and, perhaps, property-bearers)—call this the ‘infinetist strategy’.²²

²¹ For further discussion, see Morganti (2009).

²² Notice that, quite importantly, the infinitist strategy doesn’t involve an assumption that the world actually *is* infinitely composed (as Denkel, Gibb and, to some extent, Eddon—following Armstrong, seem to think), but just that it *might be*. The Gibb strategy, instead, *requires* that *every* property that resembles another actually *is* ultimately composed of basic identical simples.

While certainly a lot more can be said about this, the viability of both a) and b) suffices for re-establishing the plausibility of PIT in the face of Denkel and Gibb's objections.²³

Conclusions

With some modifications to Armstrong's original rendering of the view, all extant arguments against the account of partial similarity based on complex properties—in particular, those (due to Denkel and Gibb) that wrongly presuppose the unacceptability of infinities in ontological and explanatory analyses—can be neutralised. At least by the nominalist: the realist still has to i) provide a detailed satisfactory account of structural universals and ii) deal with Eddon's objection about point-sized objects with finite quantitative properties entering relations of partial similarity. In any event, PIT resurfaces as an important theory of properties and their similarities, and one that deserves (further) careful consideration.

Appendix

The claims in the present paper can be usefully summarised employing a slightly more rigorous approach. The simplest case is of course that of a complex property with only one determination dimension and a finite number of basic constituents that are either mere 'aggregated' or structured in simple 'linear' fashion. An example of such a property is mass. In this case, a measure of similarity is straightforwardly obtained via a count of constituents. If the number of constituents is infinite, the same holds, and the only difference is that there is no basic level of analysis and the 'units of measure' can be arbitrarily small. For complex properties with n determination dimensions (each one composed of basic constituents), similarity can be measured employing the tools of analytic geometry. Set, for instance, $n=2$: given an xy -plane and taking each axis to correspond to one determination dimension, two points (x_1, y_1) and (x_2, y_2) in it will represent two determinates of the relevant (two-dimensional) determinable, and their degree of similarity will be

²³ It is interesting to emphasise that the Gibb strategy does not turn out to be preferable upon any plausible conception of properties. Consider Schaffer (2004) distinction between the 'scientific' and the 'fundamental' view of properties (both based on the idea that empirical science is the best guide for identifying the properties that truly are 'out there'—Armstrong's 'scientific approach'—but the former allowing for several levels of genuine properties and the latter taking (some) physical properties to be ontologically basic). On the fundamental conception, at present we should consider basic the properties described by the Standard Model of elementary particles. However, although they are quantitative, these properties are *not* analysable in terms of simple basic constituents (clearly, to simply claim that even more basic properties *will be discovered* by physicists in the future is question-begging). On the scientific account, the infinitist strategy comes out at least on a par with the Gibb strategy, for (as Schaffer argues well) the assumption that there is no unique level of fundamental entities and/or properties naturally leads to the idea that there are no simple entities and/or properties at all because there is no fundamental level. As a matter of fact, it could even be legitimately argued that, all things considered, the alleged vicious regress individuated by Denkel and Gibb points to the best option available to the supporter of PIT!

given by the distance between the points in the plane, which is $d((x_1, x_2), (x_2, y_2)) = \sqrt{(x_2 - x_1)(y_2 - y_1)}$. The same holds for any value of n , of course: one just needs to add other elements to the square root to the right of the above equality ($(z_2 - z_1)$ etc.). Notice that, given what was said earlier about positive and negative quantities being determinates of different determinables, one doesn't need to employ the full Cartesian (or at any rate n -dimensional) space, but just one sub-space with positive values along all axes. Something similar works for vector properties analysed in terms of spherical coordinate systems as suggested in the paper. Considering the latter, however, one may envisage cases in which the determination dimensions are not ontologically on a par, i.e., one of them is 'more fundamental' as a component of the relevant complex property. In particular, the magnitude of a vector property may legitimately be deemed 'more important' than its two components determining spatial orientation. In this case, either one weighs the relevant constituents accordingly (which, admittedly, may be irredeemably arbitrary), or one has to posit two or more specific similarity spaces that are, strictly speaking, mutually incomparable. As we have seen, if vectors are instead understood *à la* Forrest in terms of pairs of mutually related field magnitudes, they become analogous to structural properties. As for the latter, as argued in the main text, similarity with respect to constituents must be considered alongside similarity with respect to structural facts. This second sort of similarity can be expressed, as we have seen, either in terms of relational properties that make specific reference to monadic constituents of the structural property, or of relational properties expressing purely structural facts. In both cases, the structural information is encoded in multi-dimensional properties that (independently of whether or not one takes them to be genuine constituents of structural properties) can again be analysed by having recourse to the notion of a determination dimension and to analytic geometry along the lines just sketched.

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