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VALIDITY AND ACTUALITY*

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Abstract

The notion of validity for modal languages could be defined in two slightly different ways. The first is the original definition given by S. Kripke, for which a formula φ of a modal language *L* is valid if and only if it is true in every actual world of every interpretation of *L*. The second is the definition that has become standard in most textbook presentations of modal logic, for which a formula φ of *L* is valid if and only if it is true in every world in every interpretation of *L*. For simple modal languages, "Kripkean validity" and "Textbook validity" are extensionally equivalent. According to E. Zalta, however, Textbook validity is an "incorrect" definition of validity, because: (i) it is not in full compliance with Tarski's notion of truth; (ii) in expressively richer languages, enriched by the actuality operator, some obviously true formulas count as valid only if the Kripkean notion is used. The purpose of this paper is to show that (i) and (ii) are not good reasons to favor Kripkean validity over Textbook validity. On the one hand, I will claim that the difference between the two should rather be seen as the result of two different conceptions on how a modal logic should be built from a non-modal basis; on the other, I will show the advantages, for the question at issue, of seeing the actuality operator as belonging to the family of two-dimensional operators.

1. Introduction

The notion of validity for modal languages could be defined in two slightly different ways. The first is the original definition given by S. Kripke, for example, in Kripke, 1963a, and Kripke, 1963b, for which, roughly, a formula φ of a modal language *L* is valid if and only if it is true in every actual world of every interpretation of *L*. The second is the definition that, for reasons better explained below, has become standard in most textbook presentations of modal logic (see, to mention just a few, Chellas, 1980, Hughes & Cresswell, 1996, Blackburn, Rijike, & Venema, 2001 and Garson, 2006) for which,

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roughly, a formula φ of L is valid if and only if it is true in every world in every interpretation of L. For simple modal languages, "Kripkean validity" and "Textbook validity" are extensionally equivalent.

However, at least two reasons are sometimes presented in favor of Kripke Validity:

1. According to Zalta, 1988 — but see also the recent reprise in Nelson and Zalta (2012) — only the Kripkean definition of validity is in full compliance with Tarski's notion of truth. Textbook validity is thus an "incorrect" definition of validity; furthermore, Textbook validity conflates a semantic notion with a metaphysical one.
2. In expressively richer languages, most notably modal languages augmented with the so-called actuality operator, some obviously true formulas count as valid only if the Kripkean notion is used.[1]

The purpose of this paper is to show that 1 and 2 are not good reasons to favor Kripkean validity over Textbook validity. In particular, I will argue that even Textbook validity is a legitimate generalization to the modal case of Tarski's definition of validity for extensional languages and I will also argue that even languages with Textbook validity, once properly enriched, could account for the validity of some formulas with the actuality operator.

2. Two notions of validity

According to Kripke, a model for a *normal modal propositional calculus* L consists of two associated elements:[2] (i) a *normal modal structure* $\langle G, K, R\rangle$, where K is an arbitrary set ("possible worlds"), $G \in K$, and R is a binary, reflexive relation defined on K, and (ii) a binary function $\Phi(P, H)$, where 'P' is a variable that ranges over propositional atoms of L and $H \in K$. The range Φ is the set $\{0, 1\}$, i.e., for every P and $H \in K$, either $\Phi(H, P) = 0$ or $\Phi(H, P) = 1$. To deal with complex formulas, the valuation function is defined recursively in the usual way. A given modal structure will be an $S4$ model structure, if the relation R is transitive, a Brouwersche modal structure, if R is symmetric and an $S5$ modal structure, if R is Euclidean. By extension, an interpretation containing an $S5$ model structure is an $S5$ interpretation.

[1] Even though not exactly in this form, this second reason to prefer Kripke validity is also traceable in Zalta (1988) and Nelson and Zalta (2012). See footnote 8 for an exposition of Zalta's overall dialectical position.

[2] In Kripke (1963a), a modal propositional calculus is *normal*, if it contains the Axiom schemes $\Box A \to A$ and $\Box(A \to B) \to \Box A \to \Box B$ and the rules of Modus Ponens and Necessitation. Note that this definition is different from the one usually given today, for which the first Axiom scheme (usually called 'T') is not required (cf. Hughes and Cresswell (1996, p. 40).

From this basis, Kripke defines an intermediate notion, namely, "truth in a model":

- A is true in a model M iff $\Phi(A, G) = 1$.

From this intermediate notion, Kripke defines the highest degree of validity – simply called "validity" – in terms of the intermediate notion:

- A formula A is valid if, and only if, for every M, A is true in I'.

What is valid, for Kripke, is therefore what is actually true in every model.

In most textbook presentations of modal logic, the definition of validity is different. In particular, reference to the actual world of the interpretation is dropped, and new degrees of validity are defined.

Take, for example, Chellas (1980). A *standard model* M is a triple with the following structure $\langle W, R, P \rangle$, where W is, again, a set of possible worlds, R is a binary relation on W and P is an infinite sequence of subsets of W. The basic semantic notion (which Chellas calls "the lowest degree of validity") is that of "truth at α in M", recursively defined in the usual manner.[3]

Chellas then defines (what he calls) three other "degrees of validity". The first is the notion of "truth in a model" defined as truth in *every world* in the model, then there is the notion of "validity in a class of models", for which a sentence is true in a class of models iff it is true in every model of the class, and finally there is the highest degree, for which a sentence is valid *simpliciter* iff it is true in the class of all models.

A similar route, though different for some details and terminological choices, is taken by Hughes and Cresswell (1996). What Kripke called "model structure" is now called "frame", but unlike Kripke's model structure, a frame is an ordered pair $\langle W, R \rangle$, where W is again a non-empty set of objects ("possible worlds") and R is a binary (not necessarily reflexive) relation defined over members of W. A *model* is an ordered triple $\langle W, R, V \rangle$, where $\langle W, R \rangle$ is a frame and where V is a value assignment defined recursively in the usual way. As in Kripke, V plays here the role of the basic notion of "truth in a world in a model", which is explicitly defined by Chellas. The first degree of validity is that of "validity on a frame", for which a formula α is valid on a frame $\langle W, R \rangle$ iff, for every model $\langle \langle W, R \rangle, V \rangle$ based on $\langle W, R \rangle$, and for every $w \in W, V(\alpha, w) = 1$. Note that this represents a different and higher degree of validity than the notion of "truth in

[3] It is probably worth mentioning just the base clause for atomic sentences:

- $\mathbb{P}_n$ is true at α in M iff $\alpha \in P_n$

where the idea is that for each natural number n, the set P_n contains all and only those possible worlds where the atomic letter $\mathbb{P}_n$ is true.

a model" (that Hughes and Cresswell never explicitly introduce). Finally, the highest kind of validity is defined relative to a given system: for example, a formula α is $S5$-valid iff it is valid on every equivalence frame.[4] Validity in all frames corresponds to a specific type of system validity, namely, K-validity.

Blackburn et al. (2001) is, to my knowledge, the richest in variety. Like Hughes and Cresswell, a frame here is a pair $\langle W, R\rangle$, where W is a set of points (also called "states", "nodes", "worlds" or "situations" in the course of the book) and R the usual binary relation on W. A model M is a pair $\langle \mathcal{I}, V\rangle$, where $\mathcal{I}$ is a frame and V a valuation function that, unlike that of Hughes and Cresswell, assigns to each propositional atom a subset of the power set of W. The basic semantic notion is that of "satisfaction (or truth) in M at a state w", which is recursively defined as usual. Then comes the notion of "global or universal truth in a model M", for which a formula ϕ is globally or universally true in M iff it is satisfied at *all* points/states/worlds in M. Then there are various degrees of validity: "validity at a state w in a frame $\mathcal{I}$", for which a formula φ is valid at a state w in a frame $\mathcal{I}$ iff φ is true at w in every model $\langle \mathcal{I}, V\rangle$ based on $\mathcal{I}$, "validity in a frame", for which φ is valid in a frame $\mathcal{I}$ if it is valid in every state w of $\mathcal{I}$ (i.e., on every model based on $\mathcal{I}$), and "validity on a class of frames F" for which φ is valid in F if it is valid on every frame in F. Finally, a formula is valid *simpliciter* if it is valid on the class of all frames.

Here is a terminologically normalized summary of the various degrees of truth and validity found in Kripke and in the textbooks:

Degree	Notion
I	truth at w in M
II	truth in M
III	validity in w in a frame $\mathcal{F}$
IV	validity in a frame $\mathcal{F}$
V	validity in a class of frames $\mathcal{F}$
VI	validity

Kripke and the textbooks differ starting from the second degree. For Kripke, truth in a model is truth in the actual world of the model, for the textbooks, truth in a model is truth in any world of the model. This difference at degree II reverberates, of course, along all other degrees. Kripke never explicitly defined the notion of truth in a model structure (or a frame), but if he had done it, the definition would have been: a formula φ is true in a

[4] The possibility of defining validity in $S5$ in this way depends on the possibility of proving that $S5$ is complete with respect to the class of equivalence frames.

frame $\mathcal{F}$ if and only if φ is true in the actual world of every model based on $\mathcal{F}$.[5]

According to the definition given in the textbooks, the notion of truth in an interpretation corresponds to the notion of *necessary truth* (if a formula is true in an interpretation is necessary, and *vice versa*). According to the definition given by Kripke, the notion of truth in an interpretation corresponds instead to the notion of *actual truth*.

In Kripke, unlike the textbooks, there is a distinction between necessity and truth in an interpretation, even though necessity and truth converge at the very last degree of validity. The reason is the following. validity in Kripke is truth in the actual world of every interpretation. The set of all actual worlds of every interpretation, however, is just the set of absolutely all possible worlds. This is because each possible world happens to be the actual world of some interpretation or other. Therefore, if a formula Φ is true in every actual world of every interpretation, such formula is true in every possible world, i.e., a necessary formula. Such final convergence between validity and necessity is codified by the validity of the rule of Necessitation.

The identity between the set of all possible worlds and the set of actual worlds in every interpretation is the reason why Kripkean validity and Textbook validity are extensionally equivalent, *as far as simple modal languages are concerned*.[6] It is understandable why, especially in textbook presentations of modal logic, the Textbook validity is preferred. From a technical point of view, it simplifies models and recursive definitions. From a philosophical point of view, it avoids the trouble of explaining what it means for a world to be actual (having already that of explaining what it means for something to be a world).

3. Zalta's defense of Kripkean validity

According to Zalta (1988), and Nelson and Zalta (2012), Textbook validity is an "incorrect" notion of validity. To motivate their view, they present the following three, inter-connected, reasons:

[5] A terminological note. In the table above, I reserved the term "validity" only to those semantic notions that abstract or generalize over valuations. In this sense, the basic semantic notion of "truth in a model at a world" or even the notion of "truth in a model" should not be considered, as, for example Chellas does, kinds, or degrees, of *validity*. Validity is a notion that enters the scene only when valuations leave the scene. This is, in effect, why validity is so an important semantic notion, specially in modal logic. Through validity, we get a grip on the fundamental level of frames, abstracting away from the contingent and logically irrelevant informations encoded by the valuations. For this conception of validity in modal logic, see Blackburn et al. (2001, p. 24).

[6] By "simple modal language" I mean modal language with standard modal operators $\Diamond$ and $\Box$.

1. Textbook validity is defined over models where no world is distinguished as the actual world. But if no world is distinguished as actual, Zalta (and Nelson) claim, there is no way to define the notion of truth in an interpretation, which is the "most important semantic definition for a language".[7]
2. Given that the notion of truth in an interpretation cannot properly be defined, Textbook validity is defined directly from the basic notion of "truth in a world in an interpretation". This is contrary to Tarski's notion of truth, where the notion of validity (logical truth) is to be defined by means of the notion of truth in an interpretation.
3. Textbook validity, being some kind of super-necessity, represents a conflation between a semantic notion and a metaphysical notion.[8]

I think that none of these reasons is really convincing.

Let us see 1. As the somewhat painstaking exposition given in section 2 has shown, the fact that the interpretations used to define basic semantic notions in modal logic do not have a designated actual world as an element does not preclude *per se* the possibility to properly define a notion of truth in an interpretation.

The habit of defining validity directly in terms of the basic notion of truth in a world in an interpretation is surely widespread: it is the route taken, for example, by Hughes and Cresswell (1996), but also by R. Montague, E. J. Lemmon, D. Dowty, F. Wall, S. Peters[9]. I do not think, however, that all these modal logicians had "no means", like Zalta claims, to define a notion of truth in an interpretation. I do not think that they did not define the intermediate notion of truth in an interpretation because it was *impossible* for them to define it.

[7] Zalta and Nelson actually speaks of "truth in a model". I prefer the term "interpretation" to the term "model" and so I will use "truth in an interpretation". I take "truth in an interpretation" and "truth in a model" to be the very same notion.

[8] It is probably better to clarify the dialectical position of Zalta (1988), especially with respect to point 3 above. Zalta's aim is basically a defense of contingent logical truths. Such contingent logical truths emerge, as we will see in the next section, in modal languages enhanced with actuality operators (or with primitively rigid descriptions) where validity is defined *à la* Kripke. The authors present 1 and 2 above as "independent grounds" (i.e., independent on the issue whether there are such contingently logical truths) to prefer Kripkean validity over Textbook validity. They claim also that it would be question-begging to conclude that Textbook validity is a wrong conception of validity on the basis of 3. But the use of 3 would be question-begging, only if we want to show that there are logical contingent truths. Given that I am not assuming anything about logical contingent truths (because we are considering simple modal languages where such truths do not emerge), I think it is fair to attribute to Zalta the thesis that 3 is a non question-begging argument to favor Kripke validity over Textbook validity on par with 1 and 2.

[9] All mentioned in Zalta (1988, p. 64).

As shown by Chellas (1980) or Blackburn et al. (2001) and by many others, the notion of truth in every world of an interpretation seems to be a perfectly plausible definition for the notion of truth in an interpretation. It is not the presence of a designated actual world in an interpretation that allows one to define a notion of truth in an interpretation, it is not its absence to prevent its definition.[10]

As for point 2. Even those who define a notion of truth in an interpretation as truth in every world of the interpretation define validity according to an orthodox Tarskian pattern. Just as expected, they can define the notion of validity as truth in all interpretations in terms of their intermediate notion of truth in an interpretation. Zalta might object that the problem is exactly that the notion of truth in an interpretation as truth in every world of the interpretation is not a good definition of truth in an interpretation. But this is not what point 2 is about, it is a critique along the lines of point 3.

As for point 3. Here, the charge advanced by Zalta is that the notion of truth in all worlds of an interpretation is not a good notion of truth in an interpretation, because it conflates a semantic and a metaphysical notion. But what does it mean exactly that there is a "conflation" between the semantic notion of truth (or validity) and the metaphysical notion of necessity?

If by "conflation" they mean that Textbook validity preserves the somewhat traditional connection between validity and necessity, according to which all logical truths are necessary, their critique seems to be off-target. Both Kripke and Textbook validity may be charged with the accusation of conflating logical truth with necessity. For simple modal languages, a valid formula, be it Kripke or Textbook valid, is also a necessary formula. In such languages, the rule of Necessitation can be safely applied to all valid formulas.

What Zalta meant with 3 might be the following. Textbook validity is a kind of "super-necessity": a formula φ is Textbook valid if $\Box\varphi$ is true on every interpretation. What is logically true thus is not, as the orthodox Tarskian definition requires, what remains true under permutations of the non-logical vocabulary, but what remains necessarily true under such permutations.

[10] A classical, Tarskian, interpretation in classical propositional logic consists of a domain of objects and an assignment of extensions (based on such domain) to non-logical parts of the language. A sentence is true in an interpretation if it comes out true, given this assignment. In this sense, the notion of truth in an interpretation as truth in every world of the interpretation deviates from this idea. However, this notion of truth in an interpretation was designed for extensional, non-modal languages. As it will be clearer in the following sections, an interpretation of a modal language is to be conceived as a *collection* of extensional interpretations. The original Tarskian definition cannot thus be applied *literally* to this new kind of interpretations; the notion of truth in an interpretation as truth in every world of the interpretation (i.e., truth in every extensional interpretation that constitutes a modal interpretation), however, represents simply a *generalization* of the original definition and therefore it is not incompatible with it.

Textbook validity is "permutated" necessity, while Tarskian validity is "permutated" truth, therefore Kripke validity should be preferred.

But against this, at least two responses could be given.

The first is that, as already pointed out, the set of actual worlds where a formula has to remain true under permutations in Kripke validity is just the set of all possible worlds where a formula has to remain true under permutations in Textbook validity. The very same worlds and the same permutations are to be "checked" to see whether a formula is valid. Surely, Textbook validity is a little bit more redundant, because the same worlds and the same permutations will be "checked" more than one time, but the raw material over which the two notions of validity are built is exactly the same, the set of all possible worlds and all interpretations of non-logical vocabulary.

The second point is this. Zalta seems to presuppose that "truth at an actual world in an interpretation" (i.e., Kripkean truth in an interpretation) is a purely semantical notion, while, on the contrary, "truth in every world in an interpretation" (Textbook truth in an interpretation) is a non-purely semantical, metaphysical notion.

Zalta criticises Textbook validity for its metaphysical, non-purely semantical, *pedigree*. But it is wrong to claim that "truth in all worlds" is a less semantical, or a more metaphysical, notion than the notion of "truth in the actual world"; necessity is no less semantical than truth for the simple reason that necessity is a *kind* of truth. If the fact that something is true in a world (and, in particular, in the actual world) is semantical, then a mere generalization on this fact could not cause a deviation from its semantical nature. Is there something special about the actual world that makes truths in it semantical and not metaphysical? Is there something wrong with other possible worlds that make truths in them (and truths in all of them) metaphysical and not semantical?

The notion of truth in an interpretation defined as truth in the actual world of the interpretation is no more semantical than the notion of truth in an interpretation defined as truth in every world of the interpretation. Kripke validity is no more semantical than Textbook validity. Textbook validity is no less semantical than Kripke validity.

My conclusion is that 1, 2, and 3, are not good reasons to favor Kripke validity over Textbook validity and especially to deem Textbook validity an "incorrect" definition of validity for modal languages.

4. A proposed genealogy of the distinction

Far from being the distinction between a correct and an incorrect notion of validity for modal languages, the difference between Kripke and Textbook validity should be traced back to the difference between two conceptions of what a modal logic is or, to use a less committing term, two slightly different "views", "informal ideas" of how a modal logic is obtained from a non-modal one.

The difference between Kripke validity and Textbook validity is not something really deep. Yet it is interesting to make explicit the views, or informal ideas, on which such difference is based.[11]

Modal propositional logic could be seen as a generalization of non-modal propositional logic. The characterizing feature of modal propositional truth is that, while propositional non-modal truth is defined over a single interpretation, propositional modal truth is defined over a set of non-modal propositional interpretations. A modal propositional interpretation can be seen as a "cluster" of non-modal propositional interpretations.[12]

The role of the elements in W in a modal propositional interpretation, a set of arbitrary objects as far as pure semantics is concerned, is that of indexing such a cluster of non-modal propositional interpretations.[13] The truth of formulas containing modal operators, like $\Box\phi$ and $\Diamond\phi$, is then to be defined over this set of indexed non-modal propositional interpretations as, respectively, truth in every indexed propositional non-modal interpretation and truth in some indexed propositional non-modal interpretation.

The difference between Textbook validity and Kripke validity may ultimately be viewed as the by-product of two different ways of making sense of this "cluster" view of modal interpretations.

Kripke validity seems to correspond to the following procedure. Let us take a non-modal propositional interpretation $I^@$ (i.e., an assignment of truth-values to propositional atoms) and associate to $I^@$ a cluster of alternative propositional non-modal interpretations, i.e., alternative assignments of truth-values to propositional atoms. We may call this associated cluster of alternatives to $I^@$ the "variants" of $I^@$. Now, let us index this set of variants of $I^@$ with a set of arbitrary objects W. The result will be an augmented propositional interpretation, formed by the original $I^@$, the variants of $I^@$, and W. The original $I^@$ plays the role of the actual world, the indexed variants play the role of other possible worlds. Do the same for all non-modal propositional interpretations, and you will have a set of modal propositional

[11] I am not attributing to Kripke or to the authors of the various textbooks an explicit endorsement of any of the views presented here. My claim is simply that such views could be useful to explain the otherwise unjustifiable difference between the two definitions of validity.

[12] A view along these lines and the word "cluster" to define modal interpretations can be found in Menzel (1990).

[13] The set W then should not be informally interpreted as the set of possible worlds. A possible world should be rather identified with an indexed non-modal propositional interpretation.

interpretations. Within this "augmenting" strategy, the natural choice is to leave all meta-theoretical notions unchanged. In particular, a formula will be true in an interpretation if and only if it is true in the original $I^@$ of an augmented interpretation.

Informally, this seems to correspond to the idea that possible truth (what is true in the variant interpretations) does not interfere with actual truth (what is true in the original non-modal interpretation). Another informal idea involved in the procedure described above is the actualist idea for which possible worlds are to be conceived as alternative state of *the* world, alternative ways the world might have been. Under Kripke validity, modal truth is seen *sub specie actualitatis*.

Textbook validity seems instead to correspond to the following, different procedure. Let us take a certain number of non-modal propositional interpretations and index them with the members of W. In this case, there is no "original" propositional non-modal interpretation from which this cluster of interpretations is generated and no privileged non-modal propositional interpretation to be tracked down in order to define basic meta-theoretical notions. Within this "non-augmenting" view, the natural choice is to define modal meta-theoretic notions by "downgrading" non-modal meta-theoretic semantic notions: given that a modal interpretation is a cluster of non-modal interpretations, truth in a modal interpretation is non-modal propositional validity: truth in a (modal) interpretation is truth in every indexed non-modal interpretation. Informally, this procedure seems to be inspired by the possibilist idea that possibilities and actuality are on par and what possibilities there are, what possible worlds there are, is not dependent on what is actually true. Under Textbook validity, modal truth is seen *sub specie possibilitatis*.

5. The actuality operator enters the scene

In the previous section, I claimed that, for simple modal languages, there is no independent ground to favor Kripke validity over Textbook validity (or *vice versa*). Their difference may be traced back simply to two different informal views about the way in which modal logic is generated from a non-modal one.

For expressively richer modal languages, things get more complicated. In such a case, there seems to be at least one convincing reason to favor Kripke validity. In languages enriched with the so-called actuality operator, there seem to be cases of obviously true formulas that come out valid *only if* Kripke validity is used. In such languages then, the two definitions of validity are not extensionally equivalent, and Kripke validity can be credited with the advantage of better representing our informal intuitions of validity.

A Kripke valid formula that is not Textbook valid has a quite interesting feature.[14] A formula ϕ is Textbook invalid iff it is non-necessary in some interpretation. A formula ϕ is Kripke valid iff it is true in every actual world of every interpretation. A Kripke valid but Textbook invalid formula is thus a logically true formula that is not necessary.

The expressive resource needed to generate this kind of formula is the actuality operator A. From a syntactical point of view, A is a unary, sentential operator on par with $\neg$, $\Diamond$ or $\Box$.

But what is an actuality operator, and, what is more important for my purposes, what is the role of such an operator in a modal language?

Such an operator is introduced to satisfy some expressive needs mostly related to the semantics of natural language. The claim is that an actuality operator should be introduced in modal logic to express the "logical uses" of the word "actually" or "in fact" in English.[15]

Consider a sentence like:

(1*) It might be that every person who is actually honest is rich.

Why should we be interested in using such a (strange) sentence? Well, the standard story goes, we should use such a sentence to force a certain interpretation of this other sentence:

(1) It might be that every honest person should be rich.

(1) is ambiguous between a more generic reading, where we are simply envisaging a possible situation where honesty is accompanied by richness, and another, more specific reading, where we are envisaging a possible situation where the persons who happen to be honest around us are rich. In terms of possible worlds, the first reading is made true by any possible world where every honest person is rich, the second reading by any possible world where every person who is honest around us here is rich there.

With the resources of a standard predicative modal language, we can tentatively translate (1*) with one of the following formulas:

(2) $\Diamond \forall x(Hx \rightarrow Rx)$

(3) $\forall x \Diamond (Hx \rightarrow Rx)$

(4) $\forall x(Hx \rightarrow \Diamond Rx)$

[14] To say that there are formulas that are Kripke but not Textbook valid is a convenient but quite rough way of expressing ourselves. It should instead be said that there are formulas that are Kripke valid such that they *would not count* as Textbook valid when Textbook validity were the notion of validity at work.

[15] Cf. Crossley and Humberstone (1977, p. 11), Cresswell (1990, p. 34). The non-logical (i.e., rethorical) uses of "actually" or "in fact" are used to correct some misunderstanding on the part of the hearer like in "Actually, John is 35, not 37, years old" said to someone who misdescribed John as a 37 years old.

but none has the intended truth-conditions of (1), where all the individuals in the extension of H with respect to w^* need to be in the extension of R with respect to another accessible world. However, if we do not first specify the relations between quantification and modality in use, we cannot even begin to interpret such formulas.

Let first us assume that the system of quantified modal logic is one with possibilist quantification and fixed domains.[16] In such a system, (2) and (3) are equivalent and can be used to translate the first reading of (1), but not the second reading expressed by (1*). (4) seems almost right, but it is compatible with a situation where some actually honest persons are rich in some world, others in another.[17]

A language with actualist quantification and varying domains[18] does not fare any better. In such a system, (2) and (3) are not anymore equivalent, but (2) translates, again, just the first reading of (1), while (3) is now about actual individuals (even not actually honest) who are possibly honest and rich and not, as required, about actually honest individuals that are possibly rich. (4) rightly selects actually honest persons, but as it was with possibilist quantification, it distributes their possible richness across the modal space.

If we allowed ourselves a first-order predicative language with explicit quantification over possible worlds and objects, the difference between the two readings of (1) could very easily be made.[19]

A sentence like (1*) could be translated by:

(5) $\exists w(Acc(w, w^*) \wedge \forall x(H(x, w^*) \rightarrow R(x, w)))$

While a sentence like (1) could be translated by:

(6) $\exists w(Acc(w, w^*) \wedge \forall x(H(x, w) \rightarrow R(x, w)))$

Given that the meta-language of a modal logic is, basically, a first-order predicate language similar to the one in which (5) and (6) are expressed,

[16] Corresponding to the system $S + BF$ presented on ch. 13 of Hughes and Cresswell (1996).

[17] The intended reading of (1*) instead is one where the actually honest persons are possibly rich all together. (4) is the translation of something like:

(1**) Everyone actually rich might have been poor.

Cf. Cresswell (1990, p. 34). My impression, however, is that we are not really sensitive in natural language to the difference between a sentence of the form "poss (every *F* is *G*)" and another of the form "every *F* is poss *G*".

[18] Corresponding to the system presented in ch. 15 of Hughes and Cresswell (1996) or to the system famously presented in Kripke (1963b).

[19] I am assuming a first-order, two-sorted language where predicates have an extra place for possible worlds and with a special interpreted predicate Ixw for "x exists in w" and an individual constant w^* for the actual world. Acc is the accessibility relation. This is a simplified version of a language presented in D. K. Lewis (1968b).

the problem for simple modal languages is that there seems to be no formula that corresponds to (5), i.e., no formula whose truth-conditions are given by (5).

The expressive problem, for simple quantified modal languages, is that of not being capable of "quantifying out" the context of a modal operator.[20] A quantifier quantifies out the scope of a modal (or any other intensional) operator, if it is capable of binding occurrences of variables not governed by that operator. A formula like (2) could be taken as a good translation of (1*) if only the universal quantifier, in the scope of $\Diamond$, could bind the first occurrence of x as if it were not governed by $\Diamond$.

The solution has been that of introducing a brand-new unary operator, the actuality operator A, able to protect subformulas and, in particular, occurrences of variables in the scope of a modal operator and letting them to be interpreted relative to the actual world. In the propositional case, such an operator attaches to propositional letters and frees such letters from the scope of any modal operator.

Using A, a sentence like (1*) could be translated by a formula like:

(2*) $\Diamond \forall x(\mathsf{A}Hx \rightarrow Rx)$

Attention should be paid again to whether the system where (2*) is interpreted is one with possibilist quantification and fixed domains or one with actualist quantification and variable domains.

In the first case, the formula says that every individual who is honest in the actual world is rich in the world selected by $\Diamond$. In the second case, the formula says that every individual in the world selected by $\Diamond$ who is honest in the actual world is rich. The actualist interpretation is compatible with the possible non-existence of some actually existing honest persons: only those actuals that "continue" to exist in the world selected by $\Diamond$ are taken into consideration.[21]

A is integrated in the semantics of modal languages with Kripkean interpretations with the following clause, which extends the recursive definition of the basic semantic notion of truth in an interpretation in a world:

(Sem-Act) $\mathsf{A}\phi$ is true in I in a w iff ϕ is true in I in w^*

The expressive need to have a tool for breaking the scope of intensional operators, a need that *per se* is basically syntactic in nature, has been met

[20] Cf. Hazen (1995, p. 300), but the term is from D. Kaplan (1973, p. 504, fn. 12).

[21] It should be noted that the formalization of (1*) by means of (2*) is a delicate matter and this is not often noted: in a variable domain semantics with actualist quantification the formula can be true in case there exists a world none of whose inhabitants exists in the actual world. This is clearly not enough to capture the intended reading of the original sentence. The intended reading of (1*) thus can only be captured within a constant domain semantics with possibilist quantification.

by a specific *semantic* move, namely, that of defining an operator that, when applied to a sentence ϕ, sticks to ϕ the truth-value that ϕ has in the actual world.

The actuality operator defined by (Sem-Act) is a way to obtain *truth-value rigidity*. A formula like $\mathsf{A}\phi$ has the same truth-value (the one the formula has in the actual world of an interpretation) in every possible world. If one accepts the view that propositions designate their truth-values, the effect of prefixing A to ϕ could be described as that of making ϕ a rigid designator of its truth-value.[22]

Systems of modal logic with the addition of an actuality operator have been studied, at the propositional level by Crossley and Humberstone (1977) and Gregory (2001) and at the predicative level by Hodes (1984) and Stephanou (2005).[23]

The following seems to be a nice selection of axioms for A (there is a notable exception, which will be the object of the following section):

A1 $\mathsf{A}(\phi \rightarrow \psi) \rightarrow (\mathsf{A}\phi \rightarrow \mathsf{A}\psi)$

A2 $\mathsf{A}\neg\phi \leftrightarrow \neg\mathsf{A}\phi$

A3 $\Box\phi \rightarrow \mathsf{A}\phi$

A4 $\Diamond\mathsf{A}\phi \rightarrow \mathsf{A}\phi$

A5 $\mathsf{A}\mathsf{A}\phi \rightarrow \mathsf{A}\phi$

A6 $\mathsf{A}\phi \rightarrow \Box\mathsf{A}\phi$[24]

[22] According to Zalta (1988), the other expressive resource that can generate contingent logical truths in a modal language semantically *à la* Kripke is that of *primitive descriptions*. Primitive descriptions are singular terms whose semantic value is defined by a denotation function that assigns to a description like $(\imath x)\Psi$ the unique object that satisfies Ψ in w^* and it leaves it undefined otherwise. Being primitively defined, such descriptions, exactly like individual constants, are rigid designators. A formula like $P(\imath x)Qx \rightarrow (\exists y)Qy$ ("if the actual Q is P then there is a Q") is an example of a contingent logical truth that can be built with a primitive description (see Zalta, 1988, p. 61-62) and thus it is an example of a Kripke valid, but Textbook invalid formula. Viewing the actual operator (defined by (Sem-Act)) as a device to obtain actual truth-value rigid designation permits one to see the strong resemblance between the two kinds of expressive resources. Primitive descriptions could be taken simply as notational variants of definite descriptions rigidified by the actuality operator.

[23] Gregory is the only one who does not endorse (Sem-Act).

[24] I only claim that the list above is a "nice selection" of axioms for A, not a proper axiomatization for modal languages containing A. In particular, Axiom A3 requires a universal accessibility relation in a logic with Textbook validity and a reflexive accessibility relation in a logic with Kripke validity. Furthermore, if one has both A6 and A2, A4 is derivable by propositional logic. Crossley and Humberstone (1977) prove completeness for propositional systems, based on S5, containing A2, A1, A6, ACT, and:

A7 $\mathsf{A}(\mathsf{A}\phi \rightarrow \phi)$

Gregory (2001) proves completeness (and decidability) results of number of modal logics containing A1, A2, a formula equivalent to A4, A7, and ACT. Stephanou (2005) proves to which the following rule is usually added:

(ACT) If ϕ is a theorem, so is $\ulcorner \mathsf{A}\phi \urcorner$

A1–A7 explicate the behavior of A regarding the main truth-functional connectives and modal operators. A4 is especially revelatory of the key-intuition behind the actuality operator, namely, that of interrupting the scope of other modal operators. A6 is basically a by-product of (Sem-Act): if A rigidifies the truth-value of ϕ, then $\mathsf{A}\phi$ has the same truth value in every possible world; so if ϕ is true in the actual world, it is necessarily true.

It is now time to analyze the implications of the introduction of A for the Kripke *vs* Textbook validity issue.

6. Kripke, but not Textbook, valid formulas with the actuality operator

Adding an actuality operator to a standard modal language and interpreting it using a semantic clause like (Sem-Act) allows one to prove the existence of Kripke valid formulas that are also Textbook invalid. As said, cases of Kripke, but not Textbook valid formulas are interesting because they would be cases of logically, but contingently, true formulas.

Consider a formula like:

(7) $\mathsf{A}\phi \to \phi$

(7) is Kripke valid: take an arbitrary interpretation I and assume that $\mathsf{A}\phi$ is true in I. Under Kripke validity, this means that $\mathsf{A}\phi$ is true in the actual world w^* of I. But the right-to-left direction of the semantic clause for A has it that, if $\mathsf{A}\phi$ is true at w^*, then ϕ is true at w^*. The conditional is therefore true in the actual world of I and thus true in I. Given that I is arbitrary, (7) is Kripke valid.

In some Kripkean interpretations, this formula will not be necessary. Consider, for example, the following interpretation I^i (the specification of R will be omitted):

- $W = \{w_1, w_2, w_3\}$
- $w^* = w_1$
- $V(\phi, w_1) = 1$, $V(\phi, w_2) = 1$, $V(\phi, w_3) = 0$

completeness for a first-order modal system based on K with A1, A2, A5, A6, and ACT. Finally, Hodes (1984) proves completeness for an S5 propositional system enriched by A4, and by following two further axioms:

- $(\mathsf{A}\phi \to \mathsf{A}\psi) \to \mathsf{A}(\phi \to \psi)$
- $\mathsf{A}\bot \to \bot$

We know already that (7) is true in I^i (because we know already that it is a valid formula), but in I^i such a formula is also non-necessary: in w_3 the antecedent is true, but the consequent is false. Being not necessary, the formula is not, *mutatis mutandis* Textbook valid. A formula like (7), interpreted by means of I^i in the context of a Textbook modal logic would not be true in I^i and therefore would not be Textbook valid.[25]

Consider also the contrapositive of (7):

(8) $\phi \rightarrow \mathsf{A}\phi$

Even (8) is Kripke valid, non necessary and thus, *mutatis mutandis*, Textbook invalid. To show that (8) is Kripke valid, consider, again, an arbitrary, Kripke-style, interpretation and assume that ϕ is true in this interpretation, namely in the actual world of it. The left-to-right direction of the semantic clause for A has it that, if ϕ is true in the actual world of an interpretation, $\mathsf{A}\phi$ is also true. The conditional is then true in the actual world of the interpretation and thus true in the interpretation. Given that the choice was arbitrary, the formula is true in every (Kripke-style) interpretation and thus Kripke-valid.

To show that (8) is non-necessary (and thus, *mutatis mutandis*, Textbook invalid), consider the following interpretation I^{ii}:

- $w = \{w_1, w_2, w_3\}$
- $w^* = w_1$
- $V(w_1, \phi) = 0$, $V(w_2, \phi) = 1$, $V(w_3, \phi) = 1$

In w_2, the antecedent of (8) is true, but the consequent is false, and so there is at least a world where the conditional is false.

The following equivalence:

(9) $\mathsf{A}\phi \leftrightarrow \phi$

has been taken as the paradigmatic case of a Kripke valid, but Textbook invalid, formula (or of a valid, but non-necessary formula).[26]

At least *prima facie*, a formula like (9) seems an obvious truth: it seems to codify just the standard relations between actuality and truth. How could something actually true not be true? How could something true not be actually so?

[25] In order to interpret formulae containing A in a Textbook logic, the interpretations of such type of system should be enriched with designated actual worlds. Such designated worlds, however, should be used only to interpret the semantic clause governing A and not to define, like in a Kripke logic, general semantic notions, like truth in an interpretation or validity. See, for example, (Crossley & Humberstone, 1977) for a plain Textbook logic with A.

[26] … or even of a contingent, *a priori* knowable truth. Cf. Evans (1979, p. 210).

It seems, therefore, that we need to face the following dilemma: on the one hand, if Kripke validity is chosen, a system of modal logic could be enriched by the introduction of an actuality operator in a way that renders valid what seems an obviously true formula, (9). The price to be paid is a failure of Necessitation and the acceptance of contingent logical truths.[27] On the other hand, if Textbook validity is chosen, the traditional connection between necessity and validity (and thus Necessitation) is preserved, but a seemingly valid formula like (9) comes out invalid.

A defender of Textbook validity could approximate (9) with the following formula:

(10) $\mathsf{A}(\mathsf{A}\phi \leftrightarrow \phi)$

which comes out as a necessarily true formula in a Textbook logic.

A formula like (10) is true in a Textbook logic, because, if a formula ϕ is true in the actual world of some interpretation, then a formula like $\mathsf{A}\phi$ is true in every world of that interpretation and thus necessarily true. It is in fact true in every world of the interpretation that ϕ is true in the actual world of such interpretation. The necessity of any true formula governed by A is a direct effect of (Sem-Act) and it is codified by Axiom A6 above: to interpret a formula like $\mathsf{A}\phi$ in an arbitrary world w, the truth-value of ϕ in w is irrelevant, what counts is simply the truth-value of Φ in w^*. Every contingently true formula then becomes necessary if it is prefixed by the actuality operator. This fact allows us to establish a general feature of the relation between Textbook and Kripke validity in modal languages enhanced with an actuality operator, namely that a formula ϕ is Kripke valid if, and only if $\mathsf{A}\phi$ is Textbook valid.

7. Double indexing semantics for A

(Sem-Act) is not the only way to define the semantical behavior of an actuality operator in a modal language. This operator may be seen, more

[27] With the expression "price to be paid", I am not assuming that the failure of the rule of Necessitation and the consequent acceptance of contingent logical truths are necessarily bad things. As it is known, however, reactions to the failure of the rule may be different: D. Kaplan, for example, takes the failure of Necessitation in the logic of indexicals (or in what he calls "the logic of free variables") as a "delightful" aspect. He claims, however, that such a failure is a sign of a "deviant" modal logic (see (Kaplan, 1989, p. 593)). In this sense, the failure of the rule is genuine, but it is taken as an acceptable price to be paid. (Kripke, 1963b) has a more sceptical attitude: he sees failure of Necessitation as a sign of an incorrect formulation of the rule in a certain deductive environment: to correct the failures and restore the rule, he decides to assume a generality interpretation of free variables. I think that the failure of Necessitation in the case of the introduction of the actuality operator is more easily diagnosed in the Kaplan's way than in the Kripke's way.

plausibly in my opinion, as a member of a larger family of *doubly indexed*, or *two-dimensional*, operators.

In a standard, one-dimensional intensional logic, truth is relativized to only one point (time, world, etc.), and the basic semantic notion is that of "truth at w in an interpretation I". In a two-dimensional intensional logic, truth is relativized instead to two points (times, worlds, etc.), and the basic semantic notion is that of "truth at w_1 *with respect to* w_2 in an interpretation I" or, more briefly, "truth $\langle w_1, w_2\rangle$ at I". A two-dimensional logic is a logic with two kinds of operators: standard, intensional one-dimensional operators, which operate only on the first index, and two-dimensional operators, which operate also on the second index.

But what is the role of the two indices? The first index is usually called the *evaluation index* and is intended to be the point (time, world, etc.) at which the formula is evaluated. The second index is called the *reference index*, and its role is that of "keeping track" of the points (times, worlds, etc.) involved in the process of interpretation. The evaluation index is the point (time, world, etc.) at which we are evaluating a formula (or a part of it); the reference index is the point (time, world, etc.) from which we arrive at in evaluating the formula (or a part of it).

The evaluation of a formula in a bidimensional logic, $V(\varphi, w, v)$, is usually intended to capture the informal idea of a formula being true at a point (w), *from the point of view* of another point (v).[28] I am not at ease with this notion as far as it seems to suggest the idea that the truth of a formula in a possible world somewhat depends on the point of view from which the formula is "seen". Fortunately, nothing in the semantics *per se* points in this direction. Consider a formula like $\Diamond\phi$. When the process of evaluation starts in a two-dimensional logic, the world of evaluation and the world of reference coincide; this seems plausible, given that there is no world to be stored, no world from which the world of evaluation comes from. $\Diamond\varphi$ has then to be evaluated at $\langle w, w\rangle$; such a formula is then true at $\langle w, w\rangle$ if, and only if, there exists a world v such that ϕ is true at $\langle v, w\rangle$. The truth of ϕ at v "from the point of view" of w then could be simply reduced to the truth of ϕ at v and to the accessibility of v from w. This shows that what is really interesting in a two-dimensional logic lies more in its capacity of representing, in a finer-grained way, the usual semantic mechanisms of modal logic (by making them work "simultaneously"), than in its capacity of introducing and formalizing new informal ideas.

As we have already seen, in discussing the formalization of (1*), the desired behavior of an operator like A is that of breaking the scope of any intensional operator governing it and letting the rest of the formula governed by A be interpreted relative to the actual world or, at least, relative

[28] Cf. Segerberg (1973).

to the world where the intensional operator itself is to be evaluated. This process can be easily represented in a double indexing semantics by letting the role of A be that of bringing back the evaluation at the (previously stored) reference index. While interpreting a formula with a form like $\Diamond \ldots \mathsf{A}\phi$ (where ϕ is a non-modal formula) the evaluation world for $\mathsf{A}\phi$ is the one determined by $\Diamond$, and the reference world will be the one where the clause governed by $\Diamond$ is to be interpreted (or that of any other modal operator between $\Diamond$ and A). The role of A will be that of bringing back the interpretation of ϕ to the reference world in a way that eliminates the world introduced by $\Diamond$.

The semantic clause that, in a two-dimensional modal logic, codifies such semantical behavior is the following:

(Sem-Act-2) $\mathsf{A}\phi$ is true at $\langle w_1, w_2\rangle$ if, and only if ϕ is true at $\langle w_2, w_2\rangle$

The idea behind (Sem-Act-2) is that A brings back the interpretation of ϕ to the reference world by turning the reference world into an evaluation world. In this way, A "protects" ϕ from being evaluated in a world selected by an intensional operator[29].

Using (Sem-Act-2), we can capture the right truth-conditions for a formula like (2*). As said, we can assume in this case that w^* acts both as the evaluation and reference index.

> $\Diamond \forall x(\mathsf{A}Rx \to Hx)$ is true $\langle w^*, w^*\rangle$ if and only if there exists a w_1 such that $\forall x(\mathsf{A}Rx \to Hx)$ is true $\langle w_1, w^*\rangle$. $\forall x(\mathsf{A}Rx \to Hx)$ is true $\langle w_1, w^*\rangle$ if, and only if, for every x-variant v_i, $\mathsf{A}Rx \to Hx)$ is satisfied by v_i at $\langle w_1, w^*\rangle$, namely, when either $\mathsf{A}Rx$ is not satisfied by v_i at $\langle w_1, w^*\rangle$ or Hx is satisfied by v_i at $\langle w_1, w^*\rangle$. Hx is satisfied by v_i at $\langle w_1, w^*\rangle$ in case $v_i(x) \in V(P)$ in w_1, while, by (Sem-Act-2), $\mathsf{A}Rx$ is not satisfied by v_i at $\langle w_1, w^*\rangle$ just when Rx is not satisfied at $\langle w^*, w^*\rangle$.

This gives us just the right condition, because our formula will be true only when every object that is in the extension of R in w^* will be also in the extension of P in w_1.

What would happen if we use (Sem-Act-2), instead of (Sem-Act) to (9) in a two-dimensional logic?

The bidimensional truth-conditions for A now ensures that for any world and any interpretation, $\mathsf{A}\phi$ is true at $\langle w_1, w_1\rangle$ if and only if ϕ is true at $\langle w_1, w_1\rangle$.

(9) is therefore valid, under the following definition of validity:

(Bi-Val-1) A formula ϕ is valid if and only if, for every interpretation I and possible world w, ϕ is true at $\langle w, w\rangle$.

[29] Cf. Hughes and Cresswell (1996, p. 351), Cresswell (1990, p. 26).

(Bi-Val-1), like Textbook validity, is a definition of validity that universally quantifies over all possible worlds of an interpretation, but where truth is relativized to pairs of worlds of a specific form, namely, $\langle w, w \rangle$. (Bi-Val-1) thus defines validity as truth-preservation at every pair of the form $\langle w, w \rangle$.

Although valid under (Bi-Val-1), a formula like (9) is not necessary. Here is why. The effect of $\Box$ on a formula ϕ true at $\langle w_1, w_1 \rangle$ is that of a universal quantification (eventually restricted by R) on the first index. For $\Box\phi$ to be true at $\langle w_1, w_1 \rangle$, ϕ has to be true at $\langle w_i, w_1 \rangle$, for every w_i accessible from w_1. A simple interpretation where ϕ is false in at least one world is sufficient to show this:

- $w = \{w_1, w_2, w_3\}$
- $V(\phi, w_1) = 0,\ V(\phi, w_2) = 1,\ V(\phi, w_3) = 1$

In every world w_i, $\phi \leftrightarrow \mathsf{A}\phi$ is true at $\langle w_i, w_i \rangle$, but $\Box(\phi \leftrightarrow \mathsf{A}\phi)$ is false at $\langle w_1, w_1 \rangle$. This is because there is at least one world w_i, where $\phi \leftrightarrow \mathsf{A}\phi$ is false at $\langle w_i\ w_1 \rangle$. For example, in w_2 $\phi \rightarrow \mathsf{A}\phi$ is false, because ϕ is true at $\langle w_2, w_1 \rangle$, given that $V(\phi, w_2) = 1$, while $\mathsf{A}\phi$ is false at $\langle w_2, w_1 \rangle$, given that, by (Sem-Act-2), ϕ is false at $\langle w_1, w_1 \rangle$.

(9) is, therefore, non-necessary and (Bi-Val-1) valid. Does it sound like a familiar situation?

According to many bidimensionalists (Sider, 2010, p. 257, Hanson, 2006, Segerberg, 1973), such situation is not surprising at all. The reason is simply that for them (Bi-Val-1) is nothing more than the two-dimensional version of Kripke validity.

The correspondence between the two notions can be shown only in an informal way, by interpreting the notion of "truth at $\langle v, w \rangle$" in a peculiar way, namely, as "true at v from the point of view of *w as the actual world*".

What does it mean for a formula ϕ to be true at w, from the point of view of w as actual? A plausible, and quite minimalistic, interpretation is that being true at v from the point of view of w as actual means that ϕ is true at w, *in case* w happens to be the actual world. To be true at every world w, when w is actual, means that, whenever w is actual, ϕ is true in w. But this is basically equivalent to the idea, which inspires Kripke validity, for which a formula is valid when it is true in the world that, in each interpretation, happens to be the actual world of that interpretation.

Davies and Humberstone (1980) define a more general definition of validity for two-dimensional modal logics:

(Bi-Val-2) A formula ϕ is valid if and only if, for every interpretation I and any pair of possible worlds w_1, w_2 ϕ is true at $\langle w_1, w_2 \rangle$.

According to (Bi-Val-2), ϕ is valid if it is true at every pair of worlds, of whatever form. For two-dimensionalists, (Bi-Val-2) is the two-dimensional version of Textbook validity.

Under (Bi-Val-2), Necessitation is restored, but, not surprisingly, a formula like (9) comes out invalid. Given an arbitrary pair of worlds w_1 and w_2 and assuming that ϕ is true at $\langle w_1, w_2\rangle$ (true at $w_1\rangle$ and false at $\langle w_2, w_3\rangle$ (false at w_2), $\mathsf{A}\phi$ would be false at $\langle w_1, w_2\rangle$ (because ϕ would be false at $\langle w_2, w_2\rangle$).

It seems therefore that we have replicated, this time inside a single logical system, the same pattern of relations between Kripke validity, Textbook validity, and the actuality operator.

What moral should we draw from this situation?

One possible conclusion could be the following: using a finer grained logical system like two-dimensionalist modal logic, we have shown that the contrast between Textbook and Kripke validity is only illusory. Far from being a contrast between a correct and an "incorrect" definition of validity (as Zalta suggests), the distinction between Kripke and Textbook validity is simply the distinction between a very general and a less general definition of validity. (Two-dimensional) Textbook validity is not an "incorrect" notion of validity, it is just a generalized version of (two-dimensional) Kripke validity.

Although this conclusion would be congenial to my purposes, I think I will try to resist it. I have some reasons to do this.

The first is the following. If (Bi-Val-1) were simply a two-dimensionalist reshaping of Kripke validity, then (Sem-Act-2) would be simply a two-dimensionalist reshaping of (Sem-Act). The same notion of "being true at w from the point of v as actual", used to show the equivalence between (Bi-Val-1) and Kripke validity could be used to show the equivalence between the two clauses for A.

In such a case, a semantic clause like:

$\mathsf{A}\phi$ is true at w if, and only if, ϕ is true at w^*

in a Kripke logic would be just a notational variant of this clause in a two-dimensional modal logic:

$\mathsf{A}\phi$ is true at $\langle w, v\rangle$ if, and only if, ϕ is true at $\langle v, v\rangle$

The first clause claims that $\mathsf{A}\phi$ is true in w if ϕ is true in the actual world of the interpretation, the second clause that $\mathsf{A}\phi$ is true in w from the point of view of v as actual (i.e., when v is the actual world) if, and only if, ϕ is true in v when v is the actual world. The view that (Sem-Act-2) is just a redressing of (Sem-Act) is explicitly endorsed by Humberstone and Davies (1980, p. 4).

I do not think, however, that A behaves the same when it is interpreted by (Sem-Act) in a Kripke logic or by (Sem-Act-2) in a two-dimensional modal logic.

In the case of a formula with an actuality operator within iterated modalities:

$$\Diamond \ldots \Diamond \ldots \mathsf{A}\phi$$

(where ϕ is a non modal formula), the Kripkean A will interpret ϕ with respect to the world of evaluation of the entire formula, while the bidimensional A will interpret ϕ with respect to the world selected by the first possibility operator. This, of course, could make a difference in the interpretation of some formulae and so there will be formulae that will come out true in a bi-dimensional logic and false in a Kripke logic (or *vice versa*).

(Sem-Act) + Kripke logic cannot be just a reshaping of (Sem-Act-2) + two-dimensional modal logic if there are sentences with A that come out false in the former logical system and true in the latter (or *vice versa*)[30]

The second reason is the following. Consider our problematic formula $\phi \leftrightarrow \mathsf{A}\phi$. Does it really express the same thing in case it is interpreted using

[30] My point here is just (Sem-Act-2) is not just a bi-dimensional version of (Sem-Act). In a previous draft of this paper, I have tried to show the stronger claim that there are sentences that would come true under (Sem-Act-2) and false under (Sem-Act). Consider, for example, the sentence:

(11) I could have a sister that could have become a physicist but who would have actually become an art historian instead

Assuming that it is possible for me to have a sister, this sentence could be taken as true: I could easily imagine a scenario where a merely possible sister of mine, incredibly gifted in physics (and thus with the possibility of becoming a physicist) would have choosen instead to pursue a career in art history.

A formalization of (11) could be given by the following formula (where *Sxy* is "*x* is a sister of *y*", *a* is an individual constant for the author of this paper, *Mxy* is "*x* majors in *y*" *p* stands for "physics", *h* stands for "art history")

(11*) $\Diamond\exists x(Sxa \wedge \Diamond(Mpx \wedge \mathsf{A}Mxh))$

But a formula like (11*) would count as always false, if it is interpreted in a Kripke logic using (Sem-Act), while it would count as true, in the right interpretations, if interpreted in a two-dimensional modal logic using (Sem-Act-2).

In a Kripke logic, the clause governed by "actually" (or A in (11*)) would have to be interpreted in the actual world of the interpretation. In such a world, however, my merely possible sister, by hypothesis, does not exist and so it would be false that she majors in art history there. The entire sentence, contrary to intuition, would then be false. In a two-dimensional modal logic, the clause governed by "actually" would be rightly interpreted with respect to the world selected by the first modal operator which plays the role of the reference world and where, by hypothesis, my merely possible sister exists. If such a world is a world where my merely possible sister becomes an art historian and there is a world, accessible from the reference world, where she majors in physics, then the formula is true in this interpretation.

As suggested by a referee, however, the problem is that, on the one hand it is not at all clear that a proper formulation of (11) in English really requires the use of "actually", on the other (11) could easily be formalized by means of a simpler formula like:

(11**) $\Diamond\exists x(Sxa \wedge Mxh \wedge \Diamond Mpx)$

where the actuality operator is not needed at all. It is therefore not at all clear that there are convincing examples of sentences of English with iterated modalities that would come out as true within a bidimensional logic using (Sem-Act-2) and false in a Kripke logic using (Sem-Act). For the dialectical purposes of this paper, however, I am (moderately) satisfied also with the weaker point made in the text.

(Sem-Act) in a Kripke Logic or using (Sem-Act-2) in a two-dimensional modal logic?

In a two-dimensional logic with (Sem-Act-2), a formula like $\phi \leftrightarrow \mathsf{A}\phi$ seems to express the informal claim that whatever is true is in fact true. This informal claim is rendered, in terms of possible worlds, as the thesis that whatever is true in a possible world is true in *that* world, i.e, from the point of view of that world, and *vice versa*. A formula like $\phi \leftrightarrow \mathsf{A}\phi$, where no other modal operator is involved, represents a kind of "limit case" where A behaves like some sort of indexical.[31]

In support to this, consider the following: suppose that we interpret a formula like $p \leftrightarrow \mathsf{A}p$ in a two-dimensional logic by means of a two-dimensional version of (Sem-Act) like:

$\mathsf{A}\phi$ is true at $\langle w, w\rangle$ if, and only if ϕ is true at $\langle w^*, w^*\rangle$,

In such a case, our formula would indeed express a false thesis, and this would be mainly due to the rigidifying effect of two-dimensional version of (Sem-Act), which is absent in (Sem-Act-2).

On one direction, (8), the formula would say that whatever is true is actually true, but in the sense that whatever is true in some possible world is true in the actual world of the interpretation. In the other direction, (7), it would say that what is actually true is true, but in the sense that whatever is true in the actual world of the interpretation is true in some other possible world of the interpretation.

Both theses would be problematic, if not clearly false, as far as they depict modal truth shaped as a "funnel" from possible truth to actual truth. Both theses would be falsified by an interpretation where what is true in the actual world is not true in any other possible world.

What is it in a Kripke logic that prevents $\phi \leftrightarrow \mathsf{A}\phi$ from being interpreted in this implausible way? As I claimed on page 10, it is the fact that in a Kripke logic modal truth is conceived *sub specie actualitatis*: where ϕ is a non-modal formula, the truth of ϕ in an interpretation is the truth of ϕ in the actual world of the interpretation, not the truth of ϕ in some world of the interpretation. In a Kripke logic, truth is actual truth, not possible

[31] Note, however, that A under (Sem-Act-2) does not correspond, in general, to an indexical treatment of "actually". According to the indexical view, defended, for example, by D. K. Lewis (1968a, pp. 18-19), "actually" is indexical when the truth of the propositions on which it operates depends on a feature of the context of utterance, namely, the world of utterance. But as we have seen, the role of A under (Sem-Act-2) is that of "usurping" the world of evaluation in favor of another world, the reference world. The reference world, at least in typical cases, is not selected on the basis of contextual elements, but on the basis of semantic elements, having to do with the logical form of the formula. In a typical case, where A is under the scope of a modal operator, the reference world is selected by the process of interpretation of this modal operator.

truth[32]. In a Kripke logic thus, $\phi \leftrightarrow \mathsf{A}\phi$ under (Sem-Act) expresses an absolute thesis about the actual world and not, as it does in a two-dimensional logic, an indexical thesis about any world.

Another reason to resist the temptation to identify two-dimensional modal logic + (Bi-Val-1) with a Kripke logic has to do with Necessitation.

We have seen that in a two-dimensional modal logic, Necessitation fails for a formula like $\phi \leftrightarrow \mathsf{A}\phi$ where (Bi-Val-1) is the relevant notion of validity. The semantic mechanism of double indexing, however, may offer a way out. We can introduce a new operator *X* whose role is that of explicitly "storing" the reference world to be later used by A.

X and *A* would then behave like a couple of so-called Vlach Operators. Every occurrence of A in a formula should then be preceded by an occurrence of its "storing" operator *X*, which explicitly signals at what stage of the interpretation of a formula the reference world should be stored for A.

An operator like *X*, however, is particularly useful in cases of formulas with many iterated intensional operators before A[33]. Consider, for example, a sentence like (1*), but now in the scope of a counterfactual conditional:

(12) If it might have been that every actually honest person was rich, then it would be easier to raise kids

Given the usual semantics for counterfactual conditionals, (12) is true in a world w_1 if and only if, in all worlds where its antecedent is true and that are similar to w_1 as much as the truth of the antecedent permits it to, the consequent is true. For simplicity, assume that there is a single, most similar world to w_1, w_2. (12) is thus true in w_1 if, and only if, "it is easier to raise kids" is true in w_2 and the antecedent (namely, (1*)) is true in w_2. The truth of the antecedent at w_2, however, could be interpreted in two ways: in one way, there is an accessible world to w_2, w_3, such that all those honest in w_1 are rich in w_3; in another way, all those honest in w_2 are rich in w_3. The first reading could be expressed by a formula like:

(12′) $X(\Diamond \forall x(\mathsf{A}Hx \rightarrow Rx) \,\Box\!\!\rightarrow P)$

the second reading by a formula like:

(12″) $(X \Diamond \forall x(\mathsf{A}Hx \rightarrow Rx) \,\Box\!\!\rightarrow P)$

In (12′), the operator *X*, being at the beginning of the formula, stores the world w_1 as the reference world for A. In (12″), *X*, being in the antecedent

[32] This recalls the situation in Priorean temporal logic – surely an inspiration for Kripke – where the truth of ϕ is the truth of ϕ at the present time. See, for example, Prior (1957, pp. 9-10).

[33] What I call *X* is called "*Ref*" by Cresswell (1990, p. 24).

of the counterfactual conditional, stores the world w_2 as the reference world for A.

The semantic behavior of X could be nicely captured in two-dimensional modal logic by the following clause:

(X-sem) $X\phi$ is true at $\langle w, v\rangle$ if, and only if ϕ is true at $\langle w, w\rangle$

From this clause is clear that the role of X is the opposite of A: X picks the evaluation world – whatever it may be at a certain stage in the evaluation of a formula – and turns it in the reference world.

The introduction of X forces every formula with A to be reformulated with its storing operator X. A formula like $(\phi \leftrightarrow \mathsf{A}\phi)$ can be reformulated in two ways:

$$X(\mathsf{A}\phi \leftrightarrow \phi)$$

and

$$X\mathsf{A}\phi \leftrightarrow \phi$$

where the intended reading is the former. This formula, however, is also necessary:

$$\Box(X(\mathsf{A}\phi \leftrightarrow \phi))$$

Such a formula is true at $\langle w_1, w_1\rangle$ if and only if, for every world w_i accessible to w_1, $X(\mathsf{A}\phi \leftrightarrow \phi)$ is true at $\langle w_i, w_1\rangle$. The effect of X, however, is that of realigning the reference world and the evaluation world. $X\mathsf{A}\phi \leftrightarrow \phi$ is true at $\langle w_i, w_1\rangle$ if and only if $\mathsf{A}\phi \leftrightarrow \phi$ is true at $\langle w_i, w_i\rangle$. But we already know, by (Bi-Val-1), that $\mathsf{A}\phi \rightarrow \phi$ is true in whatever pair of worlds with the form $\langle w_i, w_i\rangle$. For the same reason, the formula is also valid under (Bi-Val-2); $(X(\mathsf{A}\phi \leftrightarrow \phi))$ is valid in an arbitrary pair $\langle w_i, w_n\rangle$ if and only if $\mathsf{A}\phi \leftrightarrow \phi$ is true at $\langle w_i, w_i\rangle$, but we know, by (Bi-Val-1), that such a formula is true, therefore our original formula is also (Bi-Val-2) valid.

A two-dimensional treatment of the actuality operator allows us to introduce such an operator in a language with a universally quantified definition of validity in a way that a problematic formula like $\mathsf{A}\phi \leftrightarrow \phi$ comes out valid. Furthermore, with the introduction of the X operator, we could also restore traditional connections between validity and necessity and show that our formula, once properly interpreted, is also necessarily true.

8. Conclusion

In this paper, I have defended the claim that the difference between Textbook and Kripke validity is not a difference between a correct and an incorrect definition of validity for modal languages. For simple modal languages, I have claimed, the difference could be explained by appealing to two different informal views on how a modal logic should be built from a non-modal basis. I have also claimed that a formula like $\phi \leftrightarrow \mathsf{A}\phi$, which seems to come out as valid only within a Kripkean definition of validity, is the result of a wrong conception of the role of the actuality operator (A) in a formal language. In particular, I have shown that, if A is seen as a two-dimensional operator and it is coupled with its "storing" operator X, a (bidimensional version of a) formula like $\phi \leftrightarrow \mathsf{A}\phi$ comes out as valid and necessary, even within a two-dimensional version of Textbook validity.

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